Modelling of Intensively Blasted Electric Arc

Senk J., Jakubova I., Laznickova I.

Faculty of Electrical Engineering and Communication, Brno University of Technology, Technicka 12, 616 00 Brno, Czech Rep., jakubova@feec.vutbr.cz

The paper deals with a simplified model of an intensively blasted electric arc burning in the anode channel of an arc heater. The model is based on the energy conservation law, continuity equation and Ohm's law. For computation, transport and thermodynamic properties of working medium and real experimental results describing the external manifestation of the arc are necessary. Many experimental data have been collected during numerous experiments made out with a modular arc heater operated under various experimental conditions, each experiment being characterized by the arc current, voltage, the sort and flow rate of working gas, and the flow rates and temperatures of cooling water in individual segments of the device. In the presented model a rectangular temperature profile of the arc is used. A dependency of the arc column radius r_A on the distance from the cathode is prescribed and parameters of the function $r_A(z)$ are estimated using the total power balance at the output cross-section of the anode channel. Special attention is paid to the region near the beginning. The dependencies of the arc temperature and electric field intensity on the distance from the cathode are calculated and iterations are stopped if the sums of computed increments agree with the measured values. Computed dependencies are given in diagrams and discussed.

Keywords: intensively blasted electric arc, arc heater, argon, modelling

1 INTRODUCTION

A long-term investigation of arc heaters with intensively blasted electric arc stabilized by flowing working gas has been carried out by the authors. The tested device has been designed as a modular structure to enable easy modifications both of geometry (diameter and length of a cylindrical anode channel), and of electric and cooling circuits (with/without extended anode, various segmentation of water cooling circuits) [1]. During numerous experiments the arc heaters have been operated under various conditions. Each experiment is documented by a set of measured data, including namely the arc current I (A), voltage U (V), flow rate Q_m (kgs^{-1}) of working gas (Ar, N_2, their mixtures) and flow rates of cooling water in individual segments of the device. These integral measured data characterize operational conditions and the behaviour of the device as a whole while internal distributions of temperature, electric field strength etc. inside the anode channel still remain hidden.

Simultaneously with experimental research, models of electric arc burning in the anode channel of the arc heater have been developed, capable to estimate temperature, velocity and electric field strength distribution and consequently power and mass transfer in the anode channel. Several versions of models of the electric arc in the anode channel, e.g. [2, 3] have been gradually designed, differing in the level of simplification or neglecting of some aspects, in the used approximations and in the computational method, but all of them based on the same basic principles: energy conservation law, continuity equation and Ohm's law, and all using the same dependency of the arc diameter on the axial coordinate [4]. Further details are given bellow.

Generally speaking, the presented model tries to utilize previous experience for creating a reasonably simple, fast and reproducible computational procedure giving physically credible results.

2 MODELLING

2.1 PRINCIPLES, PRESUMPTIONS

As the previous versions [2, 3], the presented model is based on the energy conservation law, continuity equation and Ohm's law.

The main simplifying presumptions are as follows: Axial symmetry is assumed in the whole anode channel. Arc plasma is supposed to be in local thermodynamic equilibrium and its kinetic energy to be small compared with its enthalpy. Only the radial component of radiative energy flow and axial component of enthalpy flow is taken into account. The conductive energy term is neglected. Mach number is assumed constant over the whole channel cross–section. The measured power loss of the anode channel is assumed to be equal to the power transferred from the arc column, without any other power exchange between the arc heater shell and the surrounding environment.

Further presumptions have been stated according to obtained prior experience. Experiments and computations have proven that voltage drops of the cathode and anode spots should not be neglected [3]. On the contrary, the radial profile of the gas temperature across the arc has been found to be almost constant and the gas temperature in the surrounding cold zone to be equal to the input gas temperature. Thus, a rectangular radial temperature profile is used in the presented model, which makes computations much simpler.

The arc boundary corresponds to the temperature where electric conductivity of the working gas falls to zero (6000 K for argon). As in the previous versions, the axial dependency of the arc radius $r_A(z)$ is as follows

$$r_A(z) = r_0\left[1+\left(\frac{z}{r_0}\right)^{1/n_r}\right] \tag{1}$$

where r_0 is the arc radius at the cathode and n_r is the parameter being searched for during the computation. For arc currents up to 2.16 kA the arc current density is taken $j_0 = 10^8$ Am^{-2} [4] which defines r_0 for the corresponding arc current I.

Transport and thermodynamic properties of the working medium are obviously necessary for the computation. In the following, electric arc burning in argon is studied [5,6,7].

