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By Professor VUJICA M.YEVDJEVICH
Colorado State University, Fort Collins,
Colorado, U.S.A. Civil Engineering Department.

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RATE OF CHANGE OF THE PEAK FOR FLOODS PROGRESSING ALONG A CHANNEL

By Professor Vujica M. Yevdjevich

Colorado State University, Fort Collins, Colorado, U. S. A.
Civil Engineering Department

The rate of change of peak discharge, dQ_o/dx, and the rate of change of peak depth, dH_o/dx, of one-peak flood hydrographs are analyzed by using the two De Saint Venant partial differential equations for unsteady free surface flow. One-peak flood hydrographs and flood profiles along the channel are approximated by Pearson Type III function, with three parameters: peak discharge (or peak depth); time-lag of the rising limb of hydrograph; and time-lag between the peak and the center of gravity of the hydrograph. The volume of flood wave is related to these parameters by an approximation. Six parameters, three for each discharge and depth hydrographs are related by using the time-lag δ between occurrences of peak discharge and peak depth, as a basic parameter. The procedure of computing δ by a trial-and-error method is outlined, as well as procedures to compute dQ_o/dx and dH_o/dx.

Les dérivées, dQ_o/dx et dH_o/dx du débit et de la profondeur de pointe d'une crue, possédant un seul maximum, sont étudiées à l'aide des équations aux dérivées partielles de Saint-Venant pour un écoulement nonpermanent à surface libre. L'hydrogramme et le profil du plan d'eau le long du canal, sont exprimés approximativement par des fonctions de Pearson du troisième genre, dépendant des trois paramètres suivants: le débit ou la profondeur de pointe, le délai de la branche ascendante de la courbe des debits de le délai entre le maximum et le centre de gravité de cette même courbe. Le volume de la crue est exprimé en fonction de ces paramètres par une approximation. Six parametres, trois pour L'hydrogramme et trois pour le profil du plan d'eau, sont reliés entre eux en utilisant comme paramètre de base, l'intervalle de temps δ séparant le débit et la profondeur de pointe. Les procédés de calcul de δ (par approximations successives), de dQ_o/dx et de dH_o/dx sont exposés.

1. Subject of paper. The rate of change of one-peak flood wave is analyzed. The wave is approximated by an analytical expression with parameters determined from the initial wave conditions. The rate of change is determined by using this expression.

Whether a wave becomes attenuated or amplified by progressing depends on three groups of parameters: (a) Characteristics of river channel; (b) Lateral outflow or inflow; and (c) Initial characteristics of flood wave. If these parameters are known, the rate of change of flood peak along the channel may be determined.

2. Basic definitions and equations. Figure 1 demonstrates a gradually varied wave at the time $t = t_o$, in the form $H_z = f(x)$, or $H_e = f_1(x)$, with H_z, H, H_o, z, x, defined in the figure. Figure 2 shows discharge and depth hydrographs at the place $x = x_o$, in the form $Q = F_1(t)$, and $H = F_2(t)$, with the base flow Q_b and the base depth H_b, with other symbols defined in the figure.

Inasmuch as the water depth is small in comparison with wave length the approximation of flood wave motion by De Saint Venant continuity and momentum partial differential equations is good. They are, respectively,

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + q = 0 \tag{1}$$

which relates to the river channel section dx, with A = cross section, q = lateral outflow per unit length (negative for lateral inflow), with $\partial A/\partial t$ positive for rising water level, and $\partial Q/\partial x$ positive for increasing discharge along the channel; and

$$\beta \frac{\partial V}{\partial t} + g \frac{\partial H_e}{\partial x} + g\, S_f - \frac{\beta\, V\, q}{A} = 0 \tag{2}$$

with: $V = Q/A$ = mean velocity; g = acceleration of gravity; S_f = friction slope taken as positive value; $\partial V/\partial t$ positive if the mean velocity with time at a given place increases; and,

$$H_e = H_z + \frac{\alpha V^2}{2g} = H + z + \frac{\alpha V^2}{2g} \tag{3}$$

with α and β velocity distribution coefficients.

The two independent variables are x and t, and discharge Q and depth H are used here as two dependent variables to facilitate the determination of rates of change of the peak: dQ_o/dx and dH_o/dx.

