Interaction of Spatially Localized LHW with Banana Particles

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The paper proposes a novel mechanism of LHW stochastic acceleration of electrons in a tokamak.

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1 INTRODUCTION

In our previous papers [1-4], where we studied the interaction of electrons with spatially localized lower hybrid waves, we considered as zero approximation the unperturbed constant velocity of electrons, for simplicity circulating along a circular magnetic field line. The basic stochastic interaction appeared at nonlinear resonant frequencies of the interaction. In these papers we anticipated the possibility of accelerating electrons to velocities higher (even relativistic) than predicted by quasi-linear theory. The paper [5] was perhaps the first paper, in which this possibility was experimentally established. In reference [6] we tried to support this phenomenon again by means of theory and numerical simulation. Despite the fact that large acceleration was found in [6], we were not able to reach the experimentally detected values due to lack of overlapping of resonances for higher velocities. In order to resolve this shortcoming, the appearance of further resonances with smaller velocity distances between them could be a solution. These may appear, abandoning the simple 1D approximation. One of the possibilities is to take into account electron banana motion in the tokamak configuration.

2 BANANA DYNAMICS

Banana motion will be characterized by the path along a magnetic field line towards stronger magnetic field values and by the mirror effect, where the particle kinetic energy will be transformed to perpendicular energy. The particle will be then reflected back into the region of smaller toroidal magnetic field.

Let us firstly summarize the basic formulae, characterizing the banana dynamics. Perhaps the most important phenomenon is the banana frequency. Generally, the most exact definition was presented by Dnestrovsky and Kostomarov [7], and also in [8] and [9]. The frequency of banana oscillations, ω_b, is derived only for small amplitudes of banana oscillations, $\theta^2 \langle\langle 1$ (θ is the poloidal angle) and is given [7] as $\omega_b = (v_\perp / qR_0)(r/2R_0)^{1/2}$.

For the analytical discussion of the interaction between banana particles and LHW localized spatially in LHW cone, the precession velocity of bananas is crucial. According to [9], this velocity is given as $v_\phi = m{v_\perp}^2/2/eBr$ and its angular frequency ω_ϕ as $\omega_\phi = m{v_\perp}^2/eBRr$. The maximal poloidal angle of banana, θ_m, is $\theta_m = v_{//0}/v_{\perp 0}\sqrt{2R_0/r}$ [8].

3 INTERACTION OF LHW FIELD WITH BANANAS

The description of banana dynamics under the effect of RF field appeared in several papers. As a representative one, see e.g. [10]. In this paper, we would like to mention our recent publications [11-14], where the stochastic interaction of bananas with alfvén waves was considered. (There, also an important effect of the stochastic radial diffusion of bananas was detected).

According to the best of our knowledge, the interaction of bananas with spatially localized LHW was not previously discussed.

Our recent results [6] were based on a 1D description of the interaction. As already mentioned, for parameters of the WEGA stellarator, the stochastic interaction of electrons with spatially localized LHW yields lower acceleration [6] than was detected [5]. The upper limit of the acceleration was given by the last harmonic amplitude, which still overlaps. The most important effect of our proposal is the addition of a new set of frequencies. These also can be considered as additional resonances to the primary resonances, generated by the spatially localized character of LHW launched by the grill. Further, the spatial extent of bananas can be larger or smaller than the toroidal length of the grill. This presents difficulties for the analytic formulation of the interaction. Moreover, estimation of the interaction of particles with LHW must take into account not only the precession velocity, but also the trace of the banana. All these effects can considerably change the picture of the interaction, based on a 1D description. Due to the mentioned problems, the most reliable approach consists in the direct numerical solution of the corresponding Hamiltonian model of this interaction.

For passing electrons on circular orbits, the Fourier analysis of the spatially localized LHW, with a rectangular envelope, the effective amplitude of the electric field is given as [2, 3, 4]

$$E_{eff} = E_0 \sum \frac{\sin\left(n\pi L_0 / L\right)}{\pi n} \times \cos\left[\left(k_{//} + n\frac{2\pi}{L}\right)Q_3 - \omega t\right] \tag{1}$$

Now, we would like to generalize this model including banana motion. Under the assumption that each mode of the foregoing expansion acts independently, we insert the induced change of the trajectory in each member of the expansion as follows:

$$E_{tot} = E_0 \sum \frac{sin\left(n\pi L_0 / L\right)}{\pi n} \times cos\left[\left(k_{//} + n\frac{2\pi}{L}\right)Q_3 + k_{//}\Delta Q_3(t) + v_\phi t - \omega t\right].$$

The expression $\Delta Q_3(t)$ represents a projection of the rather complicated banana trajectory onto the parallel coordinate Q_3. Approximating $\Delta Q_3(t) = Q_{3m}\cos\omega_b t$ for $Q_{2m} \neq 0$ for $Q_3 - v_{prc}t \subset L \pm 1/2\,\Delta Q_{3m}$, using Fourier expansion for parameter $\Delta Q_{3m}/L$ (ΔQ_{3m} is the toroidal component of banana amplitude), and introducing the result into the original expression, the resulting procedure merges into the product of two Fourier expansions and a Bessel expansion. From the point of view of resonant interaction, the effect of bananas leads to the addition of possible resonances on bananas harmonic frequencies. Considering the effect of the banana oscillations, the argument in the square bracket changes as

$$\left(k_{//} + n\,2\pi / L\right)Q_3 - \omega t + k_{//}\Delta Q_3 \sin\omega_b t + v_\phi t$$

due to banana modulation of velocity. This broadens the possible resonances of expression (1) with resonances $m\omega_b$. The basic problem is hidden in the size of bananas. To use perturbation analysis, only two extreme cases of dimensions of the grill (or LHW cone) and of size of the banana can be considered. Either the case, where the banana stays for a long period in the LHW cone, or when the intersection of trajectory with LHW cone represents only a negligible part of the whole of the banana orbit. Some analytical support can be expected in two extreme cases. For toroidally large amplitudes, well known model of a “kicked” oscillator (see e.g. [15]) can be used. For extremely short amplitudes, the resonant pumping of the energy into the global energy of the banana can be assumed.

It is interesting to note that for the cases with banana, the change of the energy concerns not only the parallel, but also the perpendicular dimensions. The introduced change of the parallel energy by LHW interaction therefore will appear in both parts of the energy. Here, some affinity with the discussion by Rax, Fisch and Laurent [16] appears.

Last but not least, the effects mentioned above can be of some interest for all cases, where bananas play an important role. (e.g., the bootstrapped current generation under the effect of induced stochastic radial diffusion of bananas represent an interesting problem).

A future paper will deal with a numerical estimate of the banana effect, including the comparison with the case of passing particles.

4 SUMMARY

The present paper offers a possible extension of nonlinear resonances, discussed in [6] by a new set of resonances, resulting from banana dynamics. In our next work, we shall estimate the broadening of resonances, using analytical formulae for bananas. Nevertheless, two basic expressions, banana frequency and banana amplitude are results of strict approximations. According to [7], e.g. the banana frequency is strongly nonlinear (depending on elliptic integrals, reminiscent of the mathematical pendulum) and its simple expression can by no means represent the whole problem presented by the banana dynamics. Moreover, since there appears a set of new parameters, such that the corresponding dynamics has a 3D character, we anticipate a computer simulation of the corresponding dynamics with a Hamiltonian in its toroidal version.

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