The boundary conditions that must be met by the computed dependencies of the arc radius $r_A(z)$ and temperature $T_A(z)$ are given by the measured integral data, namely: voltage drop over the arc column of the total length z_L

$$U_C = \int_0^{z_L} \frac{I}{2\pi\int_0^{r_A(z)}\sigma\left[T_A(r,z)\right] r\,\mathrm{d}r}\mathrm{d}z \tag{2}$$

and energy balance of the anode channel

$$UI - P_{loss} = \int_0^{z_L} \mathrm{Ma}(z) \left\{\int_0^{r_c} \rho\left[T(r,z)\right] a\left[T(r,z)\right] h\left[T(r,z)\right] 2\pi r \mathrm{d}r\right\}\mathrm{d}z \tag{3}$$

where ρ (kgm^{-3}) is the mass density, h (J kg^{-1}) the enthalpy, a (ms^{-1}) the sound velocity (velocity $c = \mathrm{Ma}\cdot a$, Ma is Mach number), σ (Sm^{-1}) the electric conductivity of the working gas, r_c (m) the anode channel radius, z the axial coordinate (m).

2.2 COMPUTATIONAL PROCEDURE

For computation, the anode channel is divided into cylindrical slices in axial direction. The step Δz of the axial net was tested between 2 and 0.5 mm with the total anode channel length z_L in order of higher tens of millimetres. Basic equations have been modified in accordance with the above mentioned presumptions, integration has been replaced by summation and iterations in individual cylinders of the anode channel volume have been made to meet the boundary conditions given by the measured integral data.

The computational complexity of the original version of the model was rather high and remained that even with a rectangular radial temperature profile. A significant decrease of computational costs has been reached with the presented model by segmenting the computation into two main steps.

First, the output arc radius $r_A(z_L)$ and temperature $T_A(z_L)$ are searched for using the following equation derived from the energy and continuity equations

$$RAH\left[T_A(z_L,n_r)\right] = \frac{UI-P_{loss}}{Q_m} \left\{RA(T_0)\left[\frac{r_c^2}{r_A^2(z_L,n_r)}-1\right] + RA\left[T_A(z_L,n_r)\right]\right\} \tag{4}$$

where $RAH(T)$ and $RA(T)$ stand for products of $\rho(T)a(T)h(T)$ and $\rho(T)a(T)$ respectively. Solving (4), pairs of $r_A(z_L)$ and $T_A(z_L)$ are obtained for which the energy balance condition (3) is fulfilled. This way, the axial dependency of the arc radius is determined (1).

In the next step, axial dependencies of the arc temperature $T_A(z)$, Mach number Ma(z) and electric field strength $E(z)$ are computed using the following equations

$$\mathrm{Ma}(z)=\frac{Q_\mathrm{m}}{\pi\left[r_\mathrm{c}^2-r_\mathrm{A}^2(z)\right]RA(T_0)+\pi r_\mathrm{A}^2(z)RA\left[T_\mathrm{A}(z)\right]}, \tag{5}$$

$$U_\mathrm{A}(z)I(1-p_\mathrm{closs})=\pi r_\mathrm{A}^2(z)\mathrm{Ma}(z)RAH\left[T_\mathrm{A}(z)\right], \tag{6}$$

$$E(z)=\frac{I}{\pi r_A^2(z)\sigma\left[T_A(z)\right]} \tag{7}$$

Here, $U_\mathrm{A}(z)$ is the voltage drop over the arc column from the beginning to z, $p_\mathrm{closs} = P_\mathrm{closs} / U_\mathrm{c}I$ is the power loss of the anode channel P_closs divided by the net input power of the arc, $U_\mathrm{c} = U - U_\mathrm{cat} - U_\mathrm{an}$ is the net voltage of the arc in the anode channel, thus the total voltage decreased by the cathode and anode spot voltage drop. The cathode voltage drop U_cat is determined from the separately measured cathode power loss while the anode voltage drop is estimated [8]. Supposing that possible mutual power exchange between the neighbouring segments does not deteriorate the power balance of each segment, iterations are performed to compute $T_\mathrm{A}(z_k)$, $\mathrm{Ma}(z_k)$ and $E(z_k)$.

