Optimization of 3-band Mean Absorption Coefficients

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In this paper we present a process for mean absorption coefficient optimal band selection applied to the 3-band model of radiation in an air electrical arc. For fixed temperature, the divergence of radiation flux in an infinite cylindrical plasma column is calculated using spectrally resolved absorption coefficient and serves as a reference value. Optimization process is used to properly select the bands of 3-band mean absorption approximation using both unmodified and limited Planck mean absorption coefficient. The accuracy of aforementioned two approximation methods is evaluated.

Keywords: radiation transfer, mean absorption coefficient, Planck mean, divergence of radiation flux

1 INTRODUCTION

Radiation transfer is one of the key factors contributing to the energy balance of an electrical arc. Accurate evaluation of energy losses due to radiation transfer is therefore critical for development of a new electrical devices including switchgears and circuit breakers. However a very complex structure of the absorption coefficient, often requiring over 100000 frequency points to proper evaluate [1], makes any calculation using this exact absorption coefficient computationally unfeasibly demanding.

A standard approach to reduce the computational demands is the usage of the mean absorption coefficients (MACs) [1-4]. The entire frequency range is divided into several bands and the radiation transfer inside each band is evaluated using grey approximation. This approach significantly reduces the computational demands at the cost of accuracy. Number of bands N is determined solely by the available computational power and required accuracy.

The value of mean absorption coefficient inside each band should reflect the general behavior of the spectral absorption coefficient in that particular frequency range. The value of spectral absorption coefficient should also not vary extensively. Three different mean values are generally proposed in the literature [1,2]:

- Natural mean

$$\kappa_n = \frac{\int_{\nu_1}^{\nu_2} \kappa_\nu d\nu}{\nu_2 - \nu_1}, \qquad (1)$$

- Planck mean

$$\kappa_P = \frac{1}{\int_{\nu_1}^{\nu_2} B_\nu d\nu} \int_{\nu_1}^{\nu_2} \kappa_\nu B_\nu d\nu, \qquad (2)$$

- Rosseland mean

$$\frac{1}{\kappa_R} = \int_{\nu_1}^{\nu_2} \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu \Big/ \int_{\nu_1}^{\nu_2} \frac{dB_\nu}{dT} d\nu, \qquad (3)$$

where ν_1 and ν_2 represents the band boundary frequencies, κ_ν is a spectral absorption coefficient and B_ν is blackbody radiation. Neither of these mean methods reflect the absorption coefficient precisely in the entire range. The Planck mean is known to behave well for low values of κ_ν whereas Rosseland mean is best suited for high values of κ_ν and is thus best suited for strong lines [1]. To compensate for the shortcoming of the Planck mean absorption coefficients the use of escape factors [1] or line limiting factors [2,4] are proposed in literature leading to accuracy enhancement.

Another important parameter is the selection of the band boundaries. Given the complexity of the spectral absorption coefficient this selection is very complicated one and fairly unintuitive. Personal experience [4] and the position of large changes in absorption continuum [1] are both used in the literature. In this proceedings we discuss the application of numerical optimization process on the selection of band boundaries for 3-band mean absorption approximation. Even with this extremely small number of bands, fairly good accuracy can be achieved with proper treatment of the band boundaries.

2 MODEL DESCRIPTION

The following model was used to evaluate the accuracy of band boundaries selection and using different mean absorption coefficients. We considered infinite cylindrical plasma column

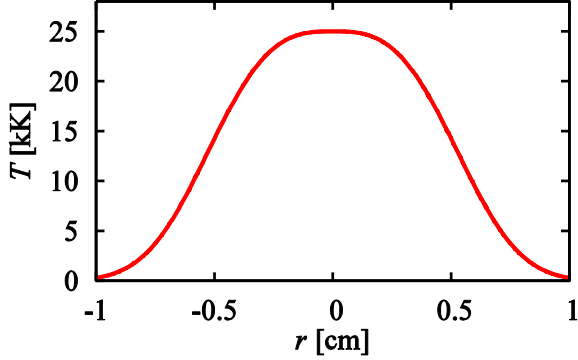with fixed axisymetrical temperature profile as shown in Fig. *1*.

Fig. 1: Assumed temperature profile

The properties of the gas inside the column was considered to be similar to the dry air. Example of calculated spectral absorption coefficient using 245000 spectral points is shown in Fig.2. More detailed information regarding the datasets used to calculate the spectral absorption coefficient and quadrature to calculate the divergence of radiation flux ∇F_ν in 20 points along the cylinder radius can be found in [5]. We used the same calculation method to calculate the divergence of radiation flux ∇F_{MAC} for the 3-band MAC approximation. The boundaries between the bands can be specified freely, however they are considered constant in the whole temperature range. Two methods were used to calculate the MAC inside each band. The first method is a classical Planck MAC as defined in (2). This approximation is referred as pure-Planck and denoted by index P. The second method uses line limiting factor

$$\kappa(r_p) = \frac{1}{r_p}\left(1 - e^{-\kappa_{\text{lines}}r_p}\right) \qquad (4)$$

