# Expanding the Applicability of Press-Brake-Formed Tub Girders Through the Extension of the Maximum Span Length and the Evaluation of Pier Continuity 

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# EXPANDING THE APPLICABILITY OF PRESS-BRAKE-FORMED TUB GIRDERS THROUGH THE EXTENSION OF THE MAXIMUM SPAN LENGTH AND THE EVALUATION OF PIER CONTINUITY 

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Dissertation submitted to the<br>Benjamin M. Statler College of Engineering and Mineral Resources<br>at<br>West Virginia University in partial fulfillment of the requirements for the degree of<br>Doctor of Philosophy<br>in<br>Civil and Environmental Engineering

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# ABSTRACT <br> EXPANDING THE APPLICABILITY OF PRESS-BRAKE-FORMED TUB GIRDERS THROUGH THE EXTENSION OF THE MAXIMUM SPAN LENGTH AND THE EVALUATION OF PIER CONTINUITY 

Robert M. Tennant

The Short Span Steel Bridge Alliance (SSSBA) is a group of bridge and buried soil steel structure industry leaders who provide educational information on the design and construction of short-span steel bridges in installations up to 140 feet in length. Within the SSSBA technical working group, a modular, shallow press-brake-formed tub girder (PBFTG) was developed to address the demand in the short-span steel bridge market for rapid infrastructure replacement solutions. PBFTGs consist of modular, shallow, trapezoidal boxes fabricated from cold-bent structural steel plate. A concrete deck, or other deck option, may be placed on the girder, and the modular unit can be shipped by truck to the bridge site.

PBFTGs perform exceptionally well in simply supported, right, straight bridges utilizing current American Association of State and Highway Transportation Officials Load Resistance and Factor Design Bridge Design Specifications' (AASHTO LRFD BDS) Live Load Distribution Factors (LLDFs). The specifications limit the use of PBFTGs outside of these scenarios, despite the expectation they would perform well in a variety of other situations. More research and data are necessary to validate the current limitations in the AASHTO LRFD BDS and increase the applicability of PBFTGs into continuous spans and skewed bridges.

The scope of this project was to expand the applicability and usability of the PBFTG system. This was performed in several stages. First, a complete understanding and background of PBFTGs, LLDFs, box-girder capacity determinations, link slabs, and the AASHTO LRFD BDS was provided. This understanding and background of the restrictions placed on PBFTGs provided insight when developing the methodologies to overcome these restrictions. Next, analytical modeling techniques were developed and refined utilizing complicated geometry and nonlinear finite element methods. These modeling techniques were benchmarked against numerous historical laboratory tests and live load field tests of in-service PBFTG bridges. Then, the analytical tools were employed in sensitivity and parametric studies on PBFTG bridge models, resulting in proposed simplified empirical LLDFs, which better predict live load distribution than those equations present in the AASHTO LRFD BDS. These tools were also used to assess the effect of bearing line skew on the capacity of PBFTGs. Finally, life cycle laboratory fatigue testing was performed on two PBFTGs joined by a full-scale link slab to assess the applicability of the joint in continuous PBFTG bridges. Results of this project demonstrate the use of PBFTGs can be expanded into continuous spans using link slabs and more accurate LLDFs may be used to increase the economic viability of the system in the short-span bridge market. In addition, the analytical tools developed during this study relating to the capacity of skewed PBFTGs will serve as the basis for future research in this field.

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## CHAPTER 1: INTRODUCTION

### 1.1 Background / Overview

The Short Span Steel Bridge Alliance (SSSBA) is a group of bridge and buried soil steel structure industry leaders who provide educational information on the design and construction of short-span steel bridges in installations up to 140 feet in length. Within the SSSBA technical working group, a modular shallow press-brake-formed tub girder (PBFTG) was developed to address the demand in the short-span steel bridge market for rapid infrastructure replacement solutions. PBFTGs consist of modular, shallow, trapezoidal boxes fabricated from cold-bent structural steel plate, as seen in Figure 1.1. A concrete deck, or other deck option, may be placed on the girder, and the modular unit can be shipped by truck to the bridge site (SSSBA, 2022).


Figure 1.1: PBFTG after Cold Bending (SSSBA, 2021)

PBFTGs have proven economically and structurally competitive through multiple laboratory experiments and field demonstrations across the country. The American Association of State Highway and Transportation Officials (AASHTO) Innovation Initiative selected the PBFTG bridge system as a 2021 Focus Technology and has invested time and resources to encourage the
adoption of the system across the nation. While this system is exceptionally efficient and economical, its applicability is limited by a lack of research in continuous spans and the AASHTO Load Resistance and Factor Bridge Design Specifications (LRFD BDS), hereafter referred to as the AASHTO LRFD BDS.

### 1.2 Project Scope \& Objectives

The scope of this project is to explore the use of PBFTGs in a broader range of applications. Specifically, link slabs are explored with modular PBFTGs in continuous span scenarios, and extensive analytical modeling will be performed to assess the validity of the restrictions placed on box section flexural members as they relate to PBFTGs. These objectives will be achieved through the following:

- Reviewing literature relating to PBFTGs, link slab details, live load distribution, and the effect of compactness on the flexural capacity of sections.
- Developing analytical tools to assess the behavior and capacity of PBFTGs with varying dimensions and properties.
- Conducting behavioral and parametric studies to assess which parameters affect the computation of live load distribution factors for PBFTGs.
- Conducting behavioral studies to assess the effect of skew on the ultimate capacity of PBFTGs.
- Performing flexural testing on modular units transversely joined by a link slab.


### 1.3 Dissertation Organization

A brief overview of the organization of this dissertation is as follows:

- Chapter 2
- This chapter summarizes previous research performed on cold-bent tub girder applications. Specific attention is given to research performed on PBFTGs in laboratory and field settings.
- Chapter 3
- This chapter summarizes the development of current live load distribution factors found in the AASHTO LRFD BDS and the influence of global bridge parameters affecting live load distribution.
- Chapter 4
- This chapter summarizes the determination of compactness of crosssections and elements. Additionally, a brief review of the effects of skew on box-girders is provided.
- Chapter 5
- This chapter summarizes previous laboratory, field, and analytical research performed on link slabs.
- Chapter 6
- This chapter provides a comprehensive review of the AASHTO LRFD BDS relating to PBFTG bridges.
- Chapter 7
- This chapter describes the analytical techniques developed using a commercial finite element software package. The analytical techniques were verified against previous experimental tests.
- Chapter 8
- This chapter documents the behavioral studies performed on the PBFTG bridge system to determine the factors affecting live load distribution. These studies were used to develop live load distribution factors benchmarked against previous live load field tests.
- Chapter 9
- This chapter documents the behavioral study performed on the PBFTG composite units to determine the effect of skew on the flexural capacity of the system.
- Chapter 10
- This chapter discusses the research methods and results obtained from experimental testing. A detailed explanation of the experiment setup, loading, and results is provided.
- Chapter 11
- This chapter provides a summary of the project and highlights key findings. Recommendations for future work and continued research to expand the applicability of the systems are also provided.
- Appendix A
- This appendix documents the results of the live load distribution sensitivity study analysis.
- Appendix B
- This appendix documents the results of the live load distribution parametric study analysis.
- Appendix C
- This appendix documents the results of the compactness sensitivity analysis.
- Appendix D
- This appendix documents the determination of the Fatigue I moment and cycle count used in the experimental testing.
- Appendix E
- This appendix documents the experimental testing data.


# CHAPTER 2: PRESS-BRAKE-FORMED TUB GIRDER LITERATURE REVIEW 

### 2.1 Introduction

This chapter presents a discussion on the previous findings regarding PBFTGs. A review of literature pertaining to other forms of cold-bent tub girders is provided, with an emphasis on research on tub girders in negative bending regions and bracing requirements for typical welded tub girder sections. A comprehensive review of the research practicum undertook by researchers at West Virginia University (WVU) and Marshall University, relating to the design of PBFTGs and their implementation, is presented.

### 2.2 Previous Applications of Cold-Bent Steel Tub Girders

Prefabricated steel tub girder systems have been studied for decades but have recently been extensively used in the short-span bridge market. As accelerated bridge construction (ABC) has become more popular in bridge design and construction, PBFTGs have been shown to be economical and competitive in spans up to 60 feet. Several researchers and organizations have conducted studies on the use of systems employing various types of cold-bent steel tub girders.

### 2.2.1 Prefabricated press-formed steel T-box girder bridge system (Taly \& Gangarao, 1979)

Taly and Gangarao (1979) proposed designs for several prefabricated superstructural systems for short-span highway bridges. One design consisted of a cold-formed steel T-box girder. The stem of the trapezoidal girder was cold formed from 3/8 inch thick A-36 steel plate shop welded to $3 / 8$ inch thick steel top flanges (Figure 2.1). Proposed girder sections consisted of either 6 foot or 8 foot top flange widths with a total depth of 2.5 feet, 3 feet, or 3.5 feet, based on the total bridge width and span. A feasibility assessment demonstrated the system to be suitable in spans of up to 65 feet.


Figure 2.1: T-Box-Girder System, Typical Girder Section with 3/8 inch Steel Plate Deck (Taly \& Gangarao, 1979)

Major advantages of this system include high strength-to-weight ratios and improved fabricability. The dead weight of the steel deck superstructure is significantly reduced compared with conventional concrete decks. Due to the closed section, a higher torsional stiffness is achieved, leading to lower live load distribution factors (LLDFs) and higher efficiency in horizontally curved structures. As the bridge system is nearly completely prefabricated, a betterquality product can be achieved. Improved quality is also achieved by the simplicity in the system, specifically the cold forming of the sections, as this reduces the amount of welding required.

A composite prefabricated cold-formed box-girder system was proposed as a modification to the all steel system. A 5 inch thick precast, prestressed concrete deck was to be utilized instead of a $3 / 8$ inch thick steel plate (Figure 2.2). Composite action is developed through shear studs welded to the top flanges of the cold-bent section.


Figure 2.2: T-Box-Girder System, Typical Girder Section with 5 inch Precast, Prestressed Concrete Slab (Taly \& Gangarao, 1979)

At the time, the AASHTO Standard Specifications did not provide design criteria for members using press-brake or composite box-girders. Therefore, the researchers evaluated their designs against the 1977 American Iron and Steel Institute specifications. The bends at the junction of the bottom flange and webs were found to provide inadequate crippling resistance over the bearings. To combat this, the designers provided a 5 inch by $1 / 2$ inch stiffener along the bottom flange and a $3 / 8$ inch thick steel plated diaphragm between the flanges and webs.

### 2.2.2 Composite girders with formed steel U-sections (Nakamura, 2002)

Nakamura (2002) proposed a continuous composite bridge system composed of cold-bent steel U -sections and a concrete deck. This bridge system, illustrated in Figure 2.3, acts compositely in the span centers where the positive bending moment is critical. As the top flange is composite with the concrete deck, buckling of the flange is restricted. However, at intermediate supports, where negative bending occurs, the bottom flange is vulnerable against buckling. To overcome
this susceptibility, the U-Section was filled with concrete and the deck was prestressed utilizing prestressed steel bars. Typical prestressed concrete beams require extensive formwork for casting. However, the steel U -section works as much of the formwork itself, significantly reducing the time and cost associated with fabricating the formwork. While extra concrete in the negative bending regions increases the reaction forces at the intermediate supports, it does not vastly increase the bending moment as the concrete is filled only near the supports.


Figure 2.3: Composite Girders with Steel U-Section (Nakamura, 2002)

Nakamura conducted bending tests on three different cross-sections, one of which was the system in negative bending filled with concrete and prestressed. For this cross-section, the prestress forces initially kept the concrete slab in compression, but when the concrete slab entered tension, it quickly lost strength. The filled concrete performed well until the prestressed steel bars became fully plastic and the bottom flange buckled. The researcher found the calculated maximum load, from the Bernouli-Beam equation, in negative bending was $9 \%$ lower than the measured value, likely due to the confined nature of the filled concrete and the neglection of the tensile strength of the concrete slab.


Figure 2.4: Negative Bending Test Specimen before Concrete Pour (Nakamura, 2002)

### 2.2.3 Texas Prefabricated Steel Tub-Girder System

The Texas Department of Transportation (TxDOT) explored shallow steel bridge tub girders for the FM 3267 bridge on I-35 (Chandar et al., 2010). One of the project goals was to provide a rapidly constructible and cost-efficient structure. Historically, larger steel tub girders were commonly used for horizontally curved bridge applications. Box-girders were used in this application due to their high torsional stiffness compared to wide flange sections. However, uses in straight girder applications had been limited due to economic efficiency and fabrication/handling issues.

For the FM 3267 bridge, one of the design challenges was to minimize the structural depth. The structural efficiency of trapezoidal girders allowed the designers to proportion the girders to meet the structural depth limitation. The bridge was constructed utilizing six rows of four 36 inch deep steel box-girders with a cast-in-place concrete deck poured in two stages joined by a longitudinal joint above one of the flanges (Figure 2.5). The girder sections were standardized, which saved design, fabrication, and erection costs.


Figure 2.5: TxDOT Trapezoidal Box-Girder Cross-Section (Chandar et al., 2010)

Despite the structural advantages of the system and the use of ABC in the design, current practices for welded trapezoidal box-girders often include aspects which may lead to unnecessary fabrication costs and structural inefficiencies (Armijos-Moya et al., 2019). TxDOT sponsored research projects to increase the efficiency of steel tub girders by developing and modifying superstructure design details. Many detailing practices used for steel tub girders are based in traditional practices and require unnecessary bracing. Extensive amounts of internal bracing, both in the vertical plane and the longitudinal plane, were typically required as the noncomposite system does not have the high torsional resistance seen in the composite system (Figure 2.6).


Figure 2.6: Top Lateral Bracing System (Armijos-Moya et al., 2019)

From parametric finite element analysis (FEA), researchers found removing $50 \%$ of the truss diagonals guarantees adequate torsional stiffness during construction of straight tub girder bridges. For straight tub girders, vertical K-frames could be placed at every third panel point with no loss to torsional stiffness. As found through experimental testing and FEA, offsetting the top flanges toward the inside of the tub girder allowed for simplified connections between top lateral truss elements and the top flanges of the girder. The researchers also investigated the use of shallower web slopes. The use of a lower web slope, such as $2.5: 1$, can increase the tributary width of the girder, potentially eliminating a girder line, but makes the section prone to global instability.

### 2.2.4 Con-Struct Pre-fabricated Steel Tub Girder System

The Con-Struct Prefabricated Bridge System was established in 2004 as an answer to the growing demand for ABC products (Valmont, 2022). The Con-Struct system is composed of hotdip galvanized PBFTGs. The noncomposite PBFTGs are made composite with a concrete deck
before being shipped to the bridge site (Figure 2.7). Once on site, the system is placed on the abutments and a longitudinal joint is poured between units. Designs of this system are valid up to 70 feet in length with skews up to $45^{\circ}$ and girder spacings of 7 feet.


Figure 2.7: Standard Class Composite Bridge System (Valmont, 2022)

In 2017, the St. Clair County Road Commission replaced two severely deteriorated steel bridges with new PBFTG bridges. The two bridges consisted of a 25 foot span bridge with no skew and a 35 foot span bridge with a $45^{\circ}$ skew. Each bridge consisted of hot-rolled W18 sections and a concrete deck for a total depth of approximately 2.5 feet. Due to the need for ABC, the existing abutments needed to be rehabilitated instead of replaced, requiring the new superstructure to match the existing superstructure's depth. The Con-Struct system was chosen for its light weight, low depth, and speed of construction. The composite modules were shipped to the bridge site, placed by crane and excavator, as seen in Figure 2.8, and a 6 inch wide high performance concrete deck joint was cast between the modules. The construction of both bridges was completed in 10 days.


Figure 2.8: Superstructure Placement of Composite Modules (SSSBA, 2021)

### 2.3 Previous Laboratory Testing of PBFTGs

The research practicum on the development and long-term performance of PBFTGs at WVU began in 2011. Researchers at WVU collaborated with the SSSBA to develop PBFTGs as an economical steel alternative in the short-span bridge market. This section details the experimental and analytical work performed at WVU in this area.

### 2.3.1 Development and Feasibility Assessment of Shallow Press-Brake-Formed Steel Tub

Girders for Short-Span Bridge Applications (Michaelson, 2014)
The SSSBA developed a modular PBFTG, as seen in Figure 2.9, as an alternative to adjacent concrete box beams in short-span bridge installations up to 140 feet in length. The trapezoidal steel shape is fabricated from standard plate sizes and cold-bent using a large capacity press-brake. After the steel is bent into the desired shape, shear studs are welded to the top flange,
and a reinforced concrete deck is compositely cast in a fabrication shop. Then, the composite PBFTG system can be shipped to the bridge site where the modular units will be joined with longitudinal closure pours.


Figure 2.9: Modular Press-Brake-Formed Tub Girder (Michaelson, 2014)

Michaelson (2014) first developed a program to generate section properties of any configuration of PBFTG. Certain geometrical properties were held constant in the calculation: The slope of the webs were held at a constant 1:4 ratio, the inside bend radii of the bends were held to five times the thickness, the top flanges were kept at a constant 6 inches, the concrete deck dimensions were kept at 7.5 feet wide by 8 inches thick, the concrete's compressive strength was held at a constant 4 ksi , and the yield stress of the steel was held at 50 ksi . Three different standard plate thicknesses were considered: $7 / 16,1 / 2$, and $5 / 8$ inch. Six different standard plate widths were evaluated: $60,72,84,96,108$, and 120 inch. An optimum depth was chosen for each plate width corresponding to the maximum yield moment.

Following the design of the system, physical flexural testing was performed on two separate composite PBFTGs and two separate noncomposite PBFTGs. The dimensions of the steel
specimens were uniform across all testing. Each specimen was constructed from 84 inch wide by 7/16 inch thick by 35 foot long plate. This plate was chosen because the composite PBFTGs formed from this plate were the largest the 330-kip servo-hydraulic actuator could test to ultimate failure. The deck thickness was shortened from 8 inches to 6 inches to ensure ultimate failure could be reached. Each girder was subjected to three-point bending.

The composite modules were loaded to failure at approximately 300 kip and a midspan vertical deflection of 3.1 inches. The result was crushing of the concrete deck and loss of composite action (Figure 2.10). The noncomposite girders were loaded until the girders exhibited excessive lateral deflection and twist under relatively small loads, 90 kip and 30 kip (Figure 2.11).


Figure 2.10: Typical Failure Mode for Composite Specimens (Michaelson, 2014)


Figure 2.11: Typical Failure Mode for Noncomposite Specimens (Michaelson, 2014)

In addition to laboratory testing, Michaelson developed a three-dimensional nonlinear finite element modeling procedure to capture the behavior and ultimate capacity of composite and noncomposite PBFTGs. Michaelson showed the analysis accurately captured the behavior of the composite system through ultimate failure (Figure 2.12). This was used to assess the applicability of the AASHTO LRFD BDS for the system. The AASHTO LRFD BDS were found to be slightly conservative in computing the nominal capacity of the system and an improved expression was proposed. The noncomposite stability of the system was also assessed. The system was susceptible to lateral torsional buckling under low load levels. However, the installation of stay-in-place (SIP) metal formwork prior to girder erection was found to increase the torsional stiffness and reduce the severity of this issue.


Figure 2.12: Comparison of Experimental and Analytical Results of the Composite Specimens (Michaelson, 2014)

Michaelson assessed the validity and competitiveness of the system in the short-span bridge market. A feasibility assessment was first performed to determine the maximum span length for each plate length and thickness at the Strength I limit state and Service II limit state and to check the live load deflection. A variety of plate widths and thicknesses were reduced to a handful of sections for mainstream use. The PBFTG system composed of 120 inch wide by $5 / 8$ inch thick plate was the largest proposed system with an applicable span length of 80 feet. However, due to the current limit of large capacity press-brakes, a single PBFTG can only be fabricated up to 60 feet in length.

### 2.3.2 Experimental Evaluation of Noncomposite Shallow Press-Brake-Formed Steel Tub

## Girders (Kelly, 2014)

In conjunction with Michaelson, Kelly (2014) explored the stability and torsional behavior of noncomposite PBFTGs. The proposed PBFTG system consisted of a pre-cast concrete deck, but the option of a cast-in-place deck was explored. The critical stage of this construction method occurs while the concrete is being poured, as the noncomposite system must resist the construction loads. In this stage, the top flanges of the girder are in compression and are susceptible to torsional
buckling. To assess this stage of construction, flexural testing was performed similarly to Michaelson (2014), with load applied through a WT section bolted to the top flange at girder midspan. Prior to testing, an initial twist was noticed in one of the specimens (Figure 2.13). Measurements taken at tenth points along the span measured the flange and web inclinations. Due to these initial imperfections, Kelly calculated the first-order lateral buckling capacity of PBFTG sections and found the critical load at mid-span was 92.3 kip.


Figure 2.13: Initial Twist of Specimen \#2 (Kelly, 2014)

The first experiment consisted of an uncoated specimen and the load deflection curve was linear up to a load of approximately 94 kip, corresponding to 2.25 inches of vertical deflection at midspan (Figure 2.14). At this point, the girder suddenly failed under lateral torsional buckling and the experiment was subsequently terminated due to large lateral deflections. The second experiment consisted of a galvanized specimen, and the flexural testing was similar to the first experiment, until it was terminated due to large lateral deflections under a load of approximately 33 kip. The loss of capacity was attributed to the second-order effects relating to the initial measured imperfections. The lateral torsional buckling failure mode is illustrated in Figure 2.15.


Figure 2.14: Load-Deflection Data from Experiment \#1 (Kelly, 2014)


Figure 2.15: Lateral Torsional Buckling (Kelly, 2014)

Kelly (2014) developed modeling techniques to verify the noncomposite laboratory testing. Her focus on initial imperfections, specifically initial out-of-flatness of the web, initial tilt of the
compression flange, and initial lateral sweep of the compression flange, were used to replicate the initial state of the PBFTGs. The results of the laboratory testing and FEA modeling showed the modeling techniques accurately captured the behavior of the system. The model also adequately captured the lateral torsional buckling failure mode similar to the experimental tests.

### 2.3.3 Evaluation of Modular Press-Brake-Formed Tub Girders with UHPC Joints

 (Kozhokin, 2016)Kozhokin (2016) evaluated the performance of a longitudinal joint consisting of an ultrahigh performance concrete (UHPC) pour between two modular PBFTGs. UHPC is a cementitious material containing Portland cement, silica fume, quartz flour, fine silica sand, high-range water reducer, water, and steel fibers. Testing the joint served two purposes: to prove the capability of the UHPC to transfer loads adequately between adjacent girders and to prove the applicability of PBFTGs as modular units.

UHPC can develop a connection over an extremely small amount of exposed rebar. The connection can be strengthened if bonded to a roughened concrete surface, opposed to a typical smoothed concrete edge. Two methods were proposed to obtain the desired roughened concrete edge. The first method involved the use of a retarder on the formwork and wire brushing the concrete. Two different retarders were used: one was used with $1 / 2$ inch aggregate and smaller and the other was used with $3 / 4$ inch aggregate and larger. The second method included $3 / 4$ inch stone glued to the formwork to create voids in the concrete when the formwork was removed. The concrete used in this method was vibrated at different distances from the formwork to test the usability of the stone glued to the panel. The four connections were tested before performing large scale testing of the joint. The results of testing showed the use of retarder with larger aggregate and wire brushing provided the best shear key detail for bonding (Figure 2.16).


Figure 2.16: Concrete Surface after Wire-Brushing (Kozhokin, 2016)

The proposed joint detail was tested between two 35 foot long modular PBFTG units. Each modular unit consisted of a PBFTG formed from 84 inch wide by $7 / 16$ inch thick plate with a pretopped 6 inch thick concrete deck. After the deck concrete cured, the UHPC joint was poured, joining the two modular units (Figure 2.17). A 67.43 kip load was applied at midspan of one of the modular units over 2.7 million cycles to simulate infinite fatigue life. At a predetermined set of load cycles, a Service II load was applied statically, and strains were recorded in both the directly and indirectly loaded module to determine if the load was being adequately distributed.


Figure 2.17: Concrete Deck with the UHPC Joint (Kozhokin, 2016)

After approximately 1.6 million fatigue cycles, the deck directly under the point of load application failed. Material testing of the deck concrete revealed the compressive strength of the concrete was only 3 ksi after 28 days, contributing to the punching shear failure of the concrete (Figure 2.18). As the UHPC joint was satisfactorily transferring load from one modular unit to the other, the load was moved to the undamaged girder, and a larger plate was used to apply load from the actuator to the deck. The UHPC joint continued transferring the load satisfactorily through the remainder of the fatigue testing. A minor difference was noted in the distribution factors and deflections from before and after the load was moved to the adjacent girder, but the UHPC joint was found to be an adequate joint material.


Figure 2.18: Concrete Deck Failure (Kozhokin, 2016)

### 2.3.4 Fatigue Performance of Uncoated and Galvanized Press-Brake-Formed Tub Girders (Tennant, 2018)

This study analyzed the fatigue performance of PBFTGs with and without a steel protective system. Concerns were raised about the hot-dip galvanization process regarding residual stresses present in the bends of the PBFTGs. Tennant (2018) examined the performance of two PBFTGs consisting of ASTM A709 steel: one uncoated and the other hot-dip galvanized. Each simply supported specimen was made composite with a 6 inch concrete deck and fatigue loaded simulating a 75 -year design life in a rural environment. The PBFTGs were analyzed for rural loading due to the anticipated location of this type of short-span bridge. To avoid the localized
concrete crushing found in Kozhokin (2016), a spreader beam and elastomeric bearing pads were utilized to distribute the load more adequately (Figure 2.19).


Figure 2.19: Galvanized Modular System (Tennant, 2018)

A Service II moment was induced into each PBFTG at a predetermined number of cycles to determine if the galvanization had a negative effect on the system. The strain and deflection of each PBFTG were recorded at each induction of the Service II moment. The concrete deck of the galvanized PBFTG was found to have a significantly lower compressive strength than that of the uncoated specimen, causing slightly higher deflections and strains. However, the study concluded the galvanization process did not negatively affect the performance of the composite system. Each system performed linearly throughout the fatigue life of the system, showing the heat of galvanization did not affect the residual stresses locked into the bends of PBFTGs.

### 2.4 Previous Field Testing of PBFTGs

As the economical and long-term feasibility of the system was confirmed by the experimental and analytical research performed at WVU, PBFTGs began to be used in the field. Multiple PBFTG bridges have been constructed and field tested by researchers from WVU and Marshall University across three states. This section describes the bridges constructed and methods used to analyze the performance of the structures under live loading. Specifically, the distribution of live load across the PBFTGs is discussed.

### 2.4.1 Field Performance of Press-Brake-Formed Tub Girder Superstructures (Gibbs, 2017)

Gibbs (2017) analyzed the performance of the first bridge designed, constructed, and opened to traffic using PBFTGs. The Amish Sawmill Bridge (Figure 2.20) in Buchanan County, Iowa consists of four galvanized PBFTGs made from 96 inch wide by $1 / 2$ inch thick plate. The contractors chose to use a cast-in-place concrete deck. The goal of the research was to compare the results from a live load field test performed on site to analytical results obtained from finite element modeling.


Figure 2.20: New Amish Sawmill Bridge (Gibbs, 2017)

For the live load field test, researchers applied Bridge Diagnostics, Inc. strain gauges to the bottom flange of each PBFTG. Each girder was equipped with a minimum of three gauges at midspan for redundancy. Axle weights and distances of the loading truck were recorded to adequately model later using FEA. As the bridge was symmetric and not skewed, only five truck runs were necessary to complete the field test. One run was placed to maximize the load in an exterior girder and another run was placed to maximize the load in an interior girder. Two more loads were placed 12 feet away from either of the aforementioned runs to maximize the load in an interior or exterior girder from the two-lane loaded condition. The final truck was placed in the center of the bridge to determine if symmetrical results were produced.

LLDFs were calculated by dividing the strain in the girder in question by the sum of strain in all girders. LLDFs were also generated for two-lane loaded scenarios by superimposing strain values from two truck runs. The distribution factors matched closely, but the bottom flange strains generated by FEA were higher than those found from the field test. This discrepancy was attributed to differing boundary conditions. The Amish Sawmill Bridge utilized integral abutments, where the ends of each girder were encased in concrete, leading to much stiffer supports than traditional hinge-roller supports. However, integral abutments were not used in the finite element model as not enough information exists to adequately model this type of boundary condition. LLDFs from the experimental and analytical data were compared to LLDFs calculated according to the AASHTO LRFD BDS (Figure 2.21).


Figure 2.21: FEA vs. Experimental vs. AASHTO LLDFs for Truck Run 2 (Gibbs, 2017)

As seen in Figure 2.21, the analytical and experimental LLDFs are similar, while the AASHTO LRFD BDS provided significantly higher LLDFs for one-lane loaded scenarios. LLDFs for two-lane loaded scenarios for the experimental and analytical analysis showed less variation from the AASHTO LRFD BDS LLDFs but were still found to be conservative. The research proved PBFTGs can exhibit consistent performance and LLDFs provided by the AASHTO LRFD BDS can be used conservatively. Further research was proposed to provide less conservative distribution factor equations in the AASHTO LRFD BDS which more precisely simulate load distribution.

### 2.4.2 Field Performance and Rating Evaluation of Modular Press-Brake-Formed Steel Tub

 Girder with a Steel Sandwich Plate Deck (Underwood, 2019)The Cannelville Road Bridge in Muskingum County, Ohio was the second bridge built utilizing PBFTGs and the first to be constructed using modular units. The superstructure was composed of four hot-dip galvanized PBFTGs constructed from 5/8 inch thick plate. The girders were both internally and externally braced every 6 feet. The girders were delivered to the construction site as two modular units consisting of two girders each, bolted to a sandwich plate
steel (SPS®) deck. The use of the modular superstructure, as well as some other innovative ABC methods, allowed for expedient construction. Erection of the superstructure was completed in approximately 20 minutes, and the entire project from bridge demolition to opening of the new bridge was complete in 26 days.

Underwood (2019) worked with researchers from WVU and Marshall University to perform a live load field test on the Cannelville Road Bridge. The field test assessed the applicability of AASHTO LRFD BDS' LLDFs with regards to PBFTGs combined with an SPS® deck. Bridge Diagnostics, Inc. strain gauges were applied to the midspan of each girder in a manner similar to Gibbs (2017), and a tandem axle load truck was placed at predetermined grid points. The strain readings were not immediately recorded, allowing any vibrations to settle for a couple moments to negate any impact effects.

A finite element model of the bridge was developed to verify the results from the live load field test. Equivalent loads were applied to the structure to replicate the loads produced by the tandem axle load truck. The stresses were queried at each gauge location in the field and were compared. The stresses from the experimental and analytical tests were used to generate LLDFs and live load girder ratings. LLDFs were compared to equations present in the AASHTO LRFD BDS, and it was found, as in other tests, the AASHTO LRFD BDS tended to be overly conservative and underpredict the performance of PBFTGs (Figure 2.22).


Figure 2.22: Field v. FEA v. AASHTO LLDFs (Underwood, 2019)

### 2.4.3 Field Evaluation of a Modular Press-Brake-Formed Steel Tub Girder in an Application that Includes Skew and Superelevation (Roh, 2020)

The Fourteen Mile Bridge in Lincoln County, West Virginia is a 58 foot long single span PBFTG bridge (Roh, 2020). The bridge has a skew angle of $10^{\circ}$ and a superelevation of $8 \%$. As this bridge has significant skew and superelevation, special attention was paid to detailing shear studs, end diaphragms, and mounting angles for interior formwork. Specifically, the use of sacrificial interior wooden formwork in conjunction with varying length shear studs was used to address the superelevation. Skew was addressed by cutting the plate at the ends of the girder and offsetting each girder from the previous (Figure 2.23). Once all five composite modules were completed, they were transported to the bridge site and placed onto the abutments by crane. After the five modules were placed, formwork was erected around the longitudinal joints for UHPC closure pours. No further exterior or interior bracing was required due to the high torsional stiffness of the composite PBFTG modules.


Figure 2.23: PBFTG with Completed Internal Formwork (Roh, 2020)

Generally following the methodology of Gibbs (2017) and Underwood (2019), Roh (2020) conducted live load field testing on the Fourteen Mile Bridge. Gauges were placed at quarter span to allow for ease of installation. Additional care was taken to ensure gauges were placed appropriately with regards to the skew present in each girder. As load placement was much harder
to determine on site due to the superelevation and skew, the grid which the load would be applied was simplified to parallel lines at two-foot increments longitudinally and tenth points transversely. Linear interpolation would be used to generate the appropriate truck locations for worst case scenario one and two-lane loaded conditions.

Next, LLDFs from the live load field test, FEA, and empirical equations from the AASHTO LRFD BDS were compared. The strain readings from the three strain gauges on each bottom flange were averaged to account for torsion in the live load field test. Averaged girder strains and applicable multiple presence factors were used to generate LLDFs when the truck was at midspan. Midspan was chosen for comparison to the analytical model as the position would have the greatest effect on the girders. The results of the analytical modeling closely matched the results of the field testing, verifying the accuracy of both (Figure 2.24). As with previous research, Roh found the AASHTO LRFD BDS to be conservative when compared to analytical and experimental results.


Figure 2.24: Comparison of Analytical and Experimental Results (Roh, 2020)

Additionally, Roh analytically investigated the effectiveness of bracing in noncomposite PBFTGs as a continuation of research performed by Kelly (2014). Instability issues can arise in the open shape of noncomposite PBFTGs, as they are susceptible to torsional effects. These torsional effects are insignificant in composite construction, and therefore are not accurately captured in the global FEA. Different assumptions with regards to initial imperfections, material modeling, and loading must be made. Geometric imperfections, specifically flange tilt and girder twist, were considered as they had the largest effect on the results found by Kelly (2014). Elasticplastic constitutive laws were used to accurately capture the behavior of steel past the yield stress, as shown in Figure 2.25. Roh used the S4R element to model the noncomposite PBFTGs. The S4R is a 4-node general purpose shell element utilizing reduced integration with hourglass controls and is suitable for a wide range of applications The reduced integration used by the S4R elements can cause no strain at the integration points, so a small artificial stiffness associated with zero-energy deformation was introduced. Finally, the loading was applied using the modified Riks algorithm available in Abaqus/CAE to capture the complete nonlinear solution.


Figure 2.25: Multi-Linear Stress-Strain Curve (Roh, 2020)

A multitude of internal and external bracing scenarios were examined. $\mathrm{L} 4 \times 4 \times 5 / 8$ members were used as transverse and diagonal internal bracing elements between the top flanges. External bracing was modeled as transverse boundary conditions. Vertical and lateral deflections were compared from each internal and external bracing scenario to the results found from Kelly (2014). Results showed little effect of internal bracing or external bracing on vertical deflection. However,
the addition of diagonal bracing had a large reduction on the lateral displacement of the PBFTG. The addition of external braces reduced the lateral deflection to zero, but this is due to the program forcing zero lateral deflection at the point of measurement.

### 2.5 Summary

Several researchers over many decades have explored the use of prefabricated bridge elements to increase the economy in ABC. Many researchers found iterations of PBFTGs to be competitive in the short-span bridge market. Extensive laboratory research at WVU has shown PBFTGs perform exceptionally well at the ultimate limit state and under fatigue loading conditions. This research has allowed for multiple PBFTGs in three states to be constructed and field tested to assess the live load distribution characteristics of the system. However, while the system has proven its efficiency and economy in simple bridge situations, the system has not yet been thoroughly explored in more complex scenarios, including continuous spans or skewed scenarios. The empirical equations found in the AASHTO LRFD BDS have been found to be conservatively applicable to PBFTG bridges; however, the current wording of the AASHTO LRFD BDS prohibits the use of the empirical equations provided.

# CHAPTER 3: LIVE LOAD DISTRIBUTION FACTOR LITERATURE REVIEW 

### 3.1 Introduction

This chapter details the historical development of LLDFs in bridge systems and discusses the influence of various parameters on the lateral distribution of load. The primary focus of much of the research performed on LLDFs has historically been on straight beam slab bridges, but special attention is given in this chapter to studies evaluating LLDFs for box sections.

### 3.2 Historical Development of Live Load Distribution Factors

LLDFs have been included in American bridge specifications since the first edition of the American Association of State Highway Officials Standard Specifications (AASHO, 1931). The current AASHTO Standard Specifications for Highway Bridges (AASHTO, 2002) include the original distribution factors with minor modifications. The first major change in the computation of the distribution of load occurred when AASHTO adopted the LRFD BDS in 1994.

### 3.2.1 AASHTO Standard Specifications

Although AASHTO Standard Specifications and current AASHTO LRFD BDS allow the use of more refined analysis for lateral distribution of load, the use of simplified methods is permitted and frequently used, when applicable. This simplified method involves the distribution of wheel load to adjacent longitudinal elements. This lateral distribution is used in conjunction with line girder analysis (LGA) to determine the maximum possible number of wheels the girders must resist. The simplified method of the lateral distribution of wheel load generally takes the following form:

$$
\begin{equation*}
g \leq \frac{S}{D} \tag{Eq. 3.1}
\end{equation*}
$$

Where:

$$
\begin{aligned}
g & =\text { distribution factor } \\
S & =\text { beam spacing } \\
D & =\text { parameter used in determination of load fraction of wheel load }
\end{aligned}
$$

This type of equation is dependent on the bridge type and is generally valid for bridges up to a certain beam spacing. Table 3.1 shows the distribution of wheel loads, organized based on floor type. In situations with a concrete floor supported by four or more steel stringers and beam spacing less than six feet, the fraction of the wheel load shall not be less than:

$$
\begin{equation*}
g \geq \frac{S}{5.5} \tag{Eq. 3.2}
\end{equation*}
$$

Where:

$$
\begin{aligned}
& g=\text { distribution factor } \\
& S=\text { beam spacing }
\end{aligned}
$$

In situations where the concrete deck is supported by four or more steel stringers and beam spacing more than six feet, but less than fourteen feet, the minimum distribution factor is:

$$
\begin{equation*}
g \geq \frac{S}{4.0+0.25 S} \tag{Eq. 3.3}
\end{equation*}
$$

Where:
$g=$ distribution factor
$S=$ beam spacing

Table 3.1: Distribution of Wheel Loads in Longitudinal Beams (AASHTO, 2002)

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or more Traffic Lanes |
| :---: | :---: | :---: |
| Timber:" |  |  |
| Plank | S/4.0 | S13.75 |
| Nail laminated ${ }^{\text {c }}$ $4^{\prime \prime}$ thick or multiple layer ${ }^{\text {d floors over }} 5^{\prime \prime}$ thick | S/4 | S/4.0 |
| Nail laminated ${ }^{\text {c }}$ |  |  |
| $6^{\prime \prime}$ or more thick | S15.0 | S14.25 |
|  | If S exceeds $5^{\prime}$ use footnotef. | If S exceeds $6.5^{\prime}$ use footnote $f$. |
| Glued laminated ${ }^{\circ}$ |  |  |
| Panels on glued laminated stringers |  |  |
| $4^{\prime \prime}$ thick | S14.5 | S14.0 |
| $6^{\prime \prime}$ or more thick | S16.0 | S/5.0 |
|  | If $S$ exceeds $6^{\prime}$ use footnotef. | If $S$ exceeds $7.5^{\prime}$ use footnote $f$ |
| On steel stringers |  |  |
| $4^{\prime \prime}$ thick | S14.5 | S/4.0 |
| $6^{\prime \prime}$ or more thick | S15.25 | \$/4,5 |
|  | If $S$ exceeds $5.5^{\prime}$ use footnote f | If S exceeds 7' use footnote $f$ |
|  |  |  |
| On steel I-Beam stringers ${ }^{8}$ and prestressed |  |  |
| concrete girders | S/7.0 | S15.5 |
|  | If $\$$ exceeds $10^{\prime}$ use footnotef. | If 'S exceeds 14' use footnotef. |
| On concrete |  |  |
| T-Beams | S/6.5 | S/6.0 |
|  | If 'S exceeds 6' use foomote f. | If S exceeds $10^{\prime}$ use footnote f. |
| On timber |  |  |
| stringers | S/6.0 | S/5.0 |
|  | If $S$ exceeds 6' use footnote f. | If $S$ exceeds 10 use footnote f. |
| Concrete box |  |  |
| girders ${ }^{\text {h }}$ | S18.0 | S/7.0 |
|  | If S exceeds $12^{\prime}$ use footnote $f$. | If S exceeds 16 use footnote f. |
| On prestressed concrete spread box |  |  |
| Beams | See Article 3.28. |  |
| Steel grid: |  |  |
| (Less than $4^{\prime \prime}$ thick) | S14.5 | S/4.0 |
| (4' or more) | S/6.0 | S/5.0 |
|  | If S exceeds $6^{\prime}$ use footnotef. | If S exceeds $10.5^{\prime}$ use foomote f. |
| Steelbridge |  |  |
| Corrugated plank ${ }^{\text {in }}$ <br> ( $2^{\prime \prime}$ min depth) | S15.5 | S14.5 |

In situations where the beam spacing exceeds the values prescribed in Table 3.1 or the requirements in Equation 3.2 or Equation 3.3, the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between the stringers acts as a simple beam. This method of lateral distribution of load is commonly known as lever rule, and a depiction is presented in Figure 3.1. It should be noted lever rule is still used in the current edition of the AASHTO LRFD BDS for certain loading scenarios.