The described procedure fails in the region just near the cathode where the conditions for the arc burning are only formed. To avoid rough estimations used in previous versions for this region, the following consideration is used [9]. Let s be the axial distance from the cathode tip where the arc is fully developed with electric conductivity $\sigma[T_\mathrm{A}(s)] > 0$ across the arc column of the arc radius $r_\mathrm{A}(s)$ obeying the same rule (1) as in the rest of the arc column. Let $\sigma[T_\mathrm{A}(s)] > 0$ be the effective conductivity in the layer close to the cathode while the electric field intensity here $E(s) = U_\mathrm{cat}/s$. The temperature $T_\mathrm{A}(s)$ and the thickness of the near-cathode layer s are then bound by the relation

$$\sigma\left[T_\mathrm{A}(s)\right]=\frac{j_0 s}{U_\mathrm{cat}}\left[1+\left(\frac{s}{r_0}\right)^{1/n_\mathrm{r}}\right]^{-2}. \tag{8}$$

As demonstrated in the next section, a suitable $T_\mathrm{A}(s)$ can be estimated according to the shape of axial dependency of $T_\mathrm{A}(z)$ over several mesh steps near the beginning.

The voltage drop $U_\mathrm{A}(z_k)$ between the beginning and z_k is $U_\mathrm{A}(z_k) = U_\mathrm{A}(z_{k-1}) + E(z_k)\Delta z$. In the region of a very steep change of electric field strength a trapezoidal instead of rectangular approximation is used; e.g. for the first cross-section $U_\mathrm{A}(z_1) = ½[E(s) + E(z_1)](z_1 - s)$.

3 RESULTS AND DISCUSSION

In this section the results of the model are given for the following experiment: The arc was burning in the anode channel of the diameter $r_\mathrm{c} = 8 \cdot 10^{-3}$ m and the length $z_\mathrm{L} = 109 \cdot 10^{-3}$ m in argon of the flow rate $Q_\mathrm{m} = 22.5 \cdot 10^{-3}$ kgs^{-1}. The arc current was $I = 162$ A with the total voltage $U = 113.4$ V and the net arc voltage $U_\mathrm{c} = 102.4$ V. Anode channel power loss $P_\mathrm{closs} = 2339$ W.

Using (4), the range of possible $n_\mathrm{r} \in \langle 2.80, 3.80\rangle$ and the output arc temperature $T_\mathrm{A}(z_\mathrm{L}) \in \langle 10039, 13960\rangle$ K was found with the best fitting around $n_\mathrm{r} = 3.16$, $T_\mathrm{A}(z_\mathrm{L}) = 12057$ K. The iterative computation along the anode channel was performed for several estimated $T_\mathrm{A}(s)$ and the corresponding near-cathode layer thickness s determined from (8) as illustrated in Figs 1 to 4. It is obvious that the chosen value of $T_\mathrm{A}(s)$ strongly influences the curves $T_\mathrm{A}(z)$, $E(z)$ near the beginning of the anode channel while the dependencies in the rest of the anode channel remain almost unchanged (Fig.1,2). The shapes of $T_\mathrm{A}(z)$, $E(z)$ near the beginning of the anode channel clearly indicate the range of reasonable choice

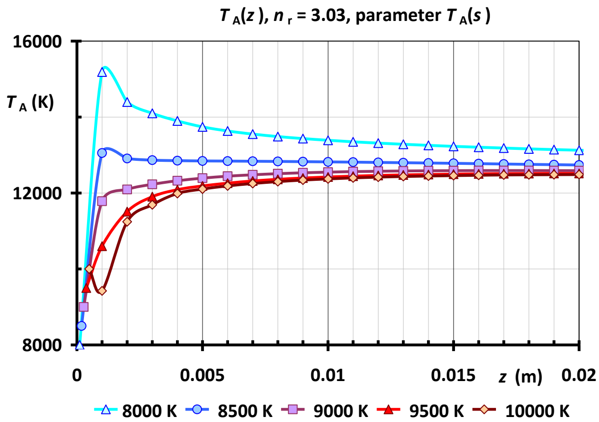

Fig.1: The influence of the estimated $T_A(s)$ on the dependency of $T_A(z)$ at the beginning of the anode channel

of $T_\mathrm{A}(s)$. Fig.3 shows how the arc voltage depends on the chosen $T_\mathrm{A}(s)$ and again confirms the relation between a properly chosen $T_A(s)$ and the total voltage drop $U_A(z_L)$. Finally, in Fig.4 the axial dependencies of all the main parameters are given.

