Magnetosonic Waves and Current Relaxation

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In this work, inertial and diffusive effects are rediscussed on the propagation of magnetosonic waves in plasmas of finite conductivity, with basis on an extension of Ohm's law. It is shown that when such effects are strong, the discussion can be pursued by defining suitably a perturbative parameter with the dimension of speed. The results presented here are part of a program which aims to investigate the influence of inertia due to charged species on the propagation of hydromagnetic waves and instabilities in resistive plasmas.

Keywords: magnetosonic waves, magnetic diffusivity, Ohm's law

1 INTRODUCTION

It is well known that the coupling of small disturbances of velocity and magnetic fields with those of density and pressure gives rise to the propagation of compressional waves in plasmas. These waves are generally referred to as magnetosonic waves. If the propagation is perpendicular to the equilibrium magnetic field, only fast modes exist. When the propagation is parallel to the field, slow modes may arise. The dispersion relation which describes the propagation of magnetosonic waves is usually derived for ideal plasmas (their resistivity is negligible) [1]. The inclusion of finite resistive effects in the problem is important because it allows the description of diffusive processes in both laboratory and astrophysical plasmas [2-9]. However, in all these references, diffusive phenomena are studied for long wavelength perturbations. Such an approach neglects the influence of charged species inertia in the plasma. In this work, diffusive and inertial effects are rediscussed on the propagation of magnetosonic waves in plasmas of finite conductivity, as can be inferred from [10].

2 THEORETICAL BASIS

It is widely accepted that the current density $\vec{J}$, induced in a plasma of finite conductivity σ, is related to the applied electric $\vec{E}$ and magnetic $\vec{B}$ fields, and to the developed fluid velocity $\vec{v}$, through the standard Ohm's law, [1]

$$\vec{J} = \sigma\left(\vec{E} + \vec{v} \times B\right). \qquad (1)$$

Eq. (1) is recognized to describe satisfactorily the dissipative processes which occur in the conductive plasma, provided that the characteristic wavelength of the electromagnetic field is sufficiently long. In this case, inertial effects due to charged species in the plasma are negligible. However, at shorter wavelengths, inertial effects become important enough for that the time rate of $\vec{J}$ may be included in Eq. (1), [11]

$$\left(1 + \tau \frac{\partial}{\partial t}\right)\vec{J} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right). \qquad (1)$$

In Eq. (2), τ is interpreted as the relaxation time of $\vec{J}$. Actually, when the electromagnetic field is suddenly removed from the presence of the plasma, Eq. (2) shows that

$$\vec{J} = \vec{J}_0 e^{-t/\tau}. \qquad (3)$$

Eq. (3) means that any initial current density $\vec{J}_0$ damps off within the plasma in a finite time scale of the order of τ. In the limit $\tau \to 0$, Eq. (2) recovers Eq. (1) and Eq. (3) shows that $\vec{J}_0$ damps off instantaneously. Clearly, such a situation can be justified only if the other time scales which are involved in the problem are much longer than τ. In this case, inertial effects can be neglected. By combining Eq. (2) with Maxwell's equations, one gets [10]

$$(1 - \delta^2\nabla^2)\frac{\partial \vec{B}}{\partial t} - \zeta\nabla^2\vec{B} = \nabla \times \left(\vec{v} \times \vec{B}\right), \qquad (4)$$

where we have introduced the standard magnetic diffusivity ζ and finite skin depth δ,

$$\zeta = \frac{1}{\mu_0\sigma} \qquad \text{and} \qquad \delta = \sqrt{\frac{\tau}{\mu_0\sigma}}, \qquad (5)$$

respectively, with μ_0 denoting the vacuum magnetic permeability.

3 RESULTS AND DISCUSSION

Consider an initially static, homogeneous and isotropic, infinite plasma with mass density ρ, subjected to a constant and uniform magnetic field $\vec{B}$. Subsequently, assume that the plasma experiences a small perturbation $\sim e^{\iota(\vec{k}\cdot\vec{r}-\omega t)}$. By adopting Cartesian coordinates, one may regard the wave vector $\vec{k}$ to be in the x-direction and the magnetic field $\vec{B}$ to lie on the xy-plane, without loss of generality. In this case, it can be shown that the angular frequency ω satisfies the dispersion relation [10]

$$(1+\delta^2k^2)\omega^4+\iota\zeta k^2\omega^3-[(1+\delta^2k^2)c_s^2+u^2]k^2\omega^2-\iota\zeta c_s^2k^4\omega+c_s^2u_x^2k^4=0, \quad (6)$$

with c_s denoting the sound speed in the medium concerned and where we have introduced the quantities

$$u_x=\frac{B_x}{\sqrt{\mu_0\rho}} \quad \text{and} \quad u=\sqrt{\frac{B_x^2+B_y^2}{\mu_0\rho}}, \quad (7)$$

both with the dimension of speed. For weak diffusive and inertial effects, the solutions of Eq. (6) may be read as

$$\omega_\pm=k\sqrt{\frac{c_s^2+u^2}{2}}\left[1\pm\sqrt{1-\left(\frac{2c_su_x}{c_s^2+u^2}\right)^2}\right]^{1/2}. \quad (8)$$

Eq. (8) recovers the standard dispersion relation for magnetosonic waves in the ideal limit [1]. The plus and minus signs describe the fast and slow magnetosonic waves, respectively. However, for strong diffusive and inertial effects, Eq. (6) may be rewritten as [10]

$$(\omega^2-c_s^2k^2)(\delta^2\omega^2+\iota\zeta\omega)-(u^2\omega^2-u_x^2c_s^2k^2)=0\,. \quad (9)$$

As it appears, Eq. (9) may be substantially simplified by considering the condition $u\sim u_x$ ($B_y\to 0$, see Eqs. (7)). Actually, in this case, Eq. (9) reduces to

$$(\omega^2-c_s^2k^2)(\delta^2\omega^2+\iota\zeta\omega-u_x^2)=0. \quad (10)$$

The solutions of Eq. (10) are immediately recognizable as the standard sound wave and modified Alfvénic wave, [10]

$$\omega_+=c_sk \quad \text{and} \quad \omega_-=\frac{\sqrt{4\delta^2u_x^2-\zeta^2}-\iota\zeta}{2\delta^2}, \quad (11)$$

respectively. Therefore, the general solutions of Eq. (9) can be obtained, at any order of approximation, by defining the perturbative parameter

$$u_y=\frac{B_y}{\sqrt{\mu_0\rho}} \quad (11)$$

with the dimension of speed. As a matter of fact, if the unperturbed solutions change slightly as $\omega_\pm\to\omega_\pm+\omega'_\pm$, then it can be shown that the first-order correction terms are given by

$$\omega'_+=\frac{\omega_+u_y^2}{2(\delta^2\omega_+^2+\iota\zeta\omega_+-u_x^2)}$$

and

$$\omega'_-=\frac{\omega_-^2u_y^2}{(\omega_-^2-c_s^2k^2)(2\delta^2\omega_-+\iota\zeta)}, \quad (12)$$

respectively.

4 CONCLUSION

In this work, inertial and diffusive effects have been rediscussed on the propagation of magnetosonic waves in plasmas of finite conductivity, with basis on the extended Ohm's law, Eq. (2). It has been shown that when such effects are strong, the discussion can be pursued by defining suitably a perturbative parameter with the dimension of speed. The results presented here are part of a program which aims to investigate the influence of inertia due to charged species on the propagation of hydromagnetic waves and instabilities in resistive plasmas [12, 13].

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