For a gradually varied wave, the friction slope S_f in unsteady flow may be replaced as approximation by the friction slope in steady flow. The friction slope S_f may be determined by either of three equations: Darcy-Weisback, Chézy and Manning. However, each of them introduces further complication in eq. (2). With given initial and boundary conditions, it is possible - at least theoretically - to determine $Q = F_1(x, t)$ and $H = F_2(x, t)$ from eqs. (1) and (2), and the two equations for differentials dQ and dH.

The continuity equation involves the cross-section area, while the momentum equation is based on the rate of change of energy line. For irregular channel shapes with changing slope S_o the bridge between these two basic equations makes the first complexity in analysis. Assuming the area-depth function in the form $A = pH^s$, the rates of change of p, s, and

S_o with x provide the needed bridge between the two equations.

3. <u>Expression for the rate of change of peak discharge.</u> Introducing $\partial A/\partial t$ from eq. (1) into eq. (2) by replacing $\partial V/\partial t$ and taking that at the peak discharge hydrograph $\partial Q/\partial t = 0$, and $\partial Q/\partial x = dQ/dx$, then

$$\frac{dQ_m}{dx} = \frac{dQ_o}{dx} = -\frac{gA_q^2}{\beta Q_m}\left[\frac{\partial H_e}{\partial x} + S_f\right] \tag{4}$$

with all values given at the time of maximum discharge Q_m of initial discharge hydrograph for a given channel position. If $(\partial H_e/\partial x + S_f)$ is positive, $dQ_o/dx < 0$, and the wave attenuates. The sum of energy slope and friction slope determines whether a wave attenuates quickly or slowly, or amplifies.

Using the relationship $A = pH^s$ and eq.(3), the rate of change of peak discharge becomes.

$$\frac{dQ_m}{dx} = \frac{dQ_o}{dx} = -\frac{gA_q^2}{(\alpha+\beta)Q_m}\left[(S_f - S_o) + (1 - \alpha\, s\, K_q^2)\frac{\partial H_q}{\partial x} - \frac{\alpha Q_m^2}{gA_q^2}\left(\frac{p'}{p} + s'\ln H\right)\right] \tag{5}$$

with $K_q = Q_m/A_q\sqrt{gH_q}$ the ratio of mean velocity to theoretical clerity for the depth H of small disturbances at the time of Q_m and given x. For a prismatic channel with $p' = s' = 0$ the last term in eq. (5) vanishes.

If the term $\left[(1 - \alpha\, s\, K_q^2)\,\partial H_q/\partial x\right]$ is negligible in comparison with $\Delta S = S_f - S_o$, a flood attenuates. Insomuch as this term may be either positive or negative, and non-negligible, a flood movement may be associated with an increase of discharge. If dQ_o/dx is positive, the peak discharge increases with distance. This increase has a limit, because the friction slope increases with an increase of Q_m, so that after a sufficient increase the difference ΔS becomes equal to the negative value of the term and the wave progresses with constant peak discharge. The change of one factor affects the others, because S_o, p', s' and $\partial H_q/\partial x$ are interrelated. For a diverging channel (S_o increasing, $p' > o$, $s' > o$), the attenuation rate dQ_o/dx decreases, and for a converging channel, there is an increase in attenuation. If these factors change sufficiently in the case of divergence the peak discharge may start to increase.

4. <u>Expression for the rate of change of maximum depth.</u> For the time of maximum depth H_m, of initial depth hydrograph at a given position x, the value $\partial H/\partial t = 0$, and $\partial H/\partial x = dH_o/dx$. As $\partial A/\partial t = 0$ for $H = H_m$, eq. (1) gives $\partial Q/\partial x = -q$. In this case eq. (2) gives

$$(\alpha\, s\, K_h^2 - 1)\frac{dH_m}{dx} = (S_f - S_o) + \frac{\beta}{gA_h}\frac{\partial Q_h}{\partial t} - \frac{\alpha Q_h^2}{gA_h^2}\left(\frac{p'}{p} + s'\ln H_m\right) - \frac{(\alpha+\beta)\,Q_h^2\, q}{gA_h^2} \tag{6}$$

with $dH_m/dx = dH_o/dx$ and various values in eq. (6) relate to the position of time occurrence of H_m for given x. In the case of a prismatic channel with no lateral flow (p', s', and q are zero), only the first three terms of eq. (6) are present. The complexity of eqs. (5) and (6) as applied to natural channels with changing factors S_o, p, s, f, q, α, β, along the channel generally justifies an approximation in the analysis of effect of these factors on the rate of change of flood peak.

