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Ahmed Saleh
Department of Basic Science, Egyptian Academy for Engineering and Advanced Technology, Cairo, Egypt, fayeznasif@yahoo.co.uk

Fayez Ibrahim
Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt, fayeznasif@yahoo.co.uk

Mohamed Hadhouda
Elgazeera High Institute for Engineering and Technology, Cairo, Egypt, fayeznasif@yahoo.co.uk

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Analytical and numerical water quality model for a sinusoidally varying pollutant discharge concentration

Ahmed Saleh[1], *Fayez Ibrahim* [2,*] *and Mohamed Hadhouda* [3]

[1]Department of Basic Science, Egyptian Academy for Engineering and Advanced Technology, Cairo, Egypt
[2]Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt
[3]Elgazeera High Institute for Engineering and Technology, Cairo, Egypt

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Abstract: Analytical solution has been obtained for one-dimensional advection-diffusion equation which includes terms of decay and increasing sources by using Laplace transformation. Also numerical solution has been obtained by using explicit finite difference scheme. In this study the boundary condition applied at $x = 0$ describes a sinusoidal variation in pollutant concentration. The analytical solution obtained produces results that are exact for any location at any time. Impact of different parameters controlling the pollutant dispersion along the river at any time has been studied separately with figures help. This publication proved mathematically the fact that the high concentration of pollutant can be reduced by releasing fresh water discharges from Delta Barrage in the Nile River. For a real situation, our simple model can give decision support for planning restrictions to be imposed on cultivating and urban practices.

Keywords: Concentration of pollutant, Advection-diffusion equation, Finite difference method, Laplace transformation, Solutions of partial differential equations.

1 Introduction

The Nile River in Egypt is polluted by a variety of chemical and biological pollutants, as well as agricultural waste therefore, it is important to accurately predict the concentration of these pollutants in order for this water to be treated. Globally, pollution is drastically increasing in water bodies, negatively influencing the surrounding areas species and grounds. Hence, mathematical models have been developed to control and reduce contamination. Such models originated in 1920s. In the modeling of the transport of pollutants, the solution of the partial differential equation and its related boundary and initial conditions play an important role. The advection-diffusion equation describes the pollutant concentration distribution in porous media as a result of the combined effects of diffusion and convection. Advection diffusion equation is applied in many disciplines like, chemical engineering, environmental sciences and petroleum engineering, bio sciences, groundwater hydrology [1].

The advection-diffusion equation's analytical and numerical solutions along with an initial condition and two boundary conditions help to explain the distribution behavior of the contaminant concentration across an open medium such as air, rivers, lakes and porous medium. On the basis of which therapeutic operations can be carried out to reduce or eliminate damage. Because only a few partial differential equations have analytical or exact solutions, anyone who wishes to construct and use models based on these equations and their related circumstances must be able to efficiently and accurately get numerical solutions [2]. Analytical solutions are provided for the one-dimensional transport of a pollutant in an open channel with steady unpolluted lateral inflow uniformly distributed over its whole length by Zoppou and Knighe [3]. Tamora and Wadham [4] made a numerical solution of advection-diffusion equation for Radial Flow. Limiting source dimensions of three-dimensional analytical point source model for solute transport studied by Ahsanuzzaman et al. [5]. Romao et al. [6] presented the finite difference methods of 3D convection diffusion equation to investigate error in the numerical solution of this equation. Agusto and Bamingbola [7] studied the numerical treatment of the mathematical model for water pollution. They employed an implicit centered difference

