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Mathematical Research in Digital Age

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Mathematical Research in Digital Age

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Abstract- The time when someone can do real, publishable mathematics completely without the aid of a computer is coming to a close; the use of computers in mathematical research is both widespread and under-acknowledged. Mathematicians use computers in a number of ways. This paper highlights the importance of mathematics and digital age in today's technological advancement; it also explains the influence of digital age on Mathematics research. Key areas where Information and Communication Technology can be applied to Mathematical research are discussed. To demonstrate the use of computer program on Mathematical analysis, some problems were solved analytically and were also solved using computer programs (Mathlab and Python). These two procedures are compared and it is clearly shown that using computer packages to solve Mathematical problems are more efficient, easier and accurate.

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I. Introduction

Mathematics is often defined as the study of quantity, magnitude, and relations of numbers or symbols. It embraces the subjects of arithmetic, geometry, algebra, calculus, probability, statistics, and many other special areas of research. It is the study of structures and pattern in large numerical sets [1]. Mathematics is an indispensable subject of study; it plays an important role in forming the basis of all other sciences which deal with the material substance of space and time. It is said that Mathematics is the gate and key of the Science.

According to [2] Mathematics is a way of thinking, a way of organizing a logical proof, a way reasoning that gives an insight into the power of human mind. It is uniquely well placed to respond to the demand of rapidly changing fields of life such as engineering, biological sciences, medicine and economic.

Research in general is a way of investigating a system, model, matter or theorem to discover hidden or previously unknown fact.

a) Research in Mathematics

In mathematics, research calls for the creation of new results, that is, either new theorems radically different or improved proofs of older results. Research comprises of creative work undertaken on a systematic basis in order to increase the stock of knowledge, including knowledge of humans, culture and society, and the use of this stock of knowledge to devise new applications. It is used to establish or confirm facts, reaffirm the results of previous work, solve new or existing problems, support theorems, or develop new theories [3]. A research project may also be an expansion on past work in the field. To test the validity of instruments, procedures, or experiments, research may replicate elements of prior projects, or the project as a whole. According to [4] research is the systematic investigative process employed to increase or revise current knowledge by discovering new facts. In Mathematics research theorem can be proved as well as taking a number of pieces and constructing a worthwhile example by putting them together in a new way [3].

b) Digital age

The digital age also known as computer age, information age, new media age; is a period in human history characterized by the shift from traditional industry that the industrial revolution brought through industrialization, to an economy based on information computerization. The digital age is the time period starting in the 1970s with the introduction of the personal computer with subsequent technology introduced providing the ability to make work easier and faster. The digital age formed by capitalizing on computer microminiaturization advances the evolution of technology in daily life, as well as educational life style. Digital age has allowed rapid global communications and networking to shape modern society which we call Information and Communication Technology (ICT) world [5].

ICT is an umbrella term that includes any communication device or application, encompassing: radio, telephone lines and wireless signals, computers as well as necessary enterprise software, hardware, storage, and audio-visual systems, satellite systems which enable users to access, store, transmit, and manipulate information. It is also encompasses various services and applications associated with them, such as videoconferencing and distance learning. It stresses the role of unified communication and the integration of telecommunications [5].

c) Benefits of Digital age to Research

The digital age (Information Age) has affected the workforce in several ways.

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- Internet as a global platform accelerates the flow of information and gives researchers access to a wealth of facts, ideals and academic assistance to other researchers. It gives access to a worldwide library of journals and articles for research literature guide.
- Information storage: With the help of digital age , researchers no longer have to pile up so many research works in form of paper, which are (papers) likely to get damaged by rodents, water etc. However, with help of digital age, so many research works can be stored in a flash [5].
- Computation: The world's technological capacity to compute information with humanly guided general-purpose computers grew from 3.0 × 108 MIPS in 1986, to 4.4 × 109 MIPS in 1993, 2.9 × 1011 MIPS in 2000 to 6.4 × 1012 MIPS in 2007 [6]. Computation is fundamental to scientific research.
- Electronic publication is gradually transforming the way we write papers. At first sight, word processing looks like just a convenient way of writing; but slowly many features of electronic versions become available that are superior to the usual printed papers.
- Presentation of research results in digital age using applications like Power Point, spreadsheets, helps the audience to understand easily.

