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# RAILWAY TRACK DEFLECTION ANALYSIS BY USING EVOLUTIONARY ALGORITHMS

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Abstract. In contrast to numerical methods, analytical modelling of the railway track is one of the less time-consuming and computationally demanding methods which, in combination with the computing power commonly available today, can form an effective tool for analysing the behaviour of the railway track.

This paper deals with the use of iterative methods of evolutionary algorithms, together with analytical modelling, for the purpose of reverse analysis of the measured deflection caused by moving loads acting on the railway track. The theoretical assumptions of the analytical model used, the data collection methodology and the method used for the reverse analysis are presented. The results of the analysis are also presented.

Keywords: Railway track deflection, reverse analysis, evolutionary algorithms.

## 1. Introduction

The analytical model presented in [1] was used for the reverse analysis. The model consists of two infinitely long beams lying on the Pasternak foundation, which, in contrast with the Winkler foundation, considers the shear interaction of adjacent elements. Graphical model interpretation is shown in Figure 1.

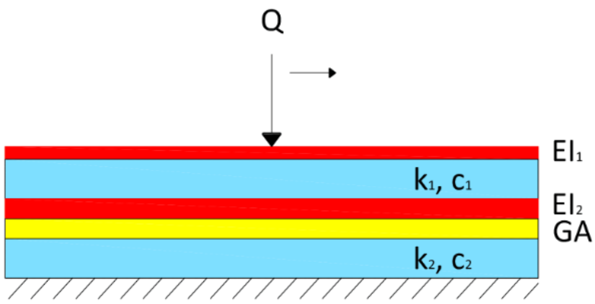


Figure 1. Schematic of the two-layer model on Pasternak foundation.

The model can be interpreted so that the first layer represents the rail; the second represents the sleeper with substructure. The influence of dynamic variables is taken into account in the same way as in the Frýba model [2].

The model can be described by the following differential equations:

$$EI_1 \frac{d^4 w_1(x,t)}{dx^4} + m_1 \frac{d^2 w_1(x,t)}{dt^2} + c_1 \frac{dw_1(x,t)}{dt} + k_1[w_1(x,t) - w_2(x,t)] = 0, \quad (1)$$

$$EI_2 \frac{d^4 w_2(x,t)}{dx^4} - GA \frac{dw_2(x,t)^2}{dx^2} + m_2 \frac{d^2 w_2(x,t)}{dt^2} + c_2 \frac{dw_2(x,t)}{dt} + (k_1 + k_2) w_2(x,t) - k_1 w_1(x,t) = 0, \quad (2)$$

where $EI_1$ is upper beam bending stiffness, $m_1$ is the mass of one meter of the upper beam. According to the paragraph above, the upper beam represents the rail, but the values of $EI_1$ and $m_1$ do not necessarily correspond to the mechanical parameters of the rail itself due to the interaction of the rail with other elements of the railway track. The parameters $k_1$ and $c_1$ are parameters that act between the first and second layers of the structure and can be interpreted as properties of the fastening system. The bottom layer of the model corresponds to the sleepers and partly to the substructure with parameters $EI_2$ - bending stiffness of the second layer, $m_2$ - mass of one meter of the second layer, $GA$ - shear stiffness of the bottom layer. Sleepers are not capable of transferring bending loads in the longitudinal direction of the track, and therefore the bending stiffness $EI_2$ can be understood as the residual ability of the sleeper layer to transfer (when the axle passes) the bending load by horizontal action on the trackbed. This stiffness, therefore, takes on a non-zero but negligible value. The parameters $k_2$ and $c_2$ represent the properties of the substructure.

It is necessary to introduce the relative coordinate $s$ to solve the above differential equations. The effect of the wheel load $Q$ is introduced during the solution through boundary conditions.

By substituting the parameters $EI_1 = 4500\ kNm^2$, $EI_2 = 0.1\ kNm^2$, $GA = 6000\ kN$, $k_1 = 250000\ kNm^{-2}$, $k_2 = 40000\ kNm-2$, $v = 30\ ms-1$, $c_1 = 90\ kNsm-2$, $c_2 = 120\ kNsm-2$, $m_1 = 60\ kg$, $m_2 = 300\ kg$, $Q = 100\ kN$ into the solution matrix, we obtain the deflection curve for the upper layer $w_1$ and the deflection curve for the bottom layer $w_2$.

