Acta Polytechnica CTU Proceedings **36**:15–24, 2022
https://doi.org/10.14311/APP.2022.36.0015

STRUCTURAL ASSESSMENT AND EXPECTED UTILITY GAIN

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Abstract. This paper contains an introduction, probabilistic formulation, and exemplification (1) of system state and utility actions, (2) system state and utility action value analysis and (3) a threshold formulation for a predicted information and action decision analysis. The approaches build upon structural condition assessment and provide a basis for the condition management by maximising the expected utility for information and actions before information acquirement and action implementation. Following the basic distinction of system state and utility actions, strengthening, replacement, repair, load reduction and consequence reduction actions are formulated. With an exemplary study encompassing an expected utility calculation and an action value analysis, it is demonstrated how expected utility optimal physical system changes can be identified before implementation. The threshold formulation for a predicted information and action decision analysis relies on the equality of the decision theoretical posterior action optimality condition. By extending the exemplary predicted action decision analyses with different structural health information (SHI), the threshold formulation is exemplified and the optimal condition management strategies are identified.

Keywords: Structural condition management, action uncertainty modelling, action and information value.

1. Introduction

Condition assessment is usually understood as a structural reliability analysis with additional information. Most commonly it is performed with obtained information, i.e., by posterior structural reliability updating. However, condition assessment constitutes a part of the structural condition management, i.e., ensuring the integrity and functionality of a system over time. The structural condition management may require measurements to learn about the structural condition and actions to adapt the structural condition. These two basic means of structural condition management should be performed efficiently, i.e., in a way that measurements and actions are planned to maximise their technological, economic and risk reduction effects.

An efficient structural condition management can be performed and planned by the utilisation, adaptation and the application of the Bayesian decision theory [1] to structures; or more widely to built environment systems. The decision theoretical context of structural condition assessment and management necessitates (1) a system state model, (2) a utility model, (3) that the information acquirement state is considered in the probabilistic modelling and (4) that the relevant actions can be modelled. Whereas points (1) to (3) have been extensively developed over the last decade, see e.g., [2–17], the action modelling remains a challenge. Relevant actions for a condition management would encompass e.g., structural modifications and utility changes. Further, it would be required to analyse the expected action utility prior to the decision to implement an action.

This paper focusses on the introduction and formulation of system state and utility actions as well as the action value analysis building upon [18] and [19] in Sections 2 and 3. The action modelling in Section 2 includes the introduction of an action uncertainty modelling and the formulation of strengthening, replacement, repair, load reduction and utility actions in conjunction with a limit state function representative for a highly correlated series system including extreme events and deterioration. Based on the action uncertainty modelling, predicted and implemented action decision analyses are formulated (Section 3). In Section 4, the action modelling is exemplified and expected utility theorem consistent action values for various actions are quantified.

Section 5 contains a decision theoretical formulation of predicted information and action decision analyses (PIPA DA) and a derivation for continuous and direct structural health information (SHI). It is shown how this SHI formulation can be discretised and solved. The in Section 4 introduced example is further developed in Section 6 and the value of the condition management is quantified and separated into measurement information and system state and utility actions. Section 7 includes and a summary and concluding remarks.

2. System performance and action modelling

This section encompasses the formulation of the system state analysis, i.e., a structural reliability analysis in a decision theoretical context, and the combination of system state reliability and a utility model to quantify the system performance (Section 2.1). In Section 2.2, a probabilistic action modelling in introduced.

2.1. System state analysis and system performance analysis

A generic component limit state function g_{X_1}, describing the component state event X_1, may be formulated with the resistance R, the damage D and the loading S, which are subjected to the resistance and loading model uncertainties M_R, M_D and M_S, respectively:

$$X_1 : g_{X_1} = M_R \cdot R\left(M_D \cdot D\right) - M_S \cdot S \leq 0 \tag{1}$$

The resistance may be decomposed in the section property Z and the material strength R_{mat}. Both may be subjected to the influence of deterioration, i.e., the design variable is subjected to section deterioration $(M_{D,Z} \cdot D_Z)$ and the material strength is influenced by degrading material properties $(M_{D,R_{mat}} \cdot D_{R_{mat}})$:

$$X_1 : g_{X_1} = M_R \cdot Z\left(M_{D,Z} \cdot D_Z\right) \cdot R_{mat}\left(M_{D,R_{mat}} \cdot D_{R_{mat}}\right) - M_S \cdot S \leq 0 \tag{2}$$

In assessing normatively an expected utility-based decision or a decision value, the benefits, costs and consequences of the system states are modelled. Based on these models, the preference of decision makers can be described using the concept of a utility function implying the context of a descriptive decision analysis [20]. In engineering decision analysis, the utility function is rather linear and usually based on monetary values, where the utilities of the system states are expressed in terms of costs or, generally, negative consequences. In the following, the utility function is thus omitted and a normative decision analysis performed.

