Adjoint Modeling to Quantify Stream Flow Changes Due to Aquifer Pumping

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ADJOINT MODELING TO QUANTIFY STREAM FLOW CHANGES DUE TO AQUIFER PUMPING

By

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COMPLETION REPORT

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Abstract

As populations grow and demand for water increases, new sources of water must be found. If groundwater resources are developed to meet these growing demands, the increased pumping of aquifers should not reduce flows in rivers to levels that would limit the availability of water for drinking water supply, irrigation, and riparian habitat. Stream depletion is the term for the change in the river flow rate due to pumping in an aquifer that is hydraulically connected to the river. In many regions of the U.S., a new well cannot be sited until it is shown that pumping the new well will not cause substantial stream depletion. Numerical simulations are typically used to quantify stream depletion. In the standard approach, two numerical simulations are run—one without pumping and one with pumping in a well at the proposed location. In both simulations, the water flux between the river and aquifer is calculated, and the difference between these fluxes is the stream depletion due to pumping at the proposed well location. If multiple well locations are considered, one addition simulation must be run for each additional potential well location; thus, this approach can be inefficient for siting new wells. The goal this research was to develop an adjoint-based modeling approach to efficiently quantify stream depletion due to aquifer pumping. In a single simulation of an adjoint model, stream depletion is calculated for a well at any location in the aquifer; thus, it is computationally efficient when the number of well locations or possible well locations is large. The adjoint approach was developed to be used with standard groundwater flow simulators, and therefore can be applied in practice. The research included rigorous development of the adjoint equation for calculating stream depletion in confined and unconfined aquifers with various models of groundwater/surface water interaction, along with numerical simulations to verify the adjoint equation. In addition, we used the adjoint method to investigate the sensitivity of stream depletion to the hydraulic conductivity of the stream channel, a parameter which is known to be uncertain.

Keywords: Stream depletion, streamflow, adjoint method, streambed conductance, non-tributary groundwater

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Justification of Work Performed

In many regions of the U.S. and the world, population growth is increasing the demand for water, while climate change may lead to a reduction in the available of water. For example, over the next thirty years, population growth of 46% and 29% is forecasted for the western and southern U.S. respectively (Hansen 2012), while climate models predict that droughts in the 21st century will last longer and be more intense (MacDonald 2010). Considering that groundwater comprises 33% of public and 99% of domestic drinking water, the increase in water demands may be met through further development of groundwater resources (Kenny et al., 2009; Vorosmarty et al., 2000). However, the increased pumping of aquifers should not reduce flows in rivers to levels that would limit the availability of water for drinking water supply, irrigation, and riparian habitat.

Stream depletion is the reduction in the flow rate in a river as a result of pumping in an aquifer that is hydraulically connected to the river. Stream depletion has many negative consequences, such as reduction in water supply for municipal, agricultural, and domestic uses; failure to satisfy existing water rights; and destruction of the ecosystems that depend on streams and rivers. Quantifying stream depletion is therefore crucial for protecting water supplies, surface water rights, and environments that depend on the streams and rivers.

The standard approach for using numerical models to calculate stream depletion is to first run one groundwater flow simulation without pumping to determine the exchange of water between the river and the aquifer, and then to run an additional simulation with pumping at one location to determine the change in the flow rate of water between the river and the stream. If the location of a new well is to be chosen, many possible well locations may be under consideration. It may be necessary to choose a location that limits depletion in a nearby stream; therefore, stream depletion must be calculated for many different well locations. Assuming that the aquifer is sufficiently complex to require numerical models to simulate stream depletion, the standard approach must be repeated for each potential well location, and can become computationally inefficient if many potential well locations are considered. In our work, we developed the adjoint method for calculation stream depletion in a river due to pumping in an adjacent aquifer. With the adjoint method, only one simulation is needed to calculate stream depletion for a well at any location in the aquifer; thus it is more efficient than the standard approach when multiple potential well locations are considered.

The accuracy of stream depletion predictions depends on the how well the parameters used in the model match the physical properties of the system. Many studies have shown that stream depletion is sensitive to the hydraulic conductivity of the streambed, K_r (e.g., Spalding and Khaleel, 1991; Sophocleous et al., 1995; Hunt 1999; Zlotnik and Huang 1999; Butler et al., 2001). Furthermore, the streambed conductance, which includes K_r, along with streambed thickness and the channel width, was identified as the most uncertain parameter in stream depletion estimations (Christensen, 2000). In addition, studies that have characterized K_r in natural channels have shown that it varies both in time (Genereux et al., 2008) and in space (Springer et al., 1999; Cardenas and Zlotnik, 2003; Chen, 2004; Chen, 2005; Ryan and Boufadel, 2006; Chen et al., 2008; Cheng et al., 2011). Unfortunately, streambed hydraulic conductivity is typically not measured in stream depletion studies, and a single assumed value of K_r is assigned to the entire streambed. In our work, we investigated the implications of using a single value of K_r, in terms of how it impacts estimations of stream depletion and the selection of feasible well locations.