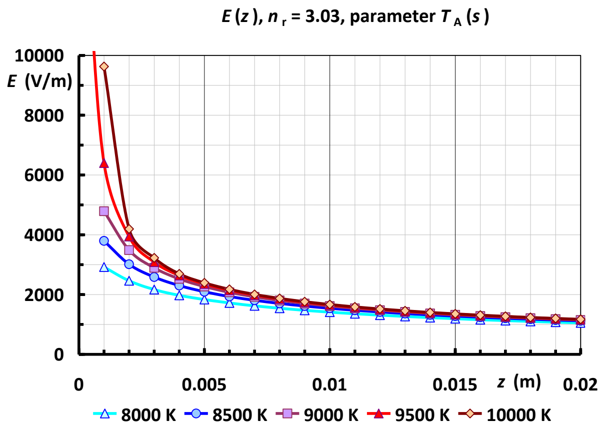

Fig.2: The influence of the estimated $T_A(s)$ on the dependency of $E(z)$ at the beginning of the anode channel

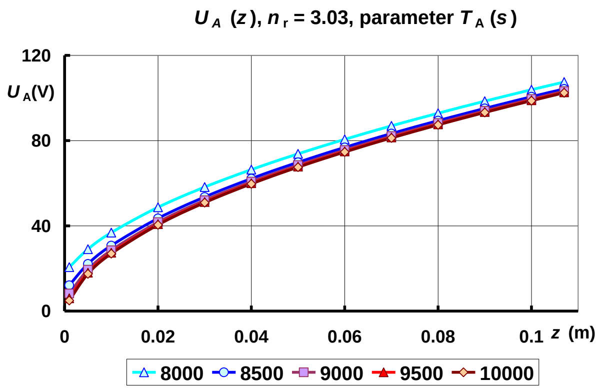

Fig.3: The influence of the estimated $T_A(s)$ on the dependency of $U_A(z)$ along the anode channel

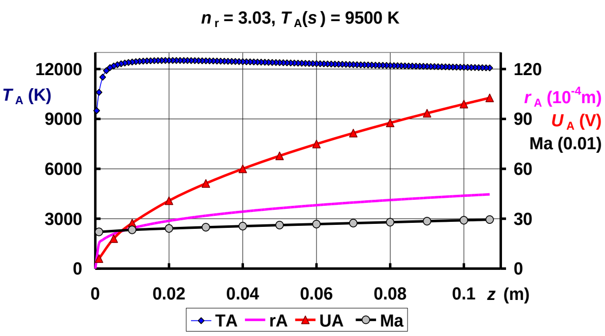

Fig.4: Computed axial dependencies of the arc temperature, arc radius (in 10^{-4} m), voltage drop over the electric arc, and Mach number (in 0.01) in the anode channel

4 CONCLUSION

The paper describes the mathematical-physical model of the intensively blasted electric arc burning in the cylindrical arc heater's anode channel. The computation has been carried out in the two steps, the first one determining the output arc radius and temperature, and the second one calculating especially the axial dependency of the arc temperature, Mach number and electric field strength. Special attention has been paid to the near-cathode region and a method of estimation of the arc temperature at the end of the near-cathode layer has been introduced. Real experimental data have been used as input data of the model and the obtained results are given in figures and discussed. The model is planned to be refined as far as more precise approximation of gas properties is concerned, and will be used with other data sets to collect further experience.

Acknowledgements

This research work has been carried out in the Centre for Research and Utilization of Renewable Energy (CVVOZE). Authors gratefully acknowledge financial support from the Ministry of Education, Youth and Sports of the Czech Republic under NPU I programme (project No. LO1210) and from the Czech Science Foundation under (project No. GA 15-14829S).

REFERENCES

[1] Jakubova I, Senk J, Laznickova I, In: Proc. of the Int Conf Electrical Power Engg. Brno: FEEC BUT, 2014, 527-531.

[2] Heinz J, Senk J, Czech J Phys 54 (2004) C702-C708.

[3] Gregor J, Jakubova I, Senk J, Chem listy 102 (2008) 1408-1413.

[4] Ramakrishnan S, Stokes A D, Lowke J, J Phys D: Appl Phys 11(1978) 2267–2280.

[5] Farmer A J D, Haddad G N, Material Functions and Equilibrium Composition of an Argon Plasma. CSIRO Division of Applied Physics 1989.

[6] Coufal O, Appl Phys 40 (2007) 3371-3385.

[7] Laznickova I, In: Proc. of the Int Conf Electrical Power Engg. Brno: FEEC BUT, 2010, 767-773.

[8] Isakaev E K, Sinkevich O A, Tyuftyaev A S, Chinnov V F, High Temperature 48 (2010) 97-125.

[9] Lowke J, J Phys IV France 7 (1997) C4-283-C4-294.