* Corresponding author e-mail: fayeznasif@yahoo.co.uk

scheme in space and a forward difference system in time to evaluate the generalized transport equation. Analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one dimensional semi-infinite media studied by Kumar et al. [8]. Wadi et al. [9] studied analytical solution for one-dimensional advection-dispersion equation of the pollution concentration and divided the river into two regions. Remediation of pollution in a river by unsteady aeration with arbitrary initial and boundary conditions was studied by Ibrahim et al. [10]. Andallah and Khatun [11] studied the numerical solution of advection-diffusion equation by using explicit centered difference scheme and Crank-Nicolson scheme for prescribed initial and boundary data. Manitcharoen and Pimpunchatt [12] studied the analytical and numerical solutions of pollution concentration with uniformly and exponentially increasing forms of sources. Azis et al. [13] used the boundary element method for solving a boundary value problem of homogeneous anisotropic media governed by diffusion-convection equation. Hesham et al. [14] proved experimentally that, the impacts of high organic loads in Rosetta branch of Nile River during the low demand period can be mitigated by releasing clean water of amount 30 million m^3/day from Nile River water at the Delta barrage. They proved that this solution reduced the concentrations of ammonia and organic nitrogen below the limits of the local guidelines. The propagation of pollution in water bodies can be studied in several ways [15], [16] and [17].

The objective of this study is to develop a mathematical model for the Nile River response to a sinusoidal varying in the concentration of the pollutant by considering advection-diffusion equation in one dimension which includes terms of decay and increasing sources. We are assuming an added pollutant sources along the river in exponentially increasing terms. We obtained an analytical solution by using Laplace transformation and numerical solution by using explicit finite difference scheme. We studied impact of different parameters controlling the pollutant dispersion along the river at any time and the effect of releasing fresh water from Delta Barrage on concentrations of pollutant in the Nile River.

2 Formulation of the problem

We consider the unsteady flow in a river as being one-dimensional characterized by a single spatial distance $x(km)$ measured from the source of pollution $(x=0)$. The concentration of the pollutant $C(x,t)(kg\,km^{-3})$ is assumed to vary with time t (day) along the length of the river. The one dimensional advection-diffusion equation can be written as ([18] and [12])

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial x^2}-u\frac{\partial C}{\partial x}-\gamma C+\mu\left(1-e^{-\lambda x}\right), \tag{1}$$

where D is the dispersion coefficient $\left(\mathrm{km}^2\ \mathrm{day}^{-1}\right)$, u is the average flow velocity $\left(\mathrm{km\,day}^{-1}\right)$, γ is the pollutant decay rate (day^{-1}), μ is the Zero-order source term $\left(\mathrm{kg\,km}^{-3}\ day^{-1}\right)$ and λ is an arbitrary constant of exponential pollution source terms $\left(\mathrm{km}^{-1}\right)$. The exponentially increasing form of pollution sources in equation (1) has been assumed because the pollution of the Nile River water at the downstream is higher than at the upstream, and this is due to chemical and biological pollutants, agricultural waste and other pollutants. In our study, we will assume that the river is initially free from the pollutant. Hence the initial and boundary conditions associated with equation (1) are:

$$C(x,0)=0, \qquad x\geq 0, \tag{2}$$

$$C(0,t)=c_a+c_b\sin(\omega t), \qquad t>0, \tag{3}$$

$$\frac{\partial C}{\partial x}=0, \qquad x\to\infty,\ t\geq 0, \tag{4}$$

where c_a is the average pollutant concentration $(kg\,km^{-3})$, c_b is a constant whose dimension is $(kg\,km^{-3})$ and ω is the unsteadiness parameter (day^{-1}). A sinusoidal variation in concentration is approximate the effect of a diurnal flow variation from a wastewater treatment plant into a stream with a constant discharge ([19] and [20]).

3 The analytical solution

Applying Laplace transformation on equations (1, 3 and 4) and using equation (2), gives:

$$\begin{aligned} &D\frac{d^2\bar{C}(x,S)}{dx^2}-u\frac{d\bar{C}(x,S)}{dx}\\ &-(S+\gamma)\bar{C}(x,S)=\frac{\mu\left(e^{-\lambda x}-1\right)}{S}, \end{aligned} \tag{5}$$

$$\bar{C}(0,S)=\frac{c_a}{S}+c_b\frac{\omega}{S^2+\omega^2}, \qquad S\geq 0, \tag{6}$$

$$\frac{d\bar{C}(x,S)}{dx}=0, \quad x\to\infty,\ S\geq 0, \tag{7}$$

where S is the Laplace transform of t and $\bar{C}(x,S)$ is Laplace transform of $C(x,t)$. Thus, the general solution of the ordinary differential equation (5) subject to conditions (6) and (7), is given by:

$$\begin{aligned} \bar{C}(x,S)=&-\frac{\mu e^{-\lambda x}\left(S-Se^{\lambda x}+\gamma-\gamma e^{\lambda x}+\lambda(u+D\lambda)e^{\lambda x}\right)}{S(S+\gamma)(S+\gamma-\lambda(u+D\lambda))}\\ &+e^{\frac{ux}{2D}}e^{-\frac{x}{\sqrt{D}}\sqrt{S+\frac{u^2}{4D}+\gamma}}\left(\begin{array}{c}\frac{\mu}{\gamma}\frac{1}{S+\gamma}\\ +\left(\frac{\lambda\mu(u+D\lambda)}{\gamma\left(\gamma-u\lambda-D\lambda^2\right)}+c_a\right)\frac{1}{S}\\ +\frac{\mu}{\left(-\gamma+u\lambda+D\lambda^2\right)\left(S+\gamma-u\lambda-D\lambda^2\right)}\\ +c_b\frac{\omega}{S^2+\omega^2}\end{array}\right) \end{aligned} \tag{8}$$

Now, applying inverse of Laplace transformation on equation (8) , hence the analytical solution of advection-dispersion equation (1) associated with the initial and boundary conditions (2-4) may be written in terms of (x,t) as:

$$
\begin{aligned}
C(x,t) = &-\frac{\mu}{\gamma(\gamma-u\lambda-D\lambda^2)}\, f_1(x,t) \\
&+\frac{\mu}{2\gamma}e^{-\gamma t} f_2(x,t) \\
&+\frac{1}{2}\left(\frac{\lambda\mu(u+D\lambda)}{\gamma(\gamma-u\lambda-D\lambda^2)}+c_a\right) f_3(x,t) \\
&+\frac{\mu}{2(-\gamma+u\lambda+D\lambda^2)} f_4(x,t) \\
&+\frac{1}{4}\, i\, c_b\, e^{-i\omega t} f_5(x,t)-\frac{1}{4}\, i\, c_b\, e^{i\omega t} f_6(x,t)
\end{aligned}
\tag{9}
$$

where:

$$
\begin{aligned}
&f_1(x,t) = \\
&\begin{pmatrix} \gamma e^{-\lambda x}-\gamma e^{\left(-\gamma+\lambda u+D\lambda^2\right)t-\lambda x} \\ -e^{-\gamma t}\left(-\gamma+u\lambda+D\lambda^2\right)-\gamma+\lambda u+D\lambda^2 \end{pmatrix} \\
&f_2(x,t) = \operatorname{erf}c\left[\tfrac{x-ut}{2\sqrt{Dt}}\right]+e^{\frac{ux}{D}}\operatorname{erf}c\left[\tfrac{x+ut}{2\sqrt{Dt}}\right],
\end{aligned}
$$

$$
f_3(x,t) = \begin{pmatrix} \mathrm{e}^{\frac{\left(u-\sqrt{u^2+4\gamma D}\right)x}{2D}}\operatorname{erfc}\left[\frac{x-\sqrt{u^2+4\gamma D}\ t}{2\sqrt{Dt}}\right] \\ +\mathrm{e}^{\frac{\left(u+\sqrt{u^2+4\gamma D}\right)x}{2D}}\operatorname{erfc}\left[\frac{x+\sqrt{u^2+4\gamma D}\ t}{2\sqrt{Dt}}\right] \end{pmatrix},
$$

$$
f_4(x,t) = \begin{pmatrix} e^{\left(-\gamma+\lambda u+D\lambda^2\right)t-\lambda x)} \\ *\operatorname{erfc}\left[\frac{x}{2\sqrt{Dt}}-\sqrt{\left(\frac{u^2}{4D}+u\lambda+D\lambda^2\right)t}\right. \\ +e^{\frac{ux}{D}}e^{\left(-\gamma+\lambda u+D\lambda^2\right)t+\lambda x} \\ *\operatorname{erfc}\left[\frac{x}{2\sqrt{Dt}}+\sqrt{\left(\frac{u^2}{4D}+u\lambda+D\lambda^2\right)t}\right] \end{pmatrix},
$$