Review of Methods Used

Forward Flow Equations and Stream Depletion Calculations

The main activity in this research was the development and verification of adjoint equations to calculate stream depletion. The adjoint equation is derived from a forward governing equation of flow in a river and aquifer system. For illustrative purposes in this report, we consider a two-dimensional, unconfined aquifer with a pumping well and a river. For this system, the forward governing equations of flow are

$$S_y \frac{\partial h}{\partial t} = \nabla \cdot \left[\mathbf{K} \left(h - \zeta \right) \nabla h \right] - Q_p \, \delta\left(\mathbf{x} - \mathbf{x}_w \right) + \frac{K_r}{b_r} \left(h_r - h \right) B\left(\mathbf{x} \right) \tag{1}$$

$$\frac{\partial Q_r}{\partial s} = - \frac{K_r}{b_r} \left(h_r - h \right) w \tag{2}$$

where h and h_r are head in the river and aquifer respectively, t is time, $\mathbf{x} = (x,y)$ is the position vector, S_y is specific yield, $\mathbf{K}$ is the hydraulic conductivity tensor for the aquifer, z is the elevation of the aquifer bottom, Q_p is the pumping rate, $\mathbf{x}_w$ is the location of the pumping well, K_r and b_r are the hydraulic conductivity and thickness of the river sediment, w is the river channel width, Q_r is the flow rate in the river, s is the spatial coordinate along the river channel in the direction of flow, and $B(\mathbf{x})$ is an indicator function that is unity at the river and zero elsewhere. Here we assume that the head in the river changes quickly relative to the change in head in the aquifer, therefore we neglect transient changes in the river volume. For illustration purposes, we use the following boundary conditions (although other boundaries conditions can be used)

$$\begin{aligned} & h\left(\mathbf{x}, 0 \right) = h_o\left(\mathbf{x} \right) \\ & h\left(\mathbf{x}, t \right) = g_1\left(\mathbf{x}, t \right) \quad \text{on} \quad \Gamma_1 \\ & \nabla h \cdot \mathbf{n} = g_2\left(\mathbf{x}, t \right) \quad \text{on} \quad \Gamma_2 \\ & Q_r\left(s = 0, t \right) = Q_r^*\left(t \right) \end{aligned} \tag{3}$$

where $h_o(\mathbf{x})$ is the initial head in the aquifer, $g_1(\mathbf{x},t)$, $g_2(\mathbf{x},t)$, and $Q_r^*(t)$ are known functions, and G_1 and G_2 are the aquifer boundaries.

Stream depletion, D$Q_r(s{=}L,t{=}t_c)$, is the decrease in the river flow rate at a compliance point at $s{=}L$ at a compliance time $t{=}t_c$ as a result of pumping. For small pumping rates, stream depletion is proportional to the pumping rate and can be expressed mathematically as

$$\Delta Q_r\left(L, t_c \right) \approx - \frac{dQ_r(L, t_c)}{dQ_p\left(\mathbf{x}_w \right)} Q_p\left(\mathbf{x}_w \right) . \tag{4}$$

Using the standard method to calculate stream depletion, (1) and (2) are solved once with $Q_p = 0$ and the flow rate in the river is recorded at $s{=}L$ for $t{=}t_c$. Then (1) and (2) are solved again with pumping at $\mathbf{x} = \mathbf{x}_w$, and the new flow rate in the river is also recorded at $s{=}L$ for $t{=}t_c$. The difference between these flow rates is stream depletion. To calculate stream depletion due to pumping at a different well location, (1) and (2) must be solved again with pumping at a different location. Note that the only component of river flow that is affected by pumping is the exchange of water between the aquifer and the river, which is represented by the last term in (1) and the term on the right hand side of (2). Thus, alternatively, one can calculate stream depletion as the change in the rate at which water is exchanged between the aquifer and the river.

Adjoint Equations and Stream Depletion Calculations

The derivative on the right hand side of (4) is the sensitivity of river flow rate to pumping, or equivalently, the stream depletion per unit pumping rate. The adjoint derivation is based on sensitivity analysis using this sensitivity as the focus of the derivation. Details of the adjoint derivation are provided in Neupauer and Griebling (2012), Griebling (2012), and Griebling and Neupauer (2013). Here we present the final adjoint equations and show how they are used to calculate stream depletion.

The adjoint of (1) – (3) is given by (Griebling and Neupauer, 2013)

$$S_y \frac{\partial \psi}{\partial \tau} = \nabla \cdot \left[\mathbf{K}(h_o - \xi) \nabla \psi \right] + \frac{K_r}{b_r}(\psi_r - \psi) B(\mathbf{x}) \tag{5}$$

$$-\frac{\partial Q_r}{\partial h_r}\frac{\partial \psi_r}{\partial s} = -\frac{K_r}{b_r}(\psi_r - \psi) w \tag{6}$$

$$\begin{aligned} &\psi(\mathbf{x}, \tau = 0) = \frac{K_r}{b_r S_y} B(\mathbf{x}) \\ &\psi(\mathbf{x}, t) = 0 \quad \text{on} \quad \Gamma_1 \\ &\nabla \psi \cdot \mathbf{n} = 0 \quad \text{on} \quad \Gamma_2 \\ &\psi_r(s = L, \tau) = 0 \end{aligned} \tag{7}$$

where t = $t - t_c$ is backward time and y and y_r are the adjoint states of h and h_r. The stream depletion in (4) can be rewritten in terms of these adjoint states as (Griebling and Neupauer, 2013)

$$\Delta Q_r(L, t_c) \approx Q_p \int_0^{t_c} \psi(\mathbf{x}, \tau)\, d\tau\,, \tag{8}$$

which implies that the sensitivity of the river flow rate to pumping rate is given by

$$\frac{dQ_r(L, t_c)}{dQ_p(\mathbf{x}_w)} = -\int_0^{t_c} \psi(\mathbf{x}, \tau)\, d\tau \quad . \tag{9}$$

Note that in (4), stream depletion is calculated for pumping at a well at a particular location $\mathbf{x}_w$, while in (8), stream depletion is calculated for pumping at a well at any location $\mathbf{x}$ in the domain. To calculate stream depletion using (8), the adjoint state y ($\mathbf{x}$,t) is obtained by solving (5) and (6) with the boundary conditions in (7). At each location $\mathbf{x}$, the adjoint state is integrated over the time domain (as shown in (8)). Multiplying this result by the pumping rate produces stream depletion due to pumping at that location $\mathbf{x}$ in the aquifer. Thus, only one simulation of the adjoint equation is needed to obtain stream depletion for pumping at a well at any location in the aquifer.