Figure 3.1: Notional Model for Applying Rule to Three-girder Bridges (AASHTO, 2020)

Slightly more complicated equations are used in AASHTO Standard Specifications for the determination of bending moment in steel box-girders. The current distribution factor in the AASHTO Standard Specifications for box-girder bridges was developed by Johnson and Mattock (1967):

$$
\begin{equation*}
W_{L}=0.1+1.7 R+\frac{0.85}{N_{w}} \tag{Eq. 3.4}
\end{equation*}
$$

In which:

$$
\begin{align*}
& \mathrm{R}=0.5 \leq \frac{N_{w}}{\text { Number of Box Girders }} \leq 1.5  \tag{Eq. 3.5}\\
& N_{w}=\frac{W_{c}}{12} \text { reduced to the nearest whole number } \tag{Eq. 3.6}
\end{align*}
$$

Where:

$$
\mathrm{W}_{c}=\text { roadway width between curbs in feet, or barriers if curbs are not used }
$$

It should be noted that while the LLDF expressions in the AASHTO Standard Specifications, specifically the S/D equations, have been applied to a wide array of bridges, the bridges considered during the development of the LLDFs were exceptionally simple. Specifically, Equation 3.1 does not consider many prominent bridge factors, such as the type and size of the bridge deck, girder stiffness, or span length, resulting in substantially conservative distribution factors. Additionally, no consideration is given in Equation 3.1 to more complicated bridge structures, including skew, horizontal curves, or continuous spans.

In the 1980s, researchers determined the AASHTO Standard Specifications should be updated and modernized. Sanders (1984) synthesized the background of then current specification criteria, provided an overview of the research being performed, and performed an evaluation of design and load rating practices as they related to the supporting structure and deck system types. He concluded an extensive study of LLDFs for highway bridges was needed as improvements to the empirical equations for the sake of accuracy were made over time and have led to many inconsistencies. Additionally, some parameters, which had been thoroughly evaluated and determined to influence live load distribution, such as the number of loaded lanes and reduction of loading intensity with increased number of loaded lanes, are not incorporated into the AASHTO Standard Specifications. Finally, Sanders (1984) introduced a study which would become the basis for an updated and unified set of LLDFs to be included in the AASHTO LRFD BDS.

### 3.2.2 AASHTO LRFD Specifications

The first edition of the AASHTO LRFD BDS was published in 1994, and with the new bridge design specifications came new methods of distributing live load to the longitudinal bridge elements. To address the issues with the AASHTO Standard Specifications, The National Cooperative Highway Research Program (NCHRP) Project 12-26 was initiated to overhaul the previous provisions relating to live load distribution.

Zokaie et al. (1991) developed comprehensive specifications for the distribution of wheel loads on highway bridges in two phases. The first phase concentrated on beam and slab and boxgirder bridges, and the second phase concentrated on slab, multi-box beam, and spread box beam bridges. For each general bridge type, three separate levels of analysis were proposed. Level One methods of analysis are the simplest and consist of equations distributing load laterally from the
wheel lines. Level Two methods of analysis include graphical methods, nomographs, influence surfaces, and grillage analysis. Level Three methods are the most accurate and involve detailed modeling of the entire bridge superstructure, such as a complete FEA.

The basis of the LLDF equations used today were developed from a parametric study of 365 bridges across multiple states comprised of three different types of bridge superstructures: prestressed T-beams, concrete I-girders, and steel I-girders. The first step was to perform a sensitivity study to determine the effect of various parameters on the lateral distribution of live load in bridges. An average bridge was generated with the average properties of each type of bridge. To determine the effect of various bridge parameters, one parameter was varied at a time with respect to the average bridge.

In the sensitivity study performed by Zokaie et al., the following set of parameters were varied to determine their effect: girder spacing/number of girders, span length, girder stiffness, slab thickness, number of loaded lanes, deck overhang, skew, load configuration, support condition, and end diaphragms. After analyzing the sensitivity of these factors, the critical parameters for the most common type of bridge, beam and slab, were found to be girder spacing, span length, girder stiffness, and deck thickness. No revisions were proposed for bridges with a concrete deck on multiple steel box-girders. The equations presented in the AASHTO LRFD BDS are generally more complex than those found the in the AASHTO Standard Specifications but are also more accurate.

### 3.3 Evaluations of Current Live Load Distribution Factors

Research has been performed by multiple investigators throughout the development of the LLDFs used in modern bridges. Investigators have examined the accuracy the AASHTO Standard Specifications and the AASHTO LRFD BDS through analytical studies of finite element models and experimental studies of existing bridges. Although the topic of live load distribution has been analyzed since the inception of a standard specification for bridge design, this section will focus on and summarize the research performed in the last forty years.

### 3.3.1 Analytical Studies

Analytical studies conducted by various researchers have been used to evaluate LLDFs for moment and shear distribution in bridges. Many of the studies performed assess the LLDFs found in the AASHTO Standard Specifications and the AASHTO LRFD BDS. Research efforts have largely been focused on the accuracy of these specifications with respect to one or more specific parameters. Some studies have been performed to develop new and simplified equations to be used in conjunction with one-dimensional LGA. A summary of selected studies is presented herein.

Johnston and Mattock (1967) developed a computer program to assess the lateral distribution of load in simple span composite box-girder bridges without transverse diaphragms or internal stiffeners. The accuracy of the computer program was confirmed with quarter-scale experimental testing. The computer program was used to evaluate the behavior of 24 composite box-girder bridges with varying span lengths, number of lanes, and number of girders. As a result of the study, two equations were proposed for the transverse distribution of load to each box-girder. As stated in Section 3.2.1, the work performed in this study led to the current equation used in the AASHTO Standard Specification.

Wallace (1976) performed finite element analyses on 51 theoretical skewed concrete boxgirder bridges using the program CELL to describe the behavior of the systems. The assessment focused on the effects of the width-to-span ratio, number of cells, skew angle, type of loading, and depth of box-girder members. Width-to-span, or aspect ratio, and skew angle were found to be the most significant factors affecting moment distribution, while span length and skew angle were found to be the most significant parameters affecting shear distribution for the exterior girder at the obtuse corner.

Hays et al. (1986) performed analysis using the computer program SALOD to evaluate lateral load distribution of simple span bridges in flexure. SALOD uses moment influence surfaces generated from the finite element system STRUDL for representative simple span bridges consisting of concrete girders, steel girders, T-beams, or flat slabs. The analytical study was verified by comparing it against field data recorded from eight bridges. A parametric study was performed to assess the effects of girder spacing and span length. The results of the study were then compared against the distribution factors calculated from the AASHTO Standard Specifications and the Ontario Bridge Design Code. The results of the study showed the effect of
span length, which at the time was neglected by the design specifications, had a considerable impact. The researchers found the distribution factors from the AASHTO Standard Specifications were unconservative for span lengths up to 60 feet and girder spacings up to 6 feet. However, as span length and girder spacings increased, the AASTHO Standard Specifications became more conservative.

Khaleel and Itani (1990) performed FEA to determine the effects of skew on lateral load distribution for continuous skewed bridges. The researchers analyzed 112 continuous span bridges consisting of five pretensioned I-girders with spans between 80 and 120 feet and girder spacings between 6 and 9 feet. Skew angles between $0^{\circ}$ and $60^{\circ}$ were evaluated. The maximum moments found in the study were then compared to the AASHTO Standard Specifications, which did not account for the effects of skew. The researchers showed the AASHTO Standard Specifications distribution of wheel loads can underestimate the design moments by $6 \%$ in some instances and overestimate the design moments by $40 \%$ in others. The researchers proposed a skew reduction factor to be used in conjunction with the AASHTO Standard Specifications based on the span length, girder spacing, skew angle, and girder location.

Tarhini and Frederick (1992) performed analytical research specifically on the distribution of loads on concrete slab on steel I-girder bridges. To determine the effects of various parameters on the distribution of load, a typical bridge was selected, and one parameter was varied within practical ranges, while the others were held constant. The parameters considered were girder spacing, girder stiffness, presence of cross-bracing, concrete slab thickness, span length, single or continuous spans, and composite or noncomposite behavior. The distribution factor was calculated in this study by dividing the maximum moment in a girder found using FEA by the maximum moment in a girder found using LGA. An equation was developed for the live load distribution based on girder spacing and span length, as they were found to be the most influential. The researchers found this equation to be applicable to single or continuous span and composite or noncomposite bridges.

Ebeido and Kennedy (1996a and 1996b) performed parametric studies on over 600 prototype continuous skew composite bridges using FEA. The analytical modeling was verified against experimental tests on three continuous and six simple span, composite, concrete deck on steel beam bridges. Empirical equations were developed for main span moment, interior support
moment, reaction, and shear distribution factors. Results from the study show increased skew decreases the main span and interior support moments while increasing the reactions and shear in the obtuse corner. Additionally, the influence of skew on shear and moment distribution factors significantly increases for skew angles above $30^{\circ}$.

Mabsout et al. (1997a, 1997b, and 1998) analytically explored the effects of various parameters on the distribution of wheel load and compared the results to the simplified equations found in the AASHTO LRFD BDS, AASHTO Standard Specifications, and previous experimental results. The researchers first explored the validity of four separate finite element modeling techniques on a simple span, two lane, composite steel I-girder bridge. Results from a parametric suite of girders with varying span length and girder spacing found each of the modeling techniques produced similar LLDFs, correlating well with the AASHTO LRFD BDS, but not with the AASHTO Standard Specifications. Following the verifications of the finite element modeling techniques, the researchers explored the effect of sidewalks, railings, and continuity on LLDFs. The researchers found when sidewalks and/or railings were constructed integrally with the bridge deck and properly reinforced, LLDFs of exterior girders increased. The researchers state the presence of sidewalks and/or railings can reduce LLDFs of interior girders by 5 to $30 \%$. When considering continuous span bridges, the researchers also recommended the addition of a reduction factor for positive and negative moments of up to $5 \%$.

Arockiasamy and Amer (1998) developed simplified equations for a multitude of varying bridge types based on analytical studies of numerous parameters. The analytical studies were based on finite element modeling using ANSYS 5.2 and were verified against field test data. The first phase of the research was to perform grillage analysis to study the effects of span length, bridge width, slab thickness, girder type, and number of lanes on moment and shear live load distribution. Generally, the researchers found LLDFs from the empirical equations found in the AASHTO LRFD BDS were slightly conservative compared to the calculated LLDFs from grillage analysis. The second phase of the research focused on the effects of skew and continuity on live load distribution. Similar to phase one, the influence of skew angle, girder spacing, span length, slab thickness, and number of lanes were studied in skewed slab-on-girder bridges. The authors explored skew angles between $0^{\circ}$ and $60^{\circ}$ and concluded the AASHTO LRFD BDS are accurate in reflecting the live load distribution in bridges, particularly for skew angles greater than $30^{\circ}$.

Additionally, the authors state the AASHTO LRFD BDS empirical equations overestimate the effect of slab thickness.

Mertz (2007) performed research to develop updated LLDFs for shear and moment. The goal of the research was to develop simplified LLDF equations with a larger range of applicability than those found in the AASHTO LRFD BDS. Three sources of data were used to develop a parametric suite of bridges: the NCHRP 12-26 bridge set, the Tennessee Technological University set, and a set of bridges from AASHTO Virtis/Opis used to compare rating procedures. Multiple simplified and rigorous analytical models were run on the suite of parametric bridges to generate LLDFs. LLDFs generated from this array of bridges was used to codify simplified moment and shear LLDFs across a wider range of applicability.

Yousif and Hindi (2007) compared LLDFs of simple span slab-on-girder concrete bridges calculated using the AASHTO LRFD BDS and several finite element linear elastic models. The researchers analyzed bridges across the full range of applicability specified by the AASHTO LRFD BDS regarding span length, slab thickness, girder spacing, and longitudinal stiffness. The deck was modeled as 4 -node quadrilateral shell elements, and the beams were modeled as space frame elements. The vehicular live load plus the lane load, as specified in the AASHTO LRFD BDS, was placed longitudinally to generate the extreme force effect and placed transversely to investigate the one-lane, two-lanes, and three-lanes loaded scenario for each applicable girder. The researchers modeled a total of 886 bridges and concluded the AASHTO LRFD BDS empirical equations significantly overestimate LLDFs when compared to the analytical modeling, reaching a maximum of approximately $55 \%$.

Michaelson (2010) developed new expressions for exterior girder LLDFs for concrete deck on steel beam bridges. Analytical modeling, benchmarked against experimental data, was used in a sensitivity study to determine the effect of key parameters on LLDFs for exterior girders. Following the results of the sensitivity study, parametric matrices of the most influential parameters were developed, and the results were compared to LLDFs from the empirical equations found in the AASHTO LRFD BDS. Finally, the results of the parametric matrices were used to develop empirical equations for exterior girder LLDFs for steel I-girder bridges. Michaelson found the key parameters for live load distribution were girder spacing, span length, deck overhang, and the number of beams in the cross-section.

Razzaq (2017) developed empirical equations for the assessment of LLDFs for composite skewed slab-on-steel I-girder bridges. A parametric study was performed on a composite bridge structure under dead and live loads for the ultimate, serviceability, and fatigue limit states. The researcher considered skew angle, girder stiffness, cross-frame layout, span length, girder spacing, number of girders, and number of design lanes in the determination of empirical equations for moment and shear LLDFs. The results of the study were compared to the equations presented in the Canadian Highway Bridge Design Code.

### 3.3.2 Experimental Studies

Many research efforts have been conducted to assess the validity of LLDF equations in existing bridge codes by performing experimental studies on in-service bridges. Researchers have devoted significant amounts of time and effort to determine LLDFs at varying loading stages, speeds, and locations.

Bakht and Jaegar (1992) performed an ultimate load test on a simply supported bridge consisting of a noncomposite concrete deck supported by six rolled steel wide flange shapes. A wooden frame was erected under midspan of the structure to allow for approximately 110 millimeters deflection to prevent the bridge from deflecting catastrophically. Uniaxial strain gauges were placed at midspan throughout the depth of each beam to measure the strain at each girder. The bridge was loaded by placing concrete blocks transversely in layers of 24 units at midspan to simulate one-lane loaded. The results of the study show: (1) the moment LLDFs improve when approaching the ultimate load; (2) any incidental composite action between the deck and the beams due to bond or friction completely breaks down approaching the ultimate load; and (3) the beams continue to carry load well past formation of first yield.

Stallings and Yoo (1993) performed a series of stationary and moving tests on three shortspan, two lane, steel girder bridges. Tandem axle dump trucks were used to load the bridge under one and two-lane loaded scenarios. LLDFs were calculated from the single and two-lanes loaded stationary tests and impact factors were calculated from the results of the moving tests. Stallings and Yoo (1993) calculated LLDFs from the actual strains measured in the bottom flange as presented in Equation 3.7.

$$
\begin{equation*}
D F_{i}=\frac{n \varepsilon_{i}}{\sum_{j=1}^{k} \epsilon_{j} w_{j}} \tag{Eq. 3.7}
\end{equation*}
$$

Where:
$D F_{i}=$ wheel load distribution factor for the $i$ th girder
$n=$ number of wheel lines of applied loading
$\varepsilon_{i}=$ bottom flange strain of the $i$ th girder
$w_{j}=$ ratio of the section modulus of the $i$ th girder to the section modulus of a typical interior girder

This equation can be simplified (Equation 3.8), assuming: the section modulus of an interior girder and an exterior girder are approximately the same; the number of wheel lines can be removed as it was a conversion factor between AASHTO Standard Specifications and AASHTO LRFD BDS; and the relationship between moment and strain is linear.

$$
\begin{equation*}
g_{i}=\frac{M_{i}}{\sum_{j=1}^{k} M_{j}} \tag{Eq. 3.8}
\end{equation*}
$$

Where:

$$
g_{i}=\text { LLDF for the } i \text { th girder }
$$

$M_{i}=$ bending moment in the $i$ th girder
$k=$ number of girders

The researchers found LLDFs calculated using the AASHTO Standard Specifications S/D equations were consistently higher than LLDFs calculated from the experimental results. The researchers state the conservatism was attributed to inaccuracies in the assumptions made using the simplified analysis presented in the AASHTO Standard Specifications.

Fu et al. (1996) performed field testing of four existing I-girder bridges under real truck loading to evaluate parameters affecting live load distribution. LLDFs were calculated from measured strain data in addition to several empirical methods. Comparison of the live load field tests results to the AASHTO LRFD BDS empirical LLDFs of straight non-skewed bridges showed
the AASHTO LRFD BDS' LLDFs to be anywhere from $7 \%$ to $42 \%$ conservative. Additionally, comparison between the live load field results of a skewed bridge to the AASHTO LRFD BDS' empirical LLDFs showed the AASHTO LRFD BDS' LLDF to be $13 \%$ unconservative.

Kim and Nowak (1997) performed live load field tests on two simply supported steel Igirder bridges to determine LLDFs and impact factors. Opposed to most other field tests, the recorded data was collected from daily traffic loads in addition to calibrated truck loads. The static strain of the daily traffic loads was measured after removing the dynamic impact component. The static strains in each girder were used to determine LLDFs following a methodology nearly identical to that used by Stallings and Yoo (1993). The strain data was further processed to obtain the mean and standard deviation of LLDFs under daily traffic. The researchers concluded the measured LLDFs were consistently lower than those found using the empirical equations found in the AASHTO LRFD BDS.

Cross et al. (2009) performed live load field tests on twelve bridges to determine the validity of shear LLDFs on typical interstate bridges. The bridges were specifically chosen to represent most interstate bridges in Illinois and to maintain a wide range of parameters. Each beam was instrumented on its web with strain gauge rosettes to measure shear stresses caused by loading vehicles. The load was run slowly across the bridge to model static loading in addition to dynamic tests at highway speeds. Analytical models were developed to verify the live load testing results. The researchers concluded AASHTO LRFD BDS shear live load distribution procedures closely approximate the actual shear live load distribution from analytical and field test results.

### 3.4 Influence of Parameters Affecting Live Load Distribution

Several previous researchers ((Newmark \& Siess, 1942), (Zokaie et al., 1991), (Tarhini \& Frederick, 1992), (Mabsout et al.,1997a and 1997b), (Arockiasamy \& Amer, 1998), (Nowak et al., 2003), (Yousif \& Hindi, 2007), (Li \& Chen, 2011), (Razzaq, 2017), (White \& Kamath, 2020)) have investigated the effect of numerous parameters on live load distribution in bridges. Two of the most comprehensive studies were conducted by Zokaie et al. (1991), as part of NCHRP 12-26, and Tarhini and Frederick (1992).

The contributions of Zokaie et al. (1991) were discussed in Section 3.2.2. Tarhini and Frederick (1992) focused their research on I-girder bridges with a concrete deck. Similar to the work performed for NCHRP 12-26, a typical bridge design was selected, and one parameter was varied at a time within practical limits. The parameters considered in the FEA included size and spacing of steel girders, presence of cross-bracing, concrete slab thickness, span length, single or continuous spans, and composite or noncomposite design. After performing FEA, girder spacing, span length and girder stiffness were determined to be the most significant parameters relating to live load distribution in slab on girder bridges. However, other parameters were investigated and were found to have a negligible effect, while some disagreement exists regarding the effects of others. This section will summarize the research performed on effects of many of those parameters.

### 3.4.1 Girder Spacing

Girder spacing has been considered the most influential factor affecting LLDFs since the early work developed for the AASHTO Standard Specifications. Newmark and Seiss (1942) originally developed LLDF empirical equations based on girder spacing, span length, and the ratio between girder and deck stiffnesses. However, later work by Newmark (1949) expressed LLDFs as linear functions of girder spacing only, removing the effects of span length and beam stiffness. This linear relationship is present in the most current AASHTO Standard Specifications with minimal changes since the adoption of the S/D factors.

However, many studies have shown the S/D factors used in the AASHTO Standard Specifications consistently produce overly conservative LLDFs. Studies performed as part of NCHRP 12-26 (Zokaie et al., 1991), and verified by Tarhini and Frederick (1992), demonstrate while the relationship between girder spacing and live load distribution is significant, it is not linear, but exponential. Many studies propose equations for beam slab bridges based on, at least in part, girder spacing. Khaloo and Mirzabozorg (2003) determined, while the effect of girder spacing is significant for the distribution of live load to exterior girders, the effects of girder spacing on the distribution of live load to interior girders is significantly less.

### 3.4.2 Span Length

Similarly to the effect of girder spacing, a nonlinear relationship between span length and LLDFs was determined by Zokaie et al. (1991) and verified by Tarhini and Frederick (1992). The relationship between span length and live load distribution was found to be more significant for moment in interior girders compared to shear in interior girders. Unlike girder spacing, span length was found to have an inverse effect, such that when span length increases, LLDFs decrease.

Bishara et. al. (1993) evaluated the distribution of live load in medium span length slab-on-girder bridges, with and without skew, to both interior and exterior girders. LLDFs were derived from FEA of 36 bridges. The results of this study showed span length had a slight effect on the distribution of live load to interior girders. However, span length was found to have a more significant effect on bridges with a smaller clear roadway width and with bridges with large skew angles. Khaloo and Mirzabozorg (2003) also determined span length has a small effect on LLDFs of interior girders, but exterior girder LLDFs increase more significantly with span length.

### 3.4.3 Girder Stiffness

The definition of girder stiffness has changed throughout the history of LLDFs. The first definition of relative stiffness comes from Newmark and Siess (1942), where the researchers compared the relative longitudinal stiffness of the girder to the relative transverse stiffness of the deck. This version of girder stiffness is expressed by the dimensionless parameter, $H$, as described by Equation 3.9:

$$
\begin{equation*}
H=\frac{E_{b} I_{b}}{a N} \tag{Eq. 3.9}
\end{equation*}
$$

In which:

$$
\begin{equation*}
N=\frac{E I}{1-\mu^{2}} \tag{Eq. 3.10}
\end{equation*}
$$

Where:
$H=$ a dimensionless coefficient which is a measure of the stiffness of the beam relative to that of the slab

$$
\begin{aligned}
E_{b} & =\text { modulus of elasticity of the material in a beam } \\
I_{b} & =\text { moment of inertia of the cross-section of a beam } \\
a & =\text { span of bridge, center to center of supports } \\
N & =\text { measure of stiffness of an element of the slab } \\
E & =\text { modulus of elasticity of the material in the slab } \\
I & =\text { moment of inertia per unit width of the cross-section of the slab } \\
\mu & =\text { Poisson's ratio, generally taken as zero in the data given here }
\end{aligned}
$$

Initial results from Newmark and Siess (1942) showed this version of the stiffness parameter had a small, but significant, effect on the distribution of live load. Later results (Newmark, 1949) found the range of applicable values for any given bridge type is small enough the stiffness parameter, in this form, is negligible. Tarhini and Frederick (1992) concluded similar results in their studies, as the girder stiffness had a small, but negligible, effect on live load distribution. The researchers performed a parametric study on the variables in Equation 3.9, such as changing the moment of inertia of the girder or the thickness of the slab. The maximum difference in LLDFs in the study was approximately 5\%, which the researchers considered insignificant.

Zokaie et al. (1991) defined girder stiffness in a different manner. The authors defined the longitudinal stiffness parameter, $K_{g}$, using Equation 3.11. The researchers confirmed an acceptable means of quantifying girder stiffness was by changing the moment of inertia, area, and eccentricity while maintaining a constant longitudinal stiffness parameter. The overall LLDF changed by approximately $1.5 \%$ with differing individual variables, confirming the longitudinal stiffness parameter is acceptable for quantifying the stiffness of the girder elements.

$$
\begin{equation*}
K_{g}=n\left(I+A e_{g}^{2}\right) \tag{Eq. 3.11}
\end{equation*}
$$

In which:

$$
\begin{equation*}
n=\frac{E_{B}}{E_{D}} \tag{Eq. 3.12}
\end{equation*}
$$

Where:
$K_{g}=$ longitudinal stiffness parameter $n=$ modular ratio between beam and deck
$I=$ moment of inertia of beam
$A=$ area of a stringer, beam, or component
$e_{g}=$ distance between the centers of gravity of the basic beam and deck
$E_{B}=$ modulus of elasticity of beam material
$E_{D}=$ modulus of elasticity of the deck material

As the accepted definition of the longitudinal stiffness parameter changed, it was found to have a major impact on live load distribution. This increase in LLDF with increased girder stiffness typically occurs with longer bridges, as longer bridges require larger, more stiff longitudinal elements. Therefore, when these two elements are combined, they tend to negate each other. Later analysis performed by Arockiasamy and Amer (1998) demonstrated girder stiffness has a negligible impact on shear live load distribution. Additionally, the researchers found the AASHTO LRFD BDS overestimates the effect of girder stiffness on moment live load distribution. Additionally, Yousif and Hindi (2007) found AASHTO LRFD BDS' LLDFs in the intermediate applicable longitudinal stiffness range compared well to three-dimensional FEA but tended to deviate at the extreme range of the specified limitations.

### 3.4.4 Deck Thickness

The consequence of the effective thickness of the concrete deck has been a subject of debate in the research. It is undeniable the effective thickness of the concrete deck has a role in both the original and modern definitions of longitudinal stiffness of the girders or beams. Newmark and Siess (1942) state deck thickness affects live load distribution, as it has a direct influence on the relative stiffness. However, later studies performed by Zokaie et al. (1991) found varying the deck thickness between six and nine inches had less than a $10 \%$ impact on live load distribution. Studies performed by Tarhini and Frederick (1992) were in agreement and found deck thickness changes between 5.5 inches and 9.5 inches had a negligible impact on live load distribution.

Nonetheless, effective thickness of the deck is included in the AASHTO LRFD BDS' LLDFs for I-Girder beam and slab bridges.

### 3.4.5 Girder Location

Walker (1987) found the location of the girder, where the force effect is maximized, had an influence on live load distribution. The researchers calculated LLDFs using a two-dimensional grid model with plate elements. The analytical model was used to calculate ' D ' constants to be used in the ASSHTO Standard Specification equations discussed in Section 3.2.1. These 'D' factors to be used in the S/D equations would be calibrated to produce the same LLDFs as the twodimensional grid model.

Zokaie (2000), following the NCHRP Report 12-26, performed a study which concluded exterior girders are more sensitive to truck placement than interior girders. Due to this finding, lever rule is used to determine exterior girder one-lane loaded LLDFs, and a correction factor is applied to the interior girder LLDFs to determine the exterior girder two-lane loaded LLDFs.

### 3.4.6 Number of Girders

Zokaie et al. (1991) considered the number of girders in the cross-section of the bridge as a variable in determining LLDFs in their initial sensitivity study for beam and slab bridges. These studies assumed one or two-lanes loaded for all scenarios as they deemed the likelihood of three or more lanes loaded to be unlikely. Additionally, the small likelihood of three or more lanes loaded at one time, combined with the girder spacing required to have three loaded lanes affecting the distribution of live load to one girder, is exceptionally rare. The results of their sensitivity study show the effect of number of girders is very small, with a negligible increase in LLDFs from three to four girder cross-sections, and an even smaller increase from four to five or more girders.

### 3.4.7 Deck Overhang

Deck overhang was found to have a negligible effect on interior girder LLDFs but a linear effect on exterior girder LLDFs (Zokaie et al., 1991). This relationship was incorporated in the

AASHTO LRFD BDS in the form of a correction factor applied to interior girders to determine exterior girder LLDFs for two or more lanes loaded scenarios. Further research by Barr and Amin (2006) demonstrated deck overhang has a more significant effect on shear LLDFs for exterior girders than shear LLDFs for interior girders, mirroring the findings by Zokaie et al. (1991).

### 3.4.8 Skew

The skew angle of the bridge has been found to be one of the most significant factors affecting live load distribution. Increased skew angle decreases longitudinal stresses and strains in interior and exterior girders and increases transverse stresses and strains in the bridge deck when compared to bridges without skew (Newmark, 1948). Increased skew angle also increases the exterior girder shear LLDFs at the obtuse corner. Results have shown for bridges with a skew angle of $60^{\circ}$ the maximum moment in interior and exterior girders is approximately $71 \%$ and $80 \%$ of those moments in a right bridge, respectively. The AASHTO LRFD BDS utilizes two correction factors, one decreasing moment LLDFs from those of a right bridge, and another increasing shear LLDFs in the obtuse corner from those of a right bridge.

### 3.5 SUMMARY

Current AASHTO LRFD BDS methodologies for static structural analysis for beam and slab bridges allow bridge engineers to consider longitudinal and transverse effects of live load separately, simplifying the analysis and design of the bridge. The current AASHTO LRFD BDS allow transverse distribution of live load to be calculated using simplified methods of analysis using LLDFs. While these factors are considerably more accurate than previous specifications, they have been shown to be overly conservative for a wide range of bridges. Specifically, through the major updates to live load distribution in the 1990s, minimal changes were made to the LLDFs used for box-girders.

A variety of research has been performed, both analytically and experimentally, on typical I-girder shaped beam and slab bridges, but very few researchers have considered box-shaped beam and slab bridges. Moving forward, more accurate and applicable LLDFs for PBFTG bridges can be developed by evaluating the parameters already shown to effect beam and slab bridges.

# CHAPTER 4: COMPACTNESS AND SKEW LITERATURE REVIEW 


#### Abstract

4.1 Introduction

This chapter presents a review and evaluation of local and global compactness limits pertaining to PBFTGs. A brief synopsis of the history and basis for many of the limits used in the AASHTO LRFD BDS is provided. This understanding will be used to evaluate the applicability of the restrictions present in the AASHTO LRFD BDS as they relate to skewed PBFTGs.


### 4.2 Stability of Plates

All steel sections, whether rolled shapes, plate girders, or box-girders, are composed of plate elements. In flexural elements in negative bending, when the cross-section is broken down into its constitutive elements, the bottom flange is essentially a plate under compression. Consideration must not only be given to buckling of the entire cross-section, but also to local buckling of the plate elements making up the cross-section. Local buckling occurs when the element cannot resist additional load prior to the onset of yielding, reducing the efficiency of the cross-section. The general approach in this section follows that of Timoshenko \& WoinowskyKrieger (1959) and Timoshenko \& Goodier (1961).

### 4.2.1 Elastic Local Buckling of Flat Plates

The buckling behavior of a plate simply supported along its edges is essential to understanding the local buckling behavior of plate assemblies (Ziemian, 2010). Ideally, the buckling stresses are derived from bifurcation of an initially perfect structure. However, in practice, the buckling response is continuous due to the presence of geometric imperfections and residual stresses. When the cross-section of a member consists of various connected elements, such as flanges and webs, a lower bound critical stress can be determined for each element assuming a simply supported boundary condition at each intersection and a free boundary condition for any other edge.

Bryan (1890) first presented the analysis of a rectangular plate simply supported along all edges subjected to a uniform compressive stress. The elastic critical stress of a plate is affected by the plate width-to-thickness ratio, restraint conditions along the longitudinal boundaries, and elastic material properties of the plate, namely the elastic modulus and Poisson's ratio. The theoretical elastic buckling stress can be expressed as:

$$
\begin{equation*}
F_{c r}=k \frac{\pi^{2} E}{12\left(1-v^{2}\right)(b / t)^{2}} \tag{Eq. 4.1}
\end{equation*}
$$

In which:

$$
\begin{equation*}
k=\left[\frac{1}{m} \frac{a}{b}+m \frac{b}{a}\right]^{2} \tag{Eq. 4.2}
\end{equation*}
$$

Where:

$$
\begin{aligned}
F_{c r} & =\text { elastic buckling stress } \\
k & =\text { plate buckling coefficient } \\
E & =\text { modulus of elasticity } \\
v & =\text { Poisson's ratio } \\
b & =\text { transverse plate width } \\
t & =\text { plate thickness } \\
a & =\text { longitudinal plate length } \\
m & =\text { number of half-sine waves that occur in the x-direction at buckling }
\end{aligned}
$$

Generally, plate compression elements can be separated into two categories: stiffened elements, those supported along two edges parallel to the direction of the compressive stress; and unstiffened elements, those supported along one edge and free on the other edge parallel to the direction of compressive stress (Ziemian, 2010). Examples of stiffened elements include I-shaped webs, bottom flanges of PBFTGs, or edges of hollow structural sections. Examples of unstiffened elements include I-shaped flanges, top flanges of noncomposite PBFTGs, or legs of an angle. Figure 4.1 shows the variation of the plate buckling coefficient with respect to the plate aspect ratio for most idealized edge conditions.


Figure 4.1: Elastic Buckling Coefficients for Compression in Flat Rectangular Plates (Ziemian, 2010)

Assuming ideal elastic-plastic material without geometric imperfections or residual stresses, the distribution of stress within the axially loaded plate remains uniform until the elastic buckling stress is reached. The load can be increased past this point, but the plate furthest from the side supports will begin to deflect out-of-plane, causing nonuniform stress distribution even though the loading is applied through rigid ends.

Figure 4.2 shows the plate strength under uniform edge compression consists of the sum of two components: the elastic or inelastic buckling stress represented by Equation 4.1 and the post-buckling strength. As seen in Figure 4.2, if the plate width-to-thickness ratio is very high, the post buckling strength becomes larger. Conversely, if the plate width-to-thickness is very low, not only does the post buckling strength drastically decrease, but the plate may have yielded and strain
hardening may have already begun. In this case, the ratio of the elastic buckling stress to the yield stress becomes greater than one. For idealized plates, plates without residual stresses or geometric imperfections, three regions must be considered when determining strength: elastic buckling, yielding, and strain hardening.


Figure 4.2: Behavior of Plate Under Edge Compression (Ziemian, 2010)

By redefining the ratio of the critical buckling stress to the yield stress, $F_{c r} / F_{y}$, as $1 / \lambda^{2}$, and substituting into Equation 4.1, the slenderness ratio of plates, $\lambda_{c}$, becomes:

$$
\begin{equation*}
\lambda_{c}=\frac{b}{t} \sqrt{\frac{F_{y} 12\left(1-v^{2}\right)(b / t)^{2}}{\pi^{2} E k}} \tag{Eq. 4.3}
\end{equation*}
$$

Haaijer and Thurlimann (1957) discovered the most important factor determining the slenderness ratio needed to achieve the elastic critical buckling stress is whether the plate is supported along the edges parallel to loading. The researchers determined the type of restraint along the loaded edge has essentially no effect. As seen in Figure 4.3, curve (b) represents the case where one edge parallel to loading is supported, with a critical slenderness value of 0.46 , and curve (c) represents the case where both edges parallel to loading are supported, with a critical slenderness value of 0.58 . Figure 4.3 also shows a transition curve between strain hardening and the Euler Hyperbola due to initial residual stresses, reducing the actual resistance of the plate.


Figure 4.3: Buckling Curve Based on Slenderness Ratio (Haaijer \& Thurlman, 1957)

When considering inelastic buckling, the generally accepted method is to extend the elastic critical buckling approximations already discussed to situations where material yielding has already occurred. Bleich \& Ramsey (1952) provided a modification to the elastic critical buckling stress of a plate under uniform compressive stress by considering the tangent modulus. The researchers proposed Equations 4.4 and 4.5 when considering inelastic buckling:

$$
\begin{equation*}
F_{c r}=k \frac{\pi^{2} E \sqrt{\eta}}{12\left(1-v^{2}\right)(b / t)^{2}} \tag{Eq. 4.4}
\end{equation*}
$$

In which:

$$
\begin{equation*}
\eta=\frac{E_{t}}{E} \tag{Eq. 4.5}
\end{equation*}
$$

Where:

$$
E_{t}=\text { tangent modulus }
$$

### 4.2.2 AISC Width / Thickness Limitations

As discussed in Section 4.2.1: For low width-to-thickness values, strain hardening occurs without buckling; for medium width-to-thickness values, inelastic buckling occurs due to residual stresses and initial geometric imperfections; and for large width-to-thickness values, buckling occurs according to Equation 4.4. Plates with large width-to-thickness ratios have strengths exceeding the buckling strength, but these plates are conservatively limited to elastic buckling. To establish design requirements for members consisting of these elements, the desired performance must be ascertained.

A logical performance criterion of an element in compression would be to prevent local buckling of an element in the cross-section prior to achieving the full strength of the cross-section. In other words, the elastic buckling stress of the component should be greater than or equal to the elastic buckling stress of the cross-section. However, this would lead to acceptable width-tothickness ratios dependent on the overall slenderness of the cross-section.

Current design limits assure the compression element reaches the yield stress without local buckling, even though the overall slenderness of the cross-section may prevent the element from reaching the yield stress. Table 4.1 provides the width-to-thickness ratio to prevent this local buckling prior to the yield stress, $\lambda_{r}$. By replacing the critical buckling stress with the yield stress in Equation 4.1, substituting known material properties of steel, and solving for the width-tothickness ratio, Equation 4.1 becomes:

$$
\begin{equation*}
\frac{b}{t} \leq 162 \sqrt{\frac{k}{F_{y}}} \tag{Eq. 4.6}
\end{equation*}
$$

The above equation assumes no residual stress is present in the cross-section. As residual stresses are present in all steel structural elements due to cold forming, plasma cutting, roll forming, and welding, a reduced slenderness ratio should be utilized to minimize the difference between idealized behavior and the assumed transition curve. An assumed reduction of $30 \%$ of the resistance is taken as a rational value, resulting in:

$$
\begin{equation*}
\frac{b}{t} \leq\left[(0.7) 162 \sqrt{\frac{k}{F_{y}}}=113 \sqrt{\frac{k}{F_{y}}}\right] \tag{Eq. 4.7}
\end{equation*}
$$

When the corresponding plate buckling coefficient, dependent on the boundary conditions parallel to the axis of loading of plate, are input into Equation 4.7, the limiting width-to-thickness values found in Table 4.1 are generated. Cross-sections with a governing width-to-thickness ratio greater than those provided in Table 4.1 are subject to local buckling limit states and associated capacity reductions found in the American Institute of Steel Construction (AISC) Steel Construction Manual (AISC, 2017) Section E7. Cross-sections where all elements have width-tothickness ratios less than those provided in Table 4.1 are not subject to local buckling limit states.

Table 4.1: Width-to-Thickness Ratios for Elements Subject to Axial Compression (AISC,
2016)

|  | $\begin{aligned} & \text { シ } \\ & \text { 心̃ } \end{aligned}$ | Description of Element | Width-toThickness Ratio | Limiting Width-to-Thickness Ratio $\lambda_{r}$ (nonslender/slender) | Examples |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees | $b / t$ | $0.56 \sqrt{\frac{E}{F_{y}}}$ |  |
|  | 2 | Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections | $b / t$ | [a] $0.64 \sqrt{\frac{k_{c} E}{F_{y}}}$ |  $\frac{\square}{\infty} \frac{1}{\infty} t$ |
|  | 3 | Legs of single angles, legs of double angles with separators, and all other unstiffened elements | $b / t$ | $0.45 \sqrt{\frac{E}{F_{y}}}$ | $\stackrel{b}{\frac{b}{4}+\frac{1}{4} t} t$ |
|  | 4 | Stems of tees | $d / t$ | $0.75 \sqrt{\frac{E}{F_{y}}}$ | $\operatorname{lic}_{-\infty} t d$ |

### 4.3 Categorization of Composite Box-Girder Bridges in the AASHTO LRFD BDS

AASHTO LRFD BDS Article 6.11.6.2.2, discussed in depth in Section 6.8.1 of this document, provides requirements on the nominal flexural resistance of box-girders. One of the requirements states the cross-section is part of a bridge satisfying the requirements of AASHTO LRFD BDS Article 6.11.2.3. AASHTO LRFD BDS Article 6.11.2.3 provides special restrictions on the use of LLDFs for multiple box sections. These restrictions, in addition to other cross-section and material restrictions, also form the basis for many other analysis and design simplifications. The first paragraph states:

Cross-sections of straight sections for straight bridges consisting of two or more single-cell box sections, for which the live load flexural moment in each box is determined in accordance with the applicable provisions of Article 4.6.2.2.2b, shall satisfy the geometric restrictions specified herein. In addition, the bearing lines shall not be skewed (AASHTO, 2020).