5. Fitting of Pearson Type III function. Flood wave functions $Q = f_1(Q, H)$ and $H = f_2(Q, H)$ are approximated here by Pearson Type III function

$$Q = Q_b + Q_o e^{-[(t-t_1)-(x-x_1)/C_1]/G} [1 + (t-t_1)/m - (x-x_1)/C_1 m]^{m/G} \tag{7}$$

and

$$H = H_b + H_o e^{-[(t-t_2)-(x-x_2)/C_2]/D} [1 + (t-t_2)/a - (x-x_2)/C_2 a]^{a/D} \tag{8}$$

where t_1 and t_2, x_1 and x_2 are values for time and channel position, respectively, at which initially the maximum discharge and the maximum depth occur, C_1 and C_2 are celerities of any discharge and any depth, respectively; and G, m, D and a are parameters of initial waves as defined in fig. 2.

If the same coordinate system (x, t) is used for both eqs. (7) and (8), then $x_2 - x_1 = - C_o \delta$.

For the position $x = x_1 = x_2$ and $t_1 = 0$, the discharge and depth hydrographs are

$$Q = Q_b + Q_o e^{-t/G} \left(1 + \frac{t}{m}\right)^{m/G} \tag{9}$$

and

$$H = H_b + H_o e^{-(t-\delta)/D} [1 + (t-\delta)/a]^{a/D} \tag{10}$$

For the time $t = t_1 = t_2$, and $x_1 = 0$, the discharge and depth profiles along the channel are

$$Q = Q_b + Q_o e^{x/C_1 G} (1 - x/C_1 m)^{m/G} \tag{11}$$

and

$$H = H_b + H_o e^{(x + C_o \delta)/C_2 D} [1 - (x + C_o \delta)/C_2 a]^{a/D} \tag{12}$$

It follows from fig. 2 that $\delta = a - m$, with $\delta > 0$, so that $a > m$.

The integral of $(Q - Q_b)\,dt$ from $t = -m$ to infinity gives the flood volume, W, above the base flow. The integration of eq. (9) gives

$$\frac{W}{Q_o G} = \left(\frac{eG}{m}\right)^{m/G} \Gamma\left(1 + \frac{m}{G}\right) \tag{13}$$

Equation (13) may be approximated by

$$\frac{W}{Q_o G} = \frac{5}{2} \sqrt{\frac{m}{G}} \qquad (14)$$

Taking $H_b = 0$, $C_2 = C_o$ = constant, and $dx = C_o\, dt$, then the integration of Adx by using eq. (12) gives for $H_b = 0$

$$\frac{Ws}{A_o C_o D} = \left(\frac{eD}{sa}\right)^{as/D} \Gamma\left(1 + \frac{as}{D}\right) \qquad (15)$$

6. Approximate relationships for parameters of discharge and depth hydrographs. Six parameters of the initial discharge and depth hydrographs are: Q_m, H_m, m, a, and G, D, all of which are interrelated. It is supposed that either Q_m, m, and G, or H_o, a and D are parameters of the hydrograph, initially known. To determine the other three parameters, it is here supposed that Q_o, m and G are known. From fig. 2

$$a = m + \delta. \qquad (16)$$

The relation of Q_o and H_o is determined by assuming that $Q_b = 0$, $H_b = 0$, or relatively small, and that at the time $t = \delta$ (peak depth) the energy slope is $S_f = S_o$, so that $Q_h = A_h \sqrt{R_h S_o}$. By putting the hydraulic radius $R_h = \eta H_o$ with $\eta \leq 1$, and taking Q_h from eq. (9) with $t = \delta$ and $Q_b = 0$, as equal to the above value of Q_h, and using $A = pH^s$, then

$$Q_o e^{-\delta/G} \left(1 + \frac{\delta}{m}\right)^{m/G} = pk\eta^{1/2} H_o^{(2s+1)/2} S_o^{1/2} \qquad (17)$$

where k is the friction coefficient in Chezy formuls.

By using eq. (15), eq. (14) as an approximation of Γ-function, then

$$D = \frac{4\, s\, W^2}{25\, A_h^2\, C_o^2\, (m + \delta)} \qquad (18)$$

with values A, s and C_o at the peak depth. Equations (16) through (18) enable the computation of H_o, a and C for given Q_o, m and G, when δ and C_o are know, and W computed by eq. (14).