The adjoint equations, (5) and (6), have similar forms as the forward equations, (1) and (2), so they can be solved using any groundwater flow simulator that solves (1) and (2). In this work, we use MODFLOW-2000 (Harbaugh et al., 2000) to solve both the forward and the adjoint equations. The left hand side of (6) has a different form than the left hand side of (2), so we modified the source code of the MODFLOW-2000 Stream package (Prudic, 1989) to solve (6). See Griebling (2012) for more details of the code modifications.

Other differences between the adjoint equations and the forward equations are:

- The state variables in the forward equations are head, which have units of length; the state variables in the adjoint equations are adjoint states, which have units of reciprocal time.
- The time variable in the forward equation is forward time, while the time variable in the adjoint equation is backward time; thus, in the adjoint simulation, information is propagated backward in time.
- The pumping term in the forward equation does not appear in the adjoint equation.
- The boundary conditions on the adjoint groundwater equations are homogeneous.
- The initial conditions are homogeneous, except where the aquifer is adjacent to the river.
- The groundwater flow term in the forward equation for the unconfined aquifer is nonlinear in head h; therefore the related term in the adjoint is linear in the adjoint state but contains the forward state variable, h. As an approximation, we assume that the saturated thickness of the unconfined aquifer can be approximated as constant, so we replace the time-dependent saturated thickness $h-z$ with h_o-z. We treat the unconfined aquifer as if it were a confined aquifer with a transmissivity of $\mathbf{K}(h_o\text{-z})$.
- In the forward river equation, the sign on the flow term is positive, while in the adjoint equation, it is negative.
- The boundary condition on the forward river equation is a specified flow at the upstream boundary, while the boundary condition for the adjoint river equation is a specified adjoint state at the downstream boundary of the river.

These differences require that the adjoint simulations with MODFLOW use somewhat different inputs than a forward simulation, and also require a somewhat different interpretation of the outputs. See Griebling and Neupauer (2013) for information on these differences.

Discussion of Results and their Significance

Adjoint Stream Depletion Calculations

The adjoint equations for calculating stream depletion were developed and tested for various aquifer and river systems, and were shown to be accurate and efficient. As an example, consider the aquifer and river system shown in Figure 1. This system is comprised of an unconfined aquifer, an underlying confining unit, and a lower confined aquitard. Two tributaries and a river are hydraulically connected to the unconfined aquifer and flow in the southward direction. The aquifer experiences natural recharge and evapotranspiration. We assume that the river and tributaries can be approximated as wide rectangular channels, an assumption that is used in the MODFLOW STR package (Prudic, 1989). For details on the model parameter values, see

Griebling and Neupauer (2013). Figure 1 shows the head distribution in the confined and unconfined aquifers in the absence of pumping.

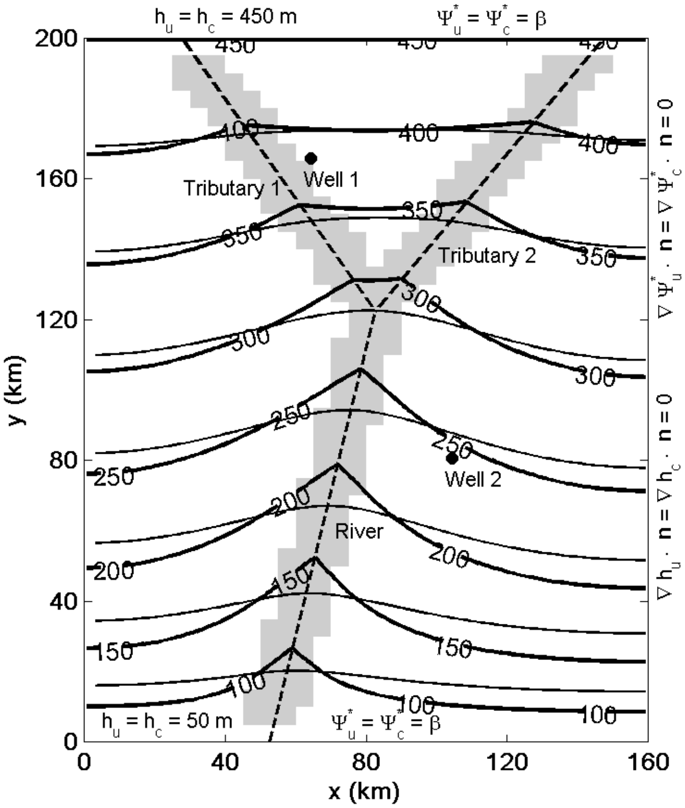

FIGURE 1. Model domain. Contours represent head (m) in the unconfined aquifer (thick line) and in the confined aquifer (thin line). Thick dashed lines represent tributaries and rivers. Boundary conditions for the forward and adjoint models are shown along the boundaries (the same boundary conditions are used at *x* = 0 km and at *x* = 160 km). Gray shaded region represents the area where evapotranspiration is occurring. The two filled circles represent the two well locations used in Figure 4.