The wording on the last sentence in this quote implies PBFTGs, which have any degree of bearing line skew, will not meet the requirements of AASHTO LRFD BDS Article 6.11.2.3, therefore not meeting the requirements of AASHTO LRFD BDS Article 6.11.6.2.3. Therefore, if the bridge contains skew, the capacity of PBFTGs is limited to the yield moment. The restrictions of AASHTO LRFD BDS Article 6.11.2.3 are based on the range of bridge characteristics conforming to the study performed by Johnston and Mattock (1967).

### 4.3.1 Johnston and Mattock

Johnston and Mattock (1967) developed a computer program to analyze steel trapezoidal girders made composite with a concrete deck with no internal or external stiffeners or bracing. The purpose of this computer program was to develop LLDFs for this type of structure. The thought at the time of the study was, due to the larger torsional stiffness of closed shape members compared to I-girders, a greater transverse distribution of load would be present with this form of construction. This increase in transverse load distribution would allow the engineer to design the structure to withstand less moment than an equivalent I-girder.

The analysis of this computer program is based on a folded plate structure consisting of adjoined thin plates rigidly connected along their edges. If the bridge is not skewed, the support diaphragms can effectively be neglected, as they can prevent displacement in their planes but can offer negligible resistance out of their plane. Similar to modern methods of three-dimensional FEA, the early computer program produces displacements and forces at each joint where coplanar plate elements are joined along their edges.

This computer program assumes isotropic linear elastic materials. It should be noted actual bridge structures will not meet these assumptions but should come close at the service level. In order to test the validity of the computer program and the structural and material assumptions, the researchers built a $1 / 4$ scale model of an 80 foot long two lane bridge. The prototype bridge crosssection and the typical girder cross-section can be seen in Figures 4.4 and 4.5. The cross-section of the $1 / 4$ scale model was made as close to $1 / 4$ scale as reasonable given the limited plate thickness availability. The scale model concrete deck used reinforced mortar to simulate the prototype bridge. While the reinforcement size and spacing were reproduced to scale, the mortar had a compressive strength of 3.34 ksi , where the prototype was designed with a compressive strength of 4 ksi .


Figure 4.4: Bridge Cross-section (Johnston \& Mattock, 1967)


Figure 4.5: Girder Cross-section (Johnston \& Mattock, 1967)

Two types of loading were conducted on the scaled bridge: influence line tests and truck loading tests. In the influence line test, the researchers individually placed a load at nine places at midspan and recorded girder deflections and bottom flange strains. The distribution of lateral load and deflection was calculated using the Stallings/Yoo methodology, discussed in Section 3.3.2. The analytical influence line testing results matched well with the experimental testing results.

The second type of loading, the truck loading, was performed by placing six concentrated loads concurrently to represent $1 / 4$ scale HS-20 loading. These loads were created using a steel frame resting on the bridge with the scaled dimensions of the HS-20 truck, and a large concrete block with $1 / 4$ of the design vehicle weight was placed on top of the steel frame. Figure 4.6 presents the $1 / 4$ scale experimental truck loading at midspan. The truck was moved transversely in both design lanes of the bridge to produce the maximum desired force effect. The results of both loaded lanes were superimposed to generate two-lane loaded scenarios. The results from the experimental truck loading matched well with the analytical testing results, verifying the reliability of the computer program.


Figure 4.6: Truck Loading Test of the Model Bridge (Johnston \& Mattock, 1967)

A matrix of 24 composite box-girder bridges was generated and analyzed using the folded plate theory computer program. The variables explored as part of the study included span length, number of loaded lanes, number of box-girders, width of lanes, and girder cross-section dimensions. The bridges analyzed in the study are provided in Figure 4.7 and Table 4.2.


Figure 4.7: Typical Midspan Cross-sections for Bridges in the Analytical Study for the: (a) Bridge and (b) Girder (Johnston \& Mattock, 1967)

Table 4.2: Dimension Summary for Bridges Considered in the Analytical Study (Johnston \& Mattock, 1967)

| Bridge | Span <br> (ft) | No. of Lanes | No. of Girders | Dimension (See Fig. 9) ${ }^{\text {a }}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | C | D | E | F | G | H | J | K | L | M | N |
| 50-1 | 50 | 2 | 2 | 34 ft 0 in, | 96 | 60 | 29 | 20 | 76 | 88 | 4 | 12 | 7 | 5/8 | 9/8 | 1/2 |
| 50-2 | 50 | 2 | 3 | 34 ft 0 in . | 63 | $461 / 2$ | 28 | 20 | 43 | 55 | 4 | 12 | 6 | 1/2 | 3/8 | $1 / 2$ |
| 50-3 | 50 | 3 | 3 | 44 ft 6 in . | 86 | 52 | 289/4 | 20 | 00 | 78 | 4 | 12 | $6 / 4$ | \% 16 | \% | 1 |
| 50-4 | 50 | 3 | 4 | 44 ft 6 in , | 63 | $46^{1 / 2}$ | 28 | 20 | 43 | 55 | 4 | 12 | 6 | $1 / 2$ | $3 / 8$ | 1/2 |
| 50-5 | 50 | 4 | 4 | 55 ft 0 in. | 80 | 50 | $28^{1 / 2}$ | 20 | 60 | 72 | 4 | 12 | $61 / 2$ | $1 / 2$ | \% | $1 /$ |
| 50-6 | 50 | 4 | 5 | 55 ft 0 in . | 63 | $46^{1 / 2}$ | 28 | 20 | 43 | 55 | 4 | 12 | 6 | 1/2 | 3/8 | 1/2 |
| $75-1$ | 75 | 2 | 2 | 34 ft 0 in . | 96 | 60 | 44 | 35 | 72 | 84 | 6 | 13 | 7 | $3 / 4$ | $7 / 16$ | 5/8 |
| 75-2 | 75 | 2 | 3 | 34 ft 0 in . | 63 | $46^{1 / 2}$ | 43 | 35 | 39 | 51 | 6 | 12 | 6 | \% | 3/8 | 5 |
| 75-3 | 75 | 3 | 3 | 44 ft 6 in . | 86 | 52 | $43^{3 / 4}$ | 35 | 62 | 74 | 6 | 13 | $63 / 4$ | $3 / 4$ | $3 / 8$ | 5/8 |
| 75-4 | 75 | 3 | 4 | 44 ft 6 in . | 63 | $461 / 2$ | 43 | 35 | 39 | 51 | 6 | 12 | 6 | $5 / 8$ | \% | 5/8 |
| 75-5 | 75 | 4 | 4 | 55 ft 0 in . | 80 | 50 | $43^{1 / 2}$ | 35 | 56 | 68 | 6 | 12 | $61 / 3$ | 3/4 | \% 8 | 5/8 |
| 75-6 | 75 | 4 | 5 | 55 ft 0 in . | 63 | $461 / 2$ | 43 | 35 | 39 | 51 | 6 | 12 | 6 | 5/8 | \% | 5/8 |
| 100-1 | 100 | 2 | 2 | 34 ft 0 in . | 96 | 60 | 58 | 49 | 72 | 84 | 6 | 14 | 7 | , | 1/2 | 3/4 |
| 100-2 | 100 | 2 | 3 | 34 ft 0 in . | 63 | $46^{1 / 2}$ | 57 | 49 | 39 | 51 | 6 | 12 | 6 | 3/4 | 7/18 |  |
| 100-3 | 100 | 3 | 3 | 44 ft 6 in . | 86 | 52 | $573 / 4$ | 49 | 62 | 74 | 6 | 13 | $63 / 4$ | 1 | $7 / 16$ | 3 |
| 100-4 | 100 | 3 | 4 | 44 ft 6 in . | 63 | $46^{1 / 2}$ | 57 | 49 | 39 | 51 | 6 | 12 | 6 | 3/4 | $7 / 16$ | $3 / 4$ |
| 100-5 | 100 | 4 | 4 | 55 ft 0 in . | 80 | 50 | $571 / 2$ | 49 | 56 | 68 | 6 | 12 | 61/9 | 1 | $7 / 16$ | 1 |
| 100-6 | 100 | 4 | 5 | 55 ft 0 in . | 63 | $46^{1 / 2}$ | 57 | 49 | 39 | 51 | 6 | 12 | 6 | 3/4 | $7 / 16$ | $3 / 4$ |
| 150-1 | 150 | 2 | 2 | 34 ft 0 in . | 96 | 60 | 85 | 76 | 72 | 84 | 6 | 14 | 7 | $11 / 2$ |  | 1 |
| 150-2 | 150 | 2 | 3 | 34 ft 0 in . | 63 | $46^{1 / 2}$ | 84 | 76 | 41 | 51 | 6 | 13 | 6 | 1 | 1/2 | 1 |
| 150-3 | 150 | 3 | 3 | 44 ft 6 in . | 86 | 52 | $84^{9 / 4}$ | 76 | 58 | 70 | 8 | 13 | $63 / 4$ | $11 / 2$ | $9 / 16$ | 1 |
| 150-4 | 150 | 3 | 4 | 44 ft 6 in . | 63 | $461 / 2$ | 84 | 76 | 41 | 51 | 6 | 13 | 6 | 1 | $1 / 2$ | 1 |
| 150-5 | 150 | 4 | 4 | 55 ft 0 in . | 80 | 50 | $84^{1 / 2}$ | 76 | 52 | 64 | 8 | 12 | $61 / 2$ | $11 / 2$ | $9 / 16$ | 1 |
| 150-6 | 150 | 4 | 5 | 55 ft 0 in . | 63 | $46^{1 / 2}$ | 84 | 76 | 41 | 51 | 6 | 13 | 6 | 1 | 1/2 | 1 |

[^0]
### 4.3.2 Compact Sections

Numerous AASHTO LRFD BDS Articles specify analysis and design simplifications based on the requirements for the use of simplified LLDF equations, and therefore conform to the dimensions analyzed by Johnston and Mattock (1967). If the simplified LLDF equations can be used, and box flanges are considered fully effective, key analysis simplifications can be made.

The first of these simplifications is distortion induced stresses in box cross-sections, due to torsion, may be neglected. These stresses include shear, warping, and plate bending stresses illustrated in Figure 4.8 (White, 2022). Further simplifications include the neglection of shear stresses due to St. Venant torsional shear in the design of box-girder webs and the shear connectors between the slab and the steel girder.


Normal stresses:


Shear stresses:


Plate bending stresses:


Figure 4.8: Stresses in a Single Box-Girder Subjected to an Eccentric Load (White, 2022)

Potentially the largest simplification the AASHTO LRFD BDS allows, when the previous restrictions are met, is that the section can be labeled compact. Compact sections are permitted to exceed the moment at first yield. The fully plastic cross-section models may be used as the basis for the member resistance calculations in composite compact sections in positive bending (White, 2022). The allowance of the fully plastic cross-section increases the nominal allowable moment on the section beyond the point of first yield.

### 4.3.3 Noncompact Sections

AASHTO LRFD BDS specify if any of the restrictions of AASHTO LRFD BDS Article 6.11.6.2.2-1, including if the cross-section does meet the requirements of AASHTO LRFD BDS Article 6.11.2.3, a more detailed analysis and design procedure for composite box-girder bridges is required. This requires a refined analysis, such as three-dimensional FEA, instead of onedimensional LGA where simplified distribution factors may be used. Additionally, when designing the webs for shear and determining the number of top flange shear connectors, the design engineer must consider shear forces from both flexure and St. Venant torsion.

When a bridge does not meet the aforementioned requirements, the bridge is also labeled noncompact. Noncompact sections cannot exceed the moment at first yield. The ability of noncompact sections to develop a nominal flexural resistance greater than the moment at first yield in the presence of potentially significant St. Venant torsional shear and cross-sectional distortion stresses has not yet been demonstrated. For noncompact sections, the elastically computed stress, considering the effect of St. Venant torsional shear, in each flange due to the factored loads is compared to the yield stress modified by the appropriate flange reduction factor. Therefore, when a bridge consisting of longitudinal box-girders has any amount of support skew, not only is a more refined analysis required, but the nominal capacity of the system dramatically decreases.

### 4.4 Behavior of Skewed Steel Girder Bridges

Geometrically, a skewed bridge contains one or more bearing lines not orientated perpendicular to the longitudinal axis of the bridge. The effect of skew is largely dependent on the magnitude of skew, the skew index, and the layout of cross-frames, if present. For non-curved
skewed I-girder bridges, the behavior of the system becomes increasingly three-dimensional with increased skew (White \& Kamath, 2020).

The structural response of skewed bridges is influenced by the end bearing line and orientation of intermediate cross-frames. Skewed bearing lines cause the girders to twist to maintain continuity between the bearing stiffeners and the main longitudinal elements. The bearings are assumed to be laterally and longitudinally restrained for ease of analysis. As the girder bends under major axis rotation, the top flange longitudinally displaces relative to the bottom flange, as can be seen in Figure 4.9. However, as the girders are supported by skewed bearing lines with bearing stiffeners with relatively high in plane stiffness, the bearing stiffeners can only achieve the longitudinal displacement required by the girder major axis bending by rotating about an axis tangent to the bearing line. This rotation of the stiffener about its weak axis in turn causes a lateral displacement between the top flange and the bottom flange and a twist rotation of the main longitudinal elements. This twist finally induces girder torsional moments in the longitudinal members. Figure 4.10, adopted from White et al. (2012), demonstrates the girder end rotations of Girder G2 from Figure 4.9. Note these figures depict the reaction of I-girder members to skew required for compatibility with bearing cross-frames, but the same behavior occurs in box-girders with compatibility with bearing diaphragms.


Figure 4.9: Relative Flange Displacement in a Skewed Bridge (Sanchez, 2011)


Figure 4.10: Girder Major Axis Bending and Twist Rotations Required for Compatibility at a Skewed Bearing Cross-Frame (White et al., 2012)

### 4.4.1 Torsional Stress Effects

Torsion induced in steel girders causes normal stresses and shear stresses throughout the girder cross-section. Typical I-girders and box-girders carry these stresses in different ways. As Igirders have low torsional stiffness, torsion is carried primarily through warping. This low torsional stiffness is a result of the open cross-sectional geometry resulting in low shear flow around the perimeter of the section. This shear flow can only generate small force couples, resulting in the ability of I-girders to carry torque via St. Venant torsion to be relatively low.

However, as box-shaped girders are closed sections, they have exceptionally high torsional stiffness. Closed cells are extremely efficient at carrying torsion by means of St. Venant torsional shear flow, because the shear flow around the circumference of the box has relatively large force couple distances (AASHTO, 2020). The final state of normal stress in box-girders is the sum of any axial stress, major axis bending stress, lateral bending stress, and warping normal stress. The final state of shear stress in box-girders is the sum of any vertical shear stress, horizontal shear stress, St. Venant torsional shear stress, and warping shear stress.

### 4.4.2 Torsional Deformation Effects

In addition to causing stresses in box shaped girders, torsion also causes deformations. Depending on the extent of the skew, span length, inclusion or exclusion of cross-frames, and type of deck casting used in PBFTGs, the deformations caused by skew can have a significant effect on the constructability of the bridge. This is not an issue with the preferred method of construction of PBFTG bridges as they are pre-topped with the concrete deck and do not have any external crossframes between adjacent members. However, if a bridge is to be constructed using a traditional cast-in-place deck, the engineer and contractor constructing the bridge must be cognizant of the erection sequence and potential differential deflection between adjacent noncomposite PBFTGs.

### 4.5 Evaluation of Bottom Flange Compactness of Box-Girders

Due to the substantial structural efficiency provided by box-girders, many designers may specify thin and slender bottom flanges in large, welded plate girders, which are the closest comparison to PBFTGs. Specifying thin bottom flanges can result in problems during fabrication, transportation, erection, and the service life of the bridge. This is particularly true in regions of negative bending when the bottom flange is in compression. White et al. (2019) surveyed bridge owners and reviewed the literature and limited analytical studies to evaluate the behavior of bottom flange width and thicknesses. Discussed in this section are the findings and proposed limits for bottom flanges of box sections.

### 4.5.1 Bottom Flange Minimum Thickness Limits

A rational minimum thickness of any plate element is 2 inches. For typical welded plate girders, this is to limit welding distortions. This is not an issue with PBFTGs as the flanges and webs are cold-bent from one piece of sheet steel. However, this limit is still viable as it also limits out-of-plane deflections under the self-weight of the PBFTG. Many owners and designers prescribe larger minimum thicknesses. The TxDOT recommends a minimum flange thickness of $3 / 4$ inch, with a preferred minimum thickness of 1 inch (TSQC, 2021). The Guidelines to Design for Constructability and Fabrication (AASHTO/NSBA Steel Bridge Collaboration, 2020) recommends a $3 / 4$ inch minimum thickness due to welding considerations. As stated previously, these limits are due to welding considerations, so they may be neglected for PBFTGs. The
minimum thickness of $1 / 2$ inch for handling and erection reasons may be considered the minimum for typical PBFTGs. Additionally, these restrictions were intended to be utilized when designing box-sections with span lengths significantly longer than those where PBFTGs can be utilized.

### 4.5.2 Bottom Flange Slenderness Limits

Longitudinally unstiffened flanges are recommended to have a width-to-thickness ratio, or $b / t$, not exceeding 90 . If the flange is longitudinally stiffened, a panel width-to-thickness ratio less than 90 is advised. Flanges exceeding these limits are prone to accidental axial compression, which may be experienced during transportation or erection, or exhibit noticeable oil canning or waviness due to welding residual stresses, which are not applicable to PBFTGs. It should be noted these limits are for flanges nominally designed for tension, but which may encounter accidental or unintended compressive loading. The researchers found the moments causing compression in the bottom flange have magnitudes comparable to those generated from the self-weight of the beam in simple span loading conditions.

### 4.5.3 Behavioral Considerations Correlated with Bottom Flange Limits

The combined limits specified in Section 4.5.1 and 4.5.2 limit the sagging of bottom flanges under their self-weight plus a transverse concentrated load of 0.3 kip. Bottom flange limits not meeting the limits specified can begin to generate undesirable effects. The following behavioral considerations can be approximately correlated with the prescribed width-to-thickness, $b / t$, ratios:

- $\quad b / t>100$
- Bridge fabricators, of welded box sections, will need to be especially cautious providing welds larger than those required for strength and/or minimum size requirements. The plate must also be adequately restrained during welding to ensure minimal distortion.
- $\quad b / t>130$
- Bottom flanges will begin to deflect out-of-plane under their self-weight with a small, concentrated transverse loads more than the maximum
deflection of $1 / 300$ times the plate width. The researchers recommend when this limit is exceeded to introduce longitudinal stiffeners to the bottom flange.
- $b / t>210$
- Dynamic excitation of a flange exceeding this limit will begin to pose issues. These sections are susceptible to fatigue damage from the bottom flange breathing under cyclic tension. The initial allowable out-of-plane bow of the flange from geometric imperfections being cyclically straightened and released under live loading causes bending moments in the thin plate.

Note, the limits of box-section flanges subject to compression are significantly more stringent than those subject to tension. The AASHTO LRFD BDS effectively limits the width-tothickness ratio for flanges subjected to compression to 24 . The strength of box section flanges subjected to compression decreases relative to the yield strength as the width-to-thickness ratio increases. The ultimate strength of the plate is approximately $0.8 F_{y}$ at $b / t=40,0.6 F_{y}$ at $b / t=60$, and $0.4 F_{y}$ at $b / t=90$.

### 4.6 Simplified Evaluation of the Effect of Skew on Box-Girders

Skew greatly complicates the behavior of steel girder bridges by introducing alternate load paths and causing greater interaction between the main girders and secondary framing members (Coletti et al., 2011). In most instances, the severity of these complications is negligible; however, in other cases, the complications can be more pronounced, including fit-up issues, distortion induced loading, and unaccounted torsional effects. Unfortunately, the line between negligible and severe effects is not clear or easy to define.

### 4.6.1 Rigid Diaphragm Behavior

The effects of skewed supports on the behavior of girders can be evaluated by considering the girder major axis bending rotations and the transverse and longitudinal constraint provided by the support diaphragms. The support diaphragms are assumed effectively rigid in their own planes
but are free to rotate with respect to the support lines. The interaction of the girder major axis bending with the restraint of displacement along the support line from the diaphragm causes the girders to twist along the major axis. This twist causes the top flanges of the box-girder to displace laterally with respect to the bottom flange. The twist between the skewed supports causes a torsional moment throughout the span.

Only one type of diaphragm is present in PBFTGs: the internal bearing diaphragm. These diaphragms are present at the ends of the girder and are welded along their sides to the bottom flange and the webs of PBFTGs. The bearing diaphragm aids in the transfer of load between the concrete deck and the bearing and prevents cross-section distortion. Most box-girder bridge diaphragms are solid-plate components with relatively thin thicknesses compared to their depth. Therefore, they are typically very stiff components able to resist loads acting on the plane of the diaphragm with small deformations when compared to their weak axis deformations.

Three-dimensional FEA performed by Chong (2012) displayed bearing diaphragm thickness has a small effect on the overall torsion due to skew. Chong performed analysis on two straight skewed and horizontally curved bridges and found the discrepancy between the maximum torque in the box-girders with a diaphragm thickness of $5 / 16$ inch and 2 inch was only $0.2 \%$.

### 4.6.2 Skew Induced Torque

In non-skewed bridges, as the main longitudinal members deflect vertically, the longitudinal member rotates about the support bearing. Compatibly, the diaphragms, acting as rigid plates, rotate about the bearing lines with the longitudinal members. In skewed bridges, as the main longitudinal members deflect vertically and attempt to rotate about their support bearing, the diaphragm, acting as a rigid plate in its own plane, forces the girder to twist to maintain compatibility (Figure 4.11). The bearing diaphragms can be idealized as rigid components in their own plane and offer no resistance out of their plane. By assuming the plate is rigid in its own plane and is rigidly connected to the box-girders, the box-girders have two components of force when a load is applied vertically: one corresponding to the major axis bending rotation of the girders and one corresponding to the twist rotation of the girders, as seen in Figure 4.12 (Chong, 2021).


Figure 4.11: Lateral Displacements due to Rotation About the Line of the Support (Chong, 2012)


Figure 4.12: Rigid Diaphragm Rotation Mechanism at a Skewed Support (Chong, 2012)

The girder twist at the supports can be mathematically approximated in terms of the major axis bending rotation and the support skew angle. From the girder twist, the girder torques along the span can be calculated by multiplying the twist by the girder torsional stiffness and summing
the results from the effects of the twist at both supports. This produces a constant torsional moment due to skewed supports represented by Equation 4.8:

$$
\begin{equation*}
T_{S}=-\frac{G J}{L}\left(\phi_{y 1} \tan \theta_{1}+\phi_{y 2} \tan \theta_{2}\right) \tag{Eq. 4.8}
\end{equation*}
$$

### 4.7 SUMMARY

This chapter presented a review of the local buckling of flat plates and their effect on the capacity of sections built from them. A brief synopsis of the specifications governing the limits on the width-to-thickness ratio values of plates making up cross-sections of longitudinal elements was provided. The categorization of box-girder sections and basis for the categorization was included to give context related to distinction between compact and noncompact sections as defined in the AASHTO LRFD BDS. Using the synopsis of the local and global compactness limits, a discussion of the force effects affecting PBFTGs and an evaluation of the bottom flange compactness limits of box-girders was provided.

## CHAPTER 5: LINK SLAB LITERATURE REVIEW

### 5.1 Introduction

Link slabs are a transverse deck level connection at piers between the decks of two adjacent spans, providing a jointless bridge without continuity. The deck is made continuous across the pier, but the supporting beams or girders are not connected. In addition -to simpler designs consisting of simple spans instead of continuous spans, link slabs can allow for prefabricated bridge elements to be implemented, further reducing the total cost of the bridge.

Approximately one third of state Department of Transportations have experience with link slab applications. Of those states, two thirds have performed research or implemented the link slab system in the field, and one third have provided design provisions or official details. North Carolina, Michigan, Virginia, and New York have been identified as significant users.

### 5.2 Previous Laboratory Testing on Link Slabs

Link slabs have been explored as a potential design solution as a replacement for expansion joints. Many previous research efforts have shown link slabs have the potential to be economical and reduce degradation associated with expansion joints. As PBFTGs have grown in popularity in recent years, the potential of this innovative system in continuous spans must be evaluated. The purpose of this section is to discuss previous research findings as they relate to laboratory testing of link slabs.

### 5.2.1 Instantaneous and Time-Dependent Response and Strength of Jointless Bridge Beams (Gastal, 1986)

Gastal (1986) explored the elimination of structural joints by casting a fully continuous deck over simply supported girders. The researcher developed an FEA model to capture the elastic and inelastic response of jointless bridge decks. The model consisted of isoperimetric beam elements representing the girders and a deck with uniaxial spring elements located at the centroid
of the deck representing the link slab. The model was verified against a suite of analytical and experimental data on simply supported and continuous beams.

Gastal applied his solution to two separate design problems. Various loadings, support conditions, continuity, and construction schemes were analyzed on a two-span bridge with steel girders and a four-span bridge with prestressed concrete girders. As no experimental data was collected, the analytical methods were benchmarked against several previous experimental tests of simply supported beams.

Based on the study, Gastal concluded the behavior of jointless deck-continuous beams was significantly influenced by the support conditions of the girders. Five support conditions were analyzed by changing the conditions at the end of each girder between hinged (H) supports and roller (R) supports. Four of the arrangements, HRHR, HRRH, HRRR, and RRRR, behaved similarly to a non-continuous beam, but with slightly smaller vertical deflections and significantly less ductility. The fifth arrangement, RHHR, behaved similarly to fully continuous beams. As with the first four arrangements, the maximum capacity is controlled by yielding in the reinforcing steel, but the ultimate capacity was significantly higher than that of a non-continuous beam.

### 5.2.2 Behavior and Design of Link Slabs (Caner \& Zia, 1998)

Caner and Zia (1998) conducted a testing program on two jointless bridge decks, one on a continuous reinforced concrete deck cast on two simple-span steel beams and the other on a similar deck cast on two simple-span precast reinforced concrete beams. The steel specimen consisted of two 20.5 foot long simply-supported W12x26 steel beams with a 2 inch gap between the adjacent ends, as seen in Figure 5.1. The 24 inch wide by 4 inch thick concrete deck was made composite to the beam with shear connectors welded to the top of the steel beam over most of the two spans. The concrete deck was debonded from each steel beam equivalent to $5 \%$ of each adjacent bridge span to reduce the stiffness and stress in the link slab. The material and geometrical properties of each bridge is provided in Table 5.1.


Figure 5.1: Details of Test Specimens (Caner \& Zia, 1998)

Table 5.1: Material and Geometrical Properties of Steel and Concrete Bridges (Caner \& Zia, 1998)

| Properties | Steel bridge | Concrete bridge |
| :--- | :---: | :---: |
| Compressive strength of concrete deck | 4200 psi | 5670 psi |
| Compressive strength of girder | - | 4580 psi |
| Girder yield strength | $52,000 \mathrm{psi}$ | - |
| Girder modulus of elasticity | $30,500,000 \mathrm{psi}$ | - |
| Girder reinforcement | - | $(4) ~ \# 8$ |
| Girder reinforcement yield strength | - | $62,000 \mathrm{psi}$ |
| Girder reinforcement modulus of elasticity | - | $29,550,000 \mathrm{psi}$ |
| Girder cross-sectional area (gross) | 7.65 sq in. | $96 \mathrm{sq} \mathrm{in}$. |
| Girder moment of inertia (gross) | $204 \mathrm{in}.{ }^{4}$ | $1152 \mathrm{in} .^{4}$ |
| Deck width | 24 in. | 24 in. |
| Deck thickness | 4 in. | 4 in. |
| Link slab reinforcement | $(3) \# 6$ | $(3) \# 6$ |
| Link slab reinforcement yield strength | $63,600 \mathrm{psi}$ | $72,400 \mathrm{psi}$ |
| Link slab reinforcement modulus of clasticity | $28,500,000 \mathrm{psi}$ | $30,300,000 \mathrm{psi}$ |

The steel bridge setup was tested with four of the support conditions used by Gastal (1986): HRHR, HRRH, RRRR, and RHHR. The goal of the testing was to observe if any differences could be observed from the differing boundary conditions. Testing was performed to a maximum of $40 \%$ of the estimated ultimate load capacity to observe the behavior of the elastic range using the same specimen for each support condition. Incremental loads, strains, deflections, and crack growth were recorded.

In the elastic range of the testing, all four test cases behaved similarly, and the loaddeflection behavior was nearly identical in both spans. The deflections measured compared closely to the predicted deflections from Gastal (1986) and El-Safty (1984) when neglecting the link slab and treating the bridge as two simply supported spans. This indicates the behavior of a steel bridge with a link slab is similar to a simply supported bridge. Under the first increment of loading, a crack developed in the top face of the link slab which did not extend to the bottom face. This showed the link slab was in bending and behaved like a beam instead of a tension member.

Under ultimate loading, the load deflection remained linear until the tensile flange of the steel beams began to yield. Following yielding of the steel sections, the tensile bars in the link slab began to yield. The final failure of the link slab occurred when crushing of the compression concrete at the bottom of the link slab was observed.

### 5.2.3 Durable Link Slabs for Jointless Bridge Decks Based on Strain-hardening

 Cementitious Composites (Li et al., 2003)Due to the unique structural demand on link slabs, Li et al. (2003) explored the use of Engineered Cementitious Composite (ECC), a high-performance cementitious composite with high tensile strain capacity, high tensile strength, and excellent post-crack strain hardening properties. Specifically, the high ductility of ECC provides small crack width in the unique loading seen by link slabs.

The researchers proposed a modification to the link slab proposed by Caner and Zia (1998), which included an additional transition zone outside the standard link slab where the concrete is poured with the link slab, but shear studs are provided. The original link slab detail included termination of the debond zone and additional reinforcement spliced to the existing reinforcement
at the deck slab/link slab interface. The abrupt termination imposed a high stress concentration and ultimately made the interface the weakest part of the link slab detail. The proposed detail, shown in Figure 5.2, included a transition zone of $2.5 \%$ of each adjacent span, in addition to a conventional debonded zone of $5 \%$. The addition of shear connectors in this transition zone allowed the development of composite action over a region instead of at the interface, reducing the stress concentration.


Figure 5.2: Improved Link Slab Configuration (Li et al, 2003)

The researchers performed monotonic and cyclic fatigue testing on three full-scale specimens utilizing the improved link slab design detail (Figure 5.2) and ECC. The specimens represented link slabs found between two identical 80 foot spans (Figure 5.3). Specimen LS-1 represented the new conventional link slab construction, where the link slab was cast with the adjacent spans' concrete decks. Specimen LS-2 was prepared by removing the concrete link slab from specimen LS-1 and replacing it with an ECC link slab. This specimen represented a retrofit to an existing concrete link slab where the reinforcement remains. Specimen LS-3 was prepared by removing the ECC from Specimen LS-2 and cutting the existing reinforcement 20 inches from link slab/deck slab interface and pouring a new ECC link slab with smaller reinforcement. Specimen LS-3 was prepared to investigate the role of reinforcement ratio on the fatigue performance of ECC link slabs.


Figure 5.3: Geometry of Link Slab Specimens (Li et al, 2003)

The physical testing of the link slab was conducted on the inverted specimens with simply supported conditions at the inflection points. This reduced the total size of the experiment as two complete spans were not necessary. The specimens were statically loaded until the stress in the rebar reached $40 \%$ of its yield strength. In subsequent cyclic loading, the static loading was chosen as the mean load with the maximum load corresponding to a maximum link slab rotation when the allowable midspan deflection reaches $\mathrm{L} / 800$. The cyclic loading was carried out to 100,000 cycles.

The researchers concluded that ECC was a suitable material in link slabs. The smaller size of rebar required in a link slab utilizing ECC allowed for a link slab with lower flexural stiffness. This, coupled with the high ductility inherent with ECC, provided highly beneficial properties for link slabs. The improved transition zone detail eliminated the cracking previously found at the link slab/concrete deck interface, further reducing the amount of cracking found in link slabs.

### 5.2.4 High Skew Link Slab Bridge System with Deck Sliding Over Backwall or Backwall Sliding Over Abutments (Aktan \& Attanayake, 2011)

A detailed analysis of skewed link slabs and calculation of the associated moment and force envelopes at the link slab section directly over the pier centerline was performed by Aktan \& Attanayake (2011). Finite element modeling was performed on a suite of bridges with consistent span length, width, and girder type while varying the support conditions and degree of skew from $0^{\circ}$ to $45^{\circ}$. The results of the finite element modeling were used to develop design recommendations for bridges with high degrees of skew.

The study determined the moment generated in a link slab under temperature gradient loads was not dependent on span length. It was also observed the moment developed in a link slab under live loads decreases with span length. This indicates the system behaves more like two independent simple spans with increased span length. Further, the researchers found the minimum amount of reinforcement required by the AASHTO LRFD BDS to be adequate for most skewed link slabs with either the HRRR or RRHR boundary conditions. Additional reinforcement in the bottom layer may be required to resist large tensile stresses which can develop near the boundaries of the debonded region.

### 5.2.5 Utilization of Ultra-High Performance Concrete in New York (Royce, 2016)

Royce (2016) presented multiple case studies on the utilization of prefabricated bridge elements and systems with field cast UHPC joints. The researcher discussed the achievement of ABC , but had concern with the durability of the structures, specifically the performance of joints between prefabricated elements. He discussed each case study with respect to the UHPC joints and the lessons learned from each. The researcher found the following to be beneficial in the utilization
of link slabs: tight leak-free formwork due to the highly flowable nature of UHPC, exposed aggregate finish on the concrete slab/joint surface to improve bonding, use of epoxy coated or stainless steel rebar to avoid macro corrosion, and application of heat for several hours during curing to increase the strength gain. Ultimately, when construction speed is needed, precast components with UHPC joints provide good value.

In addition, Royce (2016) included a section specifically on UHPC link slabs. The New York State Department of Transportation uses a detail, like Figure 5.5, to eliminate transverse deck joints whenever feasible. Girder rotations at link slabs are accommodated by micro cracking within the UHPC link slab, as the UHPC can develop ultimate tensile strains up to 0.007 . No visible cracks were reported on the link slabs in the studies, but proper design was crucial as there are several factors which influence the performance of the link slab. He also noted poor design of the link slab may cause failure of not only the link slab, but may cause structural damage to other bridge components.


Figure 5.4: Typical UHPC Link Slab Connection Detail (Graybeal, 2014)

### 5.2.6 Evaluation of High-Performance Fiber-Reinforced Concrete for Bridge Deck Connections, Closure Pours, and Joints (Hoomes et al., 2017)

The researchers evaluated the mechanical properties and performance of fiber-reinforced concrete and other cementitious composites in controlling cracking for bridge deck closure pours, such as link slabs (Hoomes et al., 2017). The high-performance fiber reinforced concretes
evaluated include: ECC, hybrid fiber-reinforce concrete with steel and synthetic fibers, hybrid fiber-reinforce concrete with only synthetic fibers, and UHPC. A multitude of fresh and hardened material tests were performed, and the results were compared against Virginia Department of Transportation Class A4 concrete, typical for bridge decks. The researchers concluded mixtures which underwent deflection hardening exhibited a series of fine cracks, instead of fewer large cracks. These small cracks reduced the amount of damaging material which penetrated the bridge deck. The researchers also concluded that UHPC attained the highest stress capacity, but the material did not undergo strain hardening, as seen in Figure 5.4, which may have been in part due to settling of the steel fibers to the bottom of the forms.


Figure 5.5: Flexural Performance of High-Performance Fiber-Reinforced Concrete Systems
(Hoomes et al., 2017)

### 5.3 Previous Field Testing of Link Slabs

Following the successful laboratory testing of link slabs, field testing could take place to demonstrate full-scale link slabs subjected to actual live loading. Most link slabs have been used for rehabilitation purposes. Currently most link slabs in the United States are designed using the methodology proposed by Caner and Zia (1998). The purpose of this section is to discuss the methodology of testing and the results of previous live load field testing.

### 5.3.1 Behavior, Analysis, and Design of an Instrumented Link Slab Bridge (Wing \&

## Kowalsky, 2005)

Wing and Kowalsky (2005) performed research to assess the long-term performance of jointless link slabs. The North Carolina Department of Transportation rehabilitated an existing bridge and installed the first link slabs in the state, which were monitored over the course of a year. The original bridge included three interior expansion joints but after rehabilitation, only the center expansion joint remained with the other two being replaced with link slabs. The primary focus of the research was to determine if the bridge design assumptions were valid, specifically if link slab bridge girders can be assumed to be simply supported for dead and live load.

The bridge was instrumented to monitor seasonal and service level loading by measuring temperature, strain, and deflections. Following a year of monitoring, including a controlled live load field test, the link slab performed well under traffic and thermal induced loads. While the thermal loads induced a higher rotation than traffic loading, both demands were much smaller than the assumed design rotational demand. As a result, it is acceptable, although conservative, to design the girders for simply supported spans. Additionally, while cracking in the link slab was found to exceed the design criteria, it did not reduce the serviceability of the bridge. It was determined the cracking in the link slab was due to a saw cut forcing all the deformation to occur in one crack. The researchers suggested a larger crack limit to be developed in conjunction to saw cuts in link slabs. Finally, a design approach based on rotation demand and crack control criteria of the bridge was proposed to size the reinforcement in the link slab.

### 5.3.2 Field Demonstration of Durable Link Slabs for Jointless Bridge Decks based on Strain-hardening Cementitious Composites (Li et al., 2005)

Li et al. (2005) performed a field demonstration of the improved design detail proposed by Li et al. (2003). The link slab was designed by utilizing a set of design guidelines produced by the Michigan Department of Transportation incorporating ECC. Large scale test mixes were poured to provide insight into the mixing of large quantities of ECC material in conventional concrete mixers. These tests showed large scale mixing of ECC is possible, and the ECC maintains its fresh material properties up to one hour. During the preparation of the link slab construction, several raw material substitutions were made addressing the availability of some raw materials. As in the
previous large-scale mixes, the final demonstration mix provided by the supplier met all the requirements set by the researchers.

Following the first phase of the partial width construction, several shrinkage cracks developed in the link slab. The cracks tended to form around the reinforcing bars and propagated radially outward. The additional cracking was attributed to higher water-to-cement ratios due to excessive washing of the concrete trucks. Changes were made to the mix design for the second half of the partial width construction, and a significant reduction in cracking occurred.

A full-scale live load field test was performed to assess the validity of the design approach proposed by Li et al. (2003). The parameters of interest included the surface strains of the link slab and the end span rotations. A strong correlation was found between girder rotations and strain measured on the link slab surface. The compatibility between the strain predicted from the measured girder rotation and the strain transducers in the link slab validate the design methodology.

### 5.4 SUMMARY

Several researchers over multiple decades have researched the economic potential of transverse link slabs in continuous span bridges. Many researchers found this technology to be a valid replacement of traditional expansion joints. However, while many researchers concluded the system is effective in removing these joints, many of these systems are limited by complex concrete materials, which would increase the total cost of the system. In addition, link slabs have not been explored in conjunction with PBFTGs. Therefore, modular PBFTGs joined by transverse link slabs would present a competitive solution in the short-span continuous bridge market.

## CHAPTER 6: OVERVIEW OF CURRENT AASHTO SPECIFICATIONS RELATING TO PBFTGS

### 6.1 Introduction

The AASHTO LRFD BDS employ the LRFD methodology, utilizing probability-based factors to achieve a specific probability of failure. AASHTO LRFD BDS Section 6 covers the design of steel structures, where AASHTO LRFD BDS Article 6.11 specifically relates to the flexure of steel tub sections. While some of sections of the AASHTO LRFD BDS do not directly apply, or are not accurate, in the analysis and capacity of PBFTGs, a review of the current governing provisions is necessary to understand the philosophy of the AASHTO LRFD BDS. This understanding will be used to propose modifications to the provisions, increasing the applicability of PBFTGs in the short-span steel bridge market.