introduced by Nordborg [2]. Based on our experience we chose $r_p = 15\text{mm}$ which seems to provide reasonable level of accuracy. The MAC is then calculated as

$$\kappa_N = \frac{1}{\int_{\nu_1}^{\nu_2} B_\nu d\nu} \int_{\nu_1}^{\nu_2} \kappa_{\text{mod}} B_\nu d\nu, \qquad (5)$$

where

$$\kappa_{\text{mod}} = \kappa_{\text{cont}} + \kappa(r_p). \qquad (5)$$

This approximation is referred as Nordborg-Planck and denoted by index P.
To evaluate the accuracy of the 3-band MAC

approximation, we defined the norm of divergence of radiation flux as

$$\Delta F = \sum_{i=1}^{20}\left(\nabla F_{\nu,i} - \nabla F_{MAC,i}\right)^2, \qquad (6)$$

where index i denotes the point along the cylinder radius. The norm in (6) also represents the objective function which will be minimized with respect to the band boundary positions. The numerical optimization procedure was carried out using low dimensional simplex method according to [6]. Since this method is capable locating the global minimum of the objective function, the resulting band boundaries can be considered the optimal ones. To ensure this, the optimization procedure was run several times.

3 RESULTS AND DISCUSSION

The optimal positions of band boundary frequencies obtained by numerical optimization are shown in Fig.2. In the case of pure-Planck MAC approximation the boundaries are located at $\nu_1 = 2.761 \times 10^{15}\text{Hz}$ and $\nu_2 = 2.766 \times 10^{15}\text{Hz}$. This creates a very narrow middle band, which contains several strong resonance lines belonging to N^+. These lines are not present at lower temperatures due to low concentration of nitrogen ions. With increasing temperature the importance of these resonance lines increases (see Fig.2) to the limit, where they justify the designation of a separate band. This is also enhanced by the fact that Planck mean is known to overestimate the importance of the strong resonance lines.
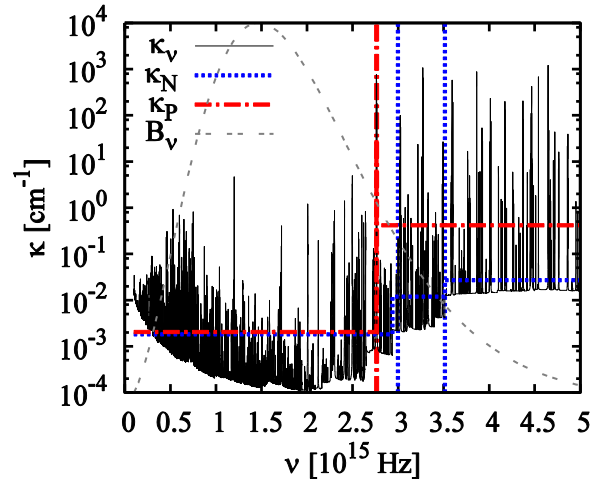

Fig.2: Mean absorption coefficient boundaries and value compared to realistic absorption coefficient at 25 kK
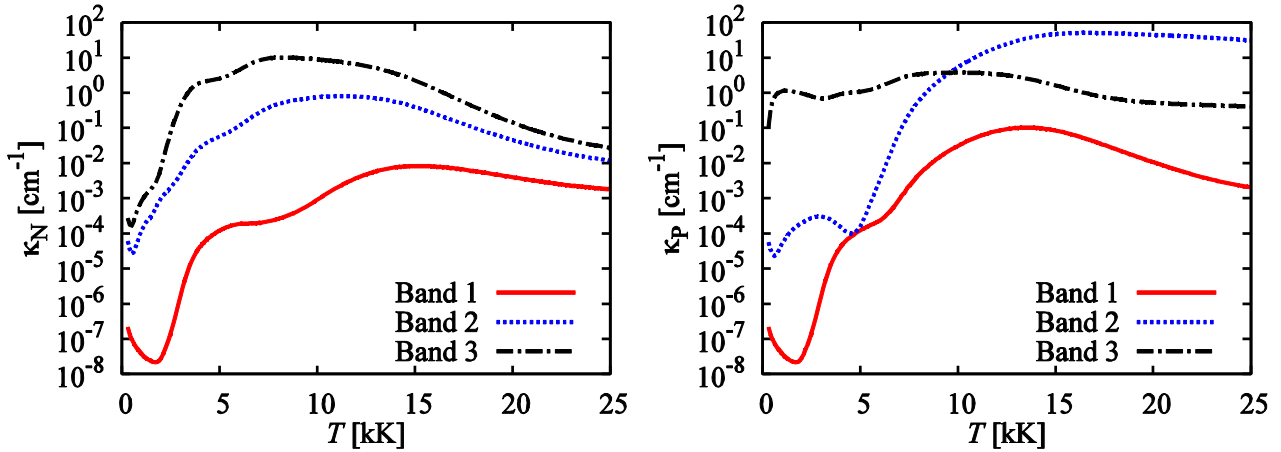
Fig.3: Temperature dependence of the MACs in each band for 3-band Nordborg-Planck approximation (left) and pure-Planck approximation (right)

