An Analysis of Data Production based on the Consistency of Decision Matrices

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Abstract. Multi-criteria decision making methods are used to solve numerous problems related to several disciplines such as engineering, management and business. Consistency of a decision making application is of crucial importance because of its dependability, reliability and sustainability. In this study, data production phenomenon is discussed based on consistency of a decision matrix. We complete different tests including (i) 16 tests (different triangular scales) for different 64, 125 and 216 decision matrix sets for four-criteria problems, (ii) 16 tests for different 256, 625 and 1296 decision matrix sets for five-criteria problems. Based on our observations from the results, we find that first level derivation is always consistent. However, the second and higher level derivations exhibit inconsistencies. The results are also valid if the expert evaluations for the same pairwise comparisons are considered as equal (1,1,1). This study is expected to improve the reliability results for the future decision making studies.

Keywords: MCDM · decision matrix · fuzzy sets · consistency · data production

1 Introduction

Industries from all different sectors such as logistics, economy, transportation, shipping business, etc. rely on data-driven in their decision making processes [22]. Users should process the input data in a well-structured form to derive meaningful inferences and outputs [20]. Multi-criteria decision making (MCDM) denotes a set of methods for ranking, selection [26], prioritization [19], allocation and so on [7, 26]. Data in MCDM are often represented in a matrix form. The matrix is designed in different forms such as (i) direct value assignments of alternatives/parameters based on criteria / attributes, (ii) pairwise comparisons of criteria or alternatives based on criteria [24] (iii) other forms in the literature.

In this study, we present a non-conventional method for data acquisition from the decision makers for their pairwise comparisons. We use the consistency control technique of Aguarón and Moreno-Jiménez (2003) [1] to observe the changes in the consistency after we produce data from the previous data obtained from the decision makers. We discuss two different approaches for the data production as first-level derivation and higher-level derivations. Here, derivation means producing data from the existing data.

Consistency of a matrix is one of the most significant phenomena to obtain information about the reliability, dependability and accuracy of the data and results [13]. In the literature, there are numerous studies that deal with either improving the centric consistency index (CCI) or applying it for the other decision making problems. For example, CCI is used for some fuzzy extended analytical hierarchy process (FAHP) modeling [2] and has found applications in several different fields such as ballast water treatment [23] and marine accident analysis [15]. Other different versions of AHP methods involving CCI include regime switching FAHP [6], rotational priority [3], intuitionistic FAHP [18] and improved Gaussian FAHP [21]. CCI can also be embedded within other techniques such as quality function deployment [5] and fuzzy fault tree analysis method [13]. Some of the most common applications are route selection [12, 10, 11, 14], technology selection [16, 17], fire protection engineering [8], marine accident analysis [15], logistics and supply chain management [25] and road traffic crashes [9]. The remainder of this paper is organized as follows. In Section 2, we present some preliminary definitions of CCI concept in order to make the paper self-contained while Section 3 provides the application use-case. Section 4 discusses the paper and provides the future directions. Finally, Section 5 concludes the paper.

2 Consistency control

Crawford and Williams (1985) proposed row geometric mean method (RGMM) to check the consistency of pairwise decision matrix [4]. Then, Aguarón and Moreno-Jiménez (2003) justified the method as geometric consistency index (GCI) [1] for the decision matrices. The GCI is given in Equation 1.

$$GCI = \frac{2}{(n-1)(n-2)} \sum_{i<j} log^2 e_{ij} \tag{1}$$

Finally, Bulut et al. (2012) developed a centric consistency index (CCI) which is the general form of the formula for the aggregated decision matrices (Equation 2) [2]. CCI can also be used for the trapezoidal fuzzy numbers [13]. CCI is the fuzzy extended version of GCI that can be defined as follows.