II. Key Roles Digital Age has Played in Mathematical Research

Before the digital age, professional mathematicians did most of their work on desks using paper and pencil. Today mathematicians still sit at a desk facing monitor screens or laptops. The paper and pencil are still there but a lot of mathematician's activities now involve the use of computer. The computer does not simply assist mathematicians in doing business as usual; rather it changes the nature of what is done. Computers then have changed the way Mathematics progresses. There are many specific forms in which digital age has contributed to mathematical research, some of these forms are in problem solving task, exploring pattern and relationships, practicing of number skills, calculators, spreadsheets, databases and online, interactive resources, automated theorem proving, symbolic computation, scientific computing.

a) Problem solving task

Problem solving task in mathematics is about solving mathematical problems. The major aim of mathematics education is to equip researchers to solve problems. Mathematics consists of skills and processes. The skills are things that researchers are familiar with. These include the basic arithmetical processes and the algorithms that go with them. They also include algebra in all its levels as well as sophisticated areas such as the calculus. Problem solving task is a mathematical process. It is the side of mathematics that enables us to use acquired skills in a wide variety of situations.

Now we shall consider some problems, by solving them manually and also using a computer program.

Example 1:

Given an initial value problem

$$y'' = -y \qquad with \qquad y(0) = 1, y'(0) = 1) \qquad (1)$$

Note [equation 1 is a linear second - order homogeneous differential equation]

Manual Solution

To solve this let assume that $y=e^{mx}$ is a solution to equation 1

Hence; finding the derivative we have,

$$y' = me^{mx}$$

Differentiating further we have,

$$y'' = m^2 e^{mx}$$

Putting these result back into equation 1,

$$m^2 e^{mx} = -e^{mx}$$

$$= 0 \qquad m^2 e^{mx} + e^{mx}$$

Factorizing,

$$e^{mx}(m^2 + 1)0$$

Divide both side by e^{mx} we have,

$$m^2 + 1 = 0 \qquad (characteristicsequation)$$

$$m^2 = -1$$

$$m = \sqrt{-1}$$

$$m = \pm i.$$

Since the root of the characteristic equation is complex, hence the general solution of equation 1 is:

$$y(x) = [A\cos x + B\sin x](2)$$

Now applying the initial conditions

$$\mathrm{y(o)} = \mathrm{A}\cos 0 + \mathrm{B}\sin 0$$
$$= 1$$

Note: $\sin 0 = 0, \quad \cos 0 = 1$

Hence, A=1

Differentiating equation 2

$$y'(x) = -A\sin x + B\cos x$$

Applying the conditions

$$y'(0) = -A\sin 0 + B\cos 0 = 1$$

Hence,

B=1

Putting the value of A and B in equation 2, hence the general solution is,

$$y(x) = \cos x + \sin x$$

Using a Computer Package (Mathlab) to Solve Equation1

The command is below;

>> G = dsolve('D2y+y=0','y(0)=1','Dy(0)=1')

G = cos(x) + sin(x)

Hence, we can conclude that using Mathlab (an I.C.T tool) has made the whole work easier and faster.

Example 2 :

Here is another example using analytical skills and Mathlab to a matrix problem.