The system states are described by X_l (see above) resulting in a utility $u\left(X_l\right)$. The system performance U_{SP}, i.e., the expected system utility, is then calculated as the expected value of a probability mass function (where the system states denote realisations of a discrete random variable), i.e.:

$$U_{SP} = E_{X_l}\left[u\left(X_l\right)\right] = \sum_{X_l} u(X_l) \cdot P(X_l) \tag{3}$$

2.2. Action modelling

The expected system performance (see above) can be influenced by physical changes affecting (1) the utility of the system states and (2) the resistance and/or the load of the system states. This basic distinction leads to the denotation of utility actions and system state actions, respectively. The latter type can be associated with actions performed by an engineer as it includes e.g., design, repair and strengthening actions. Beyond the distinction of action types, an action implementation state and uncertainty modelling is required to analyse and to optimise the expected of the utility before action implementation (see Section 1). It is thus in the further distinguished between a predicted or implemented action. The explicit modelling of the m implementation states $Y_{k,m}$ and the modelling of implementation uncertainties are introduced.

A utility action is described with the system state utilities $u(X_l, a_k, Y_{k,m})$, the implementation states $Y_{k,m}$ and the implementation uncertainties $P(Y_{k,m})$ and the costs of the action $c(a_k)$. The index m may be used for distinguishing e.g., with $m = 1 \ldots 3$ full, partial or failed action implementation. A system state action a_k is described with the system state probabilities $P(X_l(a_k, Y_{k,m}))$ beside the implementation uncertainties and the costs.

In the context of existing structures, examples for system state actions are e.g., repair, replacement, strengthening and loading actions. A repair action can be modelled by reversing deterioration effects due to mechanical reshaping of the component. The repair action including the action implementation uncertainties $(a_k, Y_{k,m})$ is then allocated to the damage, i.e., $D(a_k, Y_{k,m})$ affecting the material resistance or the section properties:

$$X_1 : g_{X_1} = M_R \cdot Z\left(M_{D,Z} \cdot D_Z\right) \cdot R_{mat}\left(M_{D,R_{mat}} \cdot D_{R_{mat}}(a_k, Y_{k,m})\right) - M_S \cdot S \leq 0 \tag{4}$$

$$X_1 : g_{X_1} = M_R \cdot Z\left(M_{D,Z} \cdot D_Z(a_k, Y_{k,m})\right) \cdot R_{mat}\left(M_{D,R_{mat}} \cdot D_{R_{mat}}\right) - M_S \cdot S \leq 0 \tag{5}$$

A replacement action would result in a damage free component with same, similar or new material properties:

$$X_1 : g_{X_1} = M_R \cdot Z \cdot R_{mat}(a_k, Y_{k,m}) - M_S \cdot S \leq 0 \tag{6}$$

A strengthening action results in an enhanced resistance of a component by e.g., section enlargement, material addition or the replacement with a stronger material, i.e., $Z(a_{S,k}, Y_{k,m}, \ldots)$ and $R_{mat}(a_{S,k}, Y_{k,m}, \ldots)$:

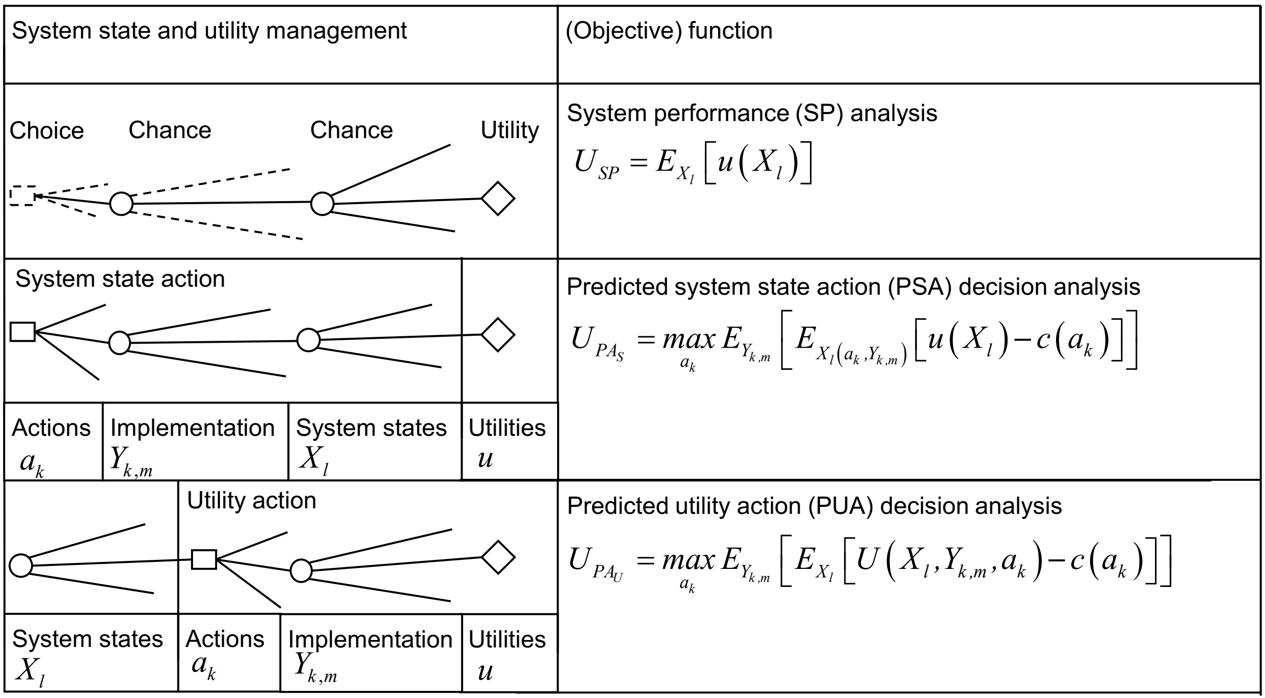

FIGURE 1. System performance functions and objective functions for predicted action decision analyses.

$$X_1 : g_{X_1} = M_R \cdot Z(a_k, Y_{k,m}, M_{D,Z} \cdot D_Z) \cdot R_{mat}(M_{D,R_{mat}} \cdot D_{R_{mat}}) - M_S \cdot S \leq 0 \tag{7}$$

$$X_1 : g_{X_1} = M_R \cdot Z(M_{D,Z} \cdot D_Z) \cdot R_{mat}(a_k, Y_{k,m}, M_{D,R_{mat}} \cdot D_{R_{mat}}) - M_S \cdot S \leq 0 \tag{8}$$

Actions on the load side can be a load application for testing purposes or the change of use leading to a modified loading, $S(a_{S,k}, Y_{k,m})$:

$$g_{X_1(a_{S,k},Y_{k,m})} = M_R \cdot Z(M_{D,Z} \cdot D_Z) \cdot R_{mat}(M_{D,R_{mat}} \cdot D_{R_{mat}}) - M_S \cdot S(a_k, Y_{k,m}) \leq 0 \tag{9}$$

Utility actions may include a benefit enhancement due to an enhanced functionality or a consequence reduction in case of failure. A utility action is subjected to action implementation uncertainties dependent on the system states, i.e., $u(X_l, Y_{k,m}, a_k)$. For illustration purposes, the expected utility quantification of an implemented action analysis can be written as:

$$U_{IA} = E_{Y_{k,m}}\left[E_{X_l}\left[U(X_l, Y_{k,m}, a_k) - c(a_k)\right]\right] = \sum_{X_l}\sum_{Y_{k,m}} P(X_l) \cdot P(Y_{k,m}) \cdot u(X_l, a_k) \tag{10}$$

A utility action can be further detailed to consequence reduction (($c(X_l, a_k)$, Equation (11)), benefit and functionality enhancement ($b(X_l, a_k)$, Equation (12)) and (Equation (13)) cost reduction actions in conjunction with the (normative) utility model characteristics:

$$U_{IA} = \sum_{X_l}\sum_{Y_{k,m}} P(X_l) \cdot (b(X_l) - P(Y_{k,m}) \cdot c(X_l, a_k) - c(a_k)) \tag{11}$$

$$U_{IA} = \sum_{X_l}\sum_{Y_{k,m}} P(X_l) \cdot (P(Y_{k,m}) \cdot b(X_l, a_k) - c(X_l) - c(a_k)) \tag{12}$$

$$U_{IA} = \sum_{X_l}\sum_{Y_{k,m}} P(X_l) \cdot (b(X_l) - c(X_l) - P(Y_{k,m}) \cdot c(a_k)) \tag{13}$$

3. PREDICTED ACTION DECISION ANALYSIS AND ACTION VALUE ANALYSIS

The objective function for a predicted action decision analysis (PA DA) can be written for system state and utility actions. For both formulations, the action is selected for maximising the expected value of the utility, i.e., for maximising the system performance.

$$U_{\mathrm{PA}} = \max_{a_k}\left(\sum_{Y_{k,m}}\sum_{X_l} u(X_l, a_k, Y_{k,m}) \cdot P(X_l\,(a_k, Y_{k,m})) - c(a_k)\right) \tag{14}$$

For the notation of the action types, an index U for a utility action and an index S for a system state action are introduced. The objective function for a predicted utility action ($\mathrm{PA_U}$) DA may be written with the action implementation uncertainty $P(Y_{k,m}$, which is assigned to the utility. The action cost is here considered as deterministic for simplicity.