To calculate stream depletion using the adjoint approach, we solved (5) and (6) using MODFLOW-2000 and the adjoint STR package to obtain the adjoint states, which were then used in (8) to calculate stream depletion. The results are shown in Figure 2a,b for the unconfined and confined aquifers, respectively. For a given location in the model domain, these plots show the amount of stream depletion in the downstream terminus of the river after 50 years of pumping at a rate of 2.5 x 10^4 m^3/d at the given location. The results show that stream depletion is highest for wells near the river and tributaries, and decreases as the distance between the well and the river or tributary increases. All of this information was obtained with just one simulation of the adjoint model.

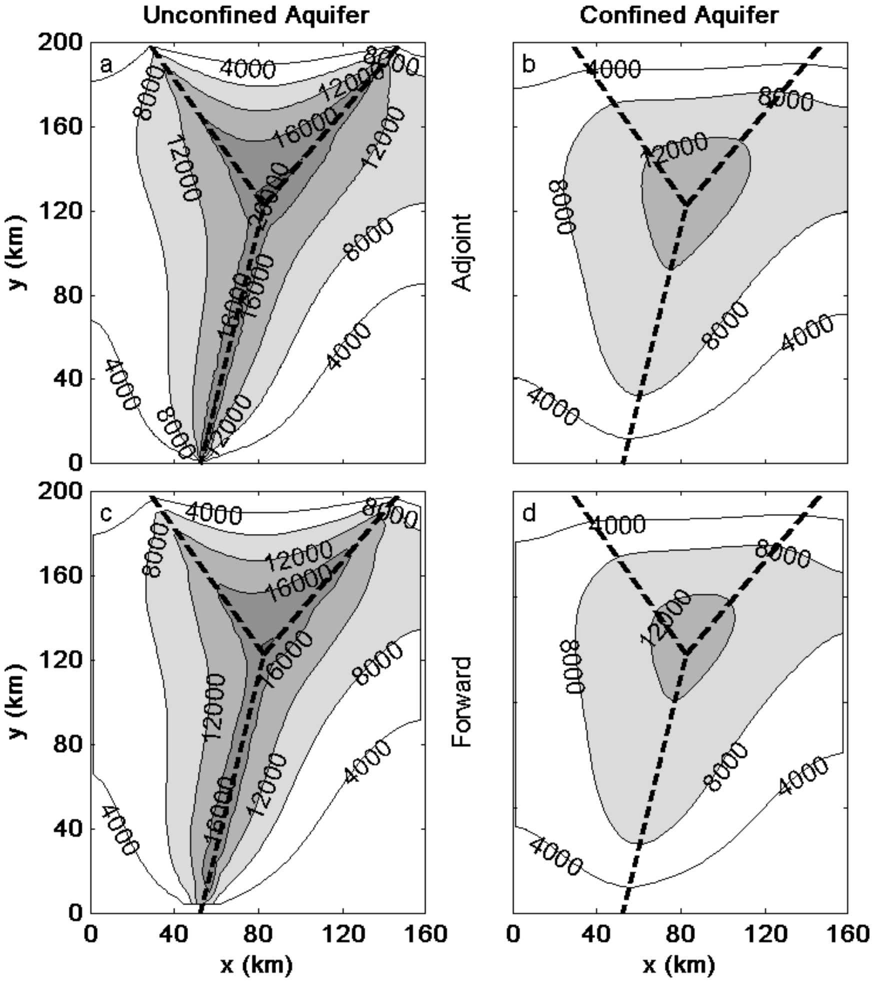

FIGURE 2. Stream depletion (m^3/d) in the river due to pumping in the unconfined (left column) and confined (right column) aquifers at a rate of 2.5 x 10^4 m^3/d, calculated using one adjoint simulation (top row) and multiple forward simulations (bottom row). Thick dashed lines represent tributaries and rivers.

For comparison, we also calculated stream depletion using the standard forward approach. The domain is discretized into 40,960 1.25 km by 1.25 km grid blocks, and each one could be considered a potential well location. We considered the potential well locations to be the cells at the intersection of every fourth row and every fourth column, for a total of 2,560 potential well locations. We ran 2,560 additional forward simulations to estimate stream depletion for each potential well. The results are shown in Figure 2c,d. Comparison with Figure 2a,b shows that the adjoint simulation produces very similar results as the standard approach. For both aquifers, the patterns are very similar between the results of the two methods, with some slight variations near the tributaries and rivers. Figure 3 shows the percent difference between the stream depletion values calculated for the adjoint and forward simulations. The differences are less than 4% throughout most of the domain, except where the stream depletion is low. In these areas, the absolute error is low.

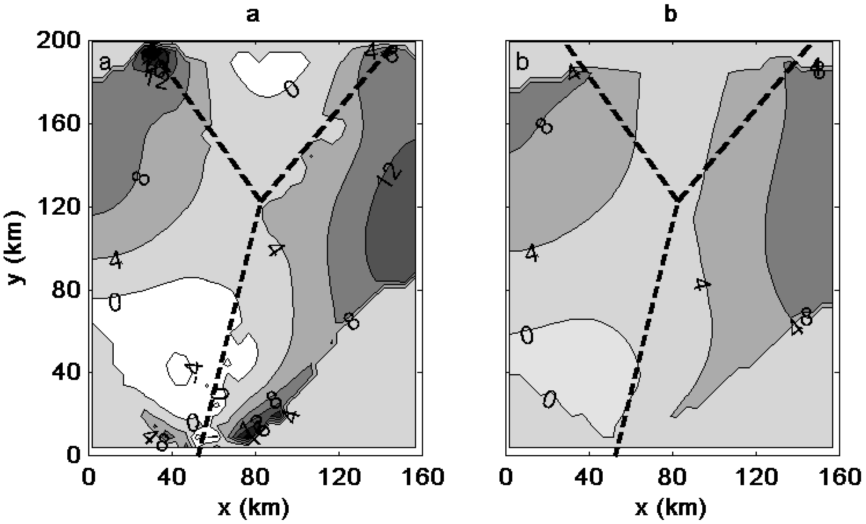

FIGURE 3. Percent difference in stream depletion values calculated using the adjoint method and the standard method in the (a) unconfined aquifer and (b) confined aquifer. Wherever stream depletion is less than 4000 m^3/d, the value is set to zero. Thick dashed lines represent tributaries and rivers.