### 6.2 LoAds and Load Combinations

AASHTO LRFD BDS Sections 1 and 3 discuss the various aspects of loads, and Section 4 discusses the combinations of loads for which the designer must consider. This section will discuss and review the various applicable loads and limit state load combinations to avoid any nongoverning load combinations.

### 6.2.1 Structural Loads

Bridge loads are divided into two main categories: permanent loads and transient loads. Permanent loads are assumed to be either constant upon completion of construction or varying over a long-time interval and consist of dead load and earth loads. Transient loads can vary over a short time interval relative to the lifetime of the structure and consist of vehicular load and environmental loads, such as snow, wind, or seismic. For the purposes of this review, only dead loads and live loads will be reviewed, as they are the chief components of the Strength I, Service II, and Fatigue I and II load combinations (see Section 6.2.2).

Dead loads include the weight of all components of the structure, including appurtenances, utilities, wearing surface, and future overlays. The AASHTO LRFD BDS provides traditional unit weights to calculate the total dead load, as seen in Table 6.1.

Table 6.1: Unit Weights (AASHTO, 2020)

| Material |  | Unit Weight (kcf) |
| :---: | :---: | :---: |
| Aluminum Alloys |  | 0.175 |
| Bituminous Wearing Surfaces |  | 0.140 |
| Cast Iron |  | 0.450 |
| Cinder Filling |  | 0.060 |
| Compacted Sand, Silt, or Clay |  | 0.120 |
| Concrete | Lightweight | 0.110 to 0.135 |
|  | Normal Weight with $f_{c}^{\prime} \leq 5.0 \mathrm{ksi}$ | 0.145 |
|  | Normal Weight with $5.0<f_{c}^{\prime} \leq 15.0 \mathrm{ksi}$ | $0.140+0.001 f_{c}^{\prime}$ |
| Loose Sand, Silt, or Gravel |  | 0.100 |
| Soft Clay |  | 0.100 |
| Rolled Gravel, Macadam, or Ballast |  | 0.140 |
| Steel |  | 0.490 |
| Stone Masonry |  | 0.170 |
| Wood | Hard | 0.060 |
|  | Soft | 0.050 |
| Water | Fresh | 0.0624 |
|  | Salt | 0.0640 |
| Item |  | Weight per Unit Length (klf) |
| Transit Rails, Ties, and Fastening per Track |  | 0.200 |

Dead loads can be broken down into the dead load of structural components and nonstructural attachments (DC) and the dead load of wearing surfaces and utilities (DW). Structural component dead loads can be further broken down into noncomposite dead loads ( $\mathrm{DC}_{1}$ ) and composite dead loads $\left(\mathrm{DC}_{2}\right) . \mathrm{DC}_{1}$ loads are resisted by the noncomposite section before the concrete deck is composite with the girders and include the girder self-weight, the wet concrete deck, concrete haunches, overhang tapers, stay-in-place metal formwork, shear studs, and crossframes. $\mathrm{DC}_{2}$ loads are resisted by the composite section after the concrete deck and steel girder become composite and include the weight of the traffic barriers, pedestrian railing, and sidewalks. DW loads are also resisted by the composite section but are differentiated from other dead loads because they are slightly more variable than the dead loads discussed earlier.

Vehicular live load (LL) on the roadway of bridges is designated as the HL-93. The live load model, consisting of either a design truck or tandem, applied concurrently with a design lane
load, was developed to represent typical truck loading to produce shears and moments permitted on highways. The design lane consists of a 0.64 klf distributed load present on spans maximizing the force effect in consideration. The weights and spacings of axles for the design truck are shown in Figure 6.1. The distance between the two 32-kip axles is varied between 14 feet and 30 feet to produce the maximum force effect in consideration. The design tandem consists of two 25-kip axles spaced at a constant 4 feet. Only those areas or parts of areas positively contributing to the extreme force effect should be loaded. The loaded area should be determined by the points were the influence surface meets the centerline of the design lane.


Figure 6.1: Characteristics of the Design Truck (AASHTO, 2020)

Multiple presence factors are employed to account for the probability of multiple vehicles concurrent in adjacent lanes. As noted in the AASHTO LRFD BDS commentary, the multiple presence factors are included in the approximate equations for live load distribution, but when
performing graphical analysis such as lever rule or Special Analysis, the designer should consider these factors. It should also be noted multiple presence factors are not to be used when assessing fatigue, as one design truck is used regardless of the number of possible lanes loaded. The multiple presence factors are listed in Table 6.2.

Table 6.2: Multiple Presence Factors (AASHTO, 2020)

| Number of Loaded Lanes | Multiple Presence <br> Factors, $m$ |
| :---: | :---: |
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| $>3$ | 0.65 |

The dynamic load allowance (IM) accounts for dynamic effects caused primarily by hammering effects from the wheel assembly interacting with riding surface discontinuities, such as deck joints, cracks, or potholes, and the response of the bridge due to long undulations in the roadway pavement from the resonant excitation because of loading. This effect is accounted for by modifying the static wheel loads from the design truck and design tandem. The dynamic load allowance increases the static loads for most components and limit states by $33 \%$. For the fatigue limit state, the live load is increased by $15 \%$, and for deck joints at all limit states, the live load is increased by $75 \%$.

### 6.2.2 Load Combinations

LRFD is a scheme of designing structures where both the resistance and load effects are statistically modified. The AASHTO LRFD BDS employs several resistance and load factors to account for various types of uncertainties (Galambos, 1981). Each component and connection in any given structure designed using the AASHTO LRFD BDS must satisfy the following equation:

$$
\begin{equation*}
\sum \eta_{i} \gamma_{i} Q_{i} \leq \phi R_{n}=R_{r} \tag{Eq. 6.1}
\end{equation*}
$$

Where:

$$
\begin{aligned}
\gamma_{i}= & \text { load factor: a statically based multiplier applied to force effects } \\
\phi= & \text { resistance factor: a statistically based multiplier applied to nominal resistance } \\
\eta_{i}= & \text { load modifier: a factor relating to ductility, redundancy, and operational } \\
& \text { classification } \\
Q_{i}= & \text { force effect } \\
R_{n}= & \text { nominal resistance } \\
R_{r}= & \text { factored resistance }
\end{aligned}
$$

Ductility, redundancy, and operational classification are included in the $\eta_{i}$ factor to encourage enhanced ductility and redundancy and provide additional reliability for more important bridges. As of the $9^{\text {th }}$ Edition of the AASHTO LRFD BDS (2020), modifications for the ductility and redundancy factors have not been implemented.

Service limit states account for restrictions of excessive stress, deformation, and crack width under service level loads. These load combinations are experience based and cannot always be derived from strength or statistical calculations, unlike other limit states. The service limit states are listed as follows:

- Service I: load combination relating the normal operational use of the bridge with a 55 mph wind
- Service II: load combination intended to control yielding steel structures and slip of slip critical connections due to vehicular load
- Service III: load combination for longitudinal analysis relating to tension in prestressed concrete
- Service IV: load combination relating to tension in prestressed concrete columns

Fatigue limit states account for restrictions of stress range caused by a single design truck occurring over an expected number of cycles. These limit states are intended to limit crack growth under repetitive loads to prevent fracture during the design life of the bridge. The fatigue limit states are listed as follows:

- Fatigue I: load combination related to infinite-induced fatigue life
- Fatigue II: load combination related to finite-induced fatigue life

Strength limit states ensure the local and global strength and stability of members resists the statistically significant load combination the bridge is expected to experience in its life. The stability or yielding of each structural element is considered, and if any element resistance has been exceeded, the bridge resistance has been exceeded. Excessive distress and structural damage may occur under the strength limit states provided below, but overall structural integrity is expected to be maintained:

- Strength I: load combination for normal vehicular use without wind
- Strength II: load combination for the use of owner-specified special design vehicles, evaluation permit vehicles, or both without wind
- Strength III: load combination for design wind speeds
- Strength IV: load combination for high dead to live load force effects
- Strength V: load combination for normal vehicular use with a wind speed of 80 mph

Extreme event limit states ensure the structural survival of the bridge. Such extreme events include earthquakes, floods, vehicle collisions, or ice floes. These loading situations are considered unique occurrences with severe operational impact whose return period may be significantly larger than the design life of the bridge. The extreme event limit states are listed as follows:

- Extreme Event I: load combination for earthquake
- Extreme Event II: load combination for ice load, collisions by vessels and vehicles, flooding, and other hydraulic events


### 6.3 Structural Analysis and Evaluation

AASHTO LRFD BDS Article 4.6.2.2.1 provides criteria which must be met to utilize the provided LLDFs for moment and shear in one-dimensional LGA. While performing LGA, correct distribution of the live load to individual girders is paramount to establish the total moment and shear on interior and exterior girders. There are several restrictions on the use of multiple steel
box-girders in LGA, and bridges not meeting these restrictions must be analyzed using more refined methods of analysis. These restrictions will be discussed in depth in Section 6.4. For bridges constructed of multiple steel box-girders topped with a concrete deck, only one equation is used for determining LLDFs, regardless of the number of lanes loaded. The applicable expression is listed as Equation 6.2 and is valid for moment and shear in interior or exterior beams. The range of applicability of Equation 6.2 is provided by Equation 6.3. Multiple presence factors are already incorporated into the expression for the calculation of distribution factors by approximate means.

$$
\begin{align*}
& 0.05+0.85 \frac{N_{L}}{N_{b}}+\frac{0.425}{N_{L}}  \tag{Eq. 6.2}\\
& 0.5 \leq \frac{N_{L}}{N_{b}}<1.5 \tag{Eq. 6.3}
\end{align*}
$$

Where:
$N_{L}=$ number of design lanes as specified in AASHTO LRFD BDS Article 3.6.1.1.1
$N_{b}=$ number of girders

### 6.4 Cross-Section Proportion Limits

Equations 6.4 and 6.5 provide practical upper limits on the slenderness of webs without or with longitudinal stiffeners, respectfully. These equations are valid for minimum yield strengths of the web up to 100 ksi and further allow web-bend buckling to be disregarded in the design of composite sections in positive flexure. The webs of box sections must meet the following proportion limits:

Webs without longitudinal stiffeners:

$$
\begin{equation*}
\frac{D}{t_{w}} \leq 150 \tag{Eq. 6.4}
\end{equation*}
$$

Webs with longitudinal stiffeners:

$$
\begin{equation*}
\frac{D}{t_{w}} \leq 300 \tag{Eq. 6.5}
\end{equation*}
$$

Where:

$$
D=\text { depth of the web plate measured along the slope }
$$

$t_{w}=$ web thickness

Equations 6.6 through 6.8 apply to the top flanges of tub sections. Equation 6.6 provides a lower limit of flange thickness to ensure the flange will not excessively distort when welded to the web. Equation 6.7 provides a lower limit on flange width to ensure adequate strength and moment rotation characteristics. Equation 6.8 ensures the flanges provide adequate restraint against web shear buckling and the boundary conditions in the web-flange juncture used in buckling formulations are sufficiently accurate. It should be noted that all PBFTGs do not meet Equation 6.8, as the flange thickness and the web thickness are the same as they are formed from the same plate. However, this is not an issue, as the intended purpose of system is to ship the units pretopped with the concrete deck, providing adequate rigidity to the top flange.

$$
\begin{align*}
& \frac{b_{f}}{t_{f}} \leq 12.0  \tag{Eq. 6.6}\\
& b_{f} \geq D / 6  \tag{Eq. 6.7}\\
& t_{f} \geq 1.1 t_{w} \tag{Eq. 6.8}
\end{align*}
$$

Where:
$b_{f}=$ full width of the widest top flange width within the section under consideration
$t_{f}=$ flange thickness
$D=$ depth of the web plate measured along the slope
$t_{w}=$ web thickness

Cross-sections of bridges consisting of two or more box sections must meet additional specifications to utilize the LLDFs mentioned previously. First, the bearing lines must not be skewed. Second, the distance center-to-center of flanges of adjacent boxes, $a$, shall be between 80 to $120 \%$ of the distance center-to-center of the flanges of each adjacent box, $w$, as illustrated in Figure 6.2. Third, the inclination of the web plates to a plane normal to the bottom flange shall not exceed a 1 to 4 slope. Lastly, the cantilever overhang of the concrete deck shall not be greater than
$60 \%$ of the distance center-to-center of flanges of adjacent boxes, $a$. If these restrictions are met, not only are the LLDFs in the AASHTO LRFD BDS applicable, but the shear due to St. Venant torsion and secondary distortional stresses may be neglected.


Figure 6.2: Center-to-Center Flange Distance (AASHTO, 2020)

### 6.5 DESIGN FOR CONSTRUCTABILITY

Issues such as deflection, strength of steel, and stability during critical stages of construction is addressed using AASHTO LRFD BDS Article 6.11.3. Nominal yielding or reliance on post-buckling resistance is not permitted for main load-carrying members during critical stages of construction. AASHTO LRFD BDS Article 6.11.3 references AASHTO LRFD BDS Article 6.10.3 significantly but provides some key differences. Unlike plate girders, where different flange thickness may be used throughout the bridge span, tub girders must maintain constant cross-section geometry. Internal or external diaphragms or cross-frames ensure deformations of the crosssection is controlled.

For tub girders in flexure, Equations 6.9 through 6.11 must be satisfied. Although these equations apply to flanges of I-sections, they may also be applied to top flanges of tub sections where struts between the tub girder flanges may be considered brace points. The AASHTO LRFD BDS distinguishes between discretely and continually braced flanges, as flange lateral bending need not be considered if the flange is continuously braced. It should be noted that for sections with compact or noncompact webs, Equation 6.10 need not be checked.

For discretely braced flanges in compression:

$$
\begin{align*}
& f_{b u}+f_{l}<\phi_{f} R_{h} F_{y c}  \tag{Eq. 6.9}\\
& f_{b u}+\frac{1}{3} f_{l} \leq \phi_{f} F_{n c} \\
& f_{b u} \leq \phi_{f} F_{c r w}
\end{align*}
$$

Eq. 6.10
Eq. 6.11

For discretely braced flanges in tension:

$$
\begin{equation*}
f_{b u}+f_{l}<\phi_{f} R_{h} F_{y t} \tag{Eq. 6.12}
\end{equation*}
$$

For continuously braced flanges in tension or compression:

$$
\begin{equation*}
f_{b u}<\phi_{f} R_{h} F_{y t} \tag{Eq. 6.13}
\end{equation*}
$$

In addition to the above requirements specified in AASHTO LRFD BDS Article 6.10.3, the following requirements are specified in AASHTO LRFD BDS Article 6.11 .3 for noncomposite box flanges during critical stages of construction:

$$
\begin{align*}
f_{b u} & \leq \phi_{f} F_{n c}  \tag{Eq. 6.14}\\
f_{b u} & \leq \phi_{f} F_{c r w}  \tag{Eq. 6.15}\\
f_{b u} & <\phi_{f} R_{h} F_{y f} \Delta \tag{Eq. 6.16}
\end{align*}
$$

In which:

$$
\begin{align*}
& \Delta=\sqrt{1-3\left(\frac{f_{v}}{F_{y f}}\right)^{2}}  \tag{Eq. 6.17}\\
& f_{v}=\frac{T}{2 A_{o} t_{f}}
\end{align*}
$$

Eq. 6.18

Where:

$$
\begin{aligned}
\phi_{f} & =\text { resistance factor for flexure } \\
f_{b u} & =\text { flange stress calculated without consideration of flange lateral bending }
\end{aligned}
$$

$f_{l}=$ flange lateral bending stress
$F_{c r w}=$ nominal bend-buckling resistance for webs
$F_{n c}=$ nominal flexural resistance of box flanges in compression
$R_{h}=$ hybrid girder factor
$F_{y c}=$ specified minimum yield strength of the compression flange
$F_{y t}=$ specified minimum yield strength of the tension flange
$F_{y f}=$ specified minimum yield strength of the flange under consideration
$\Delta=$ reduction factor for the maximum stress in a box flange
$f_{v}=$ St. Venant torsional shear stress in the flange to the factored loads at the section under consideration
$A_{o}=$ enclosed area with the box section
$T=$ internal torque due to the factored loads

Webs shall satisfy the following shear requirements during critical stages of construction:

$$
\begin{align*}
& V_{u} \leq \phi_{v} V_{c r}  \tag{Eq. 6.19}\\
& V_{u i}=\frac{V_{u}}{\cos \theta} \tag{Eq. 6.20}
\end{align*}
$$

Where:
$\phi_{v}=$ resistance factor for shear
$V_{u}=$ vertical shear due to the factored loads on one inclined web of a box section
$V_{c r}=$ shear-buckling resistance
$V_{u i}=$ shear due to the factored loads along one inclined web of a box section
$\theta=$ the angle of inclination of the web plate to the vertical

### 6.6 Design for Serviceability

Service limit states ensure the durability and serviceability of the bridge and its components under normal traffic loading, traditionally termed service loads (Mertz, 2022a). Two of the serviceability limit states provided in the AASHTO LRFD BDS are applicable to steel bridges.

The first limit state is intended to limit elastic deformations, while the second is intended to limit permanent deformations.

The Service I limit state load combination is an optional limit state in the AASHTO LRFD BDS; however, most states require live load deflection control. It is intended to control human perception of deflection, but bridge frequency or period would be a better measure to control intended perceived deflections. Dynamic analysis could address this, but non-seismic bridge design does not typically include dynamic analysis. AASHTO LRFD BDS Article 2.5.2.6 lists suggested limits for elastic live load deflections for steel structures (Figure 6.3).
> - Vehicular load, general ............................ Span/800,
> - Vehicular and pedestrian loads ..............Span/1,000,
> - Vehicular load on cantilever arms...... Span/300, and - Vehicular and pedestrian loads on cantilever arms Span/375.

Figure 6.3: Live Load Deflection Limits (AASHTO, 2020)

The Service II limit state ensures permanent deformation caused by localized yielding does not impair the rideability of the structure. The degree of composite action between the concrete deck and the steel girder determines which sections the Service II loads are applied. Upon investigation of the limit states and their load combinations, Service II limit states could only govern if the section is compact. Note, while I-girders may redistribute negative moment at interior pier sections, box sections may not utilize moment redistribution. The following equations must be satisfied to prevent objectionable permanent deformations under severe traffic loading. It should be noted AASHTO LRFD BDS Article 6.11 .4 states the provisions of AASHTO LRFD BDS Article 6.10 .4 shall apply with some modifications. Therefore, some equations, such as Equations 6.21 and 6.22, from AASHTO LRFD BDS Article 6.10.4 have modifications.

For the top steel flange of composite sections:

$$
\begin{equation*}
f_{f}<0.95 R_{h} F_{y f} \tag{Eq. 6.21}
\end{equation*}
$$

For the bottom steel flange of composite sections:

$$
\begin{equation*}
f_{f}<0.95 R_{h} F_{y f} \tag{Eq. 6.22}
\end{equation*}
$$

Where:
$f_{f}=$ flange stress at the section under consideration due to the Service II loads calculated with consideration of flange lateral bending
$F_{y f}=$ specified minimum yield strength of the flange under consideration
$R_{h}=$ hybrid girder factor

Web bend buckling should be considered for all sections except for composite sections in positive flexure where the web satisfies the requirement of AASHTO LRFD BDS Article 6.10.2.1.1. For sections not meeting those requirements, the following restriction is provided to ensure the web has sufficient capacity to resist web bend buckling:

$$
\begin{equation*}
f_{c} \leq F_{c r w} \tag{Eq. 6.23}
\end{equation*}
$$

Where:
$f_{c}=$ compression-flange stress at the section under consideration due to the Service II loads calculated without consideration of flange lateral bending
$F_{c r w}=$ nominal bend-buckling resistance for webs

### 6.7 Design for Fatigue

Fatigue in metals is the process of initiation and growth of cracks under the action of repetitive tensile loads (Mertz, 2022(b)). Fatigue cracks can develop at stress levels significantly lower than those associated with cracking under static loading conditions. In the design of elements subject to fatigue, consideration of the total number of cycles and the element type are key. The

AASHTO LRFD BDS addresses two types of fatigue: load induced fatigue and distortion induced fatigue, discussed in AASHTO LRFD BDS Articles 6.6.1.2 and 6.6.1.3, respectively.

For load induced fatigue, only the live load plus impact needs to be considered when determining the stress range in the short-term composite section. Residual stresses need not be considered, and regions where the unfactored dead load produces compressive stress higher than the live load tensile stress also need not be considered. Connection and fabrication details are arranged by detail category where differing fatigue details can withstand differing fatigue threshold ranges as discussed in AASHTO LRFD BDS Table 6.6.1.2.3-1.

For load-induced fatigue considerations, each detail must satisfy:

$$
\begin{equation*}
\gamma(\Delta f) \leq(\Delta F)_{n} \tag{Eq. 6.24}
\end{equation*}
$$

Where:
$\gamma=$ load factor specified in ASSHTO LRFD BDS Table 3.4.1-1 for the fatigue load combination
$(\Delta f)=$ force effect, live load stress range due to the passage of the fatigue load
$(\Delta F)_{n}=$ nominal fatigue resistance

For the Fatigue I load combination and infinite life:

$$
\begin{equation*}
(\Delta F)_{n}=(\Delta F)_{T H} \tag{Eq. 6.25}
\end{equation*}
$$

Where:
$(\Delta F)_{T H}=$ constant amplitude fatigue threshold taken from AASHTO LRFD Table 6.6.1.2.5-3

For the Fatigue II load combination and finite life

$$
\begin{equation*}
(\Delta F)_{n}=\left(\frac{A}{N}\right)^{\frac{1}{3}} \tag{Eq. 6.26}
\end{equation*}
$$

In which:

$$
\begin{equation*}
N=(365)(75) n(A D T T)_{S L} \tag{Eq. 6.27}
\end{equation*}
$$

Where:
$A=$ detail category constant
$N=$ number of fatigue cycles over the design life of the structure
$n=$ number of stress range cycles per truck passage
$(A D T T)_{S L}=$ single lane $A D T T$

Distortion-induced fatigue is caused by secondary out-of-plane stresses not normally quantified in typical analysis and design of bridges. Rigid load paths must be provided to adequately transfer the forces from transverse bracing members from the web of the longitudinal element to the flanges (Mertz, 2022(b)). Load paths are established by bolting or welding connecting diaphragms, internal or external diaphragms, and floor beams or stringers to the compression and tension flanges.

Unstable crack growth, or fracture, occurs when the effects of total stress and flaw size exceed a critical value, commonly referred to as the fracture toughness (Mertz, 2022(b)). AASHTO LRFD BDS Article 6.6.2 defines the required fracture toughness based on the Charpy V-Notch impact test. All primary members and components subject to a net tensile stress under the Strength I load combination require Charpy V-Notch testing. Each member or component must be able to absorb a specified amount of energy depending on the steel grade and the applicable minimum temperature zone.

### 6.8 Design for Strength

The strength limit state ensures adequate strength and stability of the bridge against statistically predicted moments and shears over the entire life of the bridge. The AASHTO LRFD BDS typically provides strength limit state functions based on moments or shears, but in limited circumstances, such as noncompact girders, the strength limit states are defined by stress. This is mostly due to the application of loads to different sections. AASHTO LRFD BDS Article 6.11.6
describes the strength limit state for box-girders and directs the user to the appropriate articles for the design of box-girders in positive and negative flexure.

### 6.8.1 General Requirements

Sections which meet the requirements of AASHTO LRFD BDS Article 6.11.6.2.2 qualify as compact and are permitted to exceed the moment at first yield according to the provisions of AASHTO LRFD BDS Article 6.10.7. The following are the required limits:

- The section is not horizontally curved.
- The section is straight, without skew.
- The specified minimum yield strength of the flanges and web do not exceed 70 ksi .
- The web satisfies the cross-section proportion limit in AASHTO LRFD BDS Article 6.11.2.1.2.
- The section meets the special restrictions on the use of LLDFs discussed in AASHTO LRFD BDS Article 6.11.2.3.
- The box flange is fully effective as specified in AASHTO LRFD BDS Article 6.11.1.1.
- The section satisfies the following web slenderness limit:

$$
\begin{equation*}
\frac{2 D_{c p}}{t_{w}} \leq 3.76 \sqrt{\frac{E}{F_{y c}}} \tag{Eq. 6.28}
\end{equation*}
$$

Where:
$D_{c p}=$ depth of the web in compression at the plastic moment
$t_{w}=$ web thickness
$E=$ modulus of elasticity of steel
$F_{y c}=$ specified minimum yield strength of the compression flange

Compact sections shall satisfy the requirements specified in AASHTO LRFD BDS Article 6.11.7.1. Sections which do not meet the requirements listed above are labeled noncompact and must meet the requirements specified in AASHTO LRFD BDS Article 6.11.7.2. The ability of such sections to develop a nominal flexural resistance greater than the moment at first yield in the
presence of potentially significant St. Venant torsional shear and cross-sectional distortion stresses has not been demonstrated (AASHTO, 2020).

Compact and noncompact sections shall satisfy the ductility requirement as follows:

$$
\begin{equation*}
D_{p} \leq 0.42 D_{t} \tag{Eq. 6.29}
\end{equation*}
$$

Where:
$D_{p}=$ distance from the top of the concrete deck to the neutral axis of the composite section at the plastic moment
$D_{t}=$ total depth of the composite section
6.8.2 Flexural Resistance of Composite Sections

For compact sections, the section shall meet the following provisions:

$$
\begin{equation*}
M_{u} \leq \phi_{f} M_{n} \tag{Eq. 6.30}
\end{equation*}
$$

Where:
$\phi_{f}=$ resistance factor for flexure
$M_{n}=$ nominal flexural resistance of the section
$M_{u}=$ bending moment about the major axis of the cross-section due to the factored loads at the section under consideration

If $D_{p} \leq 0.1 D_{t}$, then:

$$
\begin{equation*}
M_{n} \leq M_{p} \tag{Eq. 6.31}
\end{equation*}
$$

Otherwise:

$$
\begin{equation*}
M_{n}=M_{p}\left(1.07-0.7 \frac{D_{p}}{D_{t}}\right) \tag{Eq. 6.32}
\end{equation*}
$$

Where:
$D_{p}=$ distance from the top of the concrete deck to the neutral axis of composite section at the plastic moment
$D_{t}=$ total depth of the composite section
$M_{n}=$ nominal flexural resistance of a section
$M_{p}=$ plastic moment of composite section

In a continuous span, the nominal flexural resistance is limited by:

$$
\begin{equation*}
M_{n} \leq 1.3 R_{h} M_{y} \tag{Eq. 6.33}
\end{equation*}
$$

Where:
$M_{n}=$ nominal flexural resistance of a section
$R_{h}=$ hybrid girder factor determined as specified in Article 6.10.1.10.1
$M_{y}=$ yield moment

For noncompact sections, the section shall meet the following provisions:
At the strength limit state, compression flanges shall satisfy:

$$
\begin{equation*}
f_{b u} \leq \phi_{f} F_{n c} \tag{Eq. 6.34}
\end{equation*}
$$

Where:

$$
\begin{aligned}
\phi_{f}= & \text { resistance factor for flexure } \\
f_{b u}= & \text { flange stress calculated without consideration of flange lateral bending or } \\
& \text { longitudinal warping } \\
F_{n c}= & \text { nominal flexural resistance of box flanges in compression }
\end{aligned}
$$

The nominal flexural resistance of the compression flange of closed-box sections shall be taken as:

$$
\begin{equation*}
F_{n c}=R_{b} R_{h} F_{y c} \Delta \tag{Eq. 6.35}
\end{equation*}
$$

In which:

$$
\begin{align*}
& \Delta=\sqrt{1-3\left(\frac{f_{v}}{F_{y f}}\right)^{2}}  \tag{Eq. 6.36}\\
& f_{v}=\frac{T}{2 A_{o} t_{f}} \tag{Eq. 6.37}
\end{align*}
$$

Where:

$$
\begin{aligned}
F_{n c}= & \text { nominal flexural resistance of box flanges in compression } \\
R_{b}= & \text { web load shedding factor } \\
R_{h}= & \text { hybrid girder factor } \\
F_{y c}= & \text { specified minimum yield strength of the compression flange } \\
\Delta= & \text { reduction factor for the maximum stress in a box flange } \\
f_{v}= & \text { St. Venant torsional shear stress in the flange to the factored loads at the section } \\
& \text { under consideration } \\
A_{o}= & \text { enclosed area with the box section } \\
T= & \text { internal torque due to the factored loads }
\end{aligned}
$$

At the strength limit state, tension flanges shall satisfy:

$$
\begin{equation*}
f_{b u} \leq \phi_{f} F_{n t} \tag{Eq. 6.38}
\end{equation*}
$$

Where:

$$
\phi_{f}=\text { resistance factor for flexure }
$$

$f_{b u}=$ flange stress calculated without consideration of flange lateral bending or longitudinal warping
$F_{n t}=$ nominal flexural resistance of box flanges in tension

The nominal flexural resistance of the tension flange of closed-box sections shall be taken as:

$$
\begin{equation*}
F_{n t}=R_{h} F_{y t} \Delta \tag{Eq. 6.39}
\end{equation*}
$$

In which:

$$
\begin{align*}
& \Delta=\sqrt{1-3\left(\frac{f_{v}}{F_{y f}}\right)^{2}}  \tag{Eq. 6.40}\\
& f_{v}=\frac{T}{2 A_{o} t_{f}} \tag{Eq. 6.41}
\end{align*}
$$

Where:
$F_{n t}=$ nominal flexural resistance of box flanges in tension
$R_{h}=$ hybrid girder factor
$F_{y t}=$ specified minimum yield strength of the tension flange
$\Delta=$ reduction factor for the maximum stress in a box flange
$f_{v}=$ St. Venant torsional shear stress in the flange to the factored loads at the section under consideration
$A_{o}=$ enclosed area with the box section $T=$ internal torque due to the factored loads

### 6.8.3 Flexural Resistance of Noncomposite Sections

The following provisions are applied to noncomposite sections:
At the strength limit state, compression flanges shall satisfy:

$$
\begin{equation*}
f_{b u} \leq \phi_{f} F_{n c} \tag{Eq. 6.42}
\end{equation*}
$$

Where:
$\phi_{f}=$ resistance factor for flexure
$f_{b u}=$ flange stress calculated without consideration of flange lateral bending or longitudinal warping
$F_{n c}=$ nominal flexural resistance of box flanges in compression

The nominal flexural resistance of unstiffened compression flanges shall be taken as:

$$
\begin{equation*}
F_{n c}=F_{c b} \sqrt{1-\left(\frac{f_{v}}{\phi_{v} F_{c r}}\right)^{2}} \tag{Eq. 6.43}
\end{equation*}
$$

In which:
If $\lambda_{f} \leq \lambda_{p}$, then:

$$
F_{c b}=R_{b} R_{h} F_{y c} \Delta
$$

Eq. 6.44

If $\lambda_{p}<\lambda_{f} \leq \lambda_{r}$, then:

$$
F_{c b}=R_{b} R_{h} F_{y c}\left[\Delta-\left(\Delta-\frac{\Delta-0.3}{R_{h}}\right)\left(\frac{\lambda_{f}-\lambda_{p}}{\lambda_{r}-\lambda_{p}}\right)\right]
$$

Eq. 6.45

If $\lambda_{f}>\lambda_{r}$, then:

$$
F_{c b}=\frac{0.9 E R_{b} k}{\lambda_{f}^{2}}
$$

Eq. 6.46

If $\lambda_{f} \leq 1.12 \sqrt{\frac{E k_{s}}{F_{y c}}}$, then:

$$
F_{c v}=0.58 F_{y c}
$$

Eq. 6.47

If $1.12 \sqrt{\frac{E k_{s}}{F_{y c}}} \leq \lambda_{f} \leq 1.40 \sqrt{\frac{E k_{s}}{F_{y c}}}$, then:

$$
F_{c v}=\frac{0.65 \sqrt{E k_{s}}}{\lambda_{f}}
$$

Eq. 6.48

If $\lambda_{f}>1.40 \sqrt{\frac{E k_{s}}{F_{y c}}}$, then:

$$
\begin{equation*}
F_{c v}=\frac{0.9 E k_{s}}{\lambda_{f}^{2}} \tag{Eq. 6.49}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{f}=\frac{b_{f c}}{t_{f c}} \tag{Eq. 6.50}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{p}=0.57 \sqrt{\frac{E k}{F_{y c} \Delta}} \tag{Eq. 6.51}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{r}=0.95 \sqrt{\frac{E k}{F_{y r}}} \tag{Eq. 6.52}
\end{equation*}
$$

$$
\begin{equation*}
\Delta=\sqrt{1-3\left(\frac{f_{v}}{F_{y c}}\right)^{2}} \tag{Eq. 6.53}
\end{equation*}
$$

$$
\begin{equation*}
f_{v}=\frac{T}{2 A_{O} t_{c}} \tag{Eq. 6.54}
\end{equation*}
$$

$$
\begin{equation*}
F_{y r}=(\Delta-0.3) F_{y c} \tag{Eq. 6.55}
\end{equation*}
$$

Where:
$\phi_{v}=$ resistance factor for shear
$F_{c b}=$ nominal axial compression buckling resistance of the flange under compression alone
$F_{c v}=$ nominal shear buckling resistance of the flange
$\lambda_{f}=$ slenderness ratio of the compression flange
$\lambda_{p}=$ limiting flange slenderness where the elastic buckling stress equals $R_{b} F_{y c} \Delta$
$\lambda_{r}=$ limiting flange slenderness where the elastic buckling stress equals $R_{b} F_{y r}$
$\Delta=$ reduction factor for the maximum stress in a box flange
$f_{v}=$ St. Venant torsional shear stress in the flange to the factored loads at the section under consideration
$A_{o}=$ enclosed area with the box section

$$
\begin{aligned}
T= & \text { internal torque due to the factored loads } \\
F_{y r}= & \text { smaller of the compression flange stress at the onset of nominal yield, with } \\
& \text { consideration or residual stress effects, or the specified minimum yield strength } \\
& \text { of the web } \\
k= & \text { plate-buckling coefficient for uniform normal stress }=4 \\
k_{s}= & \text { compression-flange width between webs } \\
b_{f c}= & \text { limiting flange slenderness where the elastic buckling stress equals } R_{b} F_{y r} \\
t_{f c}= & \text { compression flange thickness } \\
R_{b}= & \text { web load-shedding factor } \\
R_{h}= & \text { hybrid girder factor } \\
E= & \text { modulus of elasticity of steel }
\end{aligned}
$$

The nominal flexural resistance of stiffened compression flanges is determined in the same manner as unstiffened flanges with the following modifications:

- $\quad W$ shall be substituted for $b_{f c}$
- The plate buckling coefficient for uniform stress, $k$, shall be taken as:
- If $n=1$, then:

$$
\begin{equation*}
k=\left(\frac{8 I_{s}}{w t_{f c}^{3}}\right)^{\frac{1}{3}} \tag{Eq. 6.56}
\end{equation*}
$$

- If $n=3$, then:

$$
\begin{align*}
& k=\left(\frac{0.894 I_{s}}{w t_{f c}^{3}}\right)^{\frac{1}{3}}  \tag{Eq. 6.57}\\
& 1.0 \leq k \leq 4.0
\end{align*}
$$

- The plate buckling coefficient for shear stress, $k_{s}$, shall be taken as:
$k_{s}=\frac{5.34+2.84\left(\frac{I_{s}}{w t_{f c}^{3}}\right)^{\frac{1}{3}}}{(n+1)^{2}} \leq 5.34$
Eq. 6.58

Where:
$I_{s}=$ moment of inertia of a single longitudinal flange stiffener about an axis parallel to the flange and taken at the base of the stiffener
$n=$ number of equally spaced longitudinal flange stiffeners
$w=$ larger of the width of the flange between longitudinal flange stiffeners or the distance from a web to the nearest longitudinal flange stiffener
$t_{f c}=$ thickness of the compression flanges

At the strength limit state, tension flanges shall satisfy:

$$
\begin{equation*}
f_{b u} \leq \phi_{f} F_{n t} \tag{Eq. 6.59}
\end{equation*}
$$

Where:
$\phi_{f}=$ resistance factor for flexure
$f_{b u}=$ flange stress calculated without consideration of flange lateral bending or longitudinal warping
$F_{n t}=$ nominal flexural resistance of box flanges in tension

The nominal flexural resistance of tension flanges of tub sections shall be taken as:

$$
\begin{equation*}
F_{n t}=R_{h} F_{y t} \tag{Eq. 6.60}
\end{equation*}
$$

Where:

$$
\begin{aligned}
R_{h} & =\text { hybrid girder factor } \\
F_{y t} & =\text { specified minimum yield strength of the tension flange }
\end{aligned}
$$

### 6.8.4 Shear Resistance

At the strength limit state, each of the inclined webs will be designed for a shear due to the factored loads specified below:

$$
\begin{align*}
& V_{u} \leq \phi_{v} V_{n}  \tag{Eq. 6.61}\\
& V_{u i}=\frac{V_{u}}{\cos \theta} \tag{Eq. 6.62}
\end{align*}
$$

Where:
$V_{u i}=$ vertical shear due to the factored loads on the inclined web
$V_{u}=$ vertical shear due to the factored loads one inclined web
$\theta=$ the angle of inclination of the web plate to the vertical
$\phi_{v}=$ resistance factor for shear
$V_{n}=$ nominal shear resistance for unstiffened or stiffened webs

The nominal shear resistance of unstiffened webs shall be taken as:

$$
\begin{equation*}
V_{n}=V_{c r}=C V_{p} \tag{Eq. 6.63}
\end{equation*}
$$

In which:

$$
\begin{equation*}
V_{p}=0.58 F_{y w} D t_{w} \tag{Eq. 6.64}
\end{equation*}
$$

## Where:

$$
\begin{aligned}
V_{n} & =\text { nominal shear resistance } \\
V_{c r} & =\text { shear-buckling resistance } \\
C & =\text { ratio of the shear-buckling resistance to the shear yield strength } \\
V_{p} & =\text { plastic shear force } \\
F_{y w} & =\text { specified minimum yield strength of the web } \\
D & =\text { depth of the web } \\
t_{w} & =\text { thickness of the web }
\end{aligned}
$$

Stiffened interior web panels of nonhybrid and hybrid members satisfying Equation 6.65 can develop post buckling shear resistance due to tension field action:

$$
\begin{equation*}
\frac{2 D t_{w}}{\left(b_{f c} t_{f c}+b_{f t} t_{f t}\right)} \leq 2.5 \tag{Eq. 6.65}
\end{equation*}
$$

If Equation 6.65 is satisfied, the nominal shear resistance of a stiffened interior web panel shall be taken as:

$$
\begin{equation*}
V_{n}=V_{p}\left[C+\frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_{o}}{D}\right)^{2}}}\right] \tag{Eq. 6.66}
\end{equation*}
$$

If Equation 6.65 is not satisfied, the nominal shear resistance of a stiffened interior web panel shall be taken as:

$$
\begin{equation*}
V_{n}=V_{p}\left[C+\frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_{o}}{D}\right)^{2}}+\frac{d_{o}}{D}}\right] \tag{Eq. 6.67}
\end{equation*}
$$

In which:

$$
\begin{equation*}
V_{p}=0.58 F_{y w} D t_{w} \tag{Eq. 6.68}
\end{equation*}
$$

Where:

$$
\begin{aligned}
V_{n} & =\text { nominal shear resistance } \\
C & =\text { ratio of the shear-buckling resistance to the shear yield strength } \\
V_{p} & =\text { plastic shear force } \\
F_{y w} & =\text { specified minimum yield strength of the web } \\
d_{o} & =\text { transverse stiffener spacing } \\
D & =\text { depth of the web } \\
t_{w} & =\text { thickness of the web }
\end{aligned}
$$

The ratio, $C$, shall be determined as specified below:

- If $\frac{D}{t_{w}} \leq 1.12 \sqrt{\frac{E k}{F_{y w}}}$, then:

$$
C=1.0
$$

Eq. 6.69

- If $1.12 \sqrt{\frac{E k}{F_{y w}}} \leq \frac{D}{t_{w}} \leq 1.40 \sqrt{\frac{E k}{F_{y w}}}$, then:

$$
\begin{equation*}
C=\frac{1.12}{\frac{D}{t_{w}}} \sqrt{\frac{E k}{F_{y w}}} \tag{Eq. 6.70}
\end{equation*}
$$

- If $\frac{D}{t_{w}}>1.40 \sqrt{\frac{E k}{F_{y w}}}$, then:

$$
\begin{equation*}
C=\frac{1.12}{\left(\frac{D}{t_{w}}\right)^{2}}\left(\frac{E k}{F_{y w}}\right) \tag{Eq. 6.71}
\end{equation*}
$$

In which:

$$
k=5+\frac{5}{\left(\frac{d_{o}}{D}\right)^{2}}
$$

Eq. 6.72

Where:

$$
\begin{aligned}
D & =\text { depth of the web } \\
t_{w} & =\text { thickness of the web } \\
E & =\text { modulus of elasticity of steel } \\
k & =\text { shear buckling coefficient } \\
F_{y w} & =\text { specified minimum yield strength of the web } \\
C & =\text { ratio of the shear-buckling resistance to the shear yield strength } \\
d_{o} & =\text { transverse stiffener spacing }
\end{aligned}
$$

The nominal shear resistance of a stiffened end panel shall be taken as:

$$
V_{n}=V_{c r}=C V_{p}
$$

Eq. 6.73

In which:

$$
\begin{equation*}
V_{p}=0.58 F_{y w} D t_{w} \tag{Eq. 6.74}
\end{equation*}
$$

Where:

$$
\begin{aligned}
V_{n} & =\text { nominal shear resistance } \\
C & =\text { ratio of the shear-buckling resistance to the shear yield strength } \\
V_{p} & =\text { plastic shear force } \\
F_{y w} & =\text { specified minimum yield strength of the web } \\
d_{o} & =\text { transverse stiffener spacing } \\
D & =\text { depth of the web } \\
t_{w} & =\text { thickness of the web }
\end{aligned}
$$

### 6.9 AASHTO REFERENCES

Table 6.3 details a summary of the equations, figures, and tables referenced in this chapter, along with their respective AASHTO LRFD BDS equation reference and page numbers.