$$U_{\mathrm{PA}} = \max_{a_k}\left(\sum_{Y_{k,m}}\sum_{Y_{X_l}} u(X_l, a_k) \cdot P(Y_{k,m}) \cdot P(X_l) - c(a_k)\right) \tag{15}$$

The objective function for a predicted system state action ($\mathrm{PA_U}$) DA is written analogous with the action dependent system state probability $P(X_l(a_k, Y_{k,m}))$ and without the action influence on utility:

Variable	Mean	CoV	Characteristic value	Distribution	Reference
Load S with a reference period of 1 year	1.0	0.2	0.98 Quantile	Gumbel	E.g., JCSS Probabilistic Model Code [21], [22] representative for live and traffic loads
Load model uncertainty M_S	1.0	0.1		Logn.	E.g., JCSS Probabilistic Model Code [21], Part 3.9 (2001): Model uncertainty for moments in frames, [22]
Resistance R	1.0	0.05	0.05 Quantile	Logn.	Generic, JCSS Probabilistic Model Code [21], [22]
Resistance model uncertainty M_R	1.0	0.05	-	Logn.	JCSS Probabilistic Model Code [21], Part 3.9 (2001): Model Uncertainties, [22]
Deterioration D	0.1	0.02		Logn.	Generic
Deterioration model uncertainty M_D	1.0	0.1		Norm.	Generic
Damage resistance transfer function t_D	$t_D(M_D \cdot D) = 1 - M_D \cdot D$				Generic
Safety factor γ_S	1.5 for one dominating action				Between 1.35 and 1.5, according to EN 1990 (2010)
Safety factor γ_R	1.0 for steel				Between 1.0 and 1.5 according to material specific Eurocodes
Consequence	$C_F \sim U(20.0,\ 40.0)$				[14]

Table 1. Probabilistic system performance models.

$$U_{\mathrm{PA}} = \max_{a_k}\left(\sum_{Y_{k,m}}\sum_{Y_{X_l}} u(X_l) \cdot P(X_l(a_k, Y_{k,m})) \cdot P(Y_{k,m}) - c(a_k)\right) \tag{16}$$

The system performance and objective functions for the introduced PA DAs are visualised with decision trees, generalised and summarised in Figure 1.

An action value can be quantified in analogy to an information value as the gain of an expected utility by a predicted action, V_{PA}, i.e., as the difference between the maximised expected utility of a predicted action DA (U_{PA}) and the system performance (U_{SP}):

$$V_{PA} = U_{PA} - U_{SP} \tag{17}$$

The action value may be normalised ($\overline{V}_{PA}$) to the system performance without an action implementation, i.e.:

$$\overline{V}_{PA} = (U_{PA} - U_{SP})\,/U_{SP} \tag{18}$$

Action type and uncertainty	Space		Description	Probabilistic model	Cost and consequence model
System state actions	**a**	a_0	Do nothing	–	–
		a_1	Strengthening of section	$1.2 \cdot \gamma$	2.0%
		a_2	Replacement with higher strength	$1.2 \cdot E[R_{mat}]$	2.0%
		a_3	Repair	$M_D \cdot D = 0$	1.0%
		a_4	Load reduction	$S/1.2$	0.5%
		a_5	Consequence reduction	–	0.3%

U: Uniform distribution, Tr: Triangular distribution

TABLE 2. Action type and cost models.

Action type and uncertainty	Space		Description	Probabilistic model
Implementation uncertainty	**Y**	Y_1	Strengthening performance	$P(Y_1) \sim Tr(0.95,\ 1.05,\ 1.10)$
		Y_2	Replacement performance	–
		Y_3	Repair performance	$P(Y_3) \sim Tr(0.9,\ 0.95,\ 1.0)$
		Y_4	Load reduction uncertainty	$P(Y_3) \sim N(1.0, 0.1)$
		Y_5	Cons. reduction uncertainty	$P(Y_1) \sim Tr(0.85,\ 0.95,\ 1.05)$

N: Normal distribution, Tr: Triangular distribution

TABLE 3. Action uncertainty models.

It should be noted that per definition, the action value analysis includes the cost of the action for consistency with the expected utility theorem.

4. EXEMPLARY STUDY

The predicted system state and utility action DA is studied for an existing and deteriorated structure, for which the optimal risk reduction action is to be found. The system performance model includes the limit state function (2) representative for a highly correlated structural series system subjected to direct and indirect system failure consequences (Section 4.1). The system state actions "section strengthening", "component replacement", "component repair" and "load reduction" as well as the utility action "consequence reduction" are probabilistically modelled and exemplified.