Manual Solution

$$A=\begin{bmatrix}1 & 2 & 3\\ 6 & 4 & 2\\ 5 & 8 & 3\end{bmatrix}$$

To find the determinant of matrice A

Det(A)=1(4×3-8×2)-2(6×3-5×2)+3(6×8-5×4)

Det(A)=1(12-16)-2(18-10)+3(48-20)

Det(A)=-4-16+84

Det(A)=64

Using A Computer Package (Mathlab) To Solve The Matrix

The command is below;

>> A = [1 2 3;6 4 2;5 8 3];

>> N = det(A)

N = 64

Example 3

Giving an initial value problem

$$y'' = -y \qquad with \quad y(0) = 1, y'(0) = 1 \qquad ---------- (1)$$

Solving using the Numerical Scheme below;

Derivation Of The Scheme

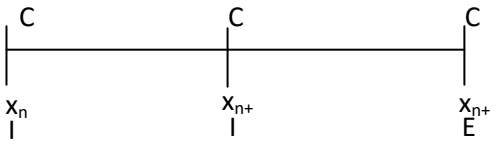


Figure 1 : diagram illustrating collocation and interpolation points

Where; I = Interpolation point, C = Collocation point and E = Evaluation point

Given

$$y''(x) = f(x, y), y(x_o) = \alpha, y(x_o) = \beta \qquad (1)$$

Let the basis solution to the special second order initial value problems (1) be the exponential function

$$y(x) = e^{\alpha x} = \sum_{j=0}^{\infty} \frac{\alpha_j x^j}{j!}, \qquad (2)$$

where α is a constant.

Expanding equation (2) gives

$$\sum_{j=0}^{r-1} \frac{\alpha_j x^j}{j!} = \alpha_o x^o + \alpha_1 x^1 + \frac{1}{2!}\alpha_2 x^2 + \frac{1}{3!}\alpha_3 x^3 + \ldots \quad (3)$$

Taking r in equation (3) to be the sum of number of interpolation points (I) and number of collocation points (C), I = 2 and C = 3, the approximate solution to equation (1) is;

$$y(x) = \sum_{j=0}^{4} \frac{\alpha_j x^j}{j!} = \alpha_o x^o + \alpha_1 x^1 + \frac{1}{2!}\alpha_2 x^2 + \frac{1}{3!}\alpha_3 x^3 + \frac{1}{4!}\alpha_2 x^2 \quad (4)$$

Differentiating equation (4) twice gives;

$$y'(x) = \alpha_1 + \alpha_2 x + \frac{1}{2}\alpha_3 x^2 + \frac{1}{6}\alpha_4 x + \ldots \qquad (5)$$

and

$$y''(x) = \alpha_2 + \alpha_3 x + \frac{1}{2}\alpha_4 x^2 + \ldots \qquad (6)$$

Interpolating equation (4) at x_n and x_{n+1}

$$y_n = \alpha_o + \alpha_1 x_n + \frac{1}{2}\alpha_2 {x_n}^2 + \frac{1}{6}\alpha_3 {x_n}^3 + \frac{1}{24}\alpha_4 {x_n}^4 \qquad (7)$$

$$y_{n+1} = \alpha_o + \alpha_1 x_{n+1} + \alpha_2 x_{n+1}^2 + \frac{1}{6}\alpha_3 x_{n+1}^3 + \frac{1}{24}\alpha_4 x_{n+1}^4 \qquad (8)$$

Collocating equation (6) at x_n, x_{n+1}, x_{n+2}.

$$f_n = \alpha_2 + \alpha_3 x_n + \frac{1}{2}\alpha_4 {x_n}^2 \qquad (9)$$

$$f_{n+1} = \alpha_2 + \alpha_3 x_{n+1} + \frac{1}{2}\alpha_4 x_{n+1}^2 \qquad (10)$$

$$f_{n+2} = \alpha_2 + \alpha_3 x_{n+2} + \frac{1}{2}\alpha_4 x_{n+2}^2 \qquad (11)$$