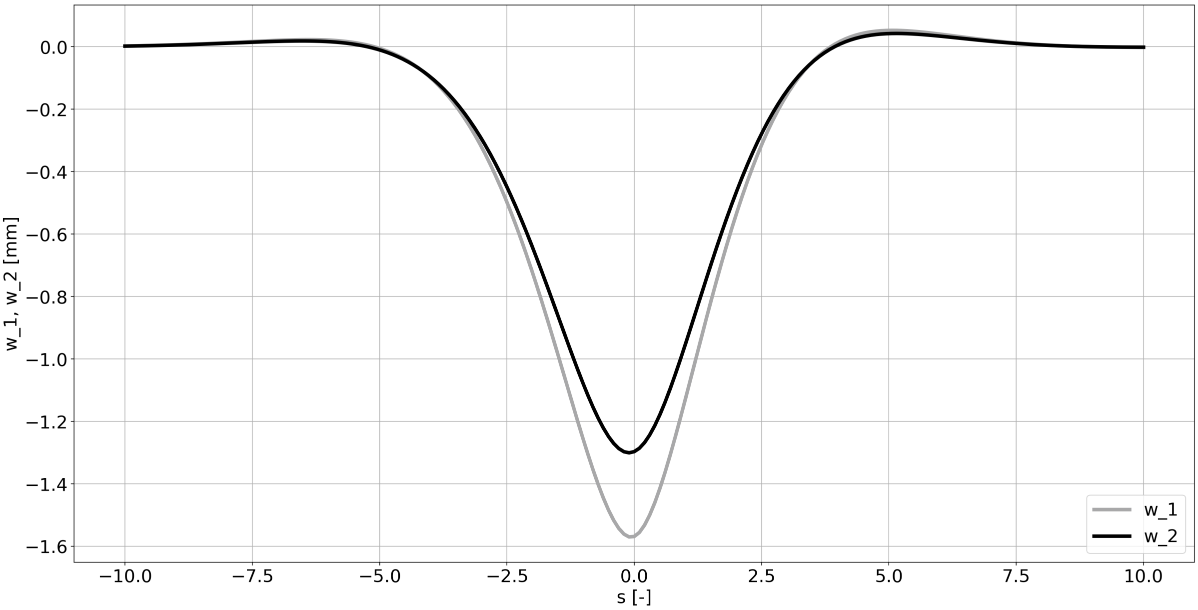


FIGURE 2. Output of the two-layer model.

The horizontal axis of both deflection curves is in the dimensionless relative coordinate $s$.

## 2. METHODS

### 2.1. TRACK VERTICAL DISPLACEMENT MEASURING METHOD

The input data for the reverse analysis are the track vertical displacements measured in-situ. Vertical displacement of railway track is measured relatively to a plain of substructure. The displacement sensors were positioned in the monitored section in such a way that it was possible to detect the pushing of the rails and sleepers. The rail foots and the sleepers in four points along longitudinal axis were monitored. The sensors were positioned as follows:

- S0 – right sleeper head;
- S1, S2 – right rail;
- S3, S4 – sleeper in thirds;
- S5, S6 – left rail;
- S7 – left sleeper head.

Using this sensor set, it is possible to obtain the deflection value of the rail unloaded by the twisting of the cross-section. The rail deflection is easily calculated as the average of the values measured by the pair of sensors S1, S2 and S5, S6. Using sensors placed on the sleeper, the sleeper deflection curve can be calculated for each measured moment using an interpolation polynomial (cubic spline). It is also possible to calculate the deflection value of the sleeper under the rail at a point that is otherwise inaccessible for sensor placement.

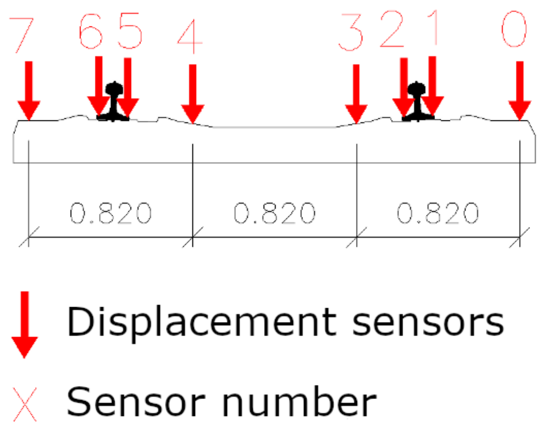


FIGURE 3. Schematic of sensor placement [3].

### 2.2. EVOLUTIONARY ALGORITHM METHOD

Reverse analysis of measured data can be classified as an optimization problem. The quantities to be optimized are the parameters of the analytical model. When solving the problem, we look for a combination of model parameters that leads to the best possible solution. The quality of the solution is assessed by the so-called fitness function, which is a mathematical expression of the similarity of the measured data and the output of the analytical model for the given parameters. In general, the higher the quality of the solution, the higher the value of the fitness function.