4.1. SYSTEM PERFORMANCE MODEL

The system state model is represented with Equation (2). For a semi-probabilistic safety concept, it can be shown that the section property can be decomposed to a central safety factor γ (see e.g., [23]), i.e., the product of the resistance and the loading safety factors (γ_R and γ_S, respectively), and the ratio r_k of the characteristic resistance and loading values (R_k and S_k, respectively). The influence of the damage on the resistance is modelled with a damage transfer function t_D:

$$Z = \gamma \cdot r_k \cdot t_D\left(M_D \cdot D\right) = \gamma_R \cdot \gamma_S \cdot \frac{S_k}{R_{mat,\,k}} \cdot t_D\left(M_D \cdot D\right) \tag{19}$$

The limit state function (2) becomes with focussing on cross section rather than material deterioration:

$$X_1 :\ g_{X_1} = M_R \cdot \gamma \cdot r_k \cdot t_D\left(M_D \cdot D\right) \cdot R_{mat} - M_S \cdot S \leq 0 \tag{20}$$

The probabilistic model and the references are summarised in Table 1.

4.2. ACTION MODEL

Following the introduction of the action type and uncertainty model in Section 2.2, a strengthening, a replacement, a repair, and a load reduction and a consequence reduction action are modelled (Table 2). The section strengthening leads to an increase of 20% of the section property, whereas the replacement with a higher strength material leads to an increase of the expected resistance by 20% with a constant variability. Both have a cost of 2.0%. The repair action sets the deterioration to 1.0 and costs 1.0%. The loading reduction action leads to decrease of the loading by 20% with the cost 0.5%. The consequence reduction sets the direct and indirect consequences in case failure to a Uniform distribution between 10.0 and 20.0 with a cost of 0.3%.

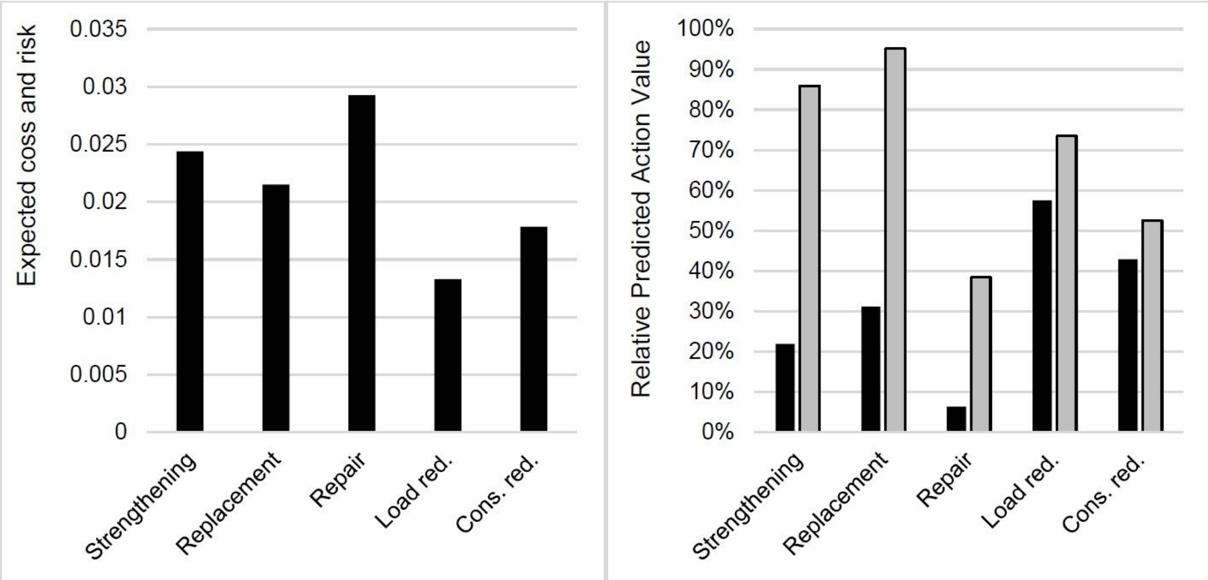

Figure 2. System performance normalised relative action values without the action costs (left side, grey) and with action costs (left side, black) and total expected action cost and risks (right).

The action implementation uncertainties are modelled continuous, i.e., with a Triangular distribution accounting for a known range and most probable value (Table 3). The strengthening action provides most probably a higher strengthening (1.05) than envisaged (Table 2) within a range between 0.95 and 1.10. The component replacement is subjected to negligible action implementation uncertainties. The repair performance is subjected to on site execution uncertainties with a mode of 0.95 and a range of 0.9 to 1.0 of repairing the deterioration. The load reduction uncertainty is Normal distributed and on average as expected and uncertain with a standard deviation of 0.1. The consequence reducing usage uncertainty is in the range of 0.85 to 1.05 with a mode of 0.95.