7. Expressions for determination of δ. Assuming that the rate of change of kinetic energy head, $\partial(\alpha V^2/2g)/\partial x$, for the position of peak discharge is small and negligible in comparison with the bottom slope S_o and with the rate of change $\partial H_q/\partial x$, then the peak discharge with the friction slope $(S_o - \partial H_q/\partial x)$ may be expressed by Chezy formula as

$$Q_o = k\, A_q \sqrt{R_q S_f} = pk\eta^{1/2} H_q^{(2s+1)/2} \left(S_o - \frac{\partial H_q}{\partial x}\right)^{1/2} \qquad (19)$$

with $R_q = \eta\, H_q$ determined from $A = pH^s$, S_o assumed to be constant

The gradient $\partial H_q/\partial x$ is determined from eq. (12). As the gradient of $(\alpha\ V^2/2g)$ is positive around Q_o, because V increases in the direction of flow at the position of maximum discharge, as shown in fig. 3, its neglect gives a value of δ somewhat different from the actual δ. The value Q_o in eq. (19) is at x = 0 of eq. (11). Taking $C_2 = C_o$ = constant, and $C_o\delta$ also a constant in eq. (12), then

$$\frac{\partial H_q}{\partial x} = - \frac{\delta H_q}{C_o D} \tag{20}$$

with C_o at the time of Q_o. The value H_q from eq. (10), for t = 0, $H_b = 0$, is

$$H_q = H_o e^{\delta/D} (1 - \delta/a)^{a/D} \tag{21}$$

Introducing the values a and D from eqs. (16) and (20), and H_o from eq. (17) into eq. (21) H_q becomes a function of known parameters δ and of $\partial H_q/\partial x$. Putting these values H_q and $\partial H_q/\partial x$ into eq. (19), for known Q_o, p, k, η, S_o, m and G, δ-value can be determined if C_o is known. Using $C_o = \sqrt{gH_q}$ and $A = pH^s$, δ can be determined by a trial-and-error procedure. Assuming a value δ, H_o is computed by eq. (17), a is computed by eq. (16), D by eq. (18), and H_q by eq. (21). Using the expression for $C_o = \sqrt{gH_q}$, then $\partial H_q/\partial x$ is computed by eq. (20). Putting these values into eq. (19), Q value is obtained and compared with Q_o. Then the correction for δ is applied.

From eq. (9), by using Q_h at the maximum depth, and δ replacing t

$$\frac{\partial Q_h}{\partial t} = - \frac{Q_h \delta}{aG} \tag{22}$$

which is given as soon as δ is known. The rates of change of peak discharge and peak depth of eqs. (5) and (6) can be computed from the known δ by using eqs. (20) and (22), respectively.

The above expressions and computational procedures enable the approximate determination of rate of change of flood peaks along the channel, by starting from the initial hydrographs at a given channel position under the assumption that flood hydrographs and flood waves along the channel may be reasonably approximated by Pearson Type III function. The above analysis is the part of a study underway, which has as objectives of routing floods by investigating how the three parameters in Pearson Type III function change along the channel under various channel conditions.