Efficiency of Adjoint Simulations

The results in Figure 2a,b were obtained with one simulation of the adjoint model, which ran in 140 seconds on a Dell Latitude E6530 with an Intel Core i7-3720QM processor at 2.60 GHz. The adjoint simulation produced stream depletion estimates for a well at any of the 40,960 cells in the model domain, i.e., at 1.25 km by 1.25 km resolution. The results of the standard approach, shown in Figure 2c,d, were obtained at 5-km by 5-km resolution, producing 1/16th of the amount of information as obtained from the adjoint simulations. The 2560 forward simulations ran in 582 minutes, or approximately 250 times longer than the single adjoint simulation. The adjoint simulation was 250 times faster than the forward approach and produced 16 times more information.

Prior to running adjoint simulations, it is still necessary to develop and calibrate a forward model of the river and aquifer system. The parameterization of the adjoint model is developed based on the forward model, and the adjoint model requires as input the steady-state aquifer head and river head that are obtained from a forward simulation in the absence of pumping. Once the model is developed, the adjoint simulation is more efficient than the standard approach for calculating stream depletion when the number of possible well locations is large.

Typically, it is not necessary to obtain stream depletion information for well locations throughout the entire model domain, so the efficiency of the adjoint simulation may be lower than the 250-fold decrease in simulation time seen here. However, for large models with simulation times on the order of multiple hours, the time savings of a single adjoint simulation compared to just 10 or so forward simulations may be substantial. In addition, in performing a sensitivity analysis to calculate the sensitivity of stream depletion to various model parameters, a modeler may want to run simulations with several different parameter sets. Even if the sensitivity is desired for only a small subset of well locations, the computational time for running forward simulations for all combinations well locations and parameter sets may become prohibitive, and the adjoint approach may be more efficient.

Sources of Error in Adjoint Simulations

The adjoint states are related to marginal sensitivities of head to the pumping rate; thus each term in the adjoint equation represents the sensitivity of various processes to the pumping rate. The adjoint derivation requires differentiation of each term of the forward equation with respect to pumping rate. For the non-linear terms (e.g., the first term on the right hand side of (1)), these derivatives are linearized around the pre-pumping conditions. A source of error in the adjoint model as compared to the results of the standard approach is due to this linearization. The transmissivity of an unconfined aquifer depends on the saturated thickness of the aquifer, which can vary over time as the head in the aquifer varies. In a pumping scenario, the head, saturated thickness, and transmissivity would decrease over time. The adjoint approach ignores the time variation of the saturated thickness; thus, the transmissivity used in the adjoint simulation can be higher than the transmissivity used in equivalent forward simulations.

Another potential source of error is in the assumption that stream depletion varies linearly with pumping rate, as shown in (4). We ran forward simulations for a range of pumping rates ($Q_p = 0$ to 5×10^6 m^3/d) for two different well locations (Wells 1 and 2 in Figure 1) in the unconfined aquifer. Stream depletion caused by this range of pumping rates at each of the wells is shown in Figure 4. Well 1 is closer to the tributary, so drawdown is higher for pumping at Well 1 than for pumping at Well 2, for any given pumping rate. For pumping at either well, stream depletion varies approximately linearly until the simulated pumping rate exceeds a threshold pumping rate above which the model cell containing the well goes dry. If the cell goes dry, pumping ceases in the simulation causing stream depletion to drop and approach zero. This threshold pumping rate is higher for Well 1 because the saturated thickness of the aquifer is greater where the head is higher.

For comparison, the adjoint-derived stream depletion is also shown in Figure 4. In the adjoint approach, we calculate a single value of dQ_r/dQ_p from (9), which is the slope of the adjoint stream depletion curves in Figure 4. Thus, non-zero stream depletion is calculated even for pumping rates that exceed the well yield. Furthermore, the adjoint stream depletion slightly exceeds the stream depletion calculated from the forward simulations for almost all pumping rates. This discrepancy is likely caused by the assumption in the adjoint approach that the saturated thickness is unchanged during pumping.

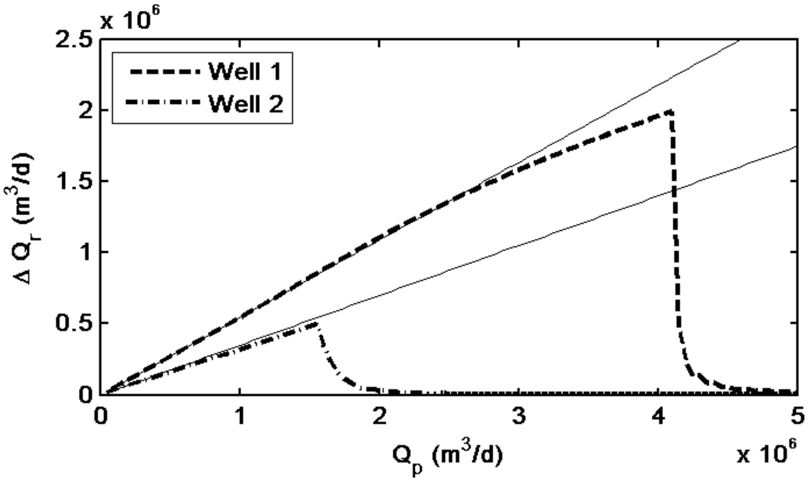

FIGURE 4. Effects of various pumping rates on stream depletion for the two well locations shown in Figure 1. Thin solid lines represent the adjoint-based stream depletion estimates.