Table 6.3: AASHTO LRFD BDS References (AASHTO, 2020)

| Chapter 6 | AASHTO LRFD BDS 9th Edition Reference | AASHTO LRFD BDS 9th Edition Page Number |
| :---: | :---: | :---: |
| Table 6.1 | Table 3.5.1-1 | 3-21 |
| Figure 6.1 | Figure 3.6.1.2.2-1 | 3-25 |
| Table 6.2 | Table 3.6.1.1.2-1 | 3-23 |
| Equation 6.1 | Equation 1.3.2.1-1 | 1-3 |
| Equation 6.2 | Table 4.6.2.2.2b-1 | 4-38 |
| Equation 6.3 | Table 4.6.2.2.2b-1 | 4-38 |
| Equation 6.4 | Equation 6.11.2.1.2-1 | 6-222 |
| Equation 6.5 | Equation 6.11.2.1.3-1 | 6-222 |
| Equation 6.6 | Equation 6.11.2.2-1 | 6-222 |
| Equation 6.7 | Equation 6.11.2.2-2 | 6-222 |
| Equation 6.8 | Equation 6.11.2.2-3 | 6-222 |
| Figure 6.2 | Figure 6.11.2.3-1 | 6-223 |
| Equation 6.9 | Equation 6.10.3.2.1-1 | 6-160 |
| Equation 6.10 | Equation 6.10.3.2.1-2 | 6-160 |
| Equation 6.11 | Equation 6.10.3.2.1-3 | 6-160 |
| Equation 6.12 | Equation 6.10.3.2.2-1 | 6-162 |
| Equation 6.13 | Equation 6.10.3.2.3-1 | 6-162 |
| Equation 6.14 | Equation 6.11.3.2-1 | 6-224 |
| Equation 6.15 | Equation 6.11.3.2-2 | 6-224 |
| Equation 6.16 | Equation 6.11.3.2-3 | 6-224 |
| Equation 6.17 | Equation 6.11.3.2-4 | 6-224 |
| Equation 6.18 | Equation 6.11.3.2-5 | 6-225 |
| Equation 6.19 | Equation 6.10.3.3-1 | 6-163 |
| Equation 6.20 | Equation 6.11.9-1 | 6-238 |
| Figure 6.3 | Article 2.5.2.6.2 | 2-12 |
| Equation 6.21 | Equation 6.10.4.2.2-1 | 6-168 |
| Equation 6.22 | Equation 6.10.4.2.2-2 | 6-168 |
| Equation 6.23 | Equation 6.10.4.2.2-4 | 6-169 |
| Equation 6.24 | Equation 6.6.1.2.2-1 | 6-40 |
| Equation 6.25 | Equation 6.6.1.2.5-1 | 6-56 |
| Equation 6.26 | Equation 6.6.1.2.5-2 | 6-57 |
| Equation 6.27 | Equation 6.6.1.2.5-3 | 6-57 |
| Equation 6.28 | Equation 6.11.6.2.2-1 | 6-230 |
| Equation 6.29 | Equation 6.10.7.3-1 | 6-182 |
| Equation 6.30 | Equation 6.11.7.1.1-1 | 6-231 |

Table 6.3: AASHTO LRFD BDS References (cont'd) (AASHTO, 2020)

| Chapter 6 | AASHTO LRFD BDS <br> 9th Edition Reference | AASHTO LRFD BDS 9th Edition Page Number |
| :---: | :---: | :---: |
| Equation 6.31 | Equation 6.10.7.1.2-1 | 6-179 |
| Equation 6.32 | Equation 6.10.7.1.2-2 | 6-179 |
| Equation 6.33 | Equation 6.10.7.1.2-3 | 6-179 |
| Equation 6.34 | Equation 6.11.7.2.1-1 | 6-231 |
| Equation 6.35 | Equation 6.11.7.2.2-2 | 6-232 |
| Equation 6.36 | Equation 6.11.7.2.2-3 | 6-232 |
| Equation 6.37 | Equation 6.11.7.2.2-4 | 6-232 |
| Equation 6.38 | Equation 6.11.7.2.1-2 | 6-232 |
| Equation 6.39 | Equation 6.11.7.2.2-5 | 6-233 |
| Equation 6.40 | Equaiton 6.11.7.2.2-6 | 6-233 |
| Equation 6.41 | Equation 6.11.7.2.2-7 | 6-233 |
| Equation 6.42 | Equation 6.11.8.1.1-1 | 6-233 |
| Equation 6.43 | Equation 6.11.8.2.2-1 | 6-235 |
| Equation 6.44 | Equation 6.11.8.2.2-2 | 6-235 |
| Equation 6.45 | Equation 6.11.8.2.2-3 | 6-235 |
| Equation 6.46 | Equaiton 6.11.8.2.2-4 | 6-235 |
| Equation 6.47 | Equation 6.11.8.2.2-5 | 6-235 |
| Equation 6.48 | Equation 6.11.8.2.2-6 | 6-236 |
| Equation 6.49 | Equation 6.11.8.2.2-7 | 6-236 |
| Equation 6.50 | Equation 6.11.8.2.2-8 | 6-236 |
| Equation 6.51 | Equation 6.11.8.2.2-9 | 6-236 |
| Equation 6.52 | Equation 6.11.8.2.2-10 | 6-236 |
| Equation 6.53 | Equation 6.11.8.2.2-11 | 6-236 |
| Equation 6.54 | Equation 6.11.8.2.2-12 | 6-236 |
| Equation 6.55 | Equation 6.11.8.2.2-13 | 6-236 |
| Equation 6.56 | Equation 6.11.8.2.3-1 | 6-237 |
| Equation 6.57 | Equation 6.11.8.2.3-2 | 6-237 |
| Equation 6.58 | Equation 6.11.8.2.3-3 | 6-237 |
| Equation 6.59 | Equation 6.11.8.1.2-4 | 6-234 |
| Equation 6.60 | Equation 6.11.8.3-1 | 6-238 |
| Equation 6.61 | Equation 6.10.9.1-1 | 6-193 |
| Equation 6.62 | Equation 6.11.9-1 | 6-238 |
| Equation 6.63 | Equation 6.10.9.2-1 | 6-194 |
| Equation 6.64 | Equation 6.10.9.2-2 | 6-194 |
| Equation 6.65 | Equation 6.10.9.3.2-1 | 6-195 |

Table 6.3: AASHTO LRFD BDS References (cont'd) (AASHTO, 2020)

| Chapter 6 | AASHTO LRFD BDS <br> 9th Edition Reference | AASHTO LRFD BDS <br> 9th Edition Page Number |
| :---: | :---: | :---: |
| Equation 6.66 | Equation 6.10.9.3.2-2 | $6-195$ |
| Equation 6.67 | Equation 6.10.9.3.2-8 | $6-196$ |
| Equation 6.68 | Equation 6.10.9.3.2-3 | $6-195$ |
| Equation 6.69 | Equation 6.10.9.3.2-4 | $6-195$ |
| Equation 6.70 | Equation 6.10.9.3.2-5 | $6-196$ |
| Equation 6.71 | Equation 6.10.9.3.2-6 | $6-196$ |
| Equation 6.72 | Equation 6.10.9.3.2-7 | $6-196$ |
| Equation 6.73 | Equation 6.10.9.3.3-1 | $6-196$ |
| Equation 6.74 | Equation 6.10.9.3.3-2 | $6-196$ |

### 6.10 SUMMARY

This chapter summarized the applicable articles and equations from the AASHTO LRFD BDS related to PBFTGs. As PBFTGs are a relatively new type of system, specifications directly relating to them in the AASHTO LRFD BDS do not exist. The system was designed using the applicable articles found in AASHTO LRFD BDS Article 6.11 as they relate to large, welded boxgirders. However, several sections are not applicable to the design of most PBFTG bridges. For example, as written, the LLDFs found in AASHTO LRFD BDS Article 4.6.2.2 are not directly applicable to PBFTGs, as they inherently violate special restrictions on the use of LLDFs for multiple box sections found in AASHTO LRFD BDS Article 6.11.2.3. Additionally, the limit of skew on the compactness, and therefore ultimate capacity of PBFTGs, greatly reduces the applicability of PBFTGs in skewed bridge configurations. Based on the findings of this chapter, a more refined analysis was performed on PBFTGs with the focus of improving the AASHTO LRFD BDS relating to PBFTGs.

## CHAPTER 7: FINITE ELEMENT MODELING TECHNIQUES

### 7.1 Introduction

This chapter details the analytical modeling techniques developed to assess the capacity of PBFTG systems. FEA was performed utilizing the commercial finite element software suite Abaqus/CAE (Dassault Systèmes, 2020). An input file was generated by writing a routine in MATLAB (Mathworks, 2021) and was ran using Abaqus/CAE in the command window. A threedimensional nonlinear finite element modeling procedure was developed to determine the ultimate capacity of skewed PBFTGs. A second technique, based on the first, was used to model linear PBFTG bridges to assess live load distribution characteristics of PBFTG bridges.

### 7.2 Element Selection

Abaqus/CAE provides the user with a vast amount of element types to discretize the threedimensional model. Therefore, an investigation into the suitability of the appropriate element type is warranted. As shown by several previous researchers (Barth, 1996; Yang, 2004; Roberts, 2005: Righman, 2005), S4R elements are sufficiently accurate to model the behavior of plate girders.

### 7.2.1 Element Naming Convention

Each element utilized in Abaqus/CAE has a unique name which conveys the key attributes of the element. The element name contains five identifiers, if applicable, which describe the family, degrees of freedom, number of nodes, formulation, and integration. The first letter of the element name describes the general type, or family of element. For example, ' $S$ ' represents a shell element or 'C' represents a continuum, or solid, element. Directly related to the element family are the degrees of freedom. In a stress/displacement analysis, the degrees of freedom are translations at each node. Typically, all degrees of freedom are considered for an element and not specified in the name. However, if a degree of freedom is restrained, the degree of freedom restrained follows the family, and is in turn followed by a 'D', i.e., 'C3D8.' In this example, an 8 -node continuum element is restrained in the third degree of freedom, or the translation in direction 3. Typically, the
number of nodes in an element is clearly identified in the name, such as the ' 8 ' for 8 -node in the previous example. An element's formulation describes the mathematical theory defining the element's behavior. Unless specified with an ' $H$ ' at the end of the element name, the element is assumed to use the standard element family formulation. Abaqus/CAE uses numerical techniques to integrate quantities over the element. Gaussian quadrature is used for most elements, where the material response is evaluated at each integration point. Some elements can use reduced integration and are signified by using an ' $R$ ' at the end of the element name.

### 7.2.2 General-Purpose Shell Elements

Analytical research performed on PBFTGs (Michaelson, 2014; Kelly, 2014; Gibbs, 2017; Underwood, 2019; Roh, 2020) demonstrates the S4R element most accurately predicts the actual behavior of PBFTGs. The S4R element is a 4-node general purpose shell element utilizing reduced integration with hourglass controls and is suitable for a wide range of applications. While the S4R is robust enough to be used in thick and thin shelled problems, its use in this study is limited to thin shell behavior. Thin shell problems assume the transverse shear deformation may be neglected (Dassault Systèmes, 2020). Figure 7.1 demonstrates the difference between thin shells and thick shells where (a) illustrates material lines remain normal throughout deformation in thin wall elements and (b) illustrates material lines do not necessarily remain normal to the surface in thick wall elements. Kirchhoff shell theory is utilized in this study, that is, plane sections remain normal to the midsection and transverse shear strains are assumed to be zero.


Figure 7.1: Behavior of Transverse Shell Sections in (a) Thin Shells and (b) Thick Shells (Dassault Systèmes, 2020)

S4R elements employ reduced integration where the elements use one fewer Gauss integration point in each direction, or a total of one in the case of 4-noded elements, to determine the element stiffness matrix. There are two main advantages of reduced integration over full integration. First, the strains and stresses are calculated at locations known to provide optimal accuracy. Second, the use of fewer integration points decreases the size of the model and associated computation time. However, the mass matrix and force matrix are still integrated exactly, even though the stiffness matrix is reduced. Displacements are then calculated at each node and linear interpolation is used to determine the displacements at other locations within the element using:

$$
\begin{equation*}
u=\sum f_{i} u_{i} \tag{Eq. 7.1}
\end{equation*}
$$

In which:

$$
\begin{align*}
& f_{i}=\frac{1}{4}\left(1+\xi_{0}\right)\left(1+\eta_{0}\right)  \tag{Eq. 7.2}\\
& \xi_{0}=\xi_{i} \xi  \tag{Eq. 7.3}\\
& \eta_{0}=\eta_{i} \eta \tag{Eq. 7.4}
\end{align*}
$$

Where:

$$
\begin{aligned}
u_{i} & =\text { displacement at node } \mathrm{i} \\
f_{i} & =\text { shape function at node i } \\
\xi_{0} & =\text { local coordinate shape factor variable } \\
\eta_{0} & =\text { local coordinate shape factor variable } \\
\xi_{i} & =\text { nodal local coordinate as specified in Figure } 7.2 \\
\eta_{i} & =\text { nodal local coordinate as specified in Figure } 7.2
\end{aligned}
$$



Figure 7.2: Element Natural Coordinate System (Dassault Systèmes, 2020)

The main disadvantage of reduced integration is that deformation modes causing no strain at the integration points may be omitted. These zero-energy modes can propagate through the system causing inaccuracies in a phenomenon known as hourglassing. Hourglassing is an issue which can arise in FEA when the number of integration points is reduced where only the linearly varying part of the incremental displacement field is considered for strain calculation and the rest of the notal displacement field is neglected. Neglecting the effect of these other nodes can lead to mesh distortion. To prevent this problem, a small artificial stiffness associated with zero-energy deformation is added using the '*SECTION CONTROLS' command in the Abaqus input file.

### 7.2.3 General Purpose Continuum Elements

Through these research efforts, more accurate deck modeling was needed in nonlinear skewed scenarios. As the degree of skew increased, while using shell elements, the ultimate capacity of the system increased, but the model would terminate at a significantly lower corresponding deflection. The C3D8R element was found, as part of this research effort, to represent the behavior of the deck more accurately in skewed scenarios. The C3D8R element is an 8-node general purpose brick element utilizing reduced integration with hourglass controls suitable for complex nonlinear analysis involving contact, plasticity, and large deformations. Some of the shortcomings of this type of element including lack of stiffness in bending and less accurate results away from the integration point, or the middle of the element. This type of element is also prone to hourglassing, as discussed previously, so hourglass controls are also utilized in conjunction with this element.

### 7.3 Material Modeling

An elastic-plastic constitutive law, including strain hardening effects, is used in the singlegirder modeling portion of this work. This constitutive law is used when the loading in the system approaches the ultimate capacity of the specimen. When the system is under service level loading, such as the bridge system modeling, the steel and concrete are expected to behave elastically.

### 7.3.1 Structural Steel

Structural steel was modeled using an elastic-plastic constitutive law including strain hardening effects. Specifically, the '*PLASTIC' command was used to model the post yield behavior of the steel used in the ultimate capacity determination. An elastic-plastic constitutive model featuring standard von Mises surfaces, an associated plastic flow rule, and isotropic flow hardening has been found suitable to represent rate dependent behavior of a metal subjected to a relatively monotonic loading where creep effects are non-critical (Barth, 1996; Yang, 2004; Righman, 2005; Michaelson 2014).

A multilinear stress-strain relationship developed by Galindez (2009) was used to develop the stress-strain relationship used in the steel material modeling. The linear approximation is
derived from the equations listed in Table 7.1. As the PBFTGs used during the experimental testing are the same specimens Michaelson (2014) utilized in his research, the same structural material properties were used in these efforts (Table 7.2). In Figure 7.3, the engineering stress-strain model used in Roh's (2020) modeling efforts is presented. The engineering stress-strain model was then converted to the true stress-strain model required for input into Abaqus/CAE.

Table 7.1: Expressions for Computing Steel Stress-Strain Behavior (Galindez, 2009)
Point Strain Stress
$1 \varepsilon_{1}=\frac{\sigma_{y}}{E} \quad \sigma_{1}=\sigma_{y}$
$2 \varepsilon_{2}=\varepsilon_{s t}$
$\sigma_{2}=\sigma_{y}$
$3 \varepsilon_{3}=\frac{1}{10}\left(\varepsilon_{u}-\varepsilon_{s t}\right)+\varepsilon_{s t} \quad \sigma_{3}=\frac{E_{s t}}{10}\left(\varepsilon_{u}-\varepsilon_{s t}\right)+\sigma_{y}$
$4 \quad \varepsilon_{4}=\frac{2}{7}\left(\varepsilon_{6}-\varepsilon_{3}\right)+\varepsilon_{3} \quad \sigma_{4}=\frac{4}{7}\left(\sigma_{6}-\sigma_{3}\right)+\sigma_{3}$
$5 \quad \varepsilon_{5}=\frac{2}{7}\left(\varepsilon_{6}-\varepsilon_{3}\right)+\varepsilon_{4}$
$\sigma_{5}=\frac{2}{7}\left(\sigma_{6}-\sigma_{3}\right)+\sigma_{4}$
$6 \quad \varepsilon_{6}=\varepsilon_{u}-\frac{1}{10}\left(\varepsilon_{u}-\varepsilon_{s t}\right) \quad \sigma_{6}=\left(\frac{\sigma_{y}}{\sigma_{0.2 \%}}\right) \sigma_{u}-\frac{100}{E_{s t}}\left(\varepsilon_{u}-\varepsilon_{s t}\right)$
$7 \quad \varepsilon_{7}=\varepsilon_{u}$
$\sigma_{7}=\left(\frac{\sigma_{y}}{\sigma_{0.2 \%}}\right) \sigma_{u}$

Table 7.2: Average Steel Plate Properties (Michaelson, 2014)

| Property | Average Value |
| :--- | :--- |
| Modulus of Elasticity, $E$ (ksi) | 29559 |
| Static Yield Stress, $\sigma_{y}(\mathrm{ksi})$ | 60.962 |
| Offset Yield Stress, $\sigma_{0.2 \%}$ (ksi) | 63.05 |
| Strain at the Onset on Strain Hardening, $\varepsilon_{s t}(\%)$ | 1.7883 |
| Strain Hardening Modulus, $E_{s t}$ (ksi) | 1033.5 |
| Tensile Stress, $\sigma_{u}$ (ksi) | 84.382 |
| Strain at the Tensile Stress, $\varepsilon_{u}(\%)$ | 13.165 |



Figure 7.3: Multi-Linear Stress-Strain Curve (Roh, 2020)

### 7.3.2 Reinforced Concrete

A damaged plasticity concrete model is used in this study to model reinforced concrete elements. The damaged plasticity model provides the capability to model concrete and other quasibrittle materials. While this model can be used for plain concrete, it is primarily intended for the analysis of reinforced concrete structures. The damaged plasticity model uses isotropic damaged plasticity concepts in combination with isotropic tensile and compressive plasticity to represent the inelastic behavior of concrete (Dassault Systèmes, 2020). Specifically, the concrete in the composite deck of the PBFTG specimens and link slabs is modeled using the '*CONCRETE COMPRESSION HARDENING’ and ‘*CONCRETE TENSION STIFFENING’ commands in the Abaqus input file. The '*CONCRETE COMPRESSION HARDENING' option defines the compression damage properties, and the '*CONCRETE TENSION STIFFENING' option defines the cracking and post cracking properties of the concrete damaged plasticity material model. Note, when using these options, once a crack appears in the model, it remains for rest of the calculation.

The concrete material model used in this study, when considering nonlinearity in its response, assumes the uniaxial tensile and compressive response is characterized by damaged plasticity (Figures 7.4 and 7.5 ). Figure 7.4 shows the compressive model of concrete with a compressive strength of 4 ksi and a modulus of elasticity of $3,640 \mathrm{ksi}$. Figure 7.5 shows the tensile model of concrete with a rupture strength of 0.522 ksi . Tension stiffening is utilized in the damaged plasticity model to approximate the reinforcement interaction with concrete in a simple manner.


Figure 7.4: Stress-Strain Curve for Reinforced Concrete (Compressive Region)


Figure 7.5: Stress-Strain Curve for Reinforce Concrete (Tension Region)

The modeling of concrete elements included considerations for the steel reinforcement within the deck. Specifically, the reinforcement is considered using the '*REBAR' option in the Abaqus input file and represents a smeared layer of reinforcement at user specified locations within the concrete deck. The material model for the steel reinforcement is similar to the model used for
structural steel, with the following differences: a modulus of elasticity of $29,000 \mathrm{ksi}$, instead of $29,599 \mathrm{ksi}$, and a yield stress of 60 ksi , instead of 50 ksi .

### 7.4 Additional Modeling Considerations

Composite steel girders undergoing flexure predominately experience failure due to yielding of steel elements in tension or loss of stiffness of concrete components due to excessive compressive stress or cracking (Michaelson, 2014). In composite PBFTGs, the plastic neutral axis is in or near the top flange, and the top flange is continuously laterally supported by the concrete deck. However, in noncomposite steel girders, these conditions are not true, and girders can be susceptible to lateral torsional buckling, local flange buckling, and/or local web buckling. Therefore, initial geometric imperfections and residual stresses can have an impact on the noncomposite behavior of steel flexural elements. While the focus of these efforts is on composite behavior, attention must be paid to initial imperfections to ensure adequate modeling.

### 7.4.1 Geometric Imperfections

The nonlinear response due to initial imperfections present throughout the steel girder has a measurable impact on the girder's response to flexural loading and its susceptibility to buckling modes when the compression flange(s) are not restrained. A more refined response is necessary to model the effect of the imperfections than conventional bifurcation provides. Introducing geometric imperfections to the ideal cross-section ensures a degree of buckling occurs before the critical load in the ideal system is reached. Therefore, the introduction of these imperfections is a critical step in modeling the behavior of steel members.

In welded plate girders, initial geometric imperfections are typically generated during the welding process and/or occur due to initial plate out-of-flatness present in all plate steel. Three types of geometric imperfections are considered in these efforts to capture these characteristics: initial out of flatness of web panel(s), initial tilt of compression flange(s), and initial sweep of the compression flange(s). These imperfections are illustrated in Figure 7.6.

(a) Initial out-of-flatness of the web

(c) Initial lateral sweep of the compression flange

Figure 7.6: Initial Geometric Imperfections (Yang, 2004)

The values prescribed for these three types of imperfections are based on maximum allowable tolerances specified by the American Welding Society (AWS) and engineering judgement (Righman, 2005). AWS specifies a different tolerance for initial web out-of-flatness based on if the web is transversely stiffened. For girders with one-sided transverse stiffeners, the maximum allowable initial out-of-flatness of the web, $\delta_{o w}$, as seen in Figure 7.6a, is to be taken as $d / 67$, where $d$ is the minimum panel dimension, either the depth of the web, $D$, or the distance between stiffeners, $d_{0}$. Alternatively, for girders without stiffeners, the maximum allowable initial out-of-flatness of the web is taken as $d / 150$. In the analytical modeling performed for part of this
doctoral research, the maximum allowable initial out-of-flatness of the web shall be taken as $d / 100$, which represents a midpoint between the above requirements. This distortion is applied to the web as a sine wave in both the vertical and longitudinal directions of the web with maximum values located at the mid-height of the web and the midpoint between stiffeners. Furthermore, the longitudinal direction of distortion alternates between adjacent web panels.

AWS specifies the maximum allowable initial tilt of the compression flanges, $\delta_{o f}$, as seen in Figure 7.6 b , is to be taken as the greater of 0.25 inches or $b_{f} / 100$, where $b_{f}$ is the flange width. However, while performing relevant verification studies, it is unlikely the distortion of the flanges would be this severe for the relatively short panel lengths utilized in these girders. In the analytical modeling performed for part of this doctoral research, the maximum allowable initial tilt of the compression flange is taken to be the lesser value of $b_{f c} / 150$ or $0.3 d_{0} / 150=d_{0} / 500$. This results in values slightly smaller than those permitted by AWS for girders with long panel lengths (i.e., $b_{f c}$ $<0.3 d_{0}$ ), while resulting in values significantly smaller than permitted by AWS for short panel lengths. This distortion is applied linearly along the width of the flange with maximum distortion occurring at the flange edges and as a sine wave longitudinally along the flange. Furthermore, the longitudinal direction of distortion alternates between adjacent panels separated by stiffeners.

AWS specifies the maximum allowable sweep of the compression flange, $\delta_{o L}$, as seen in Figure 7.6 c, is to be taken as $L_{b} / 960$, where $L_{b}$ is the distance between lateral bracing. In the analytical modeling performed for part of this doctoral research, the allowable sweep of the compression flange is taken as $L_{b} / 1500$, as specified by Yang (2004) and Righman (2005). This distortion is applied linearly along the vertical axis of the member starting at unity at the tension flange-web junction and reaching its maximum at the compression flange-web junction. This distortion is applied as a sine wave longitudinally along the girder. As with other imperfections, the longitudinal direction of the distortion alternates between panels separated by stiffeners. Note, the sweep of compression flange and the tilt of the compression flange should be prescribed in the same direction in each web panel to ensure the effects of these imperfections are cumulative.

### 7.4.2 Residual Stresses

The longitudinal residual stresses in welded plate girders are primarily caused by flame cutting the plates to the dimension required and welding between the flanges and the web. The
tensile residual stresses are typically equal to the yield stress of the steel in a small area, termed the heat affect zone, while a smaller, near constant self-equilibrating compression stress is developed within the remainder of the plate. The residual stress pattern may be idealized by assuming that, when the section is free of external forces, the residual stresses over the crosssection must satisfy equilibrium.

In these efforts, residual stresses are modeled by specifying initial stress conditions prior to applying external loads. The magnitude and direction of initial residual stresses are applied depending on the location of the element. For verification studies relating to plate girders, Figure 7.7 demonstrates the residual stress pattern used. When initial stresses are provided, the initial stress state may not be in exact equilibrium for the finite element matrix. Therefore, an initial step, prior to loading, must be provided to allow Abaqus/CAE to achieve equilibrium. Specifically, the dead load is applied using the '*STATIC' option before exterior loading, ensuring equilibrium is achieved once residual stresses have been included.


Figure 7.7: Residual Stress Pattern (Righman, 2005)

### 7.5 SOLUTION ALGORITHM

To capture the behavior of specimens post-yielding and post-buckling, an unstable collapse and post-buckling analysis procedure were needed in these studies, specifically for the ultimate capacity determinations. A modified Riks algorithm available in Abaqus/CAE was chosen to be used in these studies, as it is one of the most versatile and efficient methods capable of obtaining
a complete nonlinear solution (Figure 7.8). The Riks method was initially developed by Riks (1979) and improved by several researchers, such as Crisfield (1983), Ramm (1981), and Powell and Simons (1981). The modified Riks method allows for the ability to pass the elastic limit point and trace the unloading portion of the nonlinear equilibrium path, allowing for the solution to be obtained regardless of whether the response is stable or unstable. In addition, this method provides efficient usage of computational resources during the nonlinear solution process.


Figure 7.8: Modified Riks Algorithm (Dassault Systèmes, 2020)

The modified Riks method uses the magnitude of the load as an additional unknown and solves simultaneously for loads and displacements assuming the loading is proportional, i.e., load magnitudes vary with a single scalar parameter, and the response is reasonably smooth, i.e., sudden bifurcations do not occur. As solution progress is independent of the load increment, Abaqus/CAE uses the distance along the static equilibrium path in the load-displacement space, or arc length, to control the increment size. This value is initially set by the user and is later modified by Abaqus/CAE based on the convergence rate of the solution. Development of the solution requires traveling this path as far as required. The basic algorithm specifies a finite radius of convergence. The increment size is limited by moving a given distance along the line tangent to the previous
solution and searching for equilibrium in the plane passing through the point obtained that is orthogonal to the same tangent line. In summary, the modified Riks method can be viewed as the discovery of a single equilibrium path defined by the space of displacements, rotations, and the loading parameter.

Another important change in the solution algorithm was the number of Gauss integration points through the depth of the concrete slab. This value was increased from 5 points (the default Abaqus/CAE value) to 7 points. A linear search technique was used by changing the load level during integration. These changes are well established and better capture the crushing and cracking of the concrete and improve the speed of convergence (Barth \& Wu, 2006).

### 7.6 Boundary Conditions and Multiple-Point Constraints

Boundary conditions on all models in the studies presented herein represent 'hinge-roller' conditions, which limit vertical and longitudinal displacement. The girder(s) are restrained vertically at all nodes of the bottom flange at each support location. The girders(s) are restrained longitudinally at the center node of the bottom flange at one end of each girder. The girder(s) are restrained transversely at the center of one of the webs at each support location.

In the finite element modeling, the composite action between the reinforced concrete deck and the steel girder was ensured by using beam-type multi-point constraints (MPCs). These constraints provide a rigid beam between two nodes to constrain the displacement and rotation at the first node to the displacement and rotation at the second node, corresponding to the presence of a rigid beam between the two nodes (Dassault Systèmes, 2020). To create the MPC elements, the mesh of the concrete slab was generated to have nodes vertically above the nodes in the middle of the top flange. Specifically, the '*MPC' option was used to rigidly connect the nodes in the center of the top flange, at the web-flange conjunction, to the node directly above junction in the concrete deck.

An image of one of the finite element models in the sensitivity study described in Chapter 8 is shown in Figure 7.9. Figure 7.9 displays the boundary conditions in orange and the mesh discretization. For the purpose of clarity, the MPC labels have been removed from the display, but connections can be seen between the concrete deck and the top flanges of the PBFTGs.


Figure 7.9: Abaqus/CAE Screen Capture of a Sensitivity Bridge Model

### 7.7 ApPLICATION OF LOAD

Once the PBFTGs were modeled in Abaqus/CAE, the models were loaded with the AASHTO LRFD BDS design truck, as discussed in Section 6.2.1, to determine the behavior of the single girder or bridge system. This section will provide a description of the methodology behind the truck placement on the bridge models and the methodology of applying either the concentrated truck loads to the bridges or the distributed load to the single girder system.

### 7.7.1 Placement of the AASHTO LRFD BDS Truck Loading

As part of these research efforts, a simple live load generator encompassing the Design Truck and Design Tandem, as described in the AASHTO LRFD BDS Article 3.6.1.2, for simple span bridges was developed. The design vehicular loading is placed on bridges to produce the maximum force effect to be investigated. However, the AASHTO LRFD BDS outline specific rules regarding the placement of live loads on bridges. These rules, as they relate to simple span bridges can be summarized as follows:

- AASHTO LRFD BDS Article 3.6.1.1.1
- Unless specified otherwise, the width of the design lanes should be taken as 12 feet. The number of design lanes is determined by taking the integer part of the ratio $w / 12$, where $w$ is the clear roadway width in feet between curbs, barriers, or both.
- Roadway widths from 20 to 24 feet shall have two design lanes, each equal to onehalf the roadway width.
- AASHTO LRFD BDS Article 3.6.1.3.1
- Both the design lanes and the 10 foot loaded width in each lane shall be positioned to produce extreme force effects.
- The design truck or tandem shall be positioned transversely, such that the center of any wheel load is no closer than 2 feet from the edge of the design lane.

For exterior girders, the vehicular live loads were placed laterally, as close to the edge of the bridge as possible. For one-lane-loaded scenarios, this meant placing the design vehicular live load 2 feet from the edge of the barrier. For two-lanes-loaded scenarios, if a second design lane was available, the 10 foot loaded width was placed as close to the exterior girder as possible, resulting in a lateral spacing between wheel lines of adjacent vehicular live loads of 6 feet. For interior girders, a wheel line of the vehicular live load was placed over the girder being analyzed, or as close as possible given the aforementioned rules. For two-lanes-loaded scenarios, the placement of the 10 foot loaded portion within the 12 foot design lane resulted in a lateral spacing between wheel lines of adjacent vehicular live loads between 4 and 6 feet.

Kassimalli (2015) specifies the longitudinal placement of the design vehicular live load as:
In a simply supported beam subjected to a series of moving concentrated loads, the maximum bending moment develops under a load when the midspan of the beam is located halfway between the load and the resultant of all loads on the beam.

### 7.7.2 Finite Element Model Loading

Once the loading location was determined, the point loads were linearly distributed to the neighboring nodes. A schematic of this loading is shown in Figure 7.10, and Equations 7.1 through 7.4 describe the loading. According to AASHTO LRFD BDS Article 4.6.3.3.1, nodal loads shall be statically equivalent to the actual loads being applied. It can be shown the equations corresponding to Figure 7.10, once summed, will equal the applied point load.


Figure 7.10: Schematic of Nodal Distribution of Point Loads

$$
\begin{align*}
& A=P\left(1-\frac{\xi}{x}\right)\left(1-\frac{\eta}{y}\right)  \tag{Eq. 7.1}\\
& B=P\left(\frac{\xi}{x}\right)\left(1-\frac{\eta}{y}\right) \\
& C=P\left(1-\frac{\xi}{x}\right)\left(\frac{\eta}{y}\right) \\
& D=P\left(\frac{\xi}{x}\right)\left(\frac{\eta}{y}\right)
\end{align*}
$$

Eq. 7.2

Eq. 7.3

Eq. 7.4

### 7.8 Verification of Finite Element Modeling

To assess the validity and accuracy of these modeling techniques, experimental data from previous laboratory experiments were employed as benchmarks. Discussed in this section are the laboratory and live load field tests used and the results from those experiments compared against the analytical results of these tools.

### 7.8.1 Benchmark Analysis \#1: Schilling \& Morcos (1988)

Schilling and Morcos (1988) performed experimental testing on three plate girders to determine moment-rotation curves for noncompact plate girders with improved rotation characteristics. These girders, denoted ' $S$ ' for shallow, ' $M$ ' for medium depth, and ' $D$ ' for deep,
were tested in three-point bending with simply supported conditions at the ends and a concentrated load at midspan, approximating the negative moment region over the pier in continuous-span bridges. Specimen D was chosen for verification purposes as the web slenderness value is more representative of the web slenderness found in PBFTGs. Figure 7.11 shows the plate dimensions and locations of two-sided bearing stiffeners and one-sided transverse stiffeners.


BS: Bearing Stiffener, each side
TS: Transverse Stiffener, one side

Figure 7.11: Specimen D Dimensions (Schilling \& Morcos, 1988)

The appropriate mesh density was found to be crucial as relatively large elements will result in unrealistically low predicted strengths due to stress concentration effects, while relatively small elements will result in an overestimation of the energy dissipation capacity (Righman, 2005). Yang (2004) evaluated the ideal mesh density of the plate girders represented in this study. Coarser meshes of the steel, such as four elements across the width of the flanges and six elements in the depth of the web, overestimate the strength of the girder, while finer meshes, such as ten elements across the width of the flanges and twenty elements in the depth of the web, can yield results with differences of $0.3 \%$. Thus, this finer mesh density was chosen for this verification.

The shape of longitudinal elements was made as close to square as possible. With the mesh density of the cross-section discussed above, it is not possible to achieve perfectly square
longitudinal elements for both the web and the flanges. It is prudent to have the longitudinal elements in both the web and flanges to have the same longitudinal length so these elements can share coincident nodes at the web-flange junction. To overcome this shortcoming, an approximate longitudinal element length is provided to minimize the aspect ratio for all elements. This aspect ratio was found to be approximately 1.4 for all elements.

An input file was generated using the aforementioned modeling techniques to model Specimen D, and FEA was performed using Abaqus/CAE. The load-deflection curve from experimental testing was plotted against the analytical modeling (Figure 7.12). As shown, the modeling techniques described previously capture the nonlinear behavior of this experimental test.


Figure 7.12: Comparison of the Schilling and Morcos (1988) Specimen D Experimental and Analytical Results

### 7.8.2 Benchmark Analysis \#2: Lay et al. (1964)

Lay et al. (1964) performed experimental testing on multiple steel elements to obtain rational plastic design procedures for high strength steels. Part of the testing program included calculation of the full plastic moment where beams were subjected to concentrated loads at the ends of the members and supported at third points. The 'HT-29' test was chosen for a benchmark to model nonlinear material modeling and geometric imperfections as Lay et al. (1964) presented this test as the sample of all beam tests performed as part of their research. Figure 7.13 details the experimental test setup of the 'HT-29' test.


Figure 7.13: 'HT-29’ Test Schematic (Lay et al., 1964)

An input file was generated using the aforementioned modeling techniques, including the target mesh density discussed in Section 7.6, to model the 'HT-29' test, and FEA was performed using Abaqus/CAE. The load-deflection curve from experimental testing was plotted against the analytical modeling (Figure 7.14). As shown, the modeling techniques described previously capture the nonlinear behavior of this experimental test.


Figure 7.14: Comparison of the Lay et al. (1964) ‘HT-29’ Experimental and Analytical

## Results

### 7.8.3 Benchmark Analysis \#3: Roberts (2005)

Roberts (2004) performed experimental testing on three composite girders to develop a more complete understanding of the ductility of composite positive bending sections and develop a less conservative ultimate strength equation for compact composite sections in positive bending. 'Specimen R1,' which will be used in this verification study, consisted of a W24x55 A572 Grade 50 rolled shape with a 42 inch wide by 7 inch thick reinforced concrete deck. The concrete deck had two layers of reinforcement, and full composite action was developed using 7/8 inch diameter head shear studs spaced 9 inches along the length of the girder. Figure 7.15 details the experimental test setup of the Specimen R1 test.


Figure 7.15: Geometry of Specimen R1 (Roberts, 2005)

An input file was generated using the aforementioned modeling techniques, including the target mesh density discussed in Section 7.6, to model the Specimen R1 test, and FEA was performed using Abaqus/CAE. The load-deflection curve from experimental testing was plotted against the analytical modeling performed by Roberts (2005) and the analytical modeling performed in these studies (Figure 7.16). As shown, the modeling techniques described previously capture the nonlinear behavior of this experimental test and match the modeling techniques adopted from Roberts (2005).


Figure 7.16: Comparison of the Roberts (2005) 'R1’Experimental and Analytical Results

### 7.8.4 Benchmark Analysis \#4: Michaelson (2014)

As described in Section 2.3.1, Michaelson (2014) performed experimental testing on PBFTGs to develop and refine the system for use in the short-span bridge market. An 84 inch wide by 7/16 inch thick plate was formed into a PBFTG, and a 60 inch wide by 6 inch thick reinforced concrete deck was compositely cast, creating a modular unit. Destructive flexural testing was performed on the composite specimen, and analytical procedures were developed to verify the capacity of the proposed system. Figures 7.17 and 7.18 show the cross-section of a PBFTG without and with the concrete deck, respectively.


Figure 7.17: Testing Specimen Dimensions (Michaelson, 2014)


Figure 7.18: Deck Reinforcement for Composite Sections (Michaelson, 2014)

An input file was generated using the aforementioned modeling techniques, including the target mesh density and the use of multi-point constraints discussed in Section 7.6, to model the composite experiments performed by Michaelson (2014), and FEA was performed using Abaqus/CAE. The load-deflection curve from experimental testing was plotted against the analytical modeling performed by Michaelson (2014) and the analytical modeling performed in these studies (Figure 7.19). As shown, the modeling techniques described previously capture the behavior of non-skewed PBFTGs.