Let $D = (d_{Aij}, d_{Bij}, d_{Cij})$ be a fuzzy decision matrix and w=$\Big[(\omega_{A1},\ \omega_{B1},\ \omega_{C1}),\ (\omega_{A2},\ \omega_{B2},\ \omega_{C2}),\ \ldots,\ (\omega_{An},\ \omega_{Bn},\ \omega_{Cn})\Big]$ be the priority vector obtained from the matrix D by employing the RGMM.

$$CCI = \frac{2}{(n-1)(n-2)} \sum_{i<j} \Big[log\Big(\frac{d_{Aij}+d_{Bij}+d_{Cij}}{3}\Big) - log\Big(\frac{\omega_{Ai}+\omega_{Bi}+\omega_{Ci}}{3}\Big) + log\Big(\frac{\omega_{Aj}+\omega_{Bj}+\omega_{Cj}}{3}\Big)\Big]^2 \quad (2)$$

The study of Aguarón and Moreno-Jiménez (2003) defines thresholds for $\overline{GCI}$ as 0.31, 0.35 and 0,37 if $n = 3$, $n = 4$ and $n > 4$, respectively. If $CCI(D) < \overline{GCI}$, D is consistent enough, and if $CCI(D)$ is equal to 0 it means that the D is completely consistent.

3 Application

In the conventional approach, matrix formulation is conducted by pairwise comparisons. The decision makers assign fuzzy evaluations for the decision matrices. For the four-criteria matrix (C1,C2,C3 and C4), the experts should evaluate six different comparisons as C1-C2, C1-C3, C1-C4, C2-C3, C2-C4 and C3-C4 where C1-C1, C2-C2, C3-C3 and C4-C4 are (1,1,1) and inverse fuzzy operations are done for the rest of it (C2-C1, C3-C1, C4,C1, C3-C2, C4-C2 and C4-C3).

In this study, we observe the changes in consistency of decision matrices after the data production process. The data production process is completed as follows.

1. Decision makers only evaluate the minimum number of required pairs. In this case for the four-criteria matrix, it is C1-C2, C1-C3 and C1-C4. This will also help decision makers to save time and energy.
2. We propose two types of data production: First-level derivation and higher-level derivation (multi-level derivation). For the first-level derivation, we find the values of C2-C3, C2-C4 and C3-C4 by the division operations of C1-C3 & C1-C2, C1-C4 & C1-C2 and C1-C4 & C1-C3, respectively.
3. For the multi-level derivation, we conduct first-level derivation for the second row, and we realize the second-level derivation for the third row. The values of second-level derivation are obtained from the values of the first-level derivation. For example, for the second row, we find the values of C2-C3 and C2-C4 by the division operations of x $\rightarrow$ C1-C3 & C1-C2 and y $\rightarrow$ C1-C4 & C1-C2. For the third row, we find the values of C3-C4 by the division operations of y/x.

In this study, we prefer eight different scales to observe the changes in a comprehensive way, and understand the rationale behind the changes. The scales in the form of triangular fuzzy sets are given in Table 1.

In this paper, we firstly set matrices into two groups as four-criteria and five-criteria matrices. For the four-criteria matrix, we assign all different combinations of each values of a scale. For instance, the first scale has five different values, these are E, VL, L, M and H. For the four-criteria matrix, we assign all

Table 1: The scales used for decision making

	1	2	3	4	5	6	7	8
Equal (E)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)
Very Low (VL)	(1,2,3)	(1,3,5)	(1,1,3)	(1,3,5)	(1,3,5)	(1,5/2,5)	(1,3,5)	(1,2,5)
Low (L)	(3,4,5)	(2,4,6)	(3,5,7)	(3,5,7)	(3,4,6)	(2,7/2,6)	(5,7,9)	(2,5,7)
Middle (M)	(5,6,7)	(3,5,7)	(5,7,9)	(5,7,9)	(5,6,7)	(3,9/2,7)	(1/5, 1/3, 1)	(3,8,9)
High (H)	(7,8,9)	(4,6,8)	(7,9,9)	(7,9,9)	(7,7.5,8)	(4,11/2,8)	(1/9,1/7,1/5)	
Very High (VH)		(5,7,9)			(9,9,9)	(5,13/2,9)		