The inaccuracy of the 3-band pure-Planck MAC approximation is still extremely large even with careful selection of the band boundaries. This is clearly demonstrated in Fig.4. The spectral absorption coefficient κ_ν contains way too many strong lines for pure-Planck approximation to properly capture the behavior of the spectral absorption coefficient in each band.
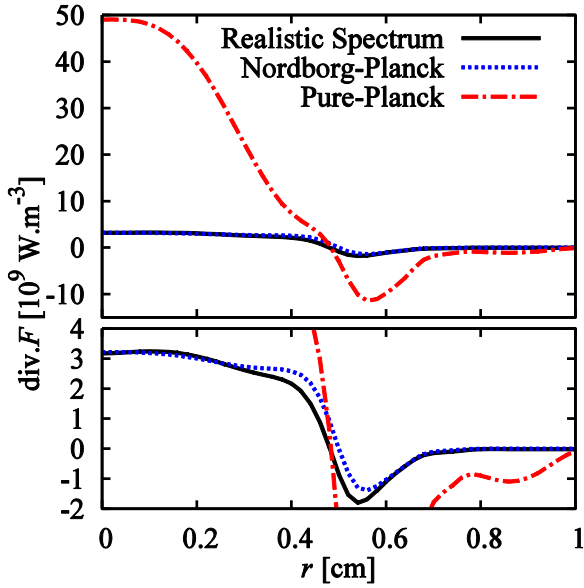
optimization procedure positioned the boundaries at the frequencies $\nu_1 = 2.938 \times 10^{15}$Hz and $\nu_2 = 3.513 \times 10^{15}$Hz. While the position of the second boundary ν_2 can clearly be linked to the absorption edge in the continuum no such link can be made for the first boundary ν_1 implying the necessity for numerical optimization process.
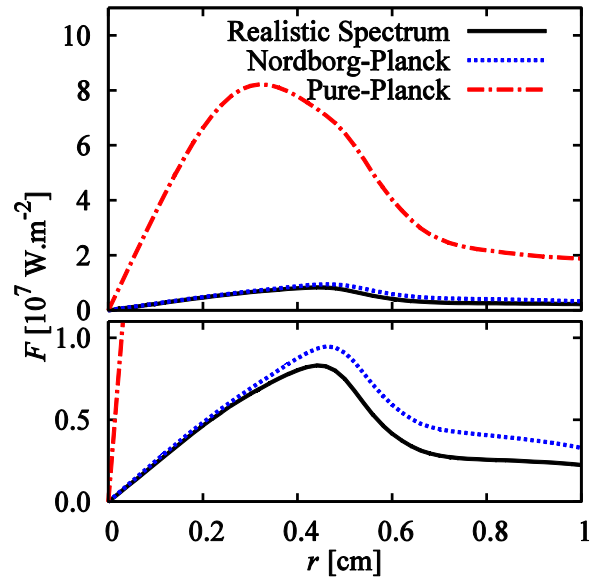

Fig.4: Divergence of radiation flux using realistic absorption coefficient and two variants of mean absorption coefficients

Fig. 5: Radiation flux in radial direction flux using realistic absorption coefficient and two variants of mean absorption coefficients

Significant improvement can be achieved by using the Nordborg-Planck MAC approximation. By limiting the lines the MACs will more precisely reflect the behavior of the spectral absorption coefficient in each band. In such case it is even more complicated to determine the optimal position of the band boundaries. The

The inclusion of line limiting factors into the MAC calculation indeed provided significant enhancement to accuracy as is shown in Fig.4. The difference between divergence of radiation flux using realistic absorption coefficient and using 3-band Nordborg-Planck approximation is less than 10% in the regions of maximum

emission. This is fairly good agreement for a very low number of bands. In the region of maximum absorption the difference is higher reaching nearly 30%. As a result the amount of radiation escaping plasma is overestimated as is shown in Fig. *5*. This can be attributed mostly to the first band which does not capture the radiation reabsorption very well. Adding one more band should probably improve the model accuracy.

The accuracy of the 3-band MAC approximation can very likely be improved further. The obvious target is the effective radius r_p in the calculation of Nordborg line limiting factor in (4). Much better agreement could probably be achieved if the effective radius r_p would be calculated by the optimization procedure.

We considered the band boundaries to be temperature independent. Additional improvement could likely be made if the band boundaries were calculated for each temperature independently. This would however significantly increase the complexity of method needed to calculate the radiation transfer.

4   CONCLUSION

For the standard definition of Planck mean absorption the position of strong lines is very important and should be considered when deciding the band boundaries. Designating one band to a very strong line is often justified. However even with proper band selection the Planck mean average remains fairly inaccurate especially in case of small number of bands.

Fairly good accuracy can be achieved with modified Planck mean absorption coefficient and proper band boundary selection for as low as 3-band approximation. The accuracy within 10% in the emitting area can be achieved. The improvement could probably be achieved by including additional band or considering band boundaries being temperature dependent. To improve the accuracy the value of limiting factor or even the MAC inside each band should be calculated by numerical optimization.

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