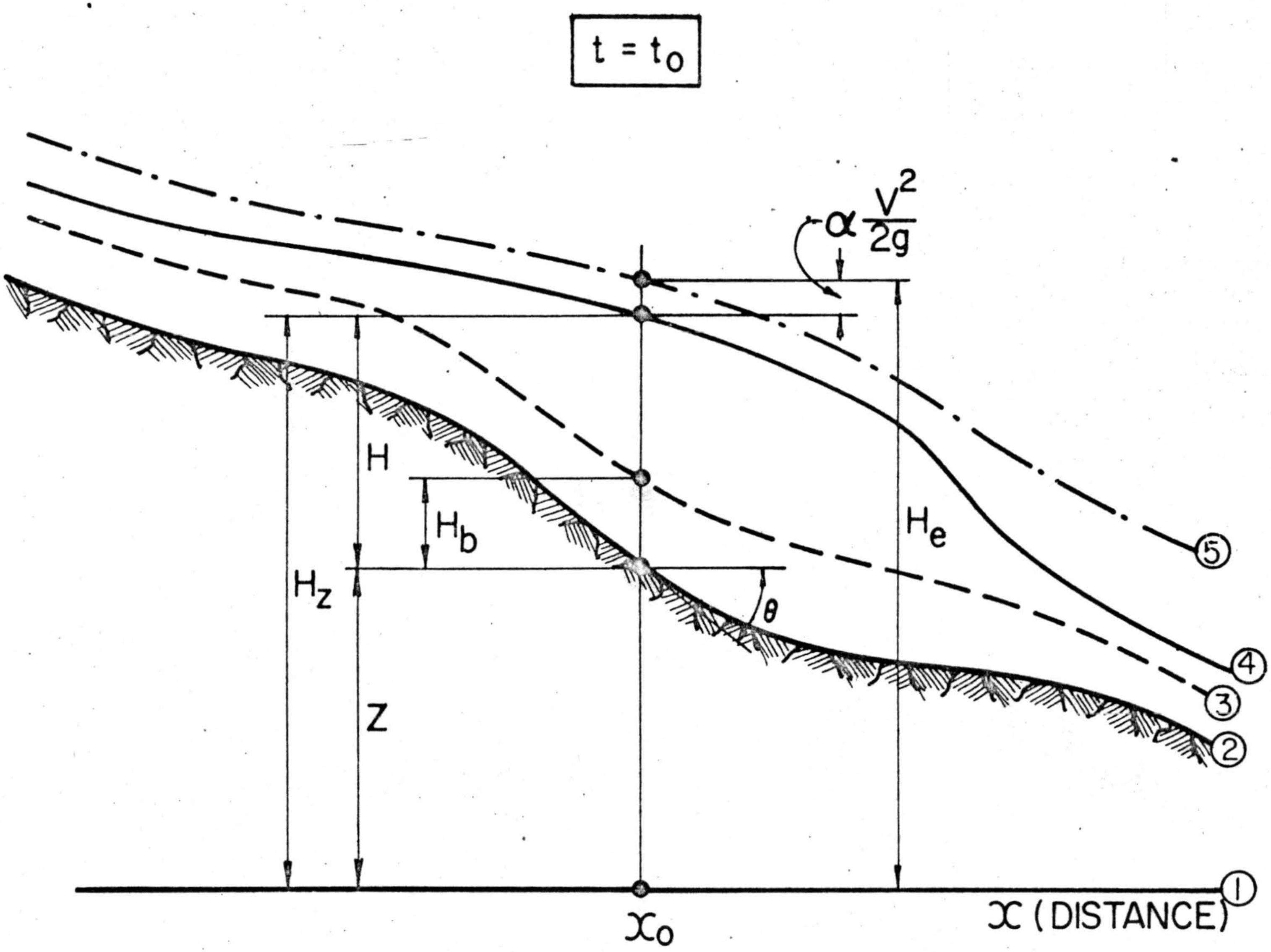

FIG. 1-- LONGITUDINAL PROFILE OF A FLOOD WAVE: (1) REFERENCE LEVEL; (2) CHANNEL BOTTOM WITH CHANGING SLOPE, $S_o = \sin\theta \approx \mathrm{tg}\,\theta$; (3) WATER SURFACE PROFILE FOR STEADY FLOW (FOR GIVEN BASE FLOW Q_b); (4) WATER SURFACE PROFILE FOR UNSTEADY FLOW AT TIME $t = t_o$; (5) ENERGY LINE FOR UNSTEADY FLOW AT TIME $t = t_o$.