For reverse analysis, algorithms have been developed that use as input a set of random combinations of parameters of the analytical model, successively test each set of parameters, evaluate their quality and from the best combinations create (by precisely defined methods) combinations of new parameters, which test repeatedly. This method of evaluation

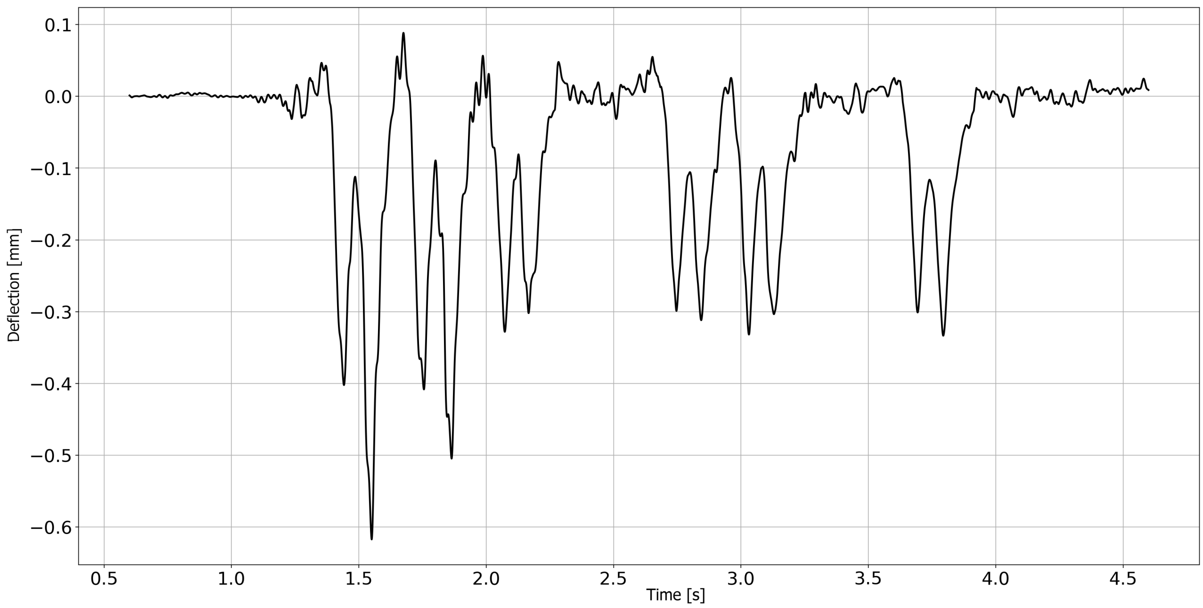


Figure 4. Measured in-situ deflection

leads to better quality solutions with each subsequent iteration. The number of iterations and the number of elements of the parameter set are the input parameters of the algorithm. The optimisation is completed when the required number of iterations is reached.

## 3. Model results

The optimization algorithms were tested on data measured in Planá nad Lužnicí, km 74,978 - section without under sleeper pads. Track superstructure consists of 60 E1 rails, W14 fastening system and B 91S/1 sleepers. Assessed data trace is measured during the passage of the passenger train. Data traces from sensors S2 and S6 were selected for further processing, and their values were averaged at the corresponding time points. The resulting data trace was used for comparison with the deflection $w_1$ of the analytical model. All axles of the passenger train were considered.

The analytical model presented above gives the deflection for only one axle. To obtain a total deflection line for multiple axles is necessary to introduce the superposition principle for several deflection lines. Figure 4 shows that the axles exert different forces on the superstructure, which is also taken to account during superposition calculation.

The displacement line obtained by the presented analytical model captures the main characteristics of the measured signal, such as the travel wave in front of the first axle, the travel waves between axles and the steepness of the drop of the track under the passing axle. Parameters of the analytical model are shown in Table 1. The track critical speed is also calculated.

## 4. Conclusion

This article discussed reverse analysis theoretical presumptions, data collection methodology, data processing methodology. The algorithm output is also compared with in-situ measured data. From the text above it can be concluded that the presented analytical model can be used for further analyses of the rail track dynamic behaviour. Despite the convincing qualitative results, the evolutionary algorithms, because of its stochastic character, can lead to misleading quantitative outputs. Examination of the applicability of evolutionary algorithms for the purposes of reverse analysis, or the incorporation of other methods, will be the subject of the author's further work.