Figure 7.19: Comparison of the Michaelson (2014) Composite Experimental and Analytical

## Results

### 7.8.5 Benchmark Analyses \#5: Amish Sawmill Bridge

To verify the validity of the finite element modeling techniques regarding bridge systems, physical load data from three field tests were compared to the analytical results of finite element models of the bridges using the modeling techniques described previously. As described in Section 2.4.1, Gibbs (2017) performed a live load field test on the Amish Sawmill Bridge in Buchanan County, Iowa. The bridge consisted of four PBFTGs fabricated from 96 inch wide by $1 / 2$ inch thick plate with a girder spacing of 7.5 feet, as seen in Figure 7.20. A loading truck was placed at points, both transversely and longitudinally, along the bridge, and strains were recorded.


Figure 7.20: Cross-section of the Amish Sawmill Bridge (Gibbs, 2017)

An input file was generated using the modeling techniques described previously, except for the nonlinear analysis. As the loading of bridge was representative of actual vehicular traffic, the magnitude of the loading would never approach the loading and material nonlinearity seen in the previous laboratory experiments. A comparison of the field recorded data and the analytical modeling to verify the accuracy of the modeling techniques of this study is shown in Table 7.3. Specifically, from the field test, the truck run maximizing the load in the first interior girder is shown. The strains, and therefore LLDFs, of Girder 2 at midspan were reported as these values directly compared with the field test data. As shown, the analytical model very accurately predicted LLDFs. The largest absolute difference in values was only 0.024 , equivalent to $8.86 \%$ difference.

Table 7.3: Amish Sawmill Bridge Analytical Model Verification

| Girder 2, Truck Run 2, LLDF Comparisons |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Longitudinal Truck Placement | Field Test <br> LLDF | Analytical <br> LLDF | Absolute <br> Difference | Absolute <br> Percent |  |
| $\mathbf{x}$ (ft) | $\mathbf{x / L}$ | --- | --- | --- | --- |
| 0 | 0 | 0.282 | 0.299 | 0.017 | $5.88 \%$ |
| 5.2 | 0.1 | 0.312 | 0.303 | 0.009 | $3.02 \%$ |
| 10.4 | 0.2 | 0.300 | 0.303 | 0.003 | $1.06 \%$ |
| 15.6 | 0.3 | 0.302 | 0.306 | 0.004 | $1.24 \%$ |
| 20.8 | 0.4 | 0.332 | 0.312 | 0.020 | $5.99 \%$ |
| 26 | 0.5 | 0.328 | 0.314 | 0.014 | $4.24 \%$ |
| 31.2 | 0.6 | 0.303 | 0.309 | 0.006 | $1.89 \%$ |
| 36.4 | 0.7 | 0.275 | 0.299 | 0.024 | $8.65 \%$ |
| 41.6 | 0.8 | 0.269 | 0.293 | 0.024 | $8.86 \%$ |
| 46.8 | 0.9 | --- | --- | --- | --- |
| 52 | 1 | $\mathbf{0 . 3 0 0}$ | $\mathbf{0 . 3 0 4}$ | $\mathbf{0 . 0 1 3}$ | $\mathbf{4 . 5 4 \%}$ |
| Average |  |  |  |  |  |

### 7.8.6 Benchmark Analyses \#6: Fourteen Mile Bridge

As discussed in Section 2.4.3, Roh (2020) performed a live load field test on the Fourteen Mile Bridge in Lincoln County, West Virginia. The bridge consisted of five PBFTGs fabricated from 96 inch wide by $1 / 2$ inch thick plate with a girder spacing of 7 feet and a span length of 58 feet. This bridge has unique characteristics, such as a skew of $10^{\circ}$ and a superelevation of $8 \%$, as seen in Figure 7.21. The loading truck, while chosen to model the HL-93 Design Load, did not exactly match, so axle weights and dimensions were recorded for use in the field test and the analytical model. The Stallings/Yoo Method was chosen to calculate the distribution factors in the analytical model, as this methodology was used in the live load field test.


Figure 7.21: Cross-section of the Fourteen Mile Bridge (Roh, 2020)

Following a methodology identical to that used for the Amish Sawmill bridge, an input file was generated and analyzed using Abaqus/CAE. The results were exported, and post-processing was completed to determine LLDFs for an exterior, the first interior girder, and the center girder. A comparison of the field recorded data and the analytical data is shown in Table 7.4. Specifically, from the field test, the truck run maximizing the load in an exterior girder is shown. The strains, and therefore LLDFs, of Girder 1 at quarter span were reported as these values directly compared with the field test data. As the values were recorded at quarter span, instead of midspan, where the strains produced by the loading vehicle would be the highest, smaller strains were recorded. It should be noted that, while percent differences have been reported, they can be somewhat deceptive as differences of a fraction can represent somewhat large percentage differences. When comparing LLDFs computed at midspan, the analytical model only overestimates by 0.046 on a bridge with significant layout complications. This slight overestimation is within the allowable error discussed in AASHTO LRFD BDS Article C4.6.2.2.1.

Table 7.4: Fourteen Mile Bridge Analytical Model Verification

| Girder 1, Truck Run 1, LLDF Comparisons |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Longitudinal Truck Placement |  | $\begin{gathered} \hline \text { Field Test } \\ \text { LLDF } \\ \hline \end{gathered}$ | Analytical LLDF | Absolute Difference | Absolute Percent |
| x (ft) | x/L |  |  |  |  |
| 0 | 0 | --- | --- | --- | --- |
| 5.8 | 0.1 | --- | --- | --- | --- |
| 11.6 | 0.2 | 0.456 | 0.347 | 0.109 | 23.91\% |
| 17.4 | 0.3 | 0.403 | 0.382 | 0.021 | 5.27\% |
| 23.2 | 0.4 | 0.406 | 0.427 | 0.021 | 5.28\% |
| 29 | 0.5 | 0.421 | 0.467 | 0.046 | 10.84\% |
| 34.8 | 0.6 | 0.420 | 0.491 | 0.071 | 16.81\% |
| 40.6 | 0.7 | 0.453 | 0.527 | 0.074 | 16.29\% |
| 46.4 | 0.8 | 0.499 | 0.578 | 0.079 | 15.89\% |
| 52.2 | 0.9 | 0.567 | 0.613 | 0.046 | 8.14\% |
| 58 | 1 | --- | --- | --- | --- |
| Average |  | 0.453 | 0.479 | 0.058 | 12.80\% |
| Maximum (in magnitude) |  | 0.567 | 0.613 | 0.109 | 23.91\% |
| Minimum (in magnitude) |  | 0.403 | 0.347 | 0.021 | 5.27\% |

### 7.8.7 Benchmark Analyses \#7: Flat Run Bridge

A third live load field test was performed concurrently with these research efforts. The test was performed on the Flat Run Bridge in Marion County, West Virginia. The bridge consisted of four PBFTGs fabricated from 96 inch wide by $1 / 2$ inch thick plate with a girder spacing of 8 feet and a span length of 56 feet. The Flat Run Bridge did not have any skew or superelevation, as seen in Figure 7.22. Similar to the Fourteen Mile Bridge live load field test, the loading truck was chosen to model the HL-93 Design Load, but as the axle weights and dimensions did not exactly match, the weights and dimensions were recorded for use in the field determined LLDFs and the analytical modeling. This difference in loading should correspond to similar LLDFs, as the actual truck was chosen to produce similar force effects. Additionally, the AASHTO LRFD BDS Design Truck is the loading used in the studies determining empirical LLDFs.


Figure 7.22: Cross-section of the Flat Run Bridge

A distinct difference for this analysis was the use of the Tarhini/Frederick method in addition to the Stallings/Yoo method of calculating LLDFs. The Stallings/Yoo calculation of LLDFs was performed as part of the field study, while the Tarhini/Frederick calculation of LLDFs was performed as part of this research. A comparison of the field recorded data and the analytical modeling maximizing load in Girder 1 with one lane loaded is shown in Table 7.5. As shown, the differences in both LLDF methodologies with the live load field test are negligible, verifying the validity of the analytical modeling techniques in determining LLDFs for PBFTG bridges.

Table 7.5: Flat Run Bridge Analytical Model Verification

|  | Field | Stallings/ | Tarhini/ | Field Calculated vs Stallings/Yoo |  | Field Calculated vs Tarhini/Frederick |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Calculated | Yoo | Frederick | Absolute Difference | Absolulte Percent | Absolute Difference | Absolulte Percent |
| Girder 1 | 0.51 | 0.47 | 0.53 | 0.04 | $7.84 \%$ | 0.02 |  |
| Girder 2 | 0.37 | 0.34 | 0.36 | 0.03 | $8.11 \%$ | $0.92 \%$ |  |
| Girder 3 | 0.2 | 0.23 | 0.19 | 0.03 | $15.00 \%$ | 0.01 |  |
| Girder 4 | 0.12 | 0.16 | 0.13 | 0.04 | $33.33 \%$ | 0.01 |  |

### 7.9 SUMMARY

This chapter detailed an explanation of the analytical modeling techniques used for assessing PBFTGs. The accuracy of these models has been benchmarked against previous experimental investigations. The results of these benchmarks show the analytical tools accurately capture the behavior of noncomposite and composite plate girder testing and capture the behavior of non-skewed PBFTGs. This chapter has shown the extensive verification of the tools used in this
research from the behavior of plates acting under point loads to behavior of PBFTGs past yielding of the steel to global behavior of PBFTGs in bridge system applications. These tools will be used in the following chapters to develop LLDFs utilizing PBFTGs and the ultimate capacity of compact PBFTGs.

# CHAPTER 8: DEVELOPMENT OF LLDFS FOR PBFTG BRIDGES 


#### Abstract

8.1 InTRODUCTION

The following chapter presents the research performed to develop LLDFs for PBFTG bridges. First, a matrix of bridges was generated and analyzed using a commercial finite element software to determine the sensitivity of commonly used parameters in the distribution of live load to longitudinal elements. Next, a modified matrix was generated based on the assessment of the key parameters most affecting on the distribution of live load in PBFTG bridges. LLDFs generated from the parametric matrix of over 50,000 hypothetical PBFTG bridges were used to generate simplified equations to be used in conjunction with LGA. Finally, the simplified equations were verified against experimental and analytical results from three in-service PBFTG bridges.


### 8.2 Sensitivity Study

The following section describes a matrix of bridges analyzed using Abaqus/CAE to determine the sensitivity of certain parameters on the live load distribution characteristics in PBFTG bridges. Specifically, this section will discuss the bridges modeled with their respective constant and varied parameters. The results of the sensitivity study are discussed, highlighting the influence of the varied parameters and the comparisons between the analytically derived LLDFs and the empirical equations presented in the AASHTO LRFD BDS.

### 8.2.1 Typical Bridge Cross-Sections

A crucial component in the development of simplified methods for live load distribution is the range of applicability. To ensure common values of various parameters were considered, four common bridges were used as the basis for the matrix of bridges to be analyzed in the sensitivity study. The four standard bridges are described in Table 8.1, and a cross-section of a typical bridge is shown in Figure 8.1.

Table 8.1: Standard Bridge Dimensions

| Bridge <br> Number | Plate Size <br> (in) | Span <br> Length <br> (ft) | Number of <br> Girders | Girder <br> Spacing <br> (ft) | Deck <br> Thickness <br> (in) | Overhang <br> Width <br> (ft) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $72 \times 1 / 2$ | 30 | 3 | 6 | 8 | 2.22 |
| 2 | $84 \times 1 / 2$ | 40 | 4 | 8 | 8 | 2.11 |
| 3 | $96 \times 1 / 2$ | 60 | 5 | 8 | 8 | 1.8 |
| 4 | $120 \times 5 / 8$ | 80 | 6 | 10 | 8 | 1.29 |



Figure 8.1: Sensitivity Bridge Cross-Section

### 8.2.2 Constant Parameters

The following parameters remained constant in the sensitivity matrix:

- The distance between the top of the top flange and the bottom of the deck was held at a constant 2 inches.
- The width of the barrier was held at a constant 15 inches.
- Normal weight reinforced concrete was used throughout. In accordance with AASHTO LRFD BDS Table 3.5.1-1, this equates to a unit weight of 0.150 kcf . Also, in accordance with the same table, the unit weight of steel was taken to be 0.490 kcf .
- The following material properties were also employed:
- For reinforced concrete, which was taken to have a compressive strength of 4 ksi , according to the provisions of AASHTO LRFD BDS Section 5.4.2.4, the modulus of elasticity of concrete was determined to be $3,640 \mathrm{ksi}$. Also, according to AASHTO LRFD BDS Section 5.4.2.5, Poisson's ratio was taken to be 0.2 .
- For steel, which was taken to have a yield strength of 50 ksi , according to the provisions of AASHTO LRFD BDS Section 6.4.1, the modulus of elasticity of steel was taken to be $29,000 \mathrm{ksi}$. Also, Poisson's ratio was taken to be 0.3 .
- The boundary conditions were kept simply supported.
- Finally, the same style and thickness of bearing stiffeners were used for all girders.


### 8.2.3 Varied Parameters

The following parameters were varied in the sensitivity matrix and investigated to determine their respective effect on the live load distribution in PBFTG bridges:

- Thirteen possible span lengths: 20 feet to 80 feet in 5 feet increments
- Four possible number of girders in the cross-section: 3, 4, 5, and 6 girders
- Four possible individual PBFTG cross-sections consisting of different plate sizes: 72 inch by 0.5 inch, 84 inch by 0.5 inch, 96 inch by 0.5 inch, and 120 inch by 0.625 inch
- Seven possible girder spacings: 4 feet to 16 feet in 2 feet increments
- Seven possible deck thicknesses: 8 inches to 9.5 inches in 0.25 inch increments
- Five overhang ratios: $0.2 \times$ Girder Spacing to $0.4 \times$ Girder Spacing in 0.05 increments
- One or two-lanes loaded
- Load placement to maximize moment in an interior or exterior girder


### 8.3 ReSults of Sensitivity Study

As the tabulated results of the sensitivity study are too large to be included in this chapter, a comprehensive summary of the results has been provided in Appendix A. Appendix A summarizes the effect of each varied parameter in tabular and graphical form. The general trends of the sensitivity study results will be discussed herein, specifically the effect of the parameters on the distribution of live load in a multitude of loading scenarios.

### 8.3.1 Comparison with AASHTO LRFD BDS LLDFs

Generally, as shown in many previous studies, such as the ones discussed in Chapter 3, the LLDFs obtained from the analytical modeling were generally lower than those calculated using the methods found in the AASHTO LRFD BDS. A comparison between LLDFs calculated from the analytical model and LLDFs calculated using empirical methods from the AASHTO LRFD BDS is shown in Figure 8.2. It should be noted the AASHTO LRFD BDS I-Girder methodology is dependent on the lever rule methodology, and any restrictions for the use of LLDFs per the AASHTO LRFD BDS have been neglected to compare results.


Figure 8.2: Comparison of Analytical and Empirical LLDF Methods

For the sensitivity matrix, the one-lane loaded LLDFs for exterior girders, averaged from the Stallings/Yoo and the Tarhini/Frederick analytical methods, are $34 \%$ lower compared to the lever rule and $22 \%$ lower compared to the box-girder simplified LLDFs. For the two-lane loaded LLDFs for exterior girders, the averaged analytical LLDFs are 7\% higher than the I-girder simplified LLDFs and $11 \%$ lower than the box-girder simplified LLDFs. A similar pattern was found for the interior girder LLDFs.

### 8.3.2 Influence of Span Length

The influence of span length on LLDFs on Standard Bridge 3, as described in Section 8.2.1, is shown in Figure 8.3. Figure 8.3 has been divided into four components for clarity, each component representing the girder being maximized and the number of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar represents a bridge with a different span length, in feet, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.3c and 8.3d.

(a)


## (b)


(c)

(d)

Figure 8.3: Comparison of the Inlfuence of Span Length on Standard Bridge 3 on an: (a) Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder One-Lane Loaded, (d) Exterior Girder Two-Lanes Loaded

From these graphs, it was concluded span length has a significant impact on PBFTG LLDFs. Span length will be considered in the parametric study. Another interesting conclusion from this data is that span length seems to have a more significant impact on one-lane loaded LLDFs. Of the potential simplified distribution methods provided by the AASHTO LRFD BDS, the I-Girder methodology most accurately reflects the trend of LLDFs with respect to span length.

### 8.3.3 Influence of Number of Beams

The influence of the number of beams in the cross-section on LLDFs on Standard Bridge 4, as described in Section 8.2.1, is shown in Figure 8.4. Figure 8.4 has been divided into four components for clarity, each component representing the girder being maximized and the number
of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar represents a bridge with a different number of beams, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.4c and 8.4d.

(a)

(b)

(c)

(d)

Figure 8.4: Comparison of the Effect of Number of Girders on Standard Bridge 4 on an: (a) Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder One-Lane Loaded, (d) Exterior Girder Two-Lanes Loaded

The number of beams in the cross-section has a small effect on PBFTG LLDFs. The number of beams will be considered in the parametric study to determine if the trend of increased number of beams corresponds to decreased effect on PBFTG live load distribution. Similar to the effect of span length, the I-Girder methodology accurately reflects the trend of LLDFs with respect to the number of beams in the cross-section. An interesting difference between the comparisons of span length and number of beams using the I-Girder methodology is the I-Girder methodology underestimates LLDFs when varying number of girders, where the same methodology overestimates LLDFs when varying span length.

### 8.3.4 Influence of PBFTG Size

The influence of the size of PBFTGs on LLDFs on Standard Bridge 4, as described in Section 8.2.1, is shown in Figure 8.5. Figure 8.5 has been divided into four components for clarity, each component representing the girder being maximized and the number of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar represents a bridge with a different PBFTG utilized, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.5c and 8.5d.

(a)

(b)

(c)

(d)

Figure 8.5: Comparison of the Influence of Girder Size on Standard Bridge 4 on an: (a) Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder One-Lane Loaded, (d) Exterior Girder Two-Lanes Loaded

The PBFTG size directly correlates to the longitudinal stiffness of the girder. As shown in Figure 8.5 , the PBFTG size, and by correlation, the longitudinal stiffness does have a minor impact on live load distribution. However, for exterior girders, this evaluation is somewhat difficult to justify based on this data, as the overhang ratio is dependent on the PBFTG size. As the size of the PBFTGs increase, not only does the longitudinal stiffness parameter increase, but the width of the PBFTGs increase. This increase in width causes the exterior wheel line to move further away from the center of the bridge, causing higher exterior girder LLDFs. This difficulty is further addressed in the assessment of overhang ratio later in the sensitivity study and in the parametric study.

### 8.3.5 Influence of Girder Spacing

The influence of girder spacing on LLDFs on Standard Bridge 1, as described in Section 8.2.1, is shown in Figure 8.6. Figure 8.6 has been divided into four components for clarity, each component representing the girder being maximized and the number of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar represents a bridge with a different girder spacing, in feet, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.6c and 8.6d.

(a)

(b)

(c)

(d)

Figure 8.6: Comparison of the Influence of Girder Spacing on Standard Bridge 1 on an: (a)

## Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder

 One-Lane Loaded, (d) Exterior Girder Two-Lanes LoadedAs anticipated, girder spacing has a significant effect on live load distribution in all cases. Girder spacing will be considered in the parametric study. Note, Figures 8.6b and 8.6d do not have girder spacings of 6 feet, as bridges with a girder spacing of 6 feet cannot be loaded with two design lanes due to the clear roadway width being below the allowable minimum of 20 feet for two design lanes. Additionally, the scale for the LLDFs was modified from a maximum of 1.0 to 1.4 to accommodate large LLDFs calculated in this study. In addition, not every bridge crosssection will utilize the entire breadth of possible girder spacings. For example, it would be a rather unique bridge to utilize PBFTGs bent out of 72 inch wide by $1 / 2$ inch thick plate with a girder spacing of 16 feet. This will be considered when developing the parametric study.

### 8.3.6 Influence of Deck Thickness

The influence of deck thickness on LLDFs on Standard Bridge 2, as described in Section 8.2.1, is shown in Figure 8.7. Figure 8.7 has been divided into four components for clarity, each component representing the girder being maximized and the number of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar represents a bridge with a different deck thickness, in inches, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.7c and 8.7d.

(a)

(b)

(c)

(d)

Figure 8.7: Comparison of the Influence of Deck Thickness on Standard Bridge 3 on an: (a) Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder One-Lane Loaded, (d) Exterior Girder Two-Lanes Loaded

The thickness of the deck has a negligible effect on LLDFs in PBFTG. As seen in Figure 8.7, all distribution methods agree, which cannot be stated for the other factors in the sensitivity matrix. However, deck thickness is a variable in the longitudinal stiffness parameter, which was not directly assessed in this study.

### 8.3.7 Influence of Overhang Ratio

The influence of overhang ratio on LLDFs on Standard Bridge 4, as described in Section 8.2.1, is shown in Figure 8.8. Figure 8.8 has been divided into four components for clarity, each component representing the girder being maximized and the number of lanes loaded. Each set of vertical bars represents a different live load distribution method, and each different color bar
represents a bridge with a different overhang ratio, as shown in the legend. Additionally, the methodology of Special Analysis is discussed in Section 8.7.1 and can only be performed on exterior girders, so values are only shown in Figure 8.8c and 8.8d.

(a)

(b)

(c)

(d)

Figure 8.8: Comparison of the Influence of Overhang Ratio on Standard Bridge 3 on an: (a) Interior Girder One-Lane Loaded, (b) Interior Girder Two-Lanes Loaded, (c) Exterior Girder One-Lane Loaded, (d) Exterior Girder Two-Lanes Loaded

Overhang ratio is defined as the ratio of the girder spacing to the distance measured from the center of the top exterior flange to the edge of the deck. As the placement of the truck, relative to the center of the girders, is also dependent on the width of the PBFTG, similar to girder size, it is difficult to evaluate the effect of this parameter solely. Additionally, this parameter is dependent on the girder spacing for any given bridge. As can be seen from Figure 8.8, the overhang ratio has a negligible effect on interior girder LLDFs but an obvious effect on exterior girder LLDFs. Due to this effect on exterior girder LLDFs, the overhang will continue to be considered in the parametric study, but the approach will be slightly different.

### 8.4 Parametric Study

The following section describes an updated matrix of bridges analyzed using Abaqus/CAE to determine the effect of key parameters on the live load distribution characteristics in PBFTG bridges. This matrix was developed based on the assessment of the results of the sensitivity matrix to further investigate the parameters found to have the most significant impact on PBFTG live load distribution. This section will additionally discuss the modified constant and varied parameters and their respective limits. The new matrix of bridges consisted of 50,312 bridges to assess the effects of a more focused set of key parameters more accurately on PBFTG LLDFs. These bridges utilize the same constant parameters discussed in Section 8.2.2.

### 8.4.1 Typical Bridge Cross-Sections

To visually ascertain the effect of the parameters used in this study, a generic cross-section was developed for comparison. The generic cross-section was developed with standard parameters to ensure the comparison was not skewed to one extreme end of the data set. The generic crosssection consists of six PBFTGs constructed from 96 inch wide by $1 / 2$ inch thick plate spaced at seven feet apart on center with a span length of 60 feet. The distance from the center of the exterior flange to the edge of the parapet is twelve inches and effective deck thickness is nine inches.

### 8.4.2 Varied Parameters

As discussed in Section 8.3, while the sensitivity study allowed for global analysis of key factors, a more detailed assessment of a refined range of variation of key variables is necessary. Based on the results of the sensitivity study, some modifications on the definitions and bounds of certain parameters were made. As PBFTGs are intended to be prefabricated bridge elements, required to be able to be fabricated in a shop and transported to the bridge site, the maximum modular width, and therefore girder spacing, was reduced from 16 feet to 9 feet. Additionally, the minimum girder spacing was increased to 5 feet, as none of the bridges in the sensitivity study were able to utilize 4 foot girder spacings.

The deck thickness variations were increased from 0.25 inch increments to 0.5 increments due to the lack of sensitivity to the variable. However, the deck thickness remained as a variable
to allow for the exploration of the longitudinal stiffness parameter. Additionally, to mirror the methodology already in place in the AASHTO LRFD BDS, the effect of width of overhang was modified from the overhang ratio to the $d_{e}$ term, as defined in the AASHTO LRFD BDS.

The number of PBFTG sizes was reduced from four to three to decrease the number of parametric variations without losing the total variety of PBFTG sizes available. The sizes remaining in the study are the ones proposed by Michaelson (2014). Additionally, as not all PBFTG size combinations are appropriate for the entire range of applicability for the PBFTG system, span ranges were provided for each PBFTG size encompassing their individual applicable span ranges.

Using the aforementioned modifications, the following parameters were varied in the parametric matrix to determine their respective effect on live load distribution in PBFTG bridges:

- Three possible individual PBFTG cross-sections consisting of different plate sizes with corresponding span lengths
- 72 inch by 0.5 inch with span lengths from 20 feet to 50 feet in 5 feet increments
- 96 inch by 0.5 inch with span lengths from 40 feet to 80 feet in 5 feet increments
- 120 inch by 0.625 inch with span lengths from 50 feet to 100 feet in 5 feet increments
- Six possible number of girders in the cross-section: 3, 4, 5, 6, 7, and 8 girders
- Five possible girder spacings: 5 feet to 9 feet in 1 foot increments
- Four possible deck thicknesses: 8 inch to 9.5 inch in $1 / 2$ inch increments
- Five possible distances from the centerline of the exterior flange to the edge of parapet: 0 inch to 24 inch in 6 inch increments
- One or two-lanes loaded
- Load placement to maximize moment in an interior or exterior girder


### 8.5 Results of Parametric Study

As the tabulated results of the parametric study are too large to be included in this chapter, a comprehensive summary of the results has been provided in Appendix B. Appendix B summarizes the effect of the varied parameter in graphical form. The general trends of the
parametric study results will be discussed herein, specifically the effect of the parameters on the distribution of live load in a multitude of loading scenarios.

### 8.5.1 Influence of Span Length

Figure 8.9 presents the comparison of different girder spacings used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge. Figure 8.9 has been divided into two components for clarity, each component representing a different analytical technique to determine LLDFs.

(a)

(b)

Figure 8.9: Comparison of the Influence of Span Length using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

As expected, span length had the same effect in the parametric study as it did in the sensitivity study. A slight difference can be seen in the magnitude of the effect of span length in the applicable range of the girder. The effect appears to be of a slightly more linear nature than found in the sensitivity study. This more linear relationship can be attributed to the larger span ranges where this beam would be utilized and removing the smaller spans were the nonlinearity would be more pronounced. Some amount of nonlinearity does exist, as seen most pronounced in the shorter span ranges when utilizing one-lane loaded scenarios.

### 8.5.2 Influence of Number of Beams

Figure 8.10 presents the comparison of different numbers of beams used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge. Figure 8.10 has been divided into two components for clarity, each component representing a different analytical technique to determine LLDFs.

(a)


## (b)

Figure 8.10: Comparison of the Influence of Number of Beams using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

As expected, the number of beams had a similar effect in the parametric study as it did in the sensitivity study. The effect of increased number of beams becomes negligible for all types of loading after the bridge contains five beams in the cross-section. The number of beams has a more significant effect for smaller numbers of beams, but it is unlikely typical bridges utilizing PBFTGs will consist of such few main longitudinal members.

### 8.5.3 Influence of PBFTG Size

Figure 8.11 presents the comparison of different PBFTG sizes used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge. Figure 8.11 has been divided into two components for clarity, each component representing a different
analytical technique to determine LLDFs. Note, for this generic cross-section, the smallest PBFTG size, the 72 inch by $1 / 2$ inch PBFTG, is not applicable for the generic cross-section.

(a)


## (b)

Figure 8.11: Comparison of the Influence of Girder Size using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

While the size, and therefore moment of inertia, of PBFTGs does have a slight effect on live load distribution, it is important to remember what these bridges represent. The 96 inch wide by $1 / 2$ inch thick plate is the basis for the bridge used in this comparison. The larger PBFTG, while usable in this instance, would represent an exceptionally conservative design. However, this evaluation may be somewhat difficult to make on its own, as a longitudinal stiffness parameter considering the concrete deck on composite specimens may be a more accurate representation of stiffness when it comes to live load distribution. However, the longitudinal stiffness parameter can be difficult to isolate as it is dependent on girder size, deck thickness, and span length.

### 8.5.4 Influence of Girder Spacing

Figure 8.12 presents the comparison of different girder spacings used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge. Figure 8.12 has been divided into two components for clarity, each component representing a different analytical technique to determine LLDFs.

(a)


## (b)

Figure 8.12: Comparison of the Influence of Girder Spacing using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

As expected, girder spacing has a similar effect on live load distribution in the parametric study as the sensitivity study. The influence of girder spacing has been verified by numerous other researchers, as noted in Section 3.4.1. Similar to the work performed by Zokaie et al. (1991), this study found girder spacing and span length may be better defined together as an aspect ratio instead of separate parameters.

### 8.5.5 Influence of Deck Thickness

Figure 8.13 presents the comparison of different deck thicknesses used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge.

Figure 8.13 has been divided into two components for clarity, each component representing a different analytical technique to determine LLDFs.

(a)

(b)

Figure 8.13: Comparison of the Influence of Thickness of Deck using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

As expected, deck thickness, on its own, has a negligible effect on the distribution of live load. As stated in Section 8.3.6, as deck thickness is a variable in the longitudinal stiffness, it was deemed appropriate to consider it as a variable when developing LLDFs.

### 8.5.6 Influence of Edge Distance

Figure 8.14 presents the comparison of different edge distances used in the parametric study. Each curve in the figure represents one of the four loading scenarios of the typical bridge. Figure 8.14 has been divided into two components for clarity, each component representing a different analytical technique to determine LLDFs.

(a)


## (b)

Figure 8.14: Comparison of the Influence of Edge Distance using the: (a) Stallings/Yoo Method, (b) Tarhini/Frederick Method

As expected, the influence of edge distance is negligible for interior girders but significant for exterior girders. The purpose of the change from overhang ratio to edge distance was to isolate the effect of the parameter. Seen in this manner, the effect of edge distance has a linear relationship with LLDFs of exterior girders. This correlates with the findings of Zokaie et al. (1991) as discussed in Section 3.4.7.

### 8.6 DEVELOPMENT OF PBFTG LLDFs

The following section describes the methodology used to develop empirical equations for interior and exterior girder, one or two-lanes loaded, LLDFs for PBFTG bridges. Specifically, a Multiple Linear Regression (MLR) relationship was used to relate the results from the parametric
study with the factors found to have the most prominent effect on live load distribution. MLR modeling was used to develop simplified empirical equations for PBFTG LLDFs. MLR is a statistical linear relationship between the dependent variable, the LLDF, and multiple independent variables, the bridge and girder parameters. Finally, the proposed equations and their correlation with the analytically determined LLDFs is presented.

### 8.6.1 Analytical Computation Technique

The first step to utilizing MLR is to determine which live load distribution methodology will be used to generate LLDFs. The Stallings/Yoo methodology calculates LLDFs for each girder in the cross-section by dividing the maximum bending strain of the girder in question by the sum of the maximum bending strains in every girder in the cross-section. The Tarhini/Frederick methodology calculates LLDFs for each girder by dividing the maximum bending strain calculated using three-dimensional FEA by the maximum bending strain calculated using LGA. The Tarhini/Frederick methodology was chosen due to lack of dependence on the strains and stresses of other girders and its use of LGA similar to methodologies found in the AASHTO LRFD BDS.

### 8.6.2 Methodology

As previously stated, MLR represents a linear relationship between a dependent variable, such as the Tarhini/Frederick LLDF, and multiple independent variables, such as key bridge parameters. To apply MLR, a model representing the relationship between the dependent and independent variables must be defined. This linear model is expressed in Equation 8.1:

$$
\begin{equation*}
y_{i}=c_{0}+c_{1} x_{i 1}+c_{2} x_{i 2}+\cdots+c_{k} x_{i k} \tag{Eq. 8.1}
\end{equation*}
$$

Where:
$y=$ dependent variable
$x=$ independent variable
$c=$ constant coefficients
$i=$ number of samples
$k=$ number of independent variables

For equations with more than one independent variable, the curve fitting process becomes more difficult. To overcome this difficulty, matrices are used to define the regression model and the subsequent analysis. As the relationship between the LLDF and key bridge parameters is not necessarily linear, and the use of the proposed equation in the AASHTO LRFD BDS is desirable, a model for the LLDF resembling those equations already in the AASHTO LRFD BDS was adopted as a working model. This model assumption was verified, as the parameters present in the AASHTO LRFD BDS for I-girder LLDFs were also the parameters found to have the most significant impact on PBFTG LLDFs. Therefore, the format for interior I-girder LLDFs form the basis of the equation as expressed in Equation 8.2.

$$
\begin{equation*}
g=c_{0}\left(x_{1}\right)^{c_{1}}\left(x_{2}\right)^{c_{2}}\left(x_{3}\right)^{c_{3}} \tag{Eq. 8.2}
\end{equation*}
$$

As the MLR is applied to a linear model, opposed to the power model shown by Equation 8.2, the natural logarithm was applied to both sides of the equation as demonstrated by Equation 8.3 and simplified into Equation 8.4.

$$
\begin{align*}
& \ln (g)=\ln \left(c_{0}\left(x_{1}\right)^{c_{1}}\left(x_{2}\right)^{c_{2}}\left(x_{3}\right)^{c_{3}}\right)  \tag{Eq. 8.3}\\
& \ln (g)=\ln \left(c_{0}\right)+c_{1} \ln \left(x_{1}\right)+c_{2} \ln \left(x_{2}\right)+c_{3} \ln \left(x_{3}\right) \tag{Eq. 8.4}
\end{align*}
$$

When applying Equation 8.4 for $k$ number of bridges, the linear equation can be formulated in matrix format, as seen in Equation 8.5.

$$
\left[\begin{array}{c}
y_{1}  \tag{Eq. 8.5}\\
y_{2} \\
\vdots \\
y_{k}
\end{array}\right]=\left[\begin{array}{ccc}
x_{11} & x_{12} & x_{13} \\
x_{21} & x_{22} & x_{23} \\
\vdots & \vdots & \vdots \\
x_{k 1} & x_{k 2} & x_{k 3}
\end{array}\right]\left[\begin{array}{c}
c_{1} \\
c_{2} \\
c_{3}
\end{array}\right]
$$

The matrices used in Equation 8.5 can be simplified into Equation 8.6:

$$
\begin{equation*}
\boldsymbol{Y}=\boldsymbol{X C} \tag{Eq. 8.6}
\end{equation*}
$$

After generating the above matrices, MLR was employed using the least square approximation procedure from Strang (2016). Strang states the solution to Equation 8.6 is expressed as Equation 8.7:

$$
\begin{equation*}
C=\left(X^{T} X\right)^{-1} X^{T} Y \tag{Eq. 8.7}
\end{equation*}
$$

Following the determination of the coefficients to the above equations, the coefficients were rounded to the number of significant figures found in the AASHTO LRFD BDS simplified equations. Another constant was added to the determined equations to shift the confidence interval to where nearly $100 \%$ of LLDFs generated using the proposed equations would result in conservative designs.

### 8.6.3 Proposed Equations

Using the key parameters found to have the most significant impact in the parametric study, including girder spacing, longitudinal stiffness, thickness of slab, and edge distance, the simplest combination of these variables producing the most accurate equations was determined. The accuracy of these equations was measured using the coefficient of multiple determination, or $R^{2}$, which is a numerical index reflecting the variation of the dependent variable with respect to two or more independent variables. Low values indicate the dependent variable is relatively unrelated to the independent variables, where high values corelate to high degrees of relation.

After analyzing the results of multiple combinations of parameters, Equations 8.8 through 8.11 were proposed to calculate LLDFs for PBFTG bridges.

For interior girders with one-lane loaded:

$$
\begin{equation*}
g=0.08+\left(\frac{S}{384}\right)^{0.1}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_{g}}{12.0 L t_{s}}\right)^{0.1} \tag{Eq. 8.8}
\end{equation*}
$$

For interior girders with two-lanes loaded:

$$
\begin{equation*}
g=0.13+\left(\frac{S}{27}\right)^{0.3}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_{g}}{12.0 L t_{s}}\right)^{0.1} \tag{Eq. 8.9}
\end{equation*}
$$

For exterior girders with one-lane loaded:

$$
\begin{equation*}
g=0.09+\left(1.07+\frac{d_{e}}{11.3}\right) g_{i n t} \tag{Eq. 8.10}
\end{equation*}
$$

For exterior girders with two-lanes loaded:

$$
\begin{equation*}
g=0.08+\left(0.86+\frac{d_{e}}{13.3}\right) g_{i n t} \tag{Eq. 8.11}
\end{equation*}
$$

Where:

$$
\begin{aligned}
g & =\text { LLDF for corresponding girder and number of loaded lanes } \\
S & =\text { girder spacing } \\
L & =\text { span length } \\
K_{g} & =\text { longitudinal stiffness parameter } \\
t_{s} & =\text { slab thickness } \\
d_{e} & =\text { edge distance } \\
g_{\text {int }} & =\text { LLDF for the interior girder for the corresponding number of loaded lanes }
\end{aligned}
$$

These equations exhibit good correlation between the key parameters identified in the parametric study and the proposed simplified LLDF equations. For Equation 8.8, the resulting $R^{2}$ value was 0.915 ; for Equation 8.9, the resulting $R^{2}$ value was 0.801 ; for Equation 8.10, the resulting $R^{2}$ value was 0.921 ; and for Equation 8.11, the resulting $R^{2}$ value was 0.838 . These $R^{2}$ values indicate these equations are fairly accurate in determining PBFTG LLDFs.

### 8.7 Comparison of Proposed Equations with AASHTO LRFD BDS Simplified Equations

In this section, comparisons are made between the proposed LLDF equations for PBFTG bridges with existing methodologies present in the AASHTO LRFD BDS. For this purpose, each of the proposed LLDFs, dependent on the girder being maximized and the number of loaded lanes, will be compared against lever rule, Special Analysis, if applicable, the LLDF for I-girders, and the LLDF for box-shaped girders. The details about each moment LLDF equation and their comparison to the proposed LLDF equation will be presented in the following subsections.

### 8.7.1 Applicable AASHTO LRFD BDS Live Load Distribution Methods

The lever rule methodology for calculating LLDFs involves summing moments about one girder line to find the reaction at another girder line, assuming the supported components are hinged at an interior support. The provisions of AASHTO LRFD BDS Article 3.6.1.1.1, regarding placement of the design lanes and the wheel lines within those lanes, should be followed when utilizing lever rule. It is important to include the applicable multiple presence factor to the LLDF calculated using lever rule when considering the strength and service limit states.

While not directly applicable to most PBFTG bridges, another LLDF methodology regularly used for exterior girder LLDFs from the AASHTO LRFD BDS is Special Analysis. Special Analysis is discussed in AASHTO LRFD BDS Article C4.6.2.2.2d and is used for steel bridge cross-sections with cross-frames or diaphragms. The LLDF for exterior girders is not to be taken as less than that which would be obtained by assuming the cross-section deflects and rotates as a rigid cross-section (Grubb et al., 2020). This methodology is included in the AASHTO LRFD BDS because it was found the simplified LLDF equations for moment were determined without consideration of cross-frames and are generally unconservative for exterior girders. Exterior girder LLDFs can be calculated by using Equation 8.12. As with lever rule, the provisions of AASHTO LRFD BDS Article 3.6.1.1.1 and the applicable multiple presence factors should be utilized.

$$
\begin{equation*}
R=\frac{N_{L}}{N_{b}}+\frac{X_{e x t} \sum_{1}^{N_{L}} e}{\sum_{1}^{N_{b}} x^{2}} \tag{Eq. 8.12}
\end{equation*}
$$

Where:

$$
\begin{aligned}
R= & \text { reaction on exterior beam in terms of lanes } \\
N_{L}= & \text { number of loaded lanes under consideration } \\
e= & \text { eccentricity of a design truck or a design lane load from the center of gravity of the } \\
& \text { pattern of girders } \\
x= & \text { horizontal distance from the center of gravity of the pattern of girder to each girder } \\
X_{e x t}= & \text { horizontal distance from the center of gravity of the pattern of girders to the exterior } \\
& \text { girder } \\
N_{b}= & \text { number of beams or girders }
\end{aligned}
$$

As discussed in Chapter 3, more complex LLDF equations were developed in the 1980s and 1990s to more accurately represent live load distribution compared to the previous S/D equations found in the AASHTO Standard Specifications. LLDFs found in the AASHTO LRFD BDS were modified from the research performed by Zokaie et al. (1991) to consider the multiple presence factor. Another difference between LLDFs in the AASHTO LRFD BDS and the AASHTO Standard Specification is that the modern LLDFs are now expressed in units of lanes rather than wheel lines.