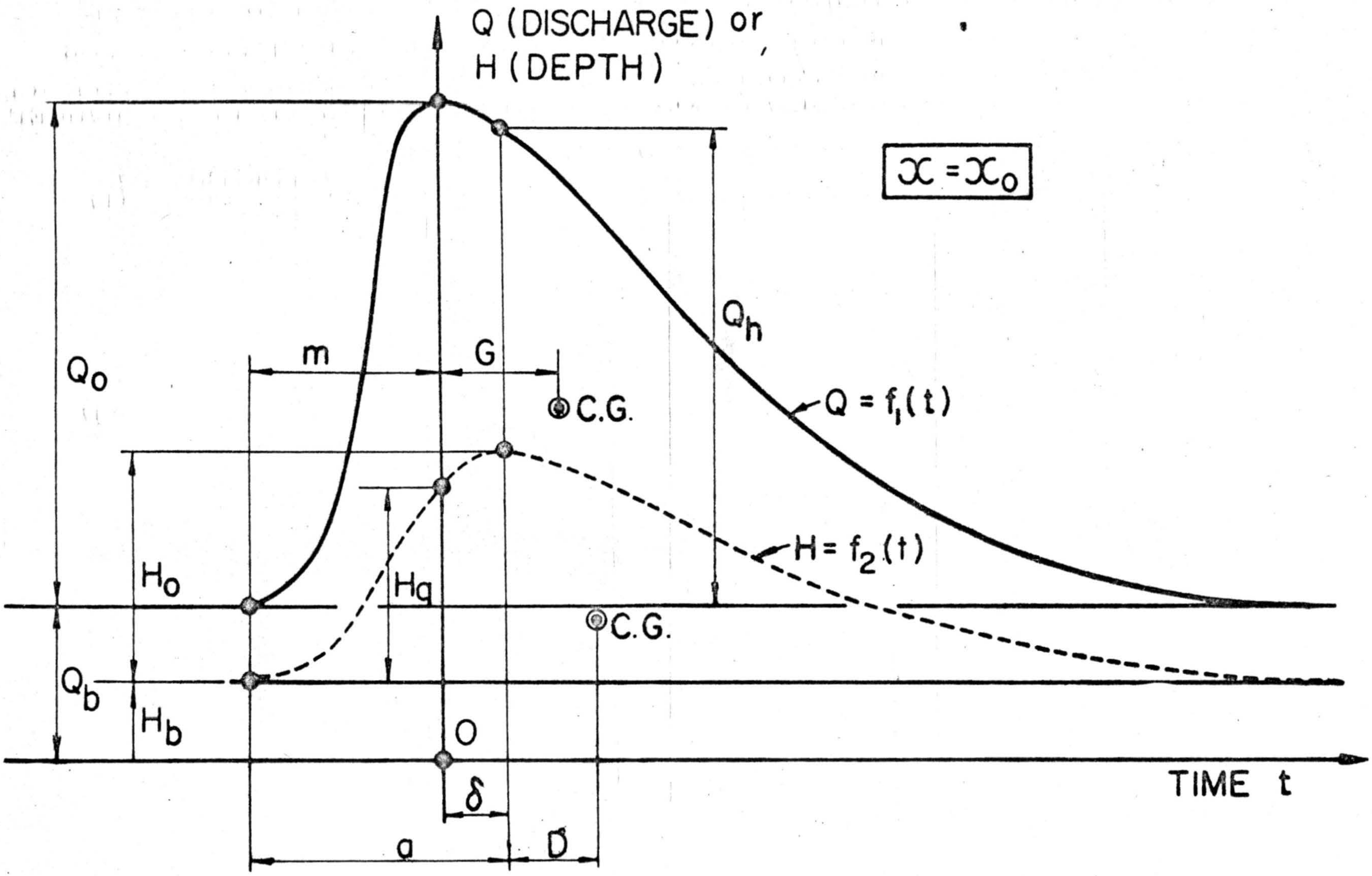

FIG. 2--SCHEMATIC DISCHARGE HYDROGRAPH AND DEPTH HYDROGRAPH AT THE PLACE $x = x_o$, WITH THE COORDINATE ORIGIN BEING AT THE TIME OF MAXIMUM DISCHARGE, WITH THE PARAMETERS DESCRIBING THE HYDROGRAPHS, AND WITH δ BEING THE TIME-LAG BETWEEN PEAK DEPTH AND PEAK DISCHARGE.