Where; $x_{n+1} = x_n + h$ and $x_{n+2} = x_n + 2h$

From the result gotten,

Now we obtain the coefficients $\alpha_o, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ by solving equations (7-11) using Guassian Elimination method

$$\alpha_o = \frac{1}{24h^2}\begin{bmatrix} 24y_n hx_n + 24y_n h^2 - 24x_n y_{n+1}h + 7x_n f_n h^3 + 12x_n^2 f_n h^2 + 6x_n^3 f_n h + x_n^4 f_n + 6x_n f_{n+1}h^3 \\ -8x_n^3 f_{n+1}h - 2x_n^4 f_{n+1} - x_n f_{n+3}h^3 + 2x_n^3 f_{n+2}h + x_n^4 f_{n+2} \end{bmatrix}$$

$$\alpha_1 = \frac{1}{24h^2}\begin{bmatrix} 24y_n h - 24x_n y_{n+1}h + 7x_n f_n h^3 + 24 f_n x_n h^2 + 18x_n^2 f_n h + 4x_n^3 f_n + 6 f_{n+1}h^3 - 24x_n^2 f_{n+1}h \\ -8x_n^3 f_{n+1} - f_{n+2}h^3 + 6x_n^2 f_{n+2}h + 4x_n^3 f_{n+2} \end{bmatrix}$$

$$\alpha_2 = -\frac{1}{2h^2}\left[2 f_n h^2 + 3x_n f_n h + x_n^2 f_n - 2x_n^2 f_{n+1} - 4x_n f_{n+1}h + x_n^2 f_{n+2} + x_n f_{n+2}h\right]$$

$$\alpha_3 = -\frac{1}{2h^2}\left[3 f_n h + 2x_n f_n - 4x_n f_{n+1}h - 4x_n f_{n+1}h - 4hf_{n+2} + 2x_n f_{n+2} + f_{n+2}h\right]$$

$$\alpha_4 = \frac{1}{h^2}\left[f_n - 2 f_{n+1} + f_{n+2}\right]$$

Putting the values of $\alpha_o, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ into equation (4) and evaluating at x_{n+2}, to get the scheme below:

$$y_{n+2} - 2y_{n+1} + y_n = \frac{h^2}{2}\left[f_{n+2} + 10 f_{n+1} + f_n\right] \qquad (12)$$

Solution

$$y_{n+2} = 2y_{n+1} - y_n + \frac{h^2}{2}[F_{n+2} + 10F_{n+1} + F_n] \qquad (13)$$

For n = 0 to 10
Solving for n = 0
The scheme becomes;

$$y_2 = 2y_1 - y_0 + \frac{h^2}{2}[F_2 + 10F_1 + F_0]$$

Solving for y_1 since the scheme is an implicit scheme
We use Taylor series as the predictor-corrector

$$y'' = -yy''' = -y'y^{iv} = -y''y^{v} = -y'''$$

$$y_{n+1} = y(x_n + h) = y(x_n) + hy'(x_n) + \frac{1}{2!}h^2 y''(x_n) + \frac{1}{3!}h^3 y'''(x_n) + \ldots$$

For n = 0, where h = 0.001

$$y_1 = 1 + (0.001)(1) + \frac{1}{2}(0.001)^2(-1) + \frac{1}{6}(0.001)^3(-1) + \ldots$$

Computing the values above, we have;

$$y_1 = 1.000995$$

$$f_{n+1} = y''(x_n + h) = y''(x_n) + hy'''(x_n) + \frac{1}{2!}h^2 y^{iv}(x_n) + \frac{1}{3!}h^3 y^{v}(x_n) + \ldots$$

For n = 0, where h = 0.001

$$f_1 = -1 + (0.001)(-1) + \frac{1}{2}(0.001)^2(1) + \frac{1}{6}(0.001)^3(1) + \ldots$$

Computing the values above, we have;

$$f_1 = -1.000995$$

$$f_{n+2} = y''(x_n + 2h) = y''(x_n) + 2hy'''(x_n) + \frac{1}{2!}(2h)^2 y^{iv}(x_n) + \frac{1}{3!}(2h)^3 y^{v}(x_n) + \ldots$$

For n = 0, where h = 0.001

$$f_2 = -1 + (0.001)(-1) + \frac{1}{2}(2x0.001)^2(1) + \frac{1}{6}(2x0.001)^3(1) + \ldots$$

Computing the values above, we have;

$$f_2 = -1.001998$$

$$f_n = y''(x_n)$$

For n = 0, where h = 0.001

$f_1 = -1$

Putting the values back into the scheme below

$$y_2 = 2y_1 - y_0 + \frac{h^2}{2}[F_2 + 10F_1 + F_0]$$

$$y_2 = 2(1.000995) - 1 + \frac{0.001^2}{2}[-1.001998 - 10(1.000995) - 1]$$

Computing the values above, we have;

$$y_2 = 1.001984$$

Going back all over again to solve for n = 1…10 will take a lot of time and computational accuracy will not be there.