4.3. Predicted action and action value analysis

The results of the predicted action DA and the action value analysis are depicted in Figure 2. The load reduction action leads to the lowest expected costs and risks due to the comparably high load reduction including the uncertainties and the low action costs (Figure 2, left). It also provides the highest action value including its costs, followed by the also relatively cheap consequence reduction action (Figure 2, right, black). The load reduction action is identified as the optimal action in consistency with the expected utility theorem. The action values without their costs would be significantly higher for strengthening, replacement and repair (Figure 2, right, grey).

5. Information and integrity management value analysis

The value of predicted information V_{PI} and the value of information and actions are quantified as the gain of an expected and optimised utility by SHI or SHI and actions, respectively [1, 18]. The value of predicted information is then the difference between the expected and maximised utilities of a predicted information and action DA (U_{PIPA}) and a predicted action DA (U_{PA}):

$$V_{PI} = U_{PIPA} - U_{PA} \tag{21}$$

The difference between the expected and maximised utilities of a PIPA DA and the system performance (U_{PA}) is referred to as the value of the structural condition management encompassing information and actions V_{PIPA}:

$$V_{PIPA} = U_{PIPA} - U_{SP} \tag{22}$$

The decision values may be normalised by the subtrahend in analogy to Equation (18):

$$\overline{V}_{PIPA} = \left(U_{PIPA} - U_{SP}\right)/U_{SP} \tag{23}$$

It should also be noted that the information value can be calculated relative to system performance. In this case, the information value is divided by the expected system performance utility:

$$\overline{V}_{PI} = \left(U_{PIPA} - U_{PA}\right)/U_{SP} \tag{24}$$

5.1. Predicted information and action DA by threshold determination

A general form of a PIPA DA can be derived for utility and system state actions (building upon the formulations in Figure 1). For the SHI acquirement strategies with the information outcomes $Z_{i,j}$ and costs $c(i_i)$, the PIPA DA objective function in extensive form constitutes:

$$U_{PIPA} = \max_{i_i} E_{Z_{i,j}}\left[\max_{a_k} E_{Y_{k,m}}\left[E_{X_{l(a_k,Y_{k,m}|Z_{i,j}}}[u(X_l,a_k,Y_{k,m}]] - c(a_k)\right] - c(i_i) \tag{25}$$

In the following it will be shown that the above formulation (25) can be solved for continuous information outcomes by discretising to two complementary outcomes.

Continuous information outcomes are basically any measurement of a random variable contained in the system state function. Per definition they are referred to direct SHI . With consideration of the model uncertainty determination process in a multiplicative formulation [21], the model uncertainty can be understood as the measurable model outcome prediction normalised with the precise measurements of a structural reliability property. For a continuous model uncertainty outcome $M_{M,i}$ of information acquirement strategy with index i, Equation (25) is readily rewritten:

$$U_{PIPA} = \max_{i_i} E_{M_{M,i}}\left[\max_{a_k} E_{Y_{k,m}}\left[E_{X_{l(a_k,Y_{k,m}|m_{M,j}}}[u(X_l,a_k,Y_{k,m}]] - c(a_k)\right] - c(i_i) \tag{26}$$

The extensive form equation (26), can be rewritten with an obtained, i.e. posterior, information and predicted action decision analysis (OIPA DA) providing U_{OIPA}:

$$U_{PIPA} = \max_{i_i} E_{M_{M,i}}[U_{OIPA}(m_{M,i})] - c(i_i) \tag{27}$$

$$U_{OIPA}(M_{M,i}) = \max_{a_k} E_{Y_{k,m}}\left[E_{X_{l(a_k,Y_{k,m})|m_{M,i}}}[u(X_l,a_k,Y_{k,m})] + c(a_k) \tag{28}$$

The expectation operator in regard to the information can be written for continuous random variables:

$$U_{OIPA} = \max_{i_i} \int\limits_{M_{M,i}} U_{OIPA}(m_{M,i}) \cdot f_{M_{M,i}}(m_{M,i})\, dm_{M,i} - c(i_i) \tag{29}$$

The measurement threshold $t_{M,i,k}$ is now found as the action-wise, i.e., a_k dependent, expected posterior utility equality of "do nothing" and the predicted action, i.e.:

$$t_{M,i,k} : E_{X_l|t_{M,i,k}}[u(X_l)] = E_{Y_{k,m}}[E_{X_{l(a_k,Y_{k,m})|t_{M,i,k}}}[u(X_l,a_k,Y_{k,m}]] + c(a_k) \tag{30}$$