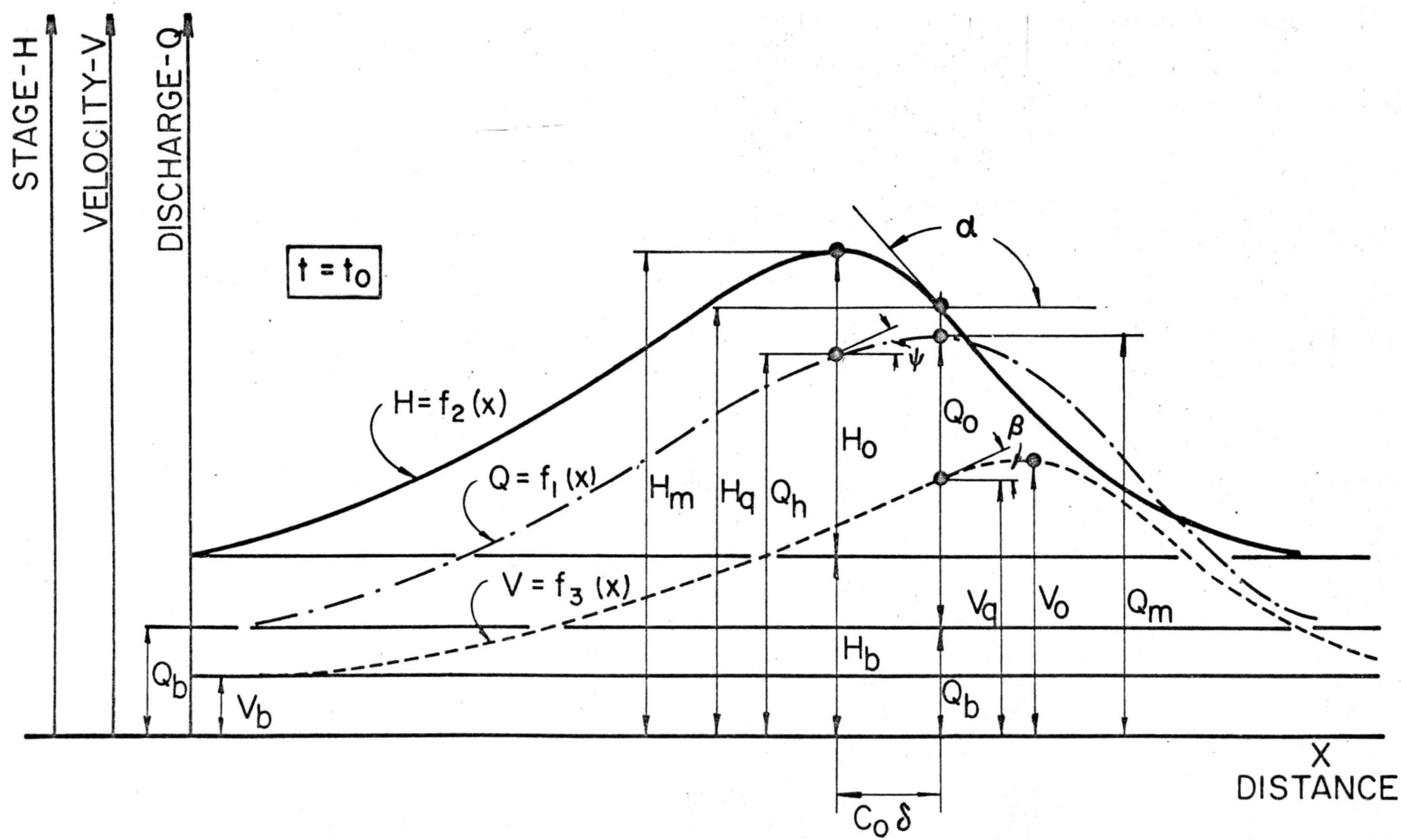

Fig. 3. SCHEMATIC PROFILES OF DISCHARGE, DEPTH, AND MEAN VELOCITY ALONG THE CHANNEL FOR $t = t_o$, SHOWING THE RELATIONSHIP OF THREE PEAKS. THE INDEX b DENOTES THE VALUES OF BASE FLOW; Q_m, H_m AND V_o ARE THE PEAK VALUES; Q_o AND H_o ARE THE PEAK VALUES FOR ZERO BASE FLOW; $C_o\ \delta$ IS THE DISTANCE-LAG BETWEEN PEAK DISCHARGE AND PEAK STAGE AT THE TIME $t = t_o$.

СКОРОСТЬ ИЗМЕНЕНИЯ ПИКА ПАВОДКА В РУСЛЕ

Аннотация

В докладе приведен анализ скорости изменения максимальных расхода dQ_0/dx, глубины dH_0/dx, однопиковых паводковых гидрографов на основе использования двух уравнения Сен-Венана в частных производных для неустановившегося потока со свободной поверхностью. Гидрографы однопиковых паводков и профили паводков вдоль русла апроксимируются по кривой Пирсона Ш типа с тремя параметрами: максимальный расход (или максимальная глубина); замедление нарастания паводка по длине русла и отставание во времени между пиком и центром тяжести гидрографа. Путем приближения установлено соотношение между объемом паводковой волны и этими параметрами.

Найдено соотношение между шестью параметрами по три для каждого гидрографа расхода и глубины путем использования в качестве основного параметра отставание δ по времени между максимальным расходом и максимальной глубиной. Описана методика расчета δ методом проб и ошибок, а также методика расчета dQ_0/dx и dH_0/dx.

Тип. № 2 Ленгорисполкома