Using A Python Program To Solve The Problem

```
#program to calculate...
Import math

y = [0,0,0,0,0,0,0,0,0,0]

f = [0,0,0,0,0,0,0,0,0,0]

h = 0.001
#h = input("Enter the value for h: ")
yp = 1
ypp = -1
yppp = -1
ypppp = 1
yppppp = 1
ypppppp = -1
yppppppp = -1
y(0) = 1
y[0] = y0 + (h * yp) + ((h**2 * ypp)/2) + ((h**3 * yppp)/6) + ((h**4 * ypppp)/24) + ((h**5 * yppppp)/120) + ((h**6 * ypppppp)/720) + ((h**7 * yppppppp)/5040)
f[0] = -(y0)
f[1] = -(y[0])
n = 2;
for i in range(1, 10):
   f[i+1] = y0 + (n*h * yp) + ((((n*h)**2) * ypp)/2) + (((n*h)**3 * yppp)/6) + (((n*h)**4 * ypppp)/24) + (((n*h)**5 * yppppp)/120) + (((n*h)**6 * ypppppp)/720) + (((n*h)**7 * yppppppp)/5040)
   y[i] = (2 * y[i-1]) - y0 + ((h**2)/2) * (f[i+1] + (10 * f[i]) + f[i-1])
   n += 1
   f[i] = f[i-1]
   f[i+1] = f[i]
   y0 = y[i-1]
```

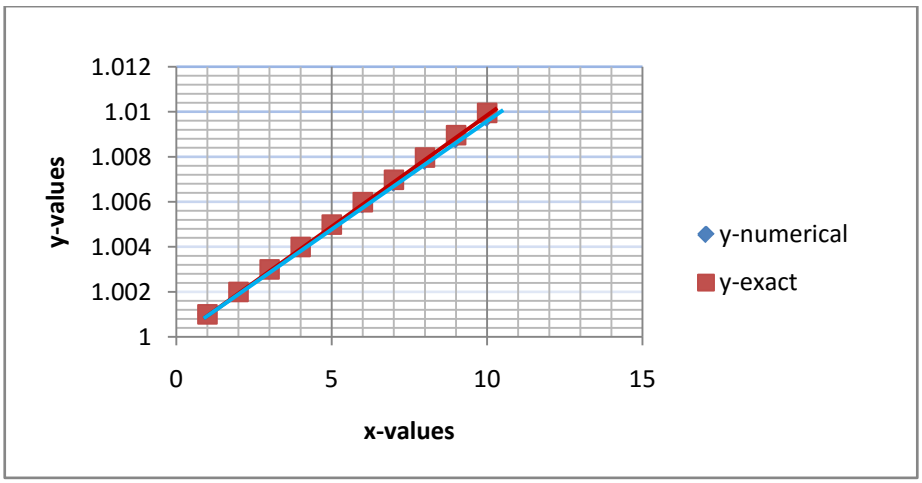
```
for j in range(10):
    print(str(y[j]) + "\t")

#Exact values
exact = [0,0,0,0,0,0,0,0,0,0]
print("------------------------[ EXACT VALUES ]------------------------")
x = h
c = 0
if (x >= 0.01):
    lastval = (x * 10)
else:
    lastval = (x * 10) + x

while x <= lastval:
    a = math.cos(x) + math.sin(x)
    print(a)
    exact[c] = a
    x += h
    c += 1
print("------------------------[ DIFFERENCE ]------------------------")
for e in range(10):
    print(y[e] - exact[e])
```