The integral in Equation (29) can then be split up into two sub-integrals and the expected utility maximisation can be performed outside the integrals:

$$U_{PIPA} = \max_{i_k,a_k}\begin{pmatrix} \int\limits_{-\infty}^{m_M=t_{M,i,k}} E_{X_l|m_M}[u(X_l)] \cdot f_{M_{M,i}}(m_{M,i})\, dm_{M,i} \\ + \int\limits_{m_M=t_{M,i,k}}^{\infty} \left[E_{Y_{k,m}}\left[E_{X_{l(a_k,Y_{k,m})|m_M}}[u(X_l,a_k,Y_{k,m}]] + c(a_k)\right] \cdot f_{M_{M,i}}(m_{M,i})\, dm_{M,i} \\ -c(i_i)\end{pmatrix} \tag{31}$$

The integration can be rewritten for the model uncertainty density functions providing:

$$U_{PIPA} = \max_{i_k,a_k}\begin{pmatrix} \int\limits_{-\infty}^{m_M=t_{M,i,k}} E_{X_l|m_M}[u(X_l)] \cdot dF_{M_{M,i}}(m_{M,i}) \\ + \int\limits_{m_M=t_{M,i,k}}^{\infty} \left[E_{Y_{k,m}}\left[E_{X_{l(a_k,Y_{k,m})|m_M}}[u(X_l,a_k,Y_{k,m}]] + c(a_k)\right] \cdot dF_{M_{M,i}}(m_{M,i}) \\ -c(i_i)\end{pmatrix} \tag{32}$$

The two complementary outcomes $Z_{i,1}$ and $Z_{i,2}$ for action $k = 1$ are then defined with Equations (33) and the integrals can be solved by calculation of the pre-posterior probability for direct information [12]:

$$\begin{aligned} Z_{i,1} : M\big|_{-\infty}^{t_{M,i,1}} \; P(Z_{i,1}) = F_{M_{M,i}}(t_{M,i,1}) \\ Z_{i,2} : M\big|_{t_{M,i,1}}^{\infty} \; P(Z_{i,2}) = 1 - F_{M_{M,i}}(t_{M,i,1}) \end{aligned} \tag{33}$$

It should be noted that there is no further assumptions nor probabilistic models about information outcomes required [24].

6. Exemplary study

The application of the threshold determination approach (Equations (30), (32) and (33)) lead for the in Section 4 introduced example to Equation (34): Given a realisation of the measurement, the threshold is to be found for which the optimal action changes to action implementation instead of do nothing. The measurement is subjected to a measurement uncertainty , which is Normal distributed with a mean of 1.0 and a coefficient of variation of 0.03. The information cost equals 0.001, e.g. according to [11].

$$t_{M,k} : P\big(X_1 \big| t_{M,k}, M_U\big) \cdot E[C_F] = P\big(X_1 \big| t_{M,k}, M_U, a_k, Y_{k,m}\big) \cdot E\big[C_F(a_k, Y_{k,m})\big] + c(a_k) \tag{34}$$

With the introduced limit state functions (see Section 4.1), the threshold dependent probabilities of the failure state are calculated for damage information (Equation (35)), loading information (Equation (36)) and resistance information (Equation (37)).

$$P\big(X_1, t_{M,D,k}, M_U\big) = P\big(M_R \cdot Z(t_{M,D,k} \cdot M_U \cdot D) \cdot R_{mat} - M_S \cdot S \le 0\big) \tag{35}$$

$$P\big(X_1, t_{M,R,k}, M_U\big) = P\big(M_R(t_{M,R,k} \cdot M_U) \cdot Z(D) \cdot R_{mat} - M_S \cdot S \le 0\big) \tag{36}$$

$$P\big(X_1, t_{M,S,k}, M_U\big) = P\big(M_R \cdot Z(D) \cdot R_{mat} - M_S(t_{M,S,k} \cdot M_U) \cdot S \le 0\big) \tag{37}$$

The solution of the first integral in Equation (31), i.e. the calculation of the pre-posterior probability, is readily performed for damage information with the to the interval $[0.0, t_{M,D,k}]$ truncated (index T) probability distribution of M_D [12]:

$$P\big(X_1, t_{M,D,k}, M_U\big) = P\Big(M_R \cdot Z\big(M_D\big[0.0, t_{M,D,k}\big]^T \cdot M_U \cdot D\big) \cdot R_{mat} - M_S \cdot S \le 0\Big) \tag{38}$$

The full solution of Equation (31) for damage, loading and resistance information is performed analogous.

The PIPA and value DA in threshold formulation (Equations (32) and (21) to (24)) result in the relative predicted action and information values for damage, load and resistance measurements (Figure 3, left side). The damage measurement has no value for damage repair nor for replacement due to the nature of the actions. For the actions "load reduction" and "consequence reduction", damage monitoring leads regardless of and dependent on the damage realisation to higher expected utility and is thus not predictable. The integrity management strategies with the highest values (38.0% to 63.1%) are load and resistance measurements for the replacement, load reduction system state actions and the "consequence reduction" utility action.