$$
f_5(x,t) = \begin{pmatrix} e^{\frac{\left(u-\sqrt{u^2+4D(\gamma-i\omega)}\right)x}{2D}} \\ *\operatorname{erfc}\left[\frac{x-\sqrt{u^2+4D(\gamma-i\omega)}\ t}{2\sqrt{Dt}}\right] \\ +e^{\frac{\left(u+\sqrt{u^2+4D(\gamma-i\omega)}\right)x}{2D}} \\ *\operatorname{erfc}\left[\frac{x+\sqrt{u^2+4D(\gamma-i\omega)}\ t}{2\sqrt{Dt}}\right] \end{pmatrix},
$$

$$
f_6(x,t) = \begin{pmatrix} e^{\frac{\left(u-\sqrt{u^2+4D(\gamma+i\omega)}\right)x}{2D}} \\ *\operatorname{erfc}\left[\frac{x-\sqrt{u^2+4D(\gamma+i\omega)}\ t}{2\sqrt{Dt}}\right] \\ +e^{\frac{\left(u+\sqrt{u^2+4D(\gamma+i\omega)}\right)x}{2D}} \\ *\operatorname{erfc}\left[\frac{x+\sqrt{u^2+4D(\gamma+i\omega)}\ t}{2\sqrt{Dt}}\right] \end{pmatrix}.
$$

where erfc is the complementary error function and $i=\sqrt{-1}$. We have confirmed that equation (9) satisfies equations (1-4).

4 Special cases

The analytical solution (equation (9)) has practical applications in many field problems as follows:
(I) The special case for which $c_a = c_b = 0$ is derived from equation (9) as:

$$
\begin{aligned}
C(x,t) = &-\frac{\mu}{\gamma(\gamma-u\lambda-D\lambda^2)}\, f_1(x,t) \\
&+\frac{\mu}{2\gamma}e^{-\gamma t} f_2(x,t)+\frac{\lambda\mu(u+D\lambda)}{2\,\gamma(\gamma-u\lambda-D\lambda^2)}\, f_3(x,t) \\
&+\frac{\mu}{2\,(-\gamma+u\lambda+D\lambda^2)} f_4(x,t).
\end{aligned}
\tag{10}
$$

Equation (10) gives $C(0,t) = 0$, this satisfies the boundary condition (3) at $c_a = c_b = 0$.
(II) The special case for which $\lambda \longrightarrow \infty$ and $c_b = 0$ is derived from equation (9) as:

$$
\begin{aligned}
C(x,t) = &\frac{\mu}{\gamma}-\frac{\mu}{\gamma}e^{-\gamma t}\left(1-\frac{1}{2}\operatorname{erfc}\left[\frac{x-ut}{2\sqrt{Dt}}\right]-\frac{1}{2}e^{\frac{ux}{D}}\operatorname{erfc}\left[\frac{x+ut}{2\sqrt{Dt}}\right]\right) \\
&+\frac{1}{2}\left(c_a-\frac{\mu}{\gamma}\right)\begin{pmatrix} \mathrm{e}^{\frac{\left(u-\sqrt{u^2+4\gamma D}\right)x}{2D}}\operatorname{erfc}\left[\frac{x-\sqrt{u^2+4\gamma D}\ t}{2\sqrt{Dt}}\right] \\ +\mathrm{e}^{\frac{\left(u+\sqrt{u^2+4\gamma D}\right)x}{2D}}\operatorname{erfc}\left[\frac{x+\sqrt{u^2+4\gamma D}\ t}{2\sqrt{Dt}}\right] \end{pmatrix}.
\end{aligned}
\tag{11}
$$

Equation (11) is the same as that obtained by Kumar [21] (when $m = 0$).