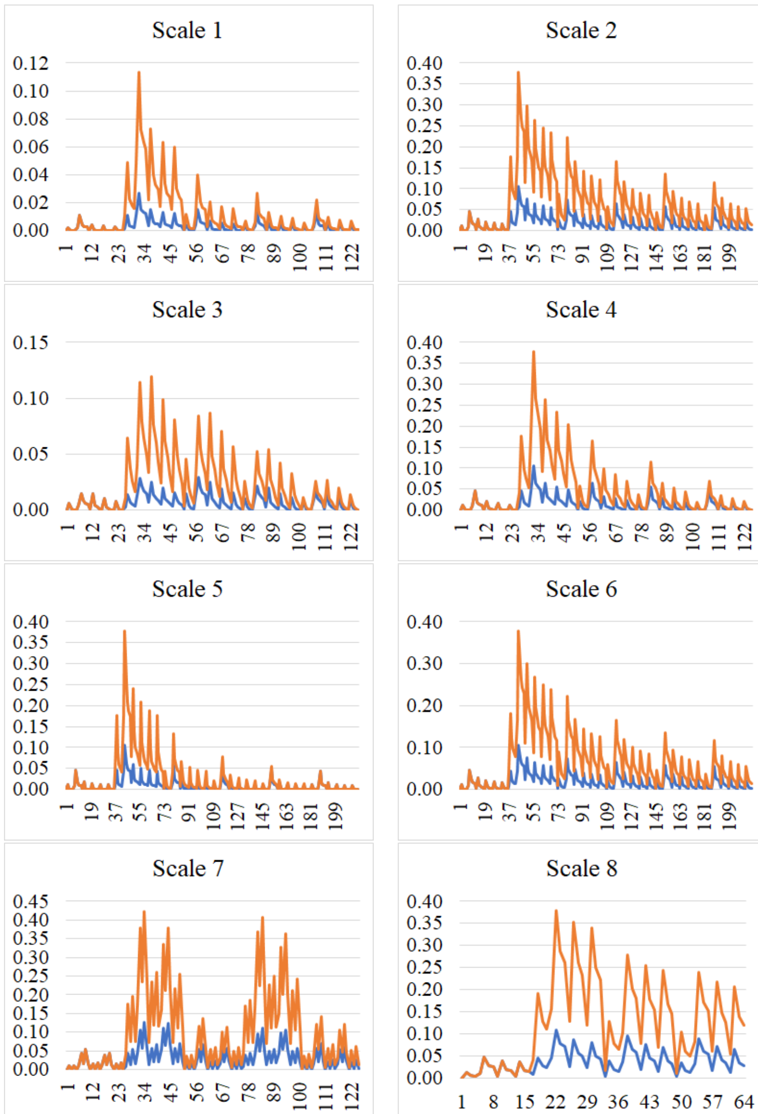
different combinations for the C1-C2, C1-C3 and C1-C4. Therefore, 5 different values for C1-C2, 5 different values for C1-C3, and 5 different values for C1-C4, $5 \times 5 \times 5 = 125$ different matrices for the four-criteria matrix based on first scale. If the scale has 6 different values, the number of matrices is $6 \times 6 \times 6 = 216$ for the four-criteria matrix, and $6 \times 6 \times 6 \times 6 = 1296$ for the five-criteria matrices. Then, we produce the data for the rest the matrix. Finally, we calculated the consistency values whether if they are under 0.37 or not. The results for the four-criteria and five-criteria matrices are provided in Figures 1 and 2.

In the second part of the study, we assume that the decision maker evaluations for two comparisons are equal to each other, in other words, we do not perform fuzzy operations, instead we take the result as (1,1,1) which means they are equal. For example, let's assume that C1-C2 is VH, and C1-C3 is also VH based on the scale values, we say C2-C3 is Equal (E) = (1,1,1) instead of (7/9, 8/8, 9/7). We make the same calculations based on this approach, and we reach the same conclusions as the first part of research which is given below:

1. First-level derivations are always consistent because the calculations stay in the boundary of expert evaluation data set. The calculations do not go beyond the data on the same row. Moreover, consistency increases if the number of elements of a matrix increases for the first-level derivations.
2. Multi-level derivations might not be consistent. The number of inconsistent four-criteria matrices is lower than the number of five-criteria matrices. Moreover, inconsistency increases if the number of elements of a matrix increases for the multi-level derivations.