### List of symbols

$EI_1$ Upper layer bending stifness $[\mathrm{N\,m^2}]$
$EI_2$ Bottom layer bending stifness $[\mathrm{N\,m^2}]$
$GA$ Bottom layer shear stifness $[\mathrm{N}]$
$k_1$ Fastening system spring stifness $[\mathrm{N\,m^{-2}}]$
$k_2$ Foundation spring stifness $[\mathrm{N\,m^{-2}}]$
$c_1$ Fastening system damping $[\mathrm{Ns\,m^{-2}}]$
$c_2$ Foundation damping $[\mathrm{Ns\,m^{-2}}]$
$m_1$ Upper layer mass $[\mathrm{kg\,m^{-1}}]$
$m_2$ Foundation layer mass $[\mathrm{kg\,m^{-1}}]$
$Q$ Wheel force $[\mathrm{N}]$
$v$ Vehicle speed $[\mathrm{km\,h^{-1}}]$
$v_{cr}$ Critical speed $[\mathrm{km\,h^{-1}}]$
$x$ Longitudinal coordinate $[\mathrm{m}]$
$t$ Time $[\mathrm{s}]$
$s$ Relative coordinate $[-]$
$w_1$ Rail layer deflection $[\mathrm{m}]$
$w_2$ Sleeper layer deflection $[\mathrm{m}]$

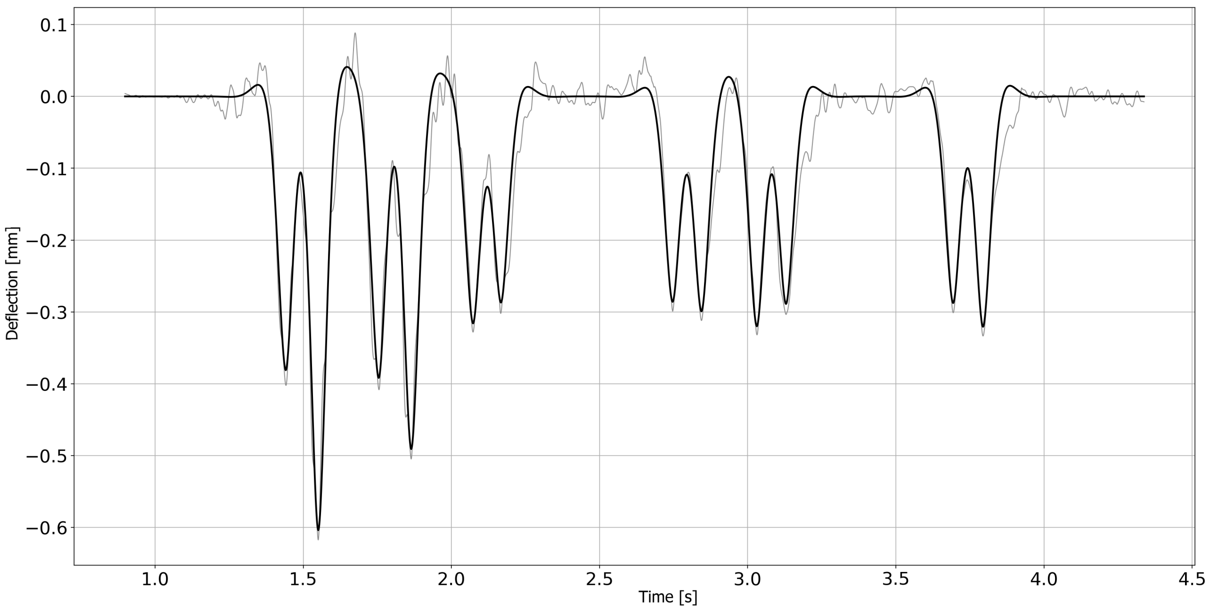


FIGURE 5. Analytical model comparison with measured data

| $EI_1$ $[kN\,m^2]$ | $EI_2$ $[kN\,m^2]$ | $GA$ $[kN]$ | $k_1$ $[kN\,m^{-2}]$ | $k_2$ $[kN\,m^{-2}]$ | $c_1$ $[kNs\,m^{-2}]$ | $c_2$ $[kNs\,m^{-2}]$ |
|---|---|---|---|---|---|---|
| 5428 | 0,739 | 549 | 403287 | 66903 | 17 | 190 |

| $m_1$ $[kg\,m^{-1}]$ | $m_2$ $[kg\,m^{-1}]$ | $v$ $[km\,h^{-1}]$ | $v_{cr}$ $[km\,h^{-1}]$ |
|---|---|---|---|
| 292 | 515 | 90 | 1018 |

TABLE 1. Analytical model parameters


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