When considering separately the system performance related information values (Figure 3, right side), it is observed that there are only positive information values for (1) load and resistance measurements in conjunction with the replacement action and (2) for load measurements for the strengthening and repair actions. Only for the replacement action, measurement information contributes significantly to the condition management strategy value (see Figure 3 and Figure 2, right side).

The optimal condition management strategies are thus replacement with load measurement ($V_{PIPA} = 63.1\,\%$ of the system performance), replacement with resistance measurement ($V_{PIPA} = 58.1\,\%$of the system performance) and load reduction ($V_{PA} = 57.5\,\%$ of the system performance).

7. Summary and concluding remarks

This study builds upon structural condition assessment and provides a basis for the condition management by maximising the expected utility for information and actions before information acquirement and action implementation. The study is intended for progress towards consistent probabilistic, decision theoretical and built environment system relevant modelling of actions, information, expected utility quantification and value.

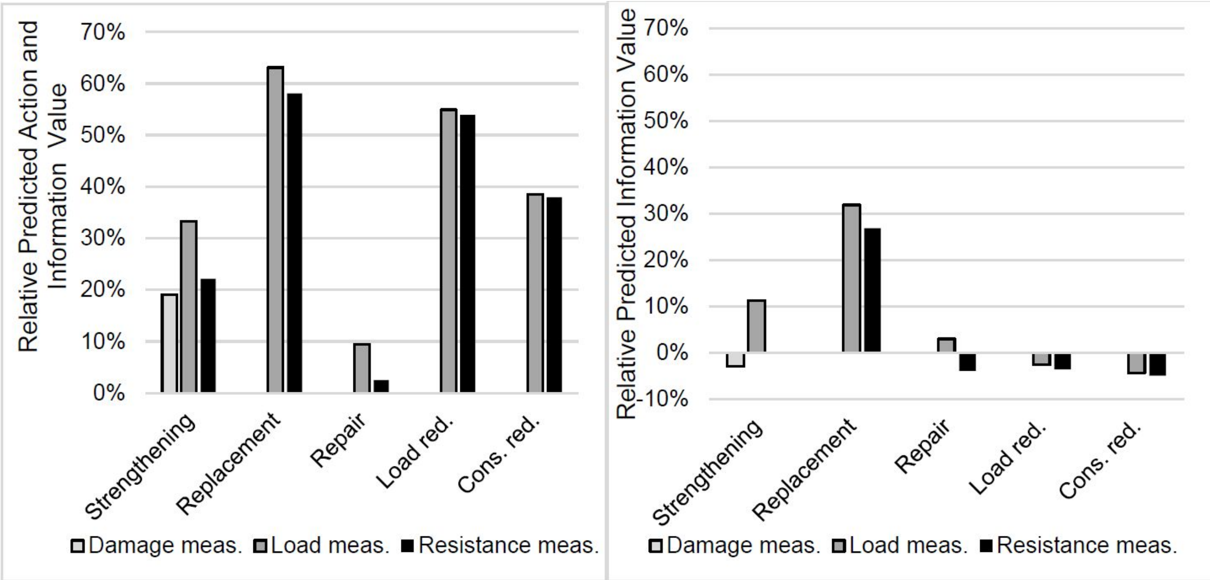

FIGURE 3. Relative predicted action and information values for damage, load and resistance measurements.

The study encompasses an introduction, probabilistic formulation, and exemplification (1) of system state and utility actions, (2) system state and utility action value analysis and (3) of a threshold formulation for predicted information and action decision analyses.

The introduced probabilistic action modelling facilitates to distinguish between system state and utility actions, i.e., between actions influencing the reliability of the structural components and actions influencing the utility model. The action uncertainty modelling includes the distinction of an implemented or predicted action, the action implementation states and the action implementation uncertainties. With an action value decision analysis as introduced, the optimal action with consideration of the action costs can be identified before action implementation in consistency with the expected utility theorem. This is demonstrated for the system state actions strengthening, replacement, repair, load reduction and the utility action: consequence reduction.

The threshold formulation for a predicted information and action decision analysis relies on the equality of the decision theoretical posterior action optimality condition, i.e., that the action is implemented according to the higher expected posterior utility. The threshold formulation is derived on the basis of direct SHI and the measurability of model uncertainties. The identification of the threshold facilitates the quantification of the SHI value for continuous measurement information and provides readily the decision rule, i.e., for which measurement outcomes the action implementation is optimal. The application of the information value threshold formulation and the action value analysis show exemplarily which structural integrity management strategies are optimal and how the action values and information values contribute to each strategy value.

Acknowledgements

The Kalix Bridge (TRV 2021/58089, EF 173793) and the CondtionValue (TRV / BBT 2021/51600) projects funded by Trafikverket, Sweden, are gratefully acknowledged.

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