The LLDF equation for bending moment in interior beams or girders with a concrete deck made composite with a steel beam with one design lane loaded is expressed by Equation 8.13, as found in AASHTO LRFD BDS Table 4.6.2.2.2b-1. However, to use this equation, a multitude of requirements must be met, which all bridges analyzed in the parametric study met. Another important note is that the I-girder equation is being compared against the proposed equation because the format of the I-girder equation is similar to the proposed equations. Additionally, when the cross-section consists of three longitudinal elements, the designer should use the lesser value of Equation 8.13 or lever rule.

$$
\begin{equation*}
g_{I G O L L}=0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_{g}}{12.0 L t_{S}^{3}}\right)^{0.1} \tag{Eq. 8.13}
\end{equation*}
$$

Where:
$g_{I G} O L L=$ LLDF for an interior girder with one-lane loaded
$S=$ spacing of beams or webs
$L=$ span length of beam
$K_{g}=$ longitudinal stiffness parameter
$t_{s}=$ depth of concrete slab

The LLDF equation for bending moment in interior beams or girders with a concrete deck made composite with a steel beam with two or more design lanes loaded is expressed by Equation 8.14, as found in AASHTO LRFD BDS Table 4.6.2.2.2b-1. Additionally, when the cross-section consists of three longitudinal elements, the designer should use the lesser value of Equation 8.14 or lever rule.

$$
\begin{equation*}
g_{I G 2 L L}=0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_{g}}{12.0 L t_{s}^{3}}\right)^{0.1} \tag{Eq. 8.14}
\end{equation*}
$$

Where:

$$
\begin{aligned}
g_{I G} 2 L L & =\text { LLDF for an interior girder with two-lanes loaded } \\
S & =\text { spacing of beams or webs } \\
L & =\text { span length of beam } \\
K_{g} & =\text { longitudinal stiffness parameter } \\
t_{s} & =\text { depth of concrete slab }
\end{aligned}
$$

The LLDF equation for bending moment in exterior beams or girders with a concrete deck made composite with a steel beam with one design lane loaded is expressed using lever rule for bridge cross-sections containing three or more longitudinal elements. The LLDF equation for bending moment in exterior beams or girders with a concrete deck made composite with a steel beam with two or more design lanes loaded is expressed by Equation 8.15, which is a modified version of the equation found in AASHTO LRFD BDS Table 4.6.2.2.2d-1. It should be noted the LLDF for two or more lanes loaded is dependent on the LLDF for the same bridge but for the interior girder. The overhang distance in Equation 8.15 shall be taken as positive if the exterior
web is inside the interior face of the parapet and negative if the exterior web is outside the interior face of the parapet. Additionally, when the cross-section consists of three longitudinal elements, the designer should use the lesser value of Equation 8.15 or lever rule.

$$
\begin{equation*}
g_{E G 2 L L}=\left(0.77+\frac{d_{e}}{9.1}\right) g_{I G 2 L L} \tag{Eq. 8.15}
\end{equation*}
$$

Where:
$g_{E G 2 L L}=$ LLDF for an exterior girder with two-lanes loaded
$d_{e}=$ horizontal distance from the centerline of the exterior web or exterior beam at deck level to the interior edge of curb or traffic barrier
$g_{I G} O L L=$ LLDF for an interior girder with two-lanes loaded

The LLDF equation for bending moment in all beams or girders with a concrete deck made composite with multiple steel box-girders, regardless of the number of loaded lanes, is expressed by Equation 8.16, as found in AASHTO LRFD BDS Table 4.6.2.2.2b-1. Note, not all bridges in the parametric study meet the range of applicability as expressed in Equation 8.17. Additionally, not all bridges in the parametric study meet the special restrictions on the use of LLDFs discussed in AASHTO LRFD BDS Article 6.11.2.3. All bridges in the parametric study were assumed to meet the restrictions to allow comparisons against the proposed equations. If the restrictions specified in the AASTHO LRFD BDS are not met, the designer must utilize a more refined analysis.

$$
\begin{align*}
& 0.05+0.85 \frac{N_{L}}{N_{b}}+\frac{0.425}{N_{L}}  \tag{Eq. 8.16}\\
& 0.5 \leq \frac{N_{L}}{N_{b}}<1.5 \tag{Eq. 8.17}
\end{align*}
$$

Where:
$N_{L}=$ number of design lanes as specified in AASHTO LRFD BDS Article 3.6.1.1.1
$N_{b}=$ number of girders

### 8.7.2 Interior Girder One-Lane Loaded LLDFs

The comparisons of the equations and methodologies found in the AASHTO LRFD BDS for LLDFs for interior girder one-lane loaded scenarios is presented in Figure 8.15 and Table 8.2. The comparison revealed the proposed moment LLDF equation more accurately predicts LLDFs than those of any other LLDF methodology or equation present in the AASHTO LRFD BDS. These comparisons serve to verify the applicability of the equation for use in PBFTG bridges.

(a)

(b)

(c)

Figure 8.15: Correlation Between Proposed LLDF Equation for Interior Girders with OneLane Loaded with: (a) Lever Rule, (b) I-Girder Equation, and (c) Box-Girder Equation

Table 8.2: Interior Girder One-Lane Loaded Statistical Analysis

| Interior Girder One Lane Loaded |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| LLDF Methodology | Proposed Eq. | Lever Rule | I-Girder Eq. | Box-Girder Eq. |
| Max Overestimation | 0.11 | 0.49 | 0.15 | 0.41 |
| Min Overestimation/Max Underestimation(*) | 0.00 | 0.17 | $0.01^{*}$ | 0.06 |
| Average | 0.07 | 0.34 | 0.09 | 0.26 |
| Standard Deviation | 0.016 | 0.070 | 0.027 | 0.062 |

### 8.7.3 Interior Girder Two-Lanes Loaded LLDFs

The comparisons of the equations and methodologies found in the AASHTO LRFD BDS for LLDFs for interior girder two-lanes loaded scenarios is presented in Figure 8.16 and Table 8.3. The comparison revealed the proposed moment LLDF equation more accurately predicts LLDFs than those of any other LLDF methodology or equation present in the AASHTO LRFD BDS. These comparisons serve to verify the applicability of the equation for use in PBFTG bridges.

(a)

(b)

(c)

Figure 8.16: Correlation Between Proposed LLDF Equation for Interior Girders with TwoLanes Loaded with: (a) Lever Rule, (b) I-Girder Equation, and (c) Box-Girder Equation

Table 8.3: Interior Girder Two-Lanes Loaded Statistical Analysis

| Interior Girder Two Lanes Loaded |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| LLDF Methodology | Proposed Eq. | Lever Rule | I-Girder Eq. | Box-Girder Eq. |
| Max Overestimation | 0.13 | 0.50 | 0.08 | 0.26 |
| Min Overestimation/Max Underestimation(*) | $0.04^{*}$ | 0.16 | $0.11^{*}$ | 0.02 |
| Average | 0.07 | 0.32 | 0.01 | 0.13 |
| Standard Deviation | 0.028 | 0.084 | 0.030 | 0.043 |

### 8.7.4 Exterior Girder One-Lane Loaded LLDFs

The comparisons of the equations and methodologies found in the AASHTO LRFD BDS for LLDFs for exterior girder one-lane loaded scenarios is presented in Figure 8.17 and Table 8.4. The comparison revealed the proposed moment LLDF equation more accurately predicts LLDFs than those of any other LLDF methodology or equation present in the AASHTO LRFD BDS. These comparisons serve to verify the applicability of the equation for use in PBFTG bridges.

(a)

(b)

(c)

(d)

Figure 8.17: Correlation Between Proposed LLDF Equation for Exterior Girders with OneLane Loaded with: (a) Lever Rule, (b) I-Girder Equation, (c) Box-Girder Equation, and (d) Special Analysis

Table 8.4: Exterior Girder One-Lane Loaded Statistical Analysis

| Exterior Girder One Lane Loaded |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| LLDF Methodology | Proposed Eq. | Lever Rule | I-Girder Eq. | Box-Girder Eq. | Special Analysis |
| Max Overestimation | 0.20 | 0.35 | 0.35 | 0.34 | 0.35 |
| Min Overestimation/Max Underestimation(*) | 0.00 | $0.16^{*}$ | $0.16^{*}$ | $0.15^{*}$ | $0.21^{*}$ |
| Average | 0.10 | 0.08 | 0.08 | 0.13 | 0.13 |
| Standard Deviation | 0.033 | 0.099 | 0.099 | 0.085 | 0.089 |

### 8.7.5 Exterior Girder Two-Lanes Loaded LLDFs

The comparisons of the equations and methodologies found in the AASHTO LRFD BDS for LLDFs for exterior girder two-lanes loaded scenarios is presented in Figure 8.18 and Table 8.5. The comparison revealed the proposed moment LLDF equation more accurately predicts LLDFs than those of any other LLDF methodology or equation present in the AASHTO LRFD BDS. These comparisons serve to verify the applicability of the equation for use in PBFTG bridges.

(a)

(b)

(c)

(d)

Figure 8.18: Correlation Between Proposed LLDF Equation for Exterior Girders with TwoLanes Loaded with: (a) Lever Rule, (b) I-Girder Equation, (c) Box-Girder Equation, and (d) Special Analysis

Table 8.5: Exterior Girder Two-Lanes Loaded Statistical Analysis

| Exterior Girder Two Lanes Loaed |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| LLDF Methodology | Proposed Eq. | Lever Rule | I-Girder Eq. | Box-Girder Eq. | Special Analysis |
| Max Overestimation | 0.18 | 0.10 | 0.12 | 0.19 | 0.43 |
| Min Overestimation/Max Underestimation(*) | 0.00 | $0.22^{*}$ | $0.12^{*}$ | $0.12^{*}$ | $0.09^{*}$ |
| Average | 0.08 | -0.08 | 0.02 | 0.06 | 0.16 |
| Standard Deviation | 0.033 | 0.061 | 0.031 | 0.055 | 0.122 |

### 8.8 VERIFICATION WITH IN-SERVICE PBFTG BRIDGES

To verify the applicability and accuracy of the proposed equation, LLDFs were computed for three in-service PBFTG bridges. The results of the live load field tests performed on each bridge were compared against the analytically derived results and the proposed equation. An important note for each comparison is the live load field test utilized an actual truck with its own unique axle spacings and wheel loads to determine LLDFs, where the analytical model, and therefore the proposed equations, are derived assuming the HL-93 axle spacings and wheel loads.

### 8.8.1 The Amish Sawmill Bridge

The Amish Sawmill Bridge live load field test and previous analytical studies, performed by Gibbs (2017) were discussed in Section 2.4.1. In Section 7.8.5, the finite element modeling techniques used throughout this study were verified against the live load field test and analytical modeling performed by Gibbs (2017) utilizing the Stallings/Yoo live load distribution methodology and the truck model used in the live load field test. Presented in Figure 8.19 is the comparison of LLDFs obtained experimentally, analytically, and with the proposed simplified equation. As seen in Figure 8.19, the results from all three methodologies closely correlate. The largest difference occurs in the interior girder one-lane loaded scenario where the proposed simplified equation underestimated the experimental LLDF by $5 \%$, but the two-lane loaded distribution factor would govern regardless.


Figure 8.19: Comparison of LLDF Methodologies for the Amish Sawmill Bridge

### 8.8.2 The Fourteen Mile Bridge

The Fourteen Mile Bridge live load field test and previous analytical studies, performed by Roh (2020) were discussed in Section 2.4.3. In Section 7.8.6, the finite element modeling techniques used throughout this study were verified against the live load field test and analytical modeling performed by Roh (2020) utilizing the Stallings/Yoo live load distribution methodology and the truck model used in the live load field test. Presented in Figure 8.20 is the comparison of LLDFs obtained experimentally, analytically, and with the proposed simplified equation. As seen in Figure 8.20, the results from all three methodologies closely correlate. The largest difference occurs in the exterior girder one-lane loaded scenario where the proposed simplified equation overestimated the experimental LLDF by $15 \%$, but the two-lane loaded distribution factor would govern regardless. Note, the Fourteen Mile Bridge is skewed approximately $10^{\circ}$. The proposed equations do not consider the effect of skew for moment LLDFs. The addition of a skew correction factor to reduce moment LLDF could account for the discrepancy, but it was not investigated in this study.


Figure 8.20: Comparison of LLDF Methodologies for the Fourteen Mile Bridge

### 8.8.3 The Flat Run Bridge

In Section 7.8.7, the finite element modeling techniques used throughout the Flat Run Bridge study were verified against the live load field test and analytical modeling performed as part of a study utilizing the Stallings/Yoo live load distribution methodology and the truck model used in the live load field test. Presented in Figure 8.21 is the comparison of LLDFs obtained experimentally, analytically, and with the proposed simplified equation. As seen in Figure 8.21, the results from all three methodologies closely correlate. The largest difference occurs in the exterior girder one-lane loaded scenario where the proposed simplified equation overestimated the experimental LLDF by 9\%, but the two-lane loaded distribution factor would govern regardless.


Figure 8.21: Comparison of LLDF Methodologies for the Flat Run Bridge

### 8.9 SUMMARY

The preceding chapter summarized the methodology and results of the study used to determine LLDFs for PBFTG bridges. First, a matrix of bridges was analyzed using a commercial finite element software package to determine the sensitivity of certain parameters on the influence of live load distribution in PBFTG bridges. From the results of the sensitivity matrix, a more refined parametric matrix was generated and analyzed to study the effect of the key parameters pertaining to PBFTG bridge LLDFs. The goal of this parametric matrix was to encapsulate the parameters deemed to have a significant impact on live load distribution and to ensure the parameters used were within reasonable limits to simulate potential real bridges. LLDFs of the parametric study were then used to generate simplified equations to be used with LGA for PBFTG bridges. Finally, the proposed equations were verified against the analytical and live load field test results of three in-service PBFTG bridges.

# CHAPTER 9: ASSESSMENT OF SKEW ON THE FLEXURAL RESISTANCE OF PBFTGS BEHAVIORAL STUDY 

### 9.1 Introduction

The purpose of this chapter is to assess the behavior of skewed PBFTG bridges. The goal of this study is to determine the applicability of the AASHTO LRFD BDS as they relate to the capacity of skewed PBFTG bridges. Specifically, previous composite laboratory testing was analytically skewed to determine if the ultimate capacity of the system was affected by the degree of support skew.

### 9.2 Importance of Study

As discussed in Section 4.3.3, per AASHTO LRFD Article 6.11.6, any box-girder section not meeting the requirements of AASHTO LRFD BDS Article 6.11.2.3 is considered noncompact for the nominal capacity of the section. AASHTO LRFD BDS Article 6.11.2.3 discusses the restrictions required to be met to use the LLDFs specified in AASHTO LRFD BDS Article 4.6.2.2. One of the restrictions is the limitation of bearing line skew; specifically, it states "bearing lines shall not be skewed." In other words, if a box-girder bridge has bearing lines skewed $1^{\circ}$, the nominal capacity of the section is limited to the yield moment, and a more rigorous analysis must be used to determine the loads resisted by the structure.

### 9.3 Refinement of the Analytical Model

The laboratory experimental tests performed by Michaelson (2014) served as a basis for the determination of the effect of skew on PBFTGs. The analytical tools discussed in Chapter 7 were applied to the PBFTGs described in Michaelson's (2014) physical laboratory experiment as discussed in Section 2.3.1. Key differences between the verification study and the behavioral study, regarding skew, are material modeling definitions and the use of a different type of element in the concrete deck.

### 9.3.1 Original Experimental Test

A brief description of the experimental test performed by Michaelson (2014) is provided herein, with a more detailed description of the entire scope of work provided in Section 2.3.1. Destructive flexural testing was performed on two composite PBFTG specimens. Each specimen was formed from 84 inch wide by $7 / 16$ inch thick by 35 foot long plate. The 6 inch thick by 60 inch wide reinforced concrete deck was made composite with the PBFTG with four rows of the $7 / 8$ inch diameter by 4 inch long shear stud connecters with a pitch of 12 inches. $3 / 4$ inch bearing stiffeners were placed 3 inches from the ends of the PBFTG, resulting in a clear span length of 34.5 feet.

The girders were subjected to three-point bending and loaded at midspan with a 330-kip servo-hydraulic actuator. The load-deflection graph of the composite specimens was recreated in Figure 9.1. As seen in Figure 9.1, eventually the load no longer increased with increased deflection, as a plastic hinge formed in the specimens at approximately 300 kip and 3 inches of deflection.


Figure 9.1: Load-Deflection Data from Flexural Testing of Composite Specimens (Michaelson, 2014)

### 9.3.2 Modifications to the Analytical Model

In the verification study, shell elements were used to model the concrete deck. When bearing skew was applied to the analytical model, the model would prematurely fail at loads close to yielding of the steel and cracking of the concrete. Shell elements were thought to not be able to adequately model this concrete cracking behavior, as a crack in the bottom face of the shell element would immediately propagate through the entire thickness of the deck, which is not accurate. To address this issue, continuum elements were used in place of shell elements in the concrete deck. As the concrete deck could now have multiple elements throughout the deck thickness, a crack could propagate through some of the deck elements and not cause failure. Figure 9.2 presents an image of the cross-section of the composite PBFTG with multiple continuum elements through the thickness of the deck.


Figure 9.2: Cross-Section of PBFTG with Continuum Elements in the Deck

The addition of continuum elements did not solve all issues with the concrete material model. In all previous studies, the smeared crack material model provided accurate modeling of PBFTG systems. However, even with the use of continuum elements, the analytical model would still prematurely fail before significant loading could be applied. Consistent error messages would
arise stating the results were diverging with skew angles as low as $5^{\circ}$. It was noticed the models would fail when the bottom face of the concrete deck directly under the load would reach the tensile rupture stress of the concrete.

To overcome this shortcoming of the material model, a damaged plasticity concrete model, as discussed in Section 7.3.2, was utilized. This material model would allow a significant reduction of the tensile capacity of the concrete without fully losing the element. When using this adjusted material model, the finite element model would behave nearly identically to the proven smeared crack model but would run out following the same general response as the $0^{\circ}$ skewed models for longer during the analysis.

### 9.4 Assessment of Composite Unit Capacity

The restrictions discussed in AASHTO LRFD BDS Article 6.11.2.3 are based on the research performed by Johnston and Mattock (1967) on the specific cross-sections analyzed in their study. Additionally, the commentary of Article 6.11.2.3 states the reason the supports shall not be skewed is because additional torsional effects occur in the box section and the lateral distribution of load is affected. No other steel section has the capacity of the section limited by the restrictions placed on live load analysis. Additionally, if the proposed equations presented in Chapter 8 are to be utilized, the restrictions of AASHTO LRFD BDS Article 6.11.2.3 would not be applicable. Therefore, the validity of the requirements of the article, specifically those relating to bearing skew, were explored. If these restrictions can be neglected, or removed entirely, the applicable range of PBFTGs can be greatly expanded, not only through skewed scenarios, but also when PBFTGs do not meet the spacing restrictions explored by Johnston and Mattock (1967).

### 9.4.1 AASHTO LRFD BDS Requirements for Sections in Positive Flexure

AASHTO LRFD BDS Article 6.11.6.2.2, as discussed in Section 6.8.1, presents the requirements for a composite box-girder section to qualify as a compact section. First, the specified minimum yield strengths of the flanges and the web do not exceed 70 ksi . This requirement is easily met by choosing standard mill plate with the required material properties. Secondly, the web slenderness ratio shall not exceed 150. If the designers utilize one of the standardized PBFTG
sections suggested by Michaelson (2014), such as the ones used in this study, this requirement will be met. The third requirement states the section must be part of a bridge that satisfies the requirements of AASHTO LRFD BDS Article 6.11.2.3. These research efforts are meant to explore the validity of this requirement. Further discussion of this article can be found in Sections 6.4 and 4.3.1. The fourth requirement is that the box flange is fully effective, as specified in AASHTO LRFD Article 6.11.1.1. Box flanges in simple spans are considered fully effective in resisting flexure if the width of flange does not exceed one-fifth of the span length. This is not an issue in this study as the smallest bridges analyzed were 20 feet long and the largest bottom flange width is less than 3 feet wide. The final requirement is the section must satisfy the web slenderness limit specified in Equation 9.1.

$$
\begin{equation*}
\frac{2 D_{c p}}{t_{w}} \leq 3.76 \sqrt{\frac{E}{F_{y c}}} \tag{Eq. 9.1}
\end{equation*}
$$

Where:
$D_{c p}=$ depth of the web in compression at the plastic moment
$t_{w}=$ web thickness
$E=$ modulus of elasticity of steel
$F_{y c}=$ specified minimum yield strength of the compression flange

This limit was thoroughly evaluated by Michaelson (2014) in a study with 450 composite PBFTG modules with varying PBFTG sizes and deck widths. The study showed the plastic neutral axis was typically located in the concrete deck or the top flange resulting in a $D_{c p}$ value of zero. Only 22 out of the 450 cases evaluated resulted in a nonzero value of $D_{c p}$, leading to web slenderness values in the plastic range significantly lower than the limit presented in Equation 9.1.

### 9.4.2 AASHTO LRFD BDS Flexural Resistance of Compact Sections

From the previous section and the assumptions made about the special limitations on the use of LLDFs from AASHTO LRFD BDS Article 6.11.2.3, the specimen analyzed was considered
compact. If a composite box-girder qualifies as compact, the nominal flexural resistance of the section is determined using Equation 9.2 or Equation 9.3:

If $D_{p} \leq 0.1 D_{t}$, then:

$$
\begin{equation*}
M_{n} \leq M_{p} \tag{Eq. 9.2}
\end{equation*}
$$

Otherwise:

$$
\begin{equation*}
M_{n}=M_{p}\left(1.07-0.7 \frac{D_{p}}{D_{t}}\right) \tag{Eq. 9.3}
\end{equation*}
$$

Where:

$$
\begin{aligned}
D_{p}= & \text { distance from the top of the concrete deck to the neutral axis of composite section } \\
& \text { at the plastic moment } \\
D_{t}= & \text { total depth of the composite section } \\
M_{n}= & \text { nominal flexural resistance of a section } \\
M_{p}= & \text { plastic moment of composite section }
\end{aligned}
$$

The yield moment of the composite section in positive flexure is calculated following a procedure specified in AASHTO LRFD BDS Section D6.2. Symbolically, the procedure is shown by solving Equations 9.4 and 9.5. Additionally, the yield moment of the section shall be taken as the lesser value calculated for the compression flange or the tension flange.

Solve for $M_{A D}$ from the equation:

$$
\begin{equation*}
F_{y f}=\frac{M_{D 1}}{S_{N C}}+\frac{M_{D 2}}{S_{L T}}+\frac{M_{A D}}{S_{S T}} \tag{Eq. 9.4}
\end{equation*}
$$

Then calculate:

$$
\begin{equation*}
M_{y}=M_{D 1}+M_{D 2}+M_{A D} \tag{Eq. 9.5}
\end{equation*}
$$

Where:
$F_{y f}=$ yield strength of the flange under consideration
$M_{D I}=$ moment due to the noncomposite dead loads
$M_{D 2}=$ moment due to the composite dead loads
$M_{A D}=$ moment due to the additional applied loads
$S_{N C}=$ noncomposite elastic section modulus
$S_{S T}=$ short-term elastic section modulus
$S_{L T}=$ long-term elastic section modulus
$M_{y}=$ yield moment

As the concrete deck was shored during construction and no additional composite dead loads were applied during testing, Equations 9.4 and 9.5 can be combined and simplified to form Equation 9.6.

$$
\begin{equation*}
M_{y}=\min \left(F_{y c} S_{f c}, F_{y t} S_{f t}\right) \tag{Eq. 9.6}
\end{equation*}
$$

Where:

$$
\begin{aligned}
M_{y} & =\text { yield moment } \\
F_{y c} & =\text { yield strength of the compression flange } \\
F_{y c} & =\text { yield strength of the compression flange } \\
S_{f c} & =\text { short-term composite compression flange elastic section modulus } \\
S_{f t} & =\text { short-term composite tension flange elastic section modulus }
\end{aligned}
$$

Solving Equation 9.6 provides a yield moment of 1768 kip-feet for the experimental composite PBFTG.

The plastic moment of the composite PBFTG was calculated following the procedure outlined in ASSHTO LRFD BDS Article D6.1. The plastic moment was calculated as the moment of the plastic forces about the plastic neutral axis. This was accomplished by first calculating the element forces and using them to determine whether the plastic neutral axis was in the webs, top bends, top flanges, or the concrete deck. Second, the location of the plastic neutral axis was determined by the equilibrium of forces from the first step by ensuring no net axial force existed
in the section. Finally, the plastic moment was calculated by summing the plastic forces of each section about the plastic neutral axis. Conservatively, the forces in the longitudinal rebar were neglected in these calculations. For the experimental composite PBFTG, the plastic moment, $M_{p}$, was determined to be 2256 kip-feet, and the distance from the top of the concrete deck to the neutral axis of composite section at the plastic moment, $D_{p}$, was determined to be 4.90 inches. By solving Equations 9.2 and 9.3, with the values of $M_{p}$ and $D_{p}$ calculated previously, the nominal flexural moment of the experimental composite PBFTG system is 1993 kip-feet.

### 9.5 BEHAVIORAL STUDY

Utilizing the analytical procedures discussed in Chapter 7, with the modifications discussed in Section 9.3.2, a series of analytical models were generated to evaluate behavior of composite PBFTGs with varying degrees of skew. The first step in the behavioral study was to verify analytical modeling techniques against experimental results. As shown in Figure 9.3, the modeling techniques efficiently capture the nonlinear behavior of the experimental composite PBFTG.


Figure 9.2: Verification of Improved Modeling Techniques Against Experimental Laboratory Results

Following the verification of the modeling techniques of a specimen without skew, the bearing skew was varied from $0^{\circ}$ to $45^{\circ}$ in $5^{\circ}$ increments. The load was applied perpendicular to the longitudinal axis of the PBFTG and distributed equally across the width of the concrete deck. Additionally, to represent the effect of the spreader beam placed under the 330-kip servo-hydraulic actuator, the load is distributed longitudinally over the width of the spreader beam flange, or approximately 12 inches.

### 9.6 Preliminary Results

Figures 9.3 and 9.4 present the load/midspan deflection results of the behavioral study. As seen in Figure 9.3, the load/deflection history of the iterations with bearing line skew under $20^{\circ}$ follows the same curve. While the load-deflection plots for PBFTGs with bearing skews under $20^{\circ}$ follows the same shape as the $0^{\circ}$ bearing line skew test, the stiffness of the system seems to increase, based on the longitudinal strains recorded from the model. However, as the degree of bearing skew increases beyond $20^{\circ}$, the analytical model terminates before the load-deflection curve can plateau in a manner similar to the low bearing skew girders. This abrupt termination in the analysis of the higher bearing line skews is presented in Figure 9.4.


Figure 9.3: Load vs. Deflection Plots for Bearing Line Skews up to $20^{\circ}$


Figure 9.4: Load vs. Deflection Plots for Bearing Line Skews above $20^{\circ}$

To compare the results of the sensitivity study against the nominal flexural resistance calculations presented in the AASHTO LRFD BDS, the yield moment, plastic moment, and nominal flexural resistance from Equation 9.3, rewritten as point loads, are presented in Figure 9.5. These comparisons show the experimental PBFTG has capacity even above the nominal plastic moment load.


Figure 9.5: Comparison of Load vs. Deflection Plots Against Point Loads Inducing Design Moments

### 9.7 SUMMARY

This chapter served to assess the behavior of skewed PBFTGs. The goal of this study was to determine the applicability of the restrictions present in AASHTO LRFD BDS Article 6.11.2.3 on the capacity of skewed PBFTGs. AASHTO LRFD BDS Article 6.11.2.3 was deemed to not be applicable to the capacity of PBFTGs up to bearing line skews of $20^{\circ}$. The nominal capacity calculations for the PBFTG analyzed, assuming the restrictions of AASHTO LRFD BDS Article 6.11.2.3 could be ignored, are still applicable in bearing line skews of up to $45^{\circ}$.

## CHAPTER 10: EXPERIMENTAL TESTING OF A LINK SLAB BETWEEN PBFTGS

### 10.1 Introduction

This chapter contains an overview of the physical experimental testing completed to assess the performance of the proposed link slab detail in conjunction with PBFTGs. A brief description of the specimen tested is provided herein, along with an instrumentation plan, a description of equipment used, and the results of the testing effort.

### 10.2 Overview of Testing Program

To verify the performance of the link slab detail in conjunction with PBFTGs, physical flexural testing was conducted at the Major Units Lab at WVU. Fatigue testing conducted on two composite PBFTGs joined with a link slab with a load at approximately midspan of each PBFTG is shown in Figures 10.1 and 10.2.


Figure 10.1: Test Setup Schematic


Figure 10.2: Isometric View of Test Setup

Each of the girders were supported under each bearing stiffener by a 2 inch round diameter bar. One support for each girder was welded to the plate beneath the roller, simulating a pinned boundary condition (Figure 10.3), and the other support was placed in a small groove allowing displacement in the longitudinal axis, simulating a roller boundary condition (Figure 10.4). The 2 inch diameter bars were supported by 6 inch by 24 inch by 2 inch steel plates. The exterior plates were each supported by a 12 inch by 24 inch by 3 inch steel plate, and the interior plates were
supported by a single 24 inch by 24 inch by 3 inch plate. Each 3 inch plate rested on two 6 inch by 6 inch by 24 inch long hollow structural sections filled with high strength concrete. Four 1 inch diameter threaded rods connect the hollow structural sections to the vertical structural support system (Figure 10.5). Lateral bracing, consisting of equal leg angles, located at the exterior supports, prevented unintentional transverse motion. $1 / 2$ inch thick steel plates welded immediately adjacent to the bearing stiffeners of each PBFTG operated as connection plates for the lateral bracing system. Figure 10.6 demonstrates the setup of the lateral force resisting system.


Figure 10.3: View of a Pinned Boundary Condition


Figure 10.4: View of a Roller Boundary Condition


Figure 10.5: Plan View of the Vertical Force Resisting System


Figure 10.6: Cross-Section View of the Lateral Force Resisting System

### 10.3 SPECIMEN DESCRIPTIONS

Michaelson (2014) proposed standardized cross-section geometry for standard mill plate widths and thicknesses. The PBFTGs in this study were fabricated from 84 inch by $7 / 16$ inch by 300 inch plate and were bent to have a 23 inch depth with 6 inch wide top flanges, as seen in Figure 10.7. Each girder was fabricated by placing the flat plate in a large capacity press-brake and coldbent with the inside bend radius equal to five time the thickness of the plate. Table 10.1 presents the noncomposite section properties of PBFTGs formed from 84 inch wide by $7 / 16$ inch thick plate. $3 / 4$ inch thick bearing stiffeners were welded 3 inches from the end of each specimen to prevent premature bearing failure during testing (Figure 10.8). Both specimens consist of HPS-50 steel. One specimen remained uncoated while the other was hot-dip galvanized prior to any further fabrication. Research has shown the hot-dip galvanization process does not affect the fatigue performance of PBFTGs (Tennant, 2018).


Figure 10.7: 84 inch by 7/16 inch PBFTG Cross-section


Figure 10.8: Bearing Diaphragm Cross-section

Table 10.1: Noncomposite PBFTG Section Properties

| Property | Value |
| :---: | :---: |
| $\mathrm{E}(\mathrm{ksi})$ | 29,000 |
| $\mathrm{G}(\mathrm{ksi})$ | 11,154 |
| $\mathrm{~A}\left(\mathrm{in}^{2}\right)$ | 36.75 |
| $\mathrm{I}_{\mathrm{x}}\left(\mathrm{in}^{4}\right)$ | 2893 |
| $\mathrm{I}_{\mathrm{y}}\left(\mathrm{in}^{4}\right)$ | 8050 |
| $\mathrm{I}_{\text {open }}\left(\mathrm{in}^{4}\right)$ | 2.345 |
| $\mathrm{I}_{\mathrm{closed}}\left(\mathrm{in}^{4}\right)$ | 69,000 |
| $\mathrm{I}_{\mathrm{w}}\left(\mathrm{in}^{6}\right)$ | 140,000 |
| $\beta_{\mathrm{x}}(\mathrm{in})$ | -19.7 |

### 10.4 Test Specimen Assembly

Following the welding of the end bearing diaphragm to the cold-bent plate, the specimens could be constructed with additional design details. To prepare the composite units, various design details surrounding the concrete deck were constructed. This section will describe the construction of the composite units from the installation of the stay-in-place formwork through the casting of the link slab.

### 10.4.1 SIP Metal Formwork

Stay-in-place (SIP) metal formwork was utilized between the top flanges of each specimen (Figure 10.9). 2 inch deep pans were utilized on the uncoated specimen and 3 in deep pans were utilized on the galvanized specimen. The SIP metal formwork ran from the exterior support of each girder longitudinally until approximately 9 inches from the interior support. This allowed for a purely unbonded region to develop between the concrete and the girders at the interior support region. 7/8 inch by 6 inch shear studs were shot through the SIP formwork, in the strong position of the bottom flute, to achieve composite action between the concrete deck and the steel specimens. 21 rows of studs, 4 studs in each row, were placed longitudinally along the girders. The first row of studs was placed directly above the bearing diaphragm, and subsequent rows were spaced 12 inches apart. The final four bottom flutes were left without shear studs to develop the transition
zone of the link slab. These final flutes were plug welded to connect the formwork to the girder without the use of studs in the transition zone.


Figure 10.9: Isometric view of the SIP Formwork and Shear Studs

### 10.4.2 Exterior Formwork

Most of the exterior formwork was reused from previous testing performed by Michaelson (2014), Kozhokin (2016), and Tennant (2018), as the cross-section and specimen height were the same (Figure 10.10). However, as the concrete deck was designed thicker to simulate the dimensions of a full-scale link slab, the wooden forms used against the concrete were replaced with 2 inch by 12 inch lumber cut to produce a total deck thickness of 10 inches. The forms allowed for a total deck width of 60 inches with overhangs on either side of the exterior of either flange measuring approximately 3 inches. The exterior formwork was braced against the structural testing frames and large concrete blocks, acting as dead-man anchors, as seen in Figure 10.11. A temporary transverse board was placed at the deck-link slab interface.


Figure 10.10: Isometric View of Vertical Deck Supports


Figure 10.11: Isometric View of Complete Exterior Deck Forms

### 10.4.3 Main Span Reinforcement

The concrete decks of the main spans were designed following the empirical deck design method described in AASHTO LRFD BDS Article 9.7.2. The bottom longitudinal reinforcement consisted of \#5 rebar spaced at 12 inches on center with an edge distance of 2 inches. 1 inch of clear cover was provided between the top of the 3 inch deep SIP metal deck and the bottom of the bottom mat of rebar, equating to 2 inches of clear cover between the 2 inch deep SIP metal deck and the bottom mat of rebar. The bottom layer of transverse reinforcement was placed directly
above the bottom layer of longitudinal reinforcement and consisted of \#5 rebar spaced 12 inches on center, which coincided with the placement of the shear studs. The top layer of longitudinal reinforcement consisted of \#4 rebar with the same spacing as the bottom layer of longitudinal reinforcement. The top layer of transverse reinforcement consisted of \#4 rebar placed directly above the top mat of longitudinal reinforcement with the same spacing as the bottom mat. With this layout, $23 / 8$ inches of top cover was provided. The deck reinforcement can be seen in Figures 10.12 and 10.13 .


Figure 10.12: Cross-section View of Concrete Deck Reinforcement


Figure 10.13: Placement of Concrete Deck Reinforcement

### 10.4.4 Main Span Concrete Pour

On the day of the concrete pour, the wooden formwork was heavily coated with form release to allow for easy removal without significant damage to the concrete deck. As seen in Figure 10.13, any gaps in the wooden formwork were filled with foam. The upper flutes of the SIP metal formwork were also filled with foam so the concrete would not flow through the flutes into the tub section. Once the concrete arrived on site, four 6 inch diameter cylinders were poured for material testing. The concrete was placed into the forms utilizing a 1 yard concrete bucket attached to an overhead ten ton crane, as seen in Figure 10.14. After each successive concrete pour for both specimens, the concrete was vibrated to minimize air pockets and honey combing.


Figure 10.14: Concrete Bucket Transporting 1 Yard of Concrete

The concrete pour for the two main slab specimens was completed in approximately two hours and can be seen in Figure 10.15. Burlap was placed over the curing concrete to control surface cracking during hydration. The burlap was rehydrated every day for two weeks following the pour to ensure the concrete surface remained moist. After one week of curing, the forms around the main slab specimens were removed with minimal localized damage where the forms were pried away from the deck.


Figure 10.15: Finished Main Span Concrete Decks

### 10.4.5 Link Slab Construction

A combination of the methodologies provided by Caner and Zia (1998) and Li et al. (2003) produced the size and layout of the link slab reinforcement. As the use of the link slab does not behave similarly to two simple spans or continuous spans, the calculation of the moment at the link slab is not easily calculated. To calculate the required reinforcement area in the link slab, the rotation at the center of the link slab was the limiting parameter chosen. Another key difference in the construction of the link slab was the need to debond the concrete deck from the underlying steel girders. This was achieved by placing plywood over the open ends of the tubs not covered by the SIP metal formwork and covering the entire bottom face of the link slab with standard roofing paper. The roofing paper allowed movement of the link slab independently from the underlying girders. The corrugation of the SIP formwork with the roofing paper allowed for the transition zone to develop between the flat link slab and the main deck (Figure 10.16).


Figure 10.16: Isometric View of Transition Zone

The same bottom mat of reinforcement used in each main span was also used for the bottom mat of the link slab reinforcement: \#5 rebar spaced 12 inches on center. The required area of steel in the top mat of rebar for the link slab was found to be 1.073 inches per 12 inches of link slab width. \#10 rebar spaced 12 inches on center would have been the optimum rebar size, but due to lack of availability, two \#7 rebar tied together were chosen to replace a single \#10 rebar. To develop the rebar in the link slab with the rebar protruding from the main spans, the link slab rebar was overlapped and tied together with the main span rebar over 5 feet. A similar development of the lower longitudinal rebar can be seen in Figure 10.17. \#4 rebar was used as transverse reinforcement in the top mat.


Figure 10.17: Completed Link Slab Reinforcement

A similar procedure to the main span concrete pour was used during the link slab concrete pour. The forms were prepared similarly, with foam filling any cracks and form release spread on the outer formwork. Four test cylinders were poured for material testing. Concrete was transported using a 1 yard concrete bucket from the mixer truck to the link slab, where it was poured and vibrated to remove any air pockets. The link slab was finished with special attention paid to the link slab-main span interface to ensure a smooth joint. The finished link slab can be seen in Figure 10.18. After the pour, it cured under wet burlap for 14 days, then dry cured for the recommended further 14 days.


Figure 10.18: Poured Link Slab

### 10.5 INSTRUMENTATION

Data was collected throughout link slab testing, including continuous load and deflection monitoring and periodical static testing. Data included strain gauges to obtain the moment at various cross-sections in the specimens and vertical deflections at the points of loading. This section will describe the instruments, layout, and installation used throughout the testing.

### 10.5.1 Instruments

Foil-resistor uniaxial strain gauges were employed throughout testing. Strain was recorded at four different cross-sections on the main span specimens and on the rebar placed in the link slab. The strain gauges were connected to a Micro-Measurements Model 5100 Scanner utilizing StrainSmart software to record strain and displacement data (Micro-Measurements, 2010). Two different hydraulic actuators applied load to the system. An MTS Model 243.70T 330-kip servohydraulic actuator applied load to the uncoated specimen and an MTS Model 243.40 110-kip
servo-hydraulic actuator applied load to the galvanized specimen. Each of the servo-hydraulic actuators are equipped with instruments to measure the deflection at the point of loading and the magnitude of loading.