In the STR package of MODFLOW, if the aquifer head is above the river channel bottom, the rate of flow across the streambed is proportional to the head difference between the river and the aquifer. However, if the aquifer head drops below the river channel bottom, the rate of flow across the stream bed is proportional to the head difference between the river and the bottom of the bed sediment, where the pressure head is assumed to be zero. Thus, in this case, the flow rate between the river and aquifer is independent of the head in the aquifer, and the river is no longer hydraulically connected to the aquifer. Since the adjoint model only solves for the adjoint state and not for the aquifer head, it is not possible to determine when the river is no longer hydraulically connected to the aquifer. Thus, the adjoint model results are only accurate for pumping scenarios for which the river remains hydraulically connected to the aquifer.

If the adjoint model is used for situations in which stream depletion is not linearly proportional to the pumping rate, the adjoint model results would overestimate stream depletion. Thus, although the results would be inaccurate, they would be conservative.

Limitations

The relationship between flow rate in the river, Q_r, and river head, h_r, is non-linear; thus the adjoint of the river flow equation (6) does not have the same form as the forward river flow equation (2). In addition, the code that solves the forward river equation must be modified to solve the adjoint river equation. We have developed the adjoint approach for two specific river models:

- The head in the river is known. With this assumption, h_r in (1) is known, so (2) is not needed. This assumption is equivalent to the assumptions of the River package in MODFLOW. The adjoint equations for this case are derived in Neupauer and Griebling (2012).
- The head in the river varies as a result of pumping, and the river channel is approximated as having a wide, rectangular cross section. This assumption is equivalent to the

assumptions of the Stream (STR) package in MODFLOW (Prudic, 1989). The adjoint equations for this case are derived in Griebling and Neupauer (2013).

Many stream depletion models use MODFLOW with either the River package or the Stream package, so although the adjoint method has only been developed for two river models, it can be applied widely. The other MODFLOW package that is commonly used to simulate stream and aquifer interaction in stream depletion simulations is the Streamflow Routing (SFR) package (Niswonger and Prudic, 2005), which allows for more complicated river channel geometries and for unsaturated flow beneath the river bottom. Developing an adjoint model for the SFR package is the subject of future work.

Investigation of the effects of streambed hydraulic conductivity on stream depletion

In the final part of the project, we investigated the effects of streambed hydraulic conductivity on stream depletion. We used the adjoint approach to calculate stream depletion in a river due to pumping at a well at any location in an unconfined aquifer. Details of the model geometry and parameter values can be found in Lackey (2013). Figure 5c,d show dQ_r/dQ_p after 200 years of pumping with $K_r = 1 \times 10^{-7}$ m/s and $K_r = 2 \times 10^{-5}$ m/s, respectively. Stream depletion is lower for a low streambed hydraulic conductivity (Figure 5c) than for a high streambed hydraulic conductivity (Figure 5d). With a low streambed hydraulic conductivity, less pumped water is taken from the stream because water cannot flow as easily across the streambed. As stated earlier, many numerical investigations of streambed hydraulic conductivity use an assumed value of K_r; these results demonstrate that choosing an incorrect value of K_r can lead to incorrect estimates of stream depletion.