5 Numerical solution

The explicit finite difference method (EFDM) is applied to solve equation (1) associated with the initial and boundary conditions (2-4). The forward difference scheme was used for $\frac{\partial C}{\partial t}$. The central difference scheme was used for $\frac{\partial^2 C}{\partial x^2}$ and $\frac{\partial C}{\partial x}$. With these substitutions, equation (1) can be written as :

$$
\begin{aligned}
C_{i,j+1} = &\, r_1\, C_{i-1,j}+r_2\, C_{i,j}+r_3\, C_{i+1,j} \\
&+\mu\,(1-e^{-\lambda x_i})\Delta t,
\end{aligned}
\tag{12}
$$

where i and j refer to the discrete step lengths Δx and Δt for the coordinate x and time t, respectively, and

$$
r_1 = \frac{D\,\Delta t}{(\Delta x)^2}+\frac{u\,\Delta t}{2(\Delta x)},
\tag{13}
$$

$$
r_2 = 1-\frac{2D\,\Delta t}{(\Delta x)^2}-\gamma\,\Delta t,
\tag{14}
$$

$$
r_3 = \frac{D\,\Delta t}{(\Delta x)^2}-\frac{u\,\Delta t}{2(\Delta x)}.
\tag{15}
$$

Equation (12) represents a formula for $C(i, j+1)$ at the $(i, j+1)^{th}$ mesh point in terms of known values along the j^{th} time row. The truncation error for equation (12) is

$O(\Delta t, (\Delta x)^2)$. Using a small-enough values of Δx and Δt, the truncation error can be reduced until the accuracy achieved is within the error tolerance [22]. The initial and boundary conditions (2-4) can be expressed in the finite difference form as

$$C_{i,0} = 0, \qquad x \geq 0,\ t = 0. \tag{16}$$

$$C_{0,j} = c_a + c_b\ \sin(\omega\, t_j), \quad x = 0,\ t > 0, \tag{17}$$

$$C_{N,j} = C_{N-1,j}\ , \qquad x \to \infty,\ t \geq 0, \tag{18}$$

where $t_j = j\Delta t$ and $x_i = i\ \Delta x$. $N = x_\infty/\Delta x$ is the grid dimension in the x direction and x_∞ is the distance in the direction x at which $\frac{\partial C}{\partial x} \to 0$.

6 Results and discussions

The analytical solution obtained in equation (9) is illustrated in figures (1 and 2). Numerical solution of equation (12) using explicit finite difference method with the initial and boundary conditions (16-18) is given in figure (1). Figures (1-3) have the common input data $0 \leq x \leq 70(\text{ km}), D = 1.2\left(\text{km}^2\text{day}^{-1}\right), \gamma = 0.4\left(\text{day}^{-1}\right), \lambda = 0.5\left(\text{ km}^{-1}\right), c_a = 37\left(\text{ kg km}^{-3}\right), c_b = 13\left(\text{ kg km}^{-3}\right)$, and $\omega = 2\pi\left(\text{day}^{-1}\right)$. Figure (1), shows the variation of $C(x,t)$ with time for the values t $= 0.5, 1$ and 1.5 (day), where $u = 26\left(\text{km day}^{-1}\right)$ and $\mu = 0.6\left(\text{ kg km}^{-3}\text{day}^{-1}\right)$. From figure (1), it is clear that:
(I) The imaginary part in equation (9) is equal to zero for all values of $C(x,t)$ this means that, when separate equation (9) into its real part and imaginary part we get the imaginary part is equal to zero. Then equation (9) can be applied to practical problems.
(II) The fluctuations of the values of $C(x,t)$ is due to the boundary condition (3) so at $t =$ constant, as x increases, $C(x,t)$ is fluctuating until it reaches zero. This result agrees with that obtained by Shukla [19].
(III) At any cross section $x =$ constant, as t increases, C is fluctuating. This result agrees with that obtained by Shukla [19].
(IV) When the time t increases, the distance between the source of pollution $(x = 0)$ and the point of zero concentration $(C(x,t) = 0)$ increases. In the numerical solution of equation (12), the step length $\Delta x = 0.1 km$ and $\Delta t = 0.002(day)$, have been used to achieve the stability of the finite difference scheme. To test the accuracy of the numerical solution, a comparison between the analytical solution given by equation (9) and numerical solution given by equation (12), is made and illustrated in figure (1). From figure (1) it is clear that there is a very good agreement between the analytical solution and numerical solution. So the explicit finite difference method is effective and accurate for solving advection-diffusion equation, which is especially important when arbitrary initial and boundary conditions are required. Figure (2) shows the variation of $C(x,t)$ with μ for the values $\mu = 0.6, 2$ and $4\left(\text{ kg km}^{-3}\text{day}^{-1}\right)$, where $t = 1.5(\text{ day })$, $u = 26\left(\text{ km day}^{-1}\right)$. From figure (2), it is clear that:
(I) At any cross section $x = constant$, $C(x,t)$ increases as μ increases.
(II) At $x \geq 42\, km$ we notice that $C(x,t)$ did not change with increasing x, and this satisfies the condition (4).
Equation (10) is illustrated in Figure (3). Let the cross section area of the river at $x = 0$ be A, then the flux of the water (the volume of water crossing A every day) will be $Q = A\, u$. Consequently increasing the value of u means increasing the value of Q for constant values of A. Let the maximum value of C be denoted by C_m. Figure (3), shows the variation of $C(x,t)$ with flow velocity for the values $u = 5, 10, 15, 20.25$ and $30\left(\text{ km day}^{-1}\right), t = 1.5(\text{ day })$ and $\mu = 0.6\left(\text{ kg km}^{-3}\text{day}^{-1}\right)$. From figure (3), it is clear that:
(I) At any cross section $x =$ constant, $C(x,t)$ decreases as u increases.
(II) The maximum value of $C(x,t)$ $(C_m = 0.68)$ is constant for $u = 5, 10, 15, 20.25$ and $30\,(km\,day^{-1})$.
(III) At $u =$ constant, as x increases, $C(x,t)$ increases until it reaches the maximum value $C_m = 0.68(kg\,km^{-3})$.
(IV) As expected, as u increases i.e. the quantity of fresh water released from the Delta Barag increases (the flux of fresh water Q increases), the maximum value of pollutant concentration C_m moves toward the downstream of the Nile River. This result agrees with that obtained by Saleh et. al [23].