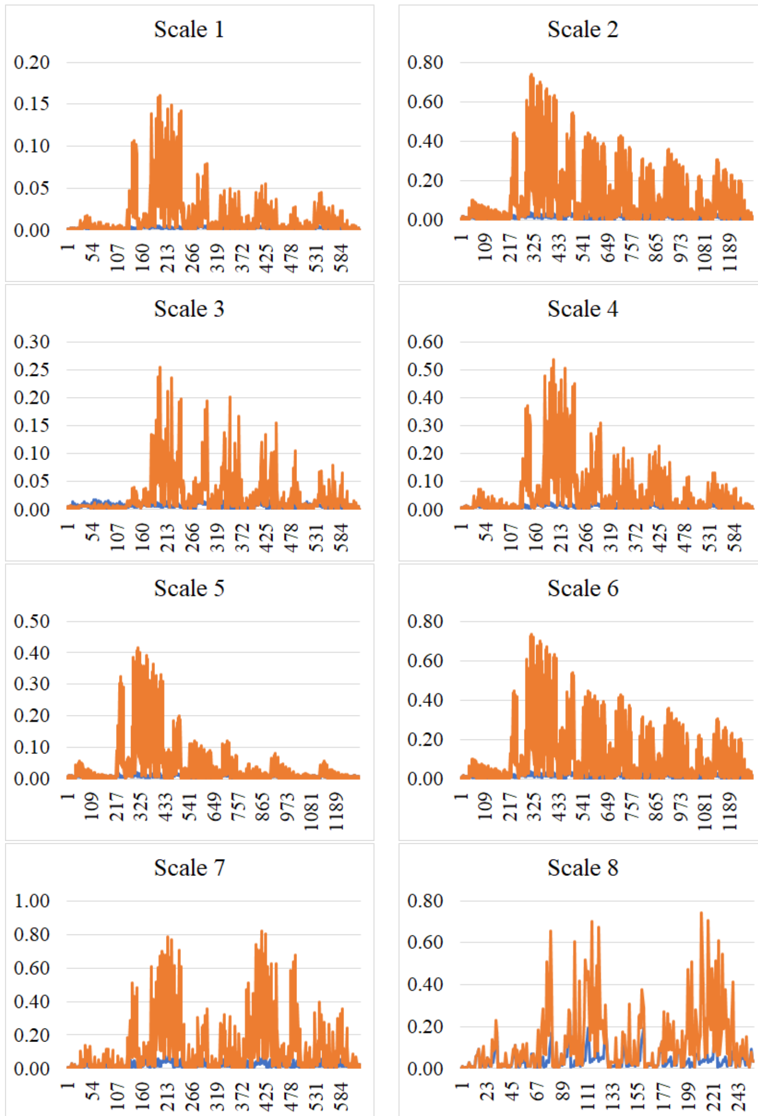
4 Discussions

For the pairwise comparisons, if a decision maker expresses two opinions for two different comparisons (C1-C2 and C1-C3), we can produce one more opinion (C2-C3) based on the first-level derivation. Similarly, if the number of opinions increases to three, we can produce three more, and so on. Therefore, this is an open research area about how much information that the opinions of decision makers carry, and the data can be produced more. Moreover, there is another concern whether these minimum number opinions of a decision maker are sufficient to understand the problem completely and solve it, even though we have



Blue: First-level derivations Orange: Multi-level derivations

Fig. 1: Consistency values for four-criteria matrices using different scales.



Blue: First-level derivations Orange: Multi-level derivations

Fig. 2: Consistency values for five-criteria matrices using different scales.

all the data that are produced. Quality of opinions of the decision makers should be discussed.

In the future, decision makers can evaluate different minimum number of required pairs. For example, for the four-criteria matrix, another alternative might be C1-C2, C2-C3 and C3-C4. The reasons for inconsistency results and the geometry of the graphics might also be discussed in the future. Different types of fuzzy numbers (trapezoidal, Gaussian, etc.) can be used.

5 Conclusions

This study deals with data production phenomenon for pairwise decision matrices. We checked the data production process by implementing centric consistency index for the decision matrices. We made tests for 2424 different four-criteria decision matrices and 13288 different five-criteria decision matrices. It was found that the first level derivations are always consistent based on combinations of eight different scales for both four-criteria and five-criteria matrices. However, multi-level derivations distort the boundaries of the problem, therefore, they are mostly inconsistent. This study helps decision makers to optimize decision making if they evaluate only minimum required comparisons in a decision problem.

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