Result Table

	Y-NUMERICAL	Y-EXACT	ERROR
1	1.0009994998333749	1.000999499833375	$2.220446049250313 \times 10^{-16}$
2	1.00199399566825	1.0019979986673335	$4.00299908354107 \times 10^{-6}$
3	1.0029834935006228	1.002995495503377	$1.2002002754307739 \times 10^{-5}$
4	1.003967994325988	1.0039919893440086	$2.399501802052839 \times 10^{-5}$
5	1.0049474991368397	1.0049874791927345	$3.998005589478382 \times 10^{-5}$
6	1.0059220089226706	1.0059819640540648	$5.9955131394229255 \times 10^{-5}$
7	1.0068915246699726	1.006975442933515	$8.391826354237075 \times 10^{-5}$
8	1.0078560473622364	1.0079679148376062	$1.118674753697313 \times 10^{-4}$
9	1.0088155779799521	1.0089593787738662	$1.4380079391407286 \times 10^{-4}$
10	1.0097701175006084	1.009949833750832	$1.7971625022350501 \times 10^{-4}$

From the table above, the use of Python (a Programming Language) has enabled us to solve the differential equation with different values of x. Furthermore, we are able to get the series of y-numerical and y-exact with their difference without much stress.

With the help of digital age, the result obtained in the table above can be further interpreted by plotting a graph that shows the difference between the y-numerical and y-exact.



b) Automated Theorem Proving

Automated Theorem Proving is an area of study to get computers to prove logical and mathematical statements. Not just enumerating instances of a theorem exhaustively, but applying logical deduction, induction, inference and search strategies (depth first, breadth first, best first, iterative deepening) to arrive at a solution. There are branches of Mathematics such as Model theory and Proof Theory which study proofs themselves. Automated theorem proving is a subfield of automated reasoning and mathematical logic dealing with proving mathematical theorem by computer programs. Example of Theorem-proving packages is Microsoft's Z3 [4].

III. Summary and Conclusion

Computers have changed the way mathematics progresses. The Digital age has made available for researchers indispensable hardware and software tools that can effortlessly assist in various ways to make research in mathematics easier, faster and motivating. This assistance covers areas such as number theory, calculus, differential equations and linear algebra among others. These resources come with many hundreds of built-in functions, extensive features for manipulating these functions, and a high-level computer language that allows one to easily create functions and procedures of their own. Mathematical research with the help of digital technology has made research to be easier and faster. Therefore mathematics research is more interesting and encouraging in digital age.

References Références Referencias

1. Monaghan, J.(2000) Some issues surrounding the use of algebraic calculators in traditional examinations. International Journal of Mathematical Education in Science & Technology,**31**(3): p. 381-392.
2. Cockcroft, W (1982). Mathematics counts. Department of Education and Skills: London.
3. Shuttleworth, M. (2008) definition of research through https//en.wikipedia.org/wiki/Research
4. Weist, L.R. (2001).The role of computers in mathematics teaching and Learning. In Using Information Technology in Mathematics Education. D.J. Tooke and N.Henderson, 41 – 55. Bing hampton, NY: Haworth Press.
5. Castells, M . (1999)The information age, Volume 1-3: economy, society and culture Https//en.m. wikipedia.org/wiki/digital age.
6. Hilbert, M. and Lopez, P. (2011), Science Journal, 332(6025),60- 65.
7. Aladjev, V.Z. (2004). Computer algebra systems*: a* new software toolbox for *Maple*. [S.l.]: Fultus Books. p.9. ISBN 9781596820005.
8. Nonweiler T. R., 1986. Computational Mathematics: An Introduction to Numerical Approximation, John Wiley and Sons.
9. Ruthven, K. (2008) The Interpretative Flexibility, Instrumental Evolution, and Institutional Adoption of Mathematical Software in Educational Practice: The examples *of* Computer Algebra *and Dynamic* Geometry. Journal of Educational Computing Research, **39**(4): p. pp. 379-394.
10. Teleman F. (1990), As Math Grows More Complex, Will Computers Reign? Through http:/ /www. wired.com/ 2013/03/computers-and math/.

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