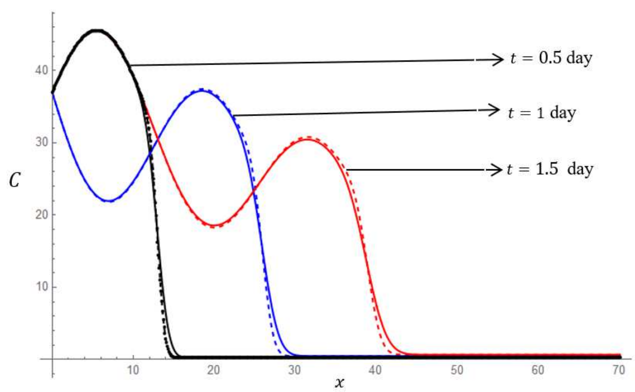

Fig. 1: The comparison between the analytical solution (equation (9)) and the numerical solution (equation (12)) for the values t $= 0.5, 1$ and 1.5 (day), $D = 1.2\left(\text{ km}^2\text{day}^{-1}\right), u = 26\left(\text{ km day}^{-1}\right), \gamma = 0.4\left(\text{day}^{-1}\right), \mu = 0.6\left(\text{ kg m}^{-3}\text{day}^{-1}\right), \lambda = 0.5\left(\text{ km}^{-1}\right), c_a = 37\left(\text{ kg km}^{-3}\right), c_b = 13\left(\text{ kg km}^{-3}\right)$ and $\omega = 2\pi\left(\text{day}^{-1}\right)$ where the dashed lines represents the numerical solution and the continuous lines represents the analytical solution.