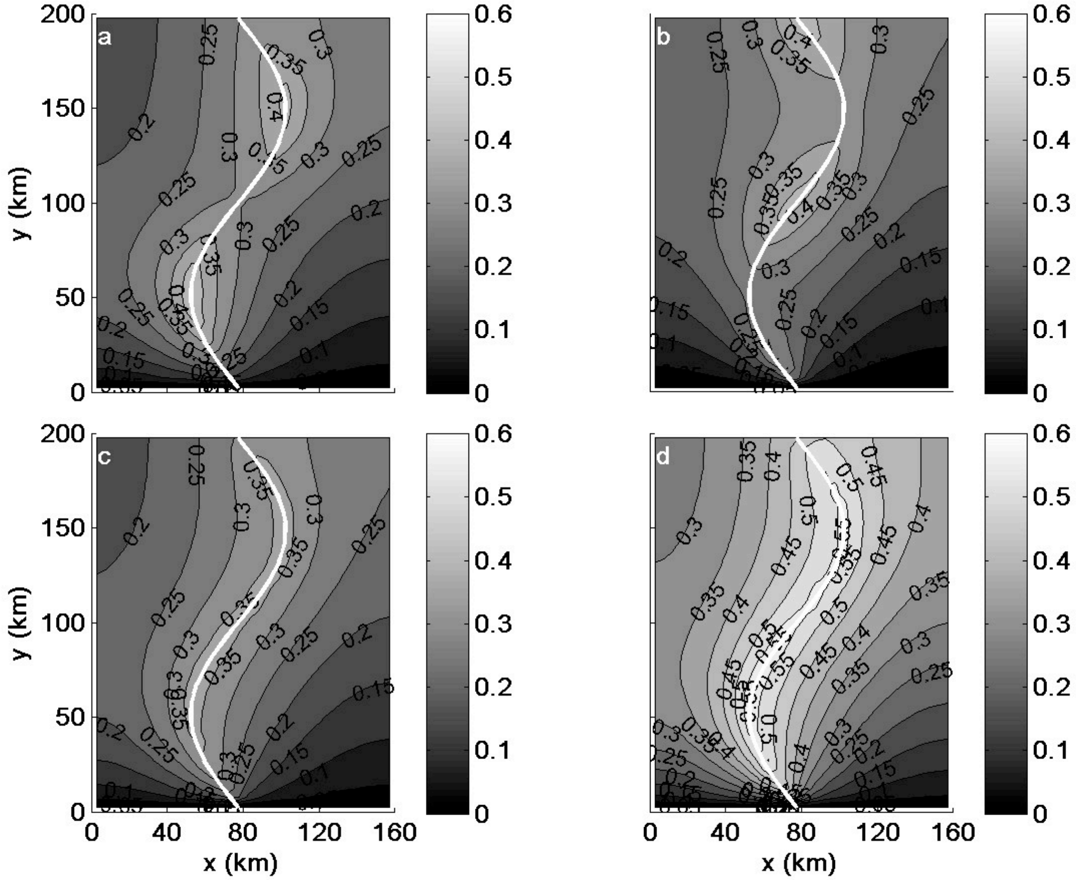

FIGURE 5. Stream depletion calculated as a fraction of pumping rate (dQ_r/dQ_p) at each potential well location for (a) a heterogeneous stream channel with high K_r near river bends and low K_r in straight river sections, (b) a heterogeneous stream channel with low K_r near river bends and high K_r in straight river sections, (c) a homogeneous stream channel with K_r equal to the mean K_r from the heterogeneous scenarios (1 x 10^{-7} m/s), and (d) a homogeneous stream channel with K_r equal to the maximum K_r from the heterogeneous scenarios (2 x 10^{-7} m/s). The white S-shaped curve is the river, which flows in the –y direction.

Although these results show that stream depletion is sensitive to K_r, we found that for a given aquifer hydraulic conductivity, a finite range of K_r exists for which stream depletion is sensitive to K_r. For example, Figure 6 shows stream depletion at a well at (x,y) = (77.5 km, 152.5 km) for K_r values ranging from 1 x 10^{-9} m/s to 1 x 10^{-2} m/s. This range represents values of stream depletion that have been used in numerous field and numerical studies (Calver, 2001). For low values of K_r (less than 1 x 10^{-8} m/s), stream depletion is very low and does not vary with K_r. For high values of K_r (greater than 1 x 10^{-6} m/s), stream depletion is high and again does not vary with K_r. For an intermediate range (from less than 1 x 10^{-8} m/s to 1 x 10 $^{-6}$ m/s), stream depletion is sensitive to K_r. We call this range the “sensitive range.” We found that the sensitive range of K_r varies with the aquifer hydraulic conductivity (Lackey, 2013).

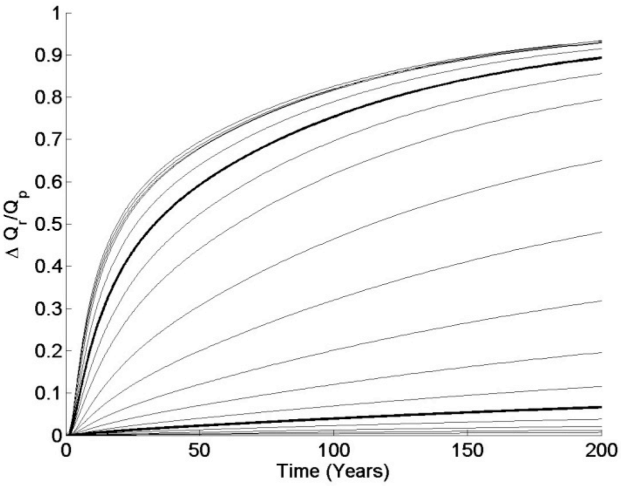

FIGURE 6. Stream depletion calculated as a fraction of pumping rate (dQ_r/dQ_p) due to pumping at a well at (x,y) = (77.5 km, 152.5 km) for K_r values ranging from 10^{-9} m/s to 10^{-2} m/s at increments of 0.25 log units. Stream depletion increases as K_r increases, i.e. the uppermost curve corresponds to $K_r = 10^{-2}$ m/s; the curve immediately below it corresponds to $K_r = 10^{-2.25}$ m/s; etc. The thick black lines represent the bounds of the sensitive range: $K_r = 1 \times 10^{-8}$ m/s for the lower bound and $K_r = 1 \times 10^{-6}$ m/s for the upper bound.

We also investigated the effects of streambed heterogeneity on stream depletion. Using a mean streambed hydraulic conductivity of 1×10^{-7} m/s, we generated a spatial distribution of K_r with high values near the bends in the river and low values in the straight sections, which is a representative pattern under high flow conditions (Andrews, 1979; Sear, 1996; Clayton and Pitlick, 2007). Figure 5a shows dQ_r/dQ_p for this scenario. In addition, we generated a spatial distribution of K_r with low values near the bends in the river and high values in the straight sections, which is a representative pattern under low flow conditions (Andrews, 1979; Sear, 1996; Clayton and Pitlick, 2007). Figure 5b shows dQ_r/dQ_p for this scenario. In both cases, stream depletion is higher for wells hear the high K_r stretches and lower for wells near the low K_r stretches. More noticeably, the spatial pattern of stream depletion (plotted as a function of well location) is different for heterogeneous streambeds (Figure 6a,b) than for homogeneous streambed (Figure 5c,d). Thus, assuming a single uniform value of K_r in simulations of stream depletion may produce incorrect estimates, unless K_r does not fall in the sensitive range (Lackey, 2013).