### 10.5.2 Layout and Installation of Girder Strain Gauges

Four separate cross-sections were chosen between the two PBFTGs to record strains, as seen in Figure 10.19. Nine strain gauges were located at each cross-section to capture the tensile strains in the steel girder. Three strain gauges were placed on each of the webs and the bottom flange. As seen in Figure 10.20, the first strain gauge on each face was located at the center of the face, and the other two strain gauges were spaced six inches away from the first in both directions along the cross-section. The strain gauges placed near midspan of each PBFTG were offset 46 inches, or two times the steel girder depth, away from the point of load application to avoid strain concentration effects.


Figure 10.19: Strain Gauge Longitudinal Layout


Figure 10.20: Strain Gauge Cross-Section Layout

### 10.5.3 Layout and Installation of Link Slab Gauges

Eight uniaxial strain gauges were placed on the underside of the top mat of longitudinal reinforcement in the link slab. Two strain gauges were placed on each of the longitudinal lines of rebar for redundancy. The first cross-section of rebar was placed directly in the center of the link slab and the second cross-section was placed two feet toward the uncoated specimen. As these strain gauges were to be exposed to the concrete of the link slab, additional protection in the form of Micro-Measurements M-Coat JA-3 Polysulfide Coating and plastic sheathing was provided. The polysulfide coating can be seen on the link slab rebar in Figure 10.21.


Figure 10.19: Polysulfide Coating on Link Slab Reinforcement

### 10.6 Material Testing

The steel and concrete used in this research were tested to obtain material properties for use in the data reduction of the composite specimens. Steel tensile coupons were taken from appropriate locations during the fabrication of the noncomposite specimens and were tested by Turner-Fairbank's Highway Research Center. As the noncomposite specimens were previously used in nondestructive testing performed by Michaelson (2014), further discussion of the material testing can be found in his dissertation. The coupon test results from Turner-Fairbanks tensile testing can be seen in Figure 10.22.


Figure 10.20: Results from Tensile Testing of Steel Coupons (Michaelson, 2014)

For this research, four concrete cylinders were cast during the main span and link slab concrete pours. Cylinders were tested 28 days after casting to obtain the in-place compressive strength of the composite specimens and the link slab. The mean of the compressive strengths was recorded to obtain the compressive strength used in the mechanistic models. The average compressive strength of the main span specimens was found to be 3.86 ksi , and the average compressive strength of the link slab was found to be 3.74 ksi .

### 10.7 LOAD CONFIGURATION

Load was applied at midspan of the composite uncoated specimen by an MTS 330-kip servo-hydraulic actuator mounted to a large, steel structural frame bolted to the floor of the Major Unit's Laboratory, as seen in Figure 10.23. An MTS 110-kip servo-hydraulic actuator applied load to the galvanized specimen in a manner similar to the load application on the uncoated specimen. Large spreader beams were used to distribute the load from the heads of the actuators to the composite specimens to avoid localized concrete crushing due to concentrated load effects. Elastomeric bearing pads, which consist of alternating layers of steel strips and neoprene in industrial grade rubber, were placed between the steel spreader beams and the concrete deck to further aid in the transfer of load in the system.


Figure 10.21: Isometric View of the PBFTG Specimens Placed Within the Structural Frames

### 10.8 Cyclic Loading Magnitude and Frequency Determination

The loading sequence was used to simulate a 75-year design life of the system. The Fatigue I limit state reflects the fatigue and fracture load combination relating to infinite load-induced fatigue life and reflects load levels found to be representative of the maximum stress range of the truck population for infinite fatigue life design. The loads required to induce the Fatigue I moment in the main span specimens were calculated prior to testing. Equation 10.1 demonstrates the load factors used in this testing, which were previously defined in Section 6.2.2.

$$
\text { Fatigue } I=1.75(L L+I M)
$$

Where:
Fatigue $I=$ force effect from the Fatigue $I$ load combination
$L L=$ force effect from vehicular live load
$I M=$ force effect from vehicular dynamic load allowance

The servo-hydraulic actuators applied a load of 87.5 kip simultaneously on both specimens to induce the Fatigue I moment in the composite specimens. Procedures describing the calculation of this load are described in Appendix D.

The AASHTO LRFD BDS were used to determine the number of cycles the system must sustain over the course of its design life. The following assumptions were made to determine the number of required fatigue cycles:

1. The average daily traffic was 850 vehicles
2. The bridge was located in a non-interstate rural environment
3. The bridge had a design life of 75 years
4. The bridge had two lanes available to trucks

The average daily truck traffic (ADTT) can be determined by multiplying the average daily traffic (ADT) by the fraction of trucks in the traffic. AASHTO LRFD BDS Table C3.6.1.4.2-1 may be used in lieu of site-specific traffic data. Assuming the bridge was located in a non-interstate rural environment, the fraction of trucks in traffic may be assumed to $15 \%$ of the ADT. The singlelane ADTT is for the traffic lane in which most truck traffic crosses the bridge. AASHTO LRFD BDS Table 3.6.1.4.2-1 is used to determine the fraction of truck traffic in a single lane, $p$. Assuming the bridge had two lanes available to trucks, the fraction of trucks in a single lane is taken as $85 \%$ of the ADTT. The number of fatigue cycles, assuming a 75 -year design life, was determined as follows:

Number of Cycles $=850($ ADT $) \times 0.15($ fraction of trucks in traffic $) \times 0.85(p) \times 365$ $($ days $/$ year $) \times 75($ years $)=2,966,766$ Cycles

### 10.9 Testing Procedure

Static testing was performed prior to fatigue loading. Five strain gauge readings were recorded prior to loading to obtain zero readings. The load was increased in stroke control in 8.75 kip intervals until ultimately reaching the Fatigue I loading. After each step in the incremental loading was reached, the load was allowed to sit on the specimens for five minutes allowing the system to settle into a constant loading state and settle any vibration effects, removing any impact on the system. After the five-minute delay, five strain readings were recorded and the process was repeated until the Fatigue I loading was reached. After the Fatigue I loading was reached, the load was removed and allowed to settle for ten minutes before the process was repeated and the readings at each load level were averaged. The static testing occurred at different numbers of cycles based on the behavior of the specimens. After any long-term stoppage of the system, which will be described in later sections, static testing occurred after 100,000 cycles. When testing was somewhat continuous, static testing occurred after 200,000 cycles. The static testing occurred at the following number of cycles: 1e5; 2e5; 3e5; 4e5; 6e5; 7e5; 8e5; 9e5; 1e6; 1.2e6; 1.3e6; 1.4e6; $1.5 \mathrm{e} 6 ; 1.7 \mathrm{e} 6 ; 1.9 \mathrm{e} 6 ; 2.1 \mathrm{e} 6 ; 2.3 \mathrm{e} 6 ; 2.5 \mathrm{e} 6$. A thorough investigation of the link slab, transition zone, slab/girder interface, and bottom flange bend regions was performed after each static test.

### 10.10 Loss of Composite Action in the Galvanized Specimen

After 300,000 fatigue cycles, composite action was lost in the galvanized specimen. Under the application of fatigue loading, a separation between the SIP metal formwork and the top flange of the PBFTG could be observed longitudinally between the point of loading and the link slab. Improper welding of the shear studs through the SIP metal formwork through the galvanization at the top flange is assumed to be a large contributing factor in the loss of composite action.

To regain composite action in the system, a methodology evaluated by Kreitman et. al. (2016) was used to install new shear connectors in the region where the pre-installed shear connectors had failed. Kreitman et. al considered many different methods of strengthening bridge systems with post-installed shear connectors and ultimately decided to pursue the use of adhesive anchors, as shown in Figure 10.24.


Adhesive Anchor
Figure 10.22: Post-Installed Shear Connectors

The adhesive anchor shear connectors were composed of 12 inch long by $7 / 8$ inch diameter ASTM A193 B7 Zinc fully threaded rods. A two-part adhesive, Hilti HIT-HY 200-R, was used in conjunction with the threaded rods. The connectors were installed using the following procedure:

1. For placement of the $7 / 8$ inch rods, measurements were marked longitudinally and transversely on each top flange. Longitudinally, the locations occurred every 12 inches on center, beginning directly under the load application and running into the link slab transition zone. This measurement was determined to ensure rods were placed in the bottom flute of the SIP formwork three inches from the failed shear studs. Transversely, at every longitudinal measure mark, two marks were made: The exterior line was 1.5 inches from the edge of the flange and the interior line was 5 inches from the edge of the flange, coinciding with the locations of the failed shear studs.
2. At the intersections of the longitudinal and transverse lines, a punch and hammer were used to create a starter hole for drilling.
3. Using a portable drill with a magnetic base, drilling through the flange began where starter holes were punched. Drilling was performed using a small drill bit to begin each hole and increasing in steps until a $11 / 16$ inch diameter hole was drilled through the flange (Figure 10.25).
4. Through the hole in the flange, a 1 inch diameter concrete bit was used to drill 8 inches into the concrete deck (Figure 10.26).
5. The finished hole in the flange was cleaned with a wire brush attachment and compressed air, as specified by the adhesive installation procedures of the HIT-HY 200 Hybrid Adhesive Anchoring System.
6. Before beginning the threaded rod installation process, all threaded rods were prepared with two washers and a heavy hex nut, as the adhesive provided limited work time.
7. Using the Hilti HDM Manual Adhesive Dispenser, the adhesive was injected into the hole from the top down to avoid air bubbles. Each hole was approximately $2 / 3$ filled with adhesive (Figure 10.27).
8. The threaded rod was inserted into the hole using a twisting motion, so the adhesive filled the threads (Figure 10.28).
9. Washers were then pressed against the girder and the nut was tightened by hand.
10. The rod was held in place until the adhesive could hold the rods in the flange without external support.
11. The adhesive was allowed to cure the recommended time from the manufacturer.
12. After the adhesive cured, the nuts were tightened to the torque specified by the manufacturer (Figure 10.29).


Figure 10.23: Drilling Through the PBFTG Top Flange


Figure 10.24: Concrete Drilling with Wet and Dry Shop Vacuum to Control Dust


Figure 10.25: Hilti HIT-HY 200-R Adhesive Injection


Figure 10.26: Insertion of Threaded Rods with a Twisting Motion


Figure 10.27: Tightening of Nut

### 10.11 Loss of Composite Action in the Uncoated Specimen

After 1,400,000 fatigue cycles, composite action was lost in the uncoated specimen. Due to user error, the concrete at the point of loading needed to be replaced. Longitudinally, 54 inches on either side of the load application point, the concrete deck was removed. The concrete deck was removed by scoring the concrete with a 14 inch diameter concrete saw and jack hammering the weakened layers, as seen in Figures 10.30 and 10.31.


Figure 10.28: Concrete Deck Scoring


Figure 10.29: Concrete Deck Removal

Care was taken to not damage the longitudinal rebar to ensure proper load transfer through the new concrete deck. Following a method similar to that discussed in Section 10.10, holes were drilled through the top flange of the uncoated specimen and threaded rods were inserted to restore composite action in the damaged portion of the deck. As the concrete was removed, a nut and washer were placed on the top of the top flange to anchor the threaded rod to the top flange.

As only a portion of the concrete deck was removed and composite action restored, the applied load was recalculated to obtain the same link slab rotation. After the concrete had cured for 28 days, a static test was performed to determine the load needed to cause the same deflection at the point of loading. Static testing was performed on the link slab specimens at small intervals of load to determine the load required to reach the same midspan deflections produced pre-concrete deck failure. The loads were increased in 5 kip increments until a load of 70 kip was reached, corresponding to the desired midspan deflection.

### 10.12 RESULTS

This section describes the results obtained during fatigue testing and the methods used to analyze the collected data. The procedure used to calculate the induced stresses and moments by the applied loading is included herein. In addition, this section includes testing summaries and comparison of the experimental versus back calculated loading at each static test time interval. The deflections of each girder throughout the fatigue life are also provided.

### 10.12.1 Gauge Configuration

The stresses throughout the system were obtained from the recorded strain data using a method developed by Helwig and Fan (2000). Longitudinal stresses in thin wall members can be calculated by considering axial forces and bending moments while neglecting warping stresses. The longitudinal stresses induced by axial forces and bending moments are assumed to be linearly distributed across the cross-section. Using this method, only three stress readings from noncollinear points are needed to determine the distribution of stresses in the cross-section. The stress distribution of the cross-section can be expressed by Equation 10.2:

$$
\begin{equation*}
f(x, y)=a+b x+c y \tag{Eq. 10.2}
\end{equation*}
$$

Where:
$f=$ longitudinal Stress (ksi)
$a, b, c=$ constants
$x, y=$ coordinate system on the cross-section of the member (in)

The strain gauge locations in terms of the transverse and vertical directions, $x$ - and $y$-, respectively, are shown in Table 10.2 where the x -datum is the center of the bottom flange and the $y$-datum is the noncomposite section centroid.

Table 10.2: $x$-, $y$-Coordinates of Strain Gauges

| Gauge Coordinates |  |  |
| :---: | :---: | :---: |
| Gauges | From Datum |  |
|  | $\mathbf{x}$ (in) | $\mathbf{y}$ (in) |
| G01, G10, G19, G28 | -16.06 | 6.87 |
| G02, G11, G20, G29 | -14.61 | 1.05 |
| G03, G12, G21, G30 | -13.15 | -4.77 |
| G04, G13, G22, G31 | -6.00 | -10.39 |
| G05, G14, G23, G32 | 0.00 | -10.39 |
| G06, G15, G24, G33 | 6.00 | -10.39 |
| G07, G16, G25, G34 | 13.15 | -4.77 |
| G08, G17, G26, G35 | 14.61 | 1.05 |
| G09, G18, G27, G36 | 16.06 | 6.87 |

### 10.12.2 Gauge Data Selection

After the system was statically loaded twice, following the methodology described in Section 10.9, the strain data was collected and sorted to only include consistent results. Typically, a gauge presenting irregularly on any given static test would continue to behave irregularly on following static tests. For the data used for further reduction, once a gauge recorded inconsistent results, it was not included in any further stress calculations. The data inclusion matrices for each girder can be seen in Tables 10.3 and 10.4 where ' 0 ' denotes data which was kept and ' 1 ' indicates data which was discarded due to inconsistency.

Table 10．3：Gauge Inclusion Matrix for the Galvanized Specimen

| Gauge | Cycle Count |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \mathrm{e} \\ & \hat{0} \\ & \hat{0} \end{aligned}$ | 气े | O． | $\begin{aligned} & \text { U } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | 8 <br> 8 <br> 8 | $\begin{aligned} & 0 . \\ & \text { O. } \\ & \text { in } \end{aligned}$ | 8ิ | $\begin{aligned} & \mathrm{O} \\ & \hat{8} \\ & \hat{\mathrm{~N}} \end{aligned}$ |  |  | 8 <br> 0 <br> -1 |  |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 . \\ & 0 \\ & \mathbf{N}_{2} \end{aligned}$ | 8 <br> 8 <br> -1 <br> -1 |  | $\stackrel{8}{8}$ |  |  |  | ¢ |
| G01 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G11 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| G14 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G15 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 10．4：Gauge Inclusion Matrix for the Uncoated Specimen

|  | Cycle Count |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gauge |  | $\begin{aligned} & \mathrm{O} \\ & 0 . \\ & 0 . \end{aligned}$ | $\begin{aligned} & \text { O. } \\ & \text { O. } \\ & \text { Ồ } \end{aligned}$ |  |  | $\begin{array}{\|l} \mathrm{O} \\ \text { O. } \\ \text { Ô } \end{array}$ | $\begin{array}{\|l\|} \hline 8 \\ 8 . \\ \text { in } \\ \text { in } \end{array}$ |  | $\begin{aligned} & \mathrm{O} \\ & \mathrm{O} \\ & \mathrm{O} \\ & \mathrm{R} \end{aligned}$ |  |  | 气ì | 8 Oi ⿳⿵人一⿲丶丶㇒一⿱⿰㇒一乂二灬 |  | $\xrightarrow{\stackrel{\rightharpoonup}{8}}$ |  |  |  |  |  |  | ¢ |
| G19 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G20 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G23 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G24 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G29 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G30 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

### 10.12.3 Linear Regression

A three-dimensional linear regression algorithm, based on least square regression, was employed to further reduce errors from physical strain measurements. The regression model is a statistical toll which does not rely on physical assumptions. To determine the constants, $b$ and $c$, in Equation 10.2, the following set of linear equations are to be solved:

$$
\begin{align*}
& {\left[\begin{array}{ll}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{array}\right]\left\{\begin{array}{l}
b \\
c
\end{array}\right\}=\left\{\begin{array}{l}
L_{10} \\
L_{20}
\end{array}\right\}}  \tag{Eq. 10.3}\\
& L_{11}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}  \tag{Eq. 10.4}\\
& L_{22}=\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2} \\
& L_{12}=L_{21}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)  \tag{Eq. 10.6}\\
& L_{10}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(f_{i}-\bar{f}\right)  \tag{Eq. 10.7}\\
& L_{20}=\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)\left(f_{i}-\bar{f}\right)  \tag{Eq. 10.8}\\
& \bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i} \\
& \bar{y}=\frac{1}{n} \sum_{i=1}^{n} y_{i}  \tag{Eq. 10.10}\\
& \bar{f}=\frac{1}{n} \sum_{i=1}^{n} f_{i}  \tag{Eq. 10.11}\\
& a=\bar{f}-b \bar{x}-c \bar{y}
\end{align*}
$$

Eq. 10.5

Eq. 10.9

Eq. 10.12

Where:

$$
\begin{aligned}
& f_{i}=\text { longitudinal stress at the } i^{\text {th }} \text { gauge (ksi) } \\
& x_{i}=\text { transverse coordinate at the } i^{\text {th }} \text { gauge (in) } \\
& y_{i}=\text { vertical coordinate at the } i^{\text {th }} \text { gauge (in) }
\end{aligned}
$$

### 10.12.4 Calculation of Induced Moment

The induced moment is calculated following a procedure developed by Imhoff (1998), where the load carrying mechanism is broken into three parts. The first part is the steel girder bending about its own neutral axis, $M_{L}$. The second part is the concrete deck bending about its own neutral axis, $M_{U}$. The final component is a moment couple induced by the composite action between the steel girder and the concrete deck, Na. Equations 10.13 through 10.16 were used to determine the total moment at each cross-section instrumented (Bertoldi, 2009). The concrete and steel properties are summarized in Section 10.6.

$$
\begin{align*}
& M_{L}=\left(\sigma_{O}-\sigma_{\text {CG }}\right) S_{\text {steel }}  \tag{Eq. 10.13}\\
& M_{u}=\left(\frac{E_{\text {conc }} I_{\text {conc }}}{E_{\text {steel }} I_{\text {steel }}}\right) M_{L}  \tag{Eq. 10.14}\\
& N a=\sigma_{C G} A_{\text {steel }}\left(d_{\text {steel }}-y_{\text {steel }}+t_{\text {haunch }}+\frac{t_{\text {conc }}}{2}\right)  \tag{Eq. 10.15}\\
& M_{T}=M_{L}+M_{u}+N a \tag{Eq. 10.16}
\end{align*}
$$

Where:

$$
S_{\text {steel }}=\text { section modulus of the noncomposite PBFTG }\left(\text { in }^{3}\right)
$$

$E_{\text {conc }}=$ modulus of elasticity of the concrete deck (ksi)
$I_{\text {conc }}=$ moment of inertia of the concrete deck about its $x$-axis $\left(\mathrm{in}^{4}\right)$
$E_{\text {steel }}=$ modulus of elasticity of the PBFTG (ksi)
$I_{\text {steel }}=$ moment of inertia of the PBFTG about its $x$-axis $\left(\right.$ in $\left.^{4}\right)$
$A_{\text {steel }}=$ cross-sectional area of the PBFTG (in ${ }^{2}$ )
$d_{\text {steel }}=$ depth of the PBFTG (in)
$y_{\text {steel }}=$ noncomposite depth of neutral axis (in)
$t_{\text {haunch }}=$ haunch thickness (in)
$t_{\text {conc }}=$ concrete deck thickness (in)

As two of the gauge locations were offset $2 d$ ( 46 inches) away from midspan during testing, the moments calculated at those locations were adjusted to calculate the moments at midspan. It is assumed each PBFTG is simply supported to calculate the moments at midspan and the quarterspan, where gauges were located. The moment calculations for each cross-section use a span length
of 24.5 feet. These values were used to back calculate the load required to induce the calculated moment. Some deviation is shown between the applied load and the back calculated load due to the assumed simply supported conditions and assumption of full composite action between the PBFTG and the concrete deck throughout testing. The percent error values and the $\mathrm{R}^{2}$ values for each cross-section at each static loading between the applied loads and the back calculated loads of 87 kip and 70 kip are shown in Tables 10.5 through 10.8. The small $\mathrm{R}^{2}$ values and erratic error values are due to the loss of gauges through testing, as seen in Table 10.3. When there are no longer three non-colinear gauges, the reduction methodology is no longer valid and produces nonrealistic results.

Table 10.5: Least Squares and Percent Error for the Galvanized Specimen at Midspan

| Cycle Count | Least Square, $\mathbf{R}^{\mathbf{2}}$ | Percent Error, $\%$ |
| :---: | :---: | :---: |
| Base Test | 1.0000 | $-7.57 \%$ |
| $0,100,000$ | 0.9989 | $-11.07 \%$ |
| $0,200,000$ | 0.9997 | $-10.91 \%$ |
| $0,300,000$ | 0.9998 | $-11.32 \%$ |
| GG LoCA Base Test | 0.9999 | $-12.97 \%$ |
| $0,400,000$ | 0.9997 | $-12.93 \%$ |
| $0,500,000$ | 0.9991 | $-13.34 \%$ |
| $0,600,000$ | 0.9995 | $-13.09 \%$ |
| $0,700,000$ | 0.9995 | $-13.45 \%$ |
| $0,800,000$ | 0.9998 | $-14.75 \%$ |
| $0,900,000$ | 0.9993 | $-11.67 \%$ |
| $1,000,000$ | 0.9996 | $-14.93 \%$ |
| $1,200,000$ | 0.9995 | $-16.00 \%$ |
| UG LoCA Base Test | 0.9999 | $-37.78 \%$ |
| $1,300,000$ | 0.9995 | $-29.68 \%$ |
| $1,400,000$ | 0.9995 | $-33.98 \%$ |
| $1,500,000$ | 0.9994 | $-34.06 \%$ |
| $1,700,000$ | 0.9981 | $-25.98 \%$ |
| $1,900,000$ | 0.9995 | $-34.73 \%$ |
| $2,100,000$ | 0.9995 | $-34.07 \%$ |
| $2,300,000$ | 0.9995 | $-33.83 \%$ |
| $2,500,000$ | 0.9989 | $-34.54 \%$ |

Table 10.6: Least Squares and Percent Error for the Galvanized Specimen at Quarter Span

| Cycle Count | Least Square, $\mathbf{R}^{\mathbf{2}}$ | Percent Error, \% |
| :---: | :---: | :---: |
| Base Test | 0.9993 | $9.98 \%$ |
| $0,100,000$ | 0.9931 | $16.55 \%$ |
| $0,200,000$ | 0.9971 | $8.25 \%$ |
| $0,300,000$ | 0.9976 | $9.40 \%$ |
| GG LoCA Base Test | 0.9995 | $9.45 \%$ |
| $0,400,000$ | 0.9986 | $10.35 \%$ |
| $0,500,000$ | 0.9927 | $21.09 \%$ |
| $0,600,000$ | 0.9975 | $17.38 \%$ |
| $0,700,000$ | 0.9971 | $18.73 \%$ |
| $0,800,000$ | 0.9995 | $17.13 \%$ |
| $0,900,000$ | 0.9975 | $16.89 \%$ |
| $1,000,000$ | 0.9979 | $18.86 \%$ |
| $1,200,000$ | 0.9972 | $16.56 \%$ |
| UG LoCA Base Test | 0.9997 | $2.28 \%$ |
| $1,300,000$ | 0.9963 | $15.42 \%$ |
| $1,400,000$ | 0.9972 | $11.07 \%$ |
| $1,500,000$ | 0.9974 | $10.05 \%$ |
| $1,700,000$ | 0.9951 | $10.68 \%$ |
| $1,900,000$ | 0.3926 | $-729.20 \%$ |
| $2,100,000$ | 0.0004 | $47.68 \%$ |
| $2,300,000$ | 0.1123 | $125.81 \%$ |
| $2,500,000$ | 0.5336 | $-725.54 \%$ |

Table 10.7: Least Squares and Percent Error for the Uncoated Specimen at Midspan

| Cycle Count | Least Square, $\mathbf{R}^{\mathbf{2}}$ | Percent Error, \% |
| :---: | :---: | :---: |
| Base Test | 1.0000 | $-3.13 \%$ |
| $0,100,000$ | 0.9994 | $-1.70 \%$ |
| $0,200,000$ | 0.9993 | $-3.15 \%$ |
| $0,300,000$ | 0.9996 | $0.31 \%$ |
| GG LoCA Base Test | 1.0000 | $0.53 \%$ |
| $0,400,000$ | 0.9998 | $0.80 \%$ |
| $0,500,000$ | 0.9991 | $3.33 \%$ |
| $0,600,000$ | 0.9994 | $4.11 \%$ |
| $0,700,000$ | 0.9995 | $2.98 \%$ |
| $0,800,000$ | 0.9999 | $3.13 \%$ |
| $0,900,000$ | 0.9994 | $1.96 \%$ |
| $1,000,000$ | 0.9995 | $3.05 \%$ |
| $1,200,000$ | 0.9996 | $3.58 \%$ |
| UG LoCA Base Test | 0.9999 | $19.31 \%$ |
| $1,300,000$ | 0.9921 | $21.38 \%$ |
| $1,400,000$ | 0.9990 | $17.48 \%$ |
| $1,500,000$ | 0.9988 | $15.70 \%$ |
| $1,700,000$ | 0.9973 | $13.82 \%$ |
| $1,900,000$ | 0.9986 | $14.42 \%$ |
| $2,100,000$ | 0.9983 | $14.75 \%$ |
| $2,300,000$ | 0.9984 | $14.70 \%$ |
| $2,500,000$ | 0.9969 | $29.72 \%$ |

Table 10.8: Least Squares and Percent Error for the Uncoated Specimen at Quarter Span

| Cycle Count | Least Square, $\mathbf{R}^{\mathbf{2}}$ | Percent Error, \% |
| :---: | :---: | :---: |
| Base Test | 0.9999 | 4.36\% |
| 0,100,000 | 0.9963 | 5.45\% |
| 0,200,000 | 0.9993 | 3.62\% |
| 0,300,000 | 0.9993 | 4.40\% |
| GG LoCA Base Test | 0.9992 | -0.27\% |
| 0,400,000 | 0.9996 | 0.80\% |
| 0,500,000 | 0.9978 | 9.28\% |
| 0,600,000 | 0.9988 | 7.60\% |
| 0,700,000 | 0.9992 | 7.04\% |
| 0,800,000 | 0.9997 | 6.57\% |
| 0,900,000 | 0.9991 | 7.16\% |
| 1,000,000 | 0.9993 | 6.44\% |
| 1,200,000 | 0.9994 | 7.30\% |
| UG LoCA Base Test | 0.9999 | 18.70\% |
| 1,300,000 | 0.9994 | 31.80\% |
| 1,400,000 | 0.9992 | 23.34\% |
| 1,500,000 | 0.9993 | 22.89\% |
| 1,700,000 | 0.9942 | 23.37\% |
| 1,900,000 | 0.9995 | 20.92\% |
| 2,100,000 | 0.9992 | 21.80\% |
| 2,300,000 | 0.9994 | 23.49\% |
| 2,500,000 | 0.9884 | 21.11\% |

Figures 10.32 and 10.33 summarize the comparisons between the loads applied to the girders at each static test and the back calculated loads computed from the strain data and threedimensional linear regression. Select static tests have been removed due to recording errors. As shown, the girders' and link slab's behavior remained constant throughout the 75-year design life. A slight discrepancy is shown in Figure 10.32 where the back calculated vertical load is higher than the applied load. This is attributed to the rigidity retained in the galvanized specimen when composite action was lost in the uncoated specimen, and the loads were adjusted accordingly. Data for each static load test can found in Appendix E.


Figure 10.30: Correlation of the Applied Load to the Back Calculated Load of the Galvanized Specimen


Figure 10.31: Correlation of the Applied Load to the Back Calculated Load of the Uncoated Specimen

Figure 10.34 presents the midspan deflections recorded throughout the cyclic loading of both specimens. As shown in Figure 10.34, the deflections remained consistent throughout the design life of the PBFTGs. This further shows the link slab continued to act as a stable structural element, even after composite action was lost in both PBFTGs at different times.


Figure 10.32: Midspan Deflection of Each Specimen Throughout Testing

### 10.13 SUMMARY

This chapter described the physical flexural testing performed on a full-scale link slab constructed between two PBFTGs. The simply supported PBFTGs were formed from 300 inch long by 84 inch wide by $1 / 2$ inch thick HPS-50 steel. Each specimen was cold-bent before delivery to the Major Units Lab where a concrete deck was cast on each specimen separately. After the specimens were fully composite, a link slab was poured longitudinally between the PBFTGs. Cyclic loading was applied to simulate a 75-year design life in a rural environment and static testing was performed throughout testing to monitor any change in the link slab behavior. Strain and deflection data were recorded at each static testing and linear regression was used to transform the strain at each given cross-section to the moment at each location. The consistent behavior shown, in both the strain and deflection data, demonstrates link slabs can adequately transfer load throughout the design life of PBFTGs.

## CHAPTER 11: CONCLUDING REMARKS

### 11.1 Project Summary

The scope of this project was to demonstrate PBFTGs can be utilized in a broader range of applications in the short-span bridge market. This was achieved by performing the following tasks:

- Reviewing literature relating to PBFTGs, live load distribution, the effect of compactness on the flexural capacity of sections, and link slab details.
- PBFTGs have been utilized successfully in multiple experimental and inservice bridge scenarios. However, multiple researchers have shown PBFTGs can be utilized more efficiently in terms of ultimate capacity and applicable LLDFs.
- Live load distribution has been extensively evaluated for bridges containing Igirder longitudinal elements but not box-girder longitudinal elements. The current restrictions on LLDFs for box-girders, and therefore PBFTGs, are based on a limited scope of analytical tests from the 1960s.
- Compactness, as it relates to the capacity of PBFTGs, is based on the same limited set of analytical studies from the 1960s. Studies have not shown the current restrictions to be scientifically derived. Instead, these limits were established because the complicated behavior of PBFTGs was not understood.
- Link slabs have been utilized successfully as a transverse bridge detail for nearly two decades, eliminating undesirable joints. However, link slabs have not been evaluated with PBFTGs and could become a competitive solution to the buckling issue of the slender bottom flange over pier regions.
- Developing analytical tools to assess the behavior and capacity of PBFTGs with varying dimensions and properties.
- Analytical modeling techniques were developed and refined to determine the behavior of modular PBFTGs loaded to ultimate capacity and PBFTG bridges under service level loads.
- These tools are sophisticated, as they can be used with complex bridge and girder geometry as well as linear or non-linear material properties.
- Loading can be applied to the model in a multitude of different ways, including linearly perpendicular to the longitudinal elements, linearly along the skew of the bridge/girder, HL-93 design truck and tandem loading, or user input vehicular loading.
- These tools have been thoroughly benchmarked against numerous experimental and field tests, including seminal noncomposite plate girder experiments, composite plate girders experiments, destructive flexural testing of PBFTGs, and multiple in-service live load field tests of PBFTGs.
- Conducting sensitivity and parametric studies to assess which parameters affect the computation of LLDFs for PBFTGs.
- A sensitivity study was developed to determine the effects of key bridge and girder parameters on live load distribution in PBFTG bridges.
- The results of the sensitivity study were used to generate a matrix of over 50,000 PBFTG bridges with different combinations of the parameters found to have the greatest effect on live load distribution.
- MLR was used on the results of the parametric study to determine simplified equations to be used with LGA.
- Based on comparisons with the parametric study and three in-service PBFTG bridges, the proposed equations prove more accurate than current simplified methodologies presented in the AASHTO LRFD BDS.
- Conducting behavioral studies to assess the effect of skew on the ultimate capacity of PBFTGs.
- An analytical model was generated, based on and verified against the results of a previous ultimate capacity PBFTG experimental test.
- The bearing skew was increased in $5^{\circ}$ increments up to $45^{\circ}$ and loaded past the ultimate theoretical strength.
- Preliminary results from the study show bearing skew does not influence the ultimate capacity of PBFTGs.
- Performing flexural testing on modular PBFTGs transversely joined by a link slab.
- A full-scale link slab was constructed between two PBFTG specimens.
- The link slab was fatigue loaded simulating the 75-year design of the specimens in a rural non-interstate environment.
- Deflection data was recorded throughout the fatigue testing, and strain data was recorded periodically throughout the design life of the link slab during static testing.
- Results showed the link slab behaved linearly throughout its design life and can adequately serve as a transverse bridge detail over piers of continuous PBFTG bridges.

This project has expanded the potential market for the innovative PBFTG system into multiple span arrangements, increasing the system's versatility. This new work will help continue to promote the commercial advancement of this technology.

### 11.2 Recommendations for Continued Research

The author recommends the following tasks for future work and/or expansions to this project:

- As more PBFTG bridges are built in the field, additional live load field testing is recommended to verify the validity of the proposed LLDF equations.
- Investigate additional parameters to determine the effect on live load distribution in PBFTG bridges. These parameters may include skew, presence of sidewalks/barriers for stiffness, continuity/support conditions, and superelevation.
- Perform sensitivity and parametric studies to determine the effects of key bridge and girder parameters on shear live load distribution in PBFTG bridges.
- Perform experimental testing to confirm the analytical result that skew does not have an impact on the capacity of PBFTGs.
- Investigate the behavior of bearing stiffeners in PBFTGs. During the behavioral study to assess the effect of skew, it was noticed the bearing stiffener stresses significantly increased with increased degree of skew.
- Perform feasibility studies to determine the maximum applicable ranges of the standardized PBFTG sections using the proposed simplified equations.
- Practices for bolted and welded splices should be assessed for PBFTGs utilized in longer spans.
- Perform live load field testing on a continuous span PBFTG bridge with a link slab over an interior pier.
- Perform destructive flexural testing on a link slab between PBFTGs.


## REFERENCES

AASHTO/NSBA Steel Bridge Collaboration. (2020). Guidelines to Design for Constructability and Fabrication (Report No. NSBAGDC-4). Washington, DC: American Association of State Highway and Transportation Officials.

Aktan, H., \& Attanayake, U. (2011). High skew link slab bridge system with deck sliding over backwall or backwall sliding over abutments (Report No. RC-1563). Lansing, MI: Michigan Department of Transportation.

American Association of State Highway and Transportation Officials. (2020). AASHTO LRFD Bridge Design Specifications, Ninth Edition. Washington, DC.

American Association of State Highway Officials. (1931). AASHO Standard Specifications for Highway Bridges, First Edition. Washington, DC.

American Institute of Steel Construction. (2017). Steel Construction Manual (15th ed). Chicago, IL.

Armijos-Moya, S., Wang, Y., Helwig, T. A., Clayton, P. M., Engelhardt, M. D., \& Williamson, E. B. (2019). Improved tub girder details: Final report (Report No. FHWA/TX-19/0-68621). Austin, TX: Texas Department of Transportation.

Arockiasamy, M., \& Amer, A. (1998). Load distribution on highway bridges based on field test data: Phase III (Report No. WPI 0510668). Tallahassee, Florida: Florida Department of Transportation.

Bakht, B., \& Jeager, L. G. (1992). Ultimate load test of slab-on-girder bridge. Journal of Structural Engineering, 118(6), 1608-1624.

Barr, P. J., \& Amin, MD. N. (2006). Shear live-load distribution factors for I-girder bridges. Journal of Bridge Engineering, 11(2), 197-204. doi: 10.1061/(ASCE)10840702(2006)11:2(197)

Barth, K. E. (1996). Moment-rotation characteristics for inelastic design of steel bridge beams and girders (Doctoral dissertation, Purdue University).

Barth, K. E., \& Wu, H. (2006). Efficient nonlinear finite element modeling of slab on steel stringer bridges. Finite Elements in Analysis and Design 42(14), 10.

Bertoldi, A. G. (2009). A strength and serviceability assessment of high performance steel Bridge 10462 (Master's thesis, West Virginia University). Available from ProQuest.

Bishara, A. G., Liu, M. C., \& El-Ali, N. D. (1993) Wheel load distribution on simply supported skew I-beam composite bridges. Journal of Structural Engineering, 119(2), 399-419.

Bleich, F., \& Ramsey, L. B. (1952). Buckling strength of metal structures. New York, NY: McGraw-Hill Book Company.

Bryan, G. H. (1890). On the stability of a plane plate under thrusts in its own plane, with applications to the "buckling" of the sides of a ship. Cambridge Philosophical Society, 6, 54-67.

Caner, A., \& Zia, P. (1998). Behavior and design of link slabs for jointless bridge decks. Journal of the Precast/Prestressed Concrete Institute, 68-80.

Chandar, G., Hyzak, M. D., \& Wolf, L. M. (2010). Rapid, economical bridge replacement. Modern Steel Construction.

Chong, J. M. J. (2012) Construction engineering of steel tub-girder bridge systems for skew effects (Doctoral dissertation, Georgia Institute of Technology). Available from ProQuest.

Coletti, D., Chavel, B., \& Gatti, W. J. (2011). Challenges of skew in bridges with steel girders. Transportation Research Record: Journal of the Transportation Research Board, 2251, 4756. doi: 10.3141/2251-05

Crisfield, M. A. (1983). An arc-length method including line searches and accelerations. International Journal for Numerical Methods in Engineering, 19, 1269-1289.

Cross, B., Vaughn, B., Panahshahi, N., Petermeier, D., Siow, Y. S., \& Domagalski, T. (2009). Analytical and experimental investigation of bridge girder shear distribution factors. Journal of Bridge Engineering, 14(3), 154-163. doi: 10.1061/(ASCE)10840702(2009)14:3(154)

Dassault Systèmes. (2020). Abaqus/CAE Version 6.20. Dassault Systèmes Simulia Corp., Providence, RI.

Ebeido, T., \& Kennedy, J. B. (1996). Girder moments in continuous skew composite bridges. Journal of Bridge Engineering, 1(1), 37-45.

Ebeido, T., \& Kennedy, J. B. (1996). Shear and reaction distributions in continuous skew composite bridges. Journal of Bridge Engineering, 1(4), 155-165.

El-Safty, A. K. (1984). Analysis of jointless bridge decks with partially debonded simple span beams (Doctoral dissertation, North Carolina State University).

Fu, C. C., Elhelbawey, M., Sahin, M. A., \& Schelling, D. R. (1996). Lateral distribution factor from bridge field testing. Journal of Structural Engineering, 122(9), 1106-1109.

Galambos, T. V. (1981). Load and resistance factor design. American Institute of Steel Construction Engineering Journal, 74-82.

Galindez, N. Y. (2009). Levels of lateral flange bending in straight, skewed and curved steel Igirder bridges during deck placement (Doctoral dissertation, West Virginia University). Available from ProQuest.

Gastal, F. P. S. L. (1986). Instantaneous and time-dependent response and strength of jointless bridge beams (Doctoral dissertation, North Carolina State University).

Gibbs, C. L. (2017). Field performance assessment of press-brake-formed steel tub girder superstructures (Master's thesis, West Virginia University). Available from ProQuest.

Graybeal, B. (2014). Design and construction of field-cast UHPC connections (Report No: FHWA-HRT-14-084). Washington, DC: US Department of Transportation Federal Highway Administration.

Grubb, M., Coletti, D., Nelson, A., \& Ream, A. (2020). NSBA Guide to Navigating Routine Steel Bridge Design. Chicago, IL: American Institute of Steel Construction.

Haaijer, G., \& Thurlimann, B. (1957). On inelastic buckling in steel (Fritz Laboratory Report No. 205E.9). Retrieved from Lehigh University, Department of Civil Engineering.

Hayes, C. O., Jr., Sessions, L. M., \& Berry, A. J. (1986). Further studies on lateral load distribution using a finite element method. Transportation Research Record, 1072, 6-14.

Helwig, T. A., \& Fan, Z. (2000). Field and computational studies of steel trapezoidal box girder bridges (Report No. 1395-3). Austin, Texas: Texas Department of Transportation.

Hoomes, L. C., Ozyildirim, H. C., \& Brown, M. C. (2017). Evaluation of high-performance fiberreinforced concrete for bridge deck connections, closure pours, and joints (Report No. FHWA