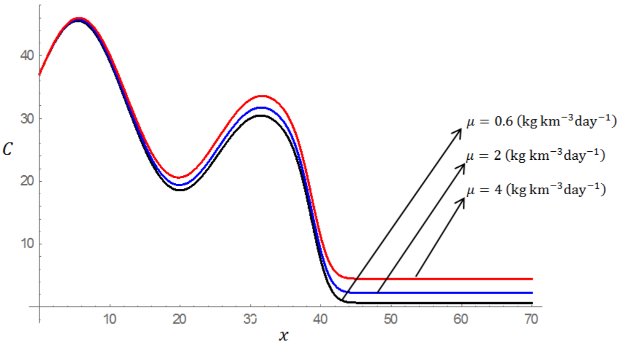

Fig. 2: The variation of $C(x,t)$ with μ in equation (9) for the values $\mu = 0.6, 2$ and $4\left(\text{kg km}^{-3}\text{day}^{-1}\right)$, t= 1.5 (day), $D = 1.2\left(\text{km}^2\text{day}^{-1}\right), u = 26\left(\text{km day}^{-1}\right), \gamma = 0.4\left(\text{day}^{-1}\right), \lambda = 0.5\left(\text{km}^{-1}\right), c_a = 37\left(\text{kg km}^{-3}\right), c_b = 13\left(\text{kg km}^{-3}\right)$ and $\omega = 2\pi\left(\text{day}^{-1}\right)$.

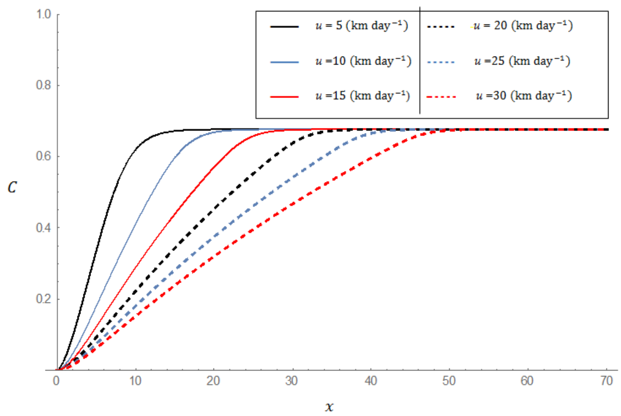

Fig. 3: The variation of $C(x,t)$ with u in equation (10) for the values $u = 5, 10, 15, 20.25$ and $30\left(\text{km day}^{-1}\right)$, t = 1.5(day), $D = 1.2\left(\text{km}^2\text{day}^{-1}\right), \gamma = 0.4\left(\text{day}^{-1}\right), \mu = 0.6\left(\text{kg m}^{-3}\text{day}^{-1}\right), \lambda = 0.5\left(\text{km}^{-1}\right), c_a = 37\left(\text{kg km}^{-3}\right), c_b = 13\left(\text{kg km}^{-3}\right)$ and $\omega = 2\pi\left(\text{day}^{-1}\right)$ $t = 1.5$(day) and $\mu = 0.6\left(\text{kg km}^{-3}\text{ day}^{-1}\right)$.

7 Conclusions

The analytical solution obtained generalize the earlier solution obtained by Kumar [21] (when $m = 0$). Numerical solution for the same problem also obtained by using explicit finite difference scheme. When comparing the analytical solution with the numerical solution, we found a very good agreement between them. Impacts of different parameters controlling the pollutant dispersion have been studied separately with the help of graphs. We found that at any cross section $x =$ constant, $C(x,t)$ is fluctuating with the increase of t and $C(x,t)$ increases as μ increases. At constant time t, as x increases, $C(x,t)$ is fluctuating until it reaches zero. Figure (3) emphasize the fact that we can reduce the high concentration of pollutant by releasing clean water discharges from barrage in a river.