We also investigated the effects of temporal changes in streambed heterogeneity. We assumed that high flow conditions exist for half of each year and low flow conditions exist for the other half of each year. Using the adjoint method, we calculated stream depletion in the river due to pumping at a well at any location in the unconfined aquifer after 200 years of pumping. Results are shown in Figure 7. Although the streambed is heterogeneous, the temporal mean value of K_r at any location along the streambed is 1×10^{-7} m/s, the same value used for the homogenous streambed in Figure 5c. These results show that the cyclic temporal variations in K_r lead to stream depletion pattern that are similar to stream depletion pattern that would be obtained if the temporal mean values of K_r are used.

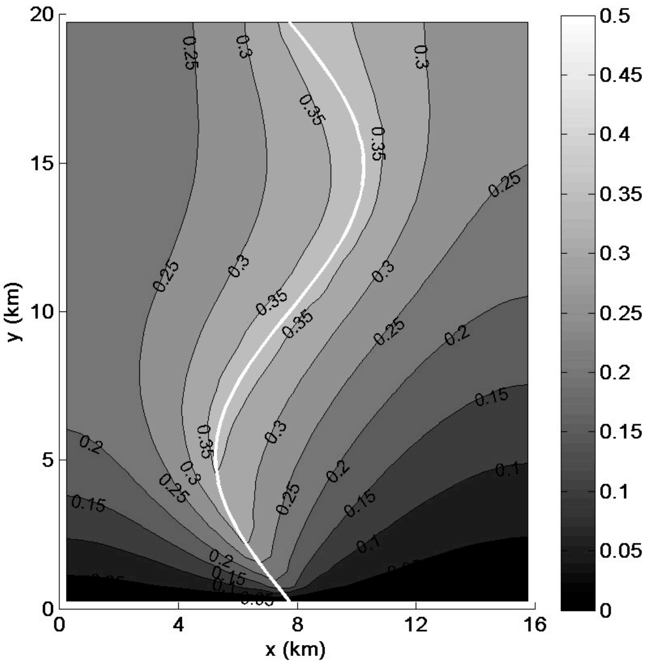

FIGURE 7. Stream depletion calculated as a fraction of pumping rate (dQ_r/dQ_p) at each potential well location for a heterogeneous stream channel with temporally-varying heterogeneity patterns. For half of each year, the patter is high K_r near river bends and low K_r in straight river sections, while for the other half of the year, the pattern is low K_r near river bends and high K_r in straight river sections. The spatial mean value at any time and the temporal mean value at any location is K_r = 1 x 10^{-7} m/s.

Principal Findings, Conclusions, and Recommendations

Principal Findings

This work developed an adjoint method to calculate stream depletion caused by pumping in an aquifer that is hydraulically connected to a river. We developed the adjoint approach for two different river models, consistent with the assumptions in the MODFLOW River package and MODFLOW Stream package, which are widely used in practice to calculate stream depletion. We tested the adjoint approach for several different scenarios using hypothetical aquifers.

The advantage of the adjoint method over the standard method for calculating stream depletion is that only one adjoint simulation is needed to calculate stream depletion due to pumping at a well at any location in the aquifers. In the standard forward method, one forward simulation must be run for each possible will location, which can be computationally prohibitive.

The results show that the adjoint solution accurately approximates stream depletion when the assumptions of small changes in river and aquifer head are satisfied, and that the adjoint approach is computationally more efficient than the standard forward method. Thus, the adjoint method is a useful approach for identifying optimal locations for new wells that minimize stream depletion or for identifying areas along a river section that are most sensitive to pumping.

Stream depletion is sensitive to the streambed hydraulic conductivity for a range of K_r values that depends on the aquifer hydraulic conductivity. Stream depletion is also sensitive to streambed heterogeneity and temporal variations. Thus, numerical modelers who are using simulations to estimate stream depletion should use measured streambed hydraulic conductivity values if available, unless it can be shown that the streambed hydraulic conductivity is outside of the sensitive range.

Recommendations

The adjoint approach has been tested using synthetic aquifers. To fully evaluate the adjoint approach, it must be tested on an actual aquifer. We are currently working on a case study for the San Pedro Basin in Arizona using a groundwater flow model that was developed for estimation of stream depletion using the standard forward modeling approach (Leake et al., 2008).

Several practitioners have expressed interest in using the adjoint method for stream depletion calculations. In order to make the method feasible for widespread application in practice, a code should be written that creates the adjoint MODFLOW input files from MODFLOW input and output files from a forward simulation in the absence of pumping. Work on the code development is currently underway. The code uses MODFLOW subroutines for reading existing MODFLOW input and out files and for writing adjoint MODFLOW input files.

In many stream depletion studies, the MODFLOW River or Stream package is used to simulate the flow between the aquifer and the surface water body, for which the adjoint method has been developed. The Streamflow Routing (SFR) package is also used in stream depletion studies; therefore, the adjoint method should be developed for the river models that are used in the SFR package.

Summary

This work has developed and tested a new adjoint approach for calculating stream depletion. With one adjoint simulation, we obtain estimates of stream depletion for pumping at a well at any location in the model domain. The adjoint approach is efficient if multiple well locations are being considered and stream depletion must be estimated for each one. We developed a procedure for using MODFLOW to solve the adjoint equations. The method has been developed and tested for two river models: (1) head is the river is known and is not impacted by pumping (equivalent to the assumptions in the MODFLOW River package), and (2) a wide rectangular river channel geometry, and negligible changes in river storage (equivalent to the MODFLOW Stream package).

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