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References

[1] W.E. Alnaser, M. Abdel-Aty and O. Al-Ubaydli, Mathematical Prospective of Coronavirus Infections in Bahrain, Saudi Arabia and Egypt, Information Sciences Letters,9,51-64(2020).
[2] G.D. Hutomo, J. Kusuma, A. Ribal, A.G. Mahie and N. Aris, Numerical solution of 2-d advection-diffusion equation with variable coefficient using du-fort frankel method, Journal of Physics: Conference Series,1180(1),012009(2019).
[3] C. Zoppou and J.H. Knighe, Analytical solutions are provided for the one-dimensional transport of a pollutant in an open channel with steady unpolluted lateral inflow uniformly distributed over its whole length, Journal of Hydraulic Engineering,132,144-148(2009).
[4] J. Tamora and C. Wadham, Numerical Solution of Advection-Diffusion Equations for Radial Flow, University of Oxford,(2002).
[5] A.N.M. Ahsanuzzama, R. Kolar and M. Zaman, Limiting source dimensions of three dimensional analytical point source model for solute transport, Hydrology Days,2003,1-15(2003).
[6] E.C. Romao, J.B. Silva and L.F. M. Moura, Error analysis in the numerical solution of 3D convection-diffusion equation by finite difference methods, J. Engenharia Termica (Thermal Engineering), 8,12-17(2009).
[7] F.B. Agusto and O.M. Bamingbola, Numerical Treatment of the Mathematical Models for Water Pollution, Research Journal of Applied Sciences,2,548-556(2007).
[8] A. Kumar, D.K. Jaiswal and N. kumar, Analytical solutions of one-dimensional advection-diffusion equation with variable coefficients in a finite domain, Journal of earth system science,118, 539-549(2009).
[9] A.S. Wadi, M.F. Dimian and F.N. Ibrahim, Analytical solutions for one-dimensional advection-dispersion equation of the pollutant concentration, Journal of Earth System Science,123, 1317-1324(2014).
[10] F.N. Ibrahim, M.F. Dimain and A.S. Wadi , Remediation of pollution in a river by unsteady aeration with arbitrary initial and boundary conditions, Journal of Hydrology,525,793-797(2015).
[11] L.S. Andallah and M.R. Khatun, Numerical solution of advection-diffusion equation using finite difference schemes, Bangladesh J. Sci. Ind. Res.,55,15-22(2020).

[12] N. Manitcharoen and B. Pimpunchatt, Analytical and Numerical Solutions of Pollution Concentration with Uniformly and Exponentially Increasing Forms of Sources, Journal of Applied Mathematics,2020,1-9,(2020).

[13] M.I. Azis, Kasbawati, A. Haddade and S.A. Thamrin, On some examples of pollutant transport problems solved numerically using the boundary element method, Journal of Physics:Conference Series 979(1),012075(2018).

[14] S. Hesham El Shazely, A. Hussein El Gammal and M. Hatem Ali, Impact of nitrogen discharges on water management in Rosetta River Nile branch , International Commission of Irrigation and Drainage (ICID) Conference, Kuala Lumpur, Malysia,(2006).

[15] M.Kh. Hadhouda and Z.S. Hassan, Mathematical Model for Unsteady Remediation of River Pollution by Aeration, Inf. Sci. Lett.,11,323-329(2022) .

[16] S.D. Kurakbayeva, Zh.R. Umarova, G.A. Besbayev and S.B. Botayeva1, Modeling and Development of a Software Complex for Calculating Pollution Propagation in a Water Bodies, Appl. Math. Inf. Sci.,9, 1699-1708(2015).

[17] A. Allahem, S. Boulaaras and S. Ghanem, A New Mathematical Model and its Application in the Pollution of Air and Water: An Application of Virtual Experience in Qassim Province in Kingdom of Saudi Arabia, Appl. Math. Inf. Sci.,11,1615-1624(2017).

[18] M.Th.V. Genuchten and W.J. Alves, Analytical solutions of the one- dimensional convective-dispersive solute,USDA Tech. Bull. 1661, U.S. Gov. Print Office, Washington, DC.,60,1982.

[19] V.P. Shukla, Analytical Solutions for Unsteady Transport Dispersion of Periodic Waste Discharge Concentration Nonconservative Pollutant with Time- Dependent, Journal Of Hydraulic Engineering, 9,866-869(2002).

[20] D. Dean Adrian, F. Xin Yu. and D. Barbe, Water quality modeling for a sinusoidally varying waste discharge concentration, Water Research,28,1167-1174(1994).

[21] L.K. Kumar, An analytical approach for one-dimensional advection-diffusion equation with temporally dependent variable coefficients of hyperbolic function in semi-infinite porous domain, International Research Journal of Engineering and Technology,4,1454-1460(2017).

[22] J.D.Jr. Anderson, Computational Fluid Dynamics, McGraw-Hill, New York,128-165,(1995).

[23] A. Saleh, F.N. Ibrahim and M.Kh. Hadhouda, Remediation of pollution in a river by releasing clean water, Inf. Sci. Lett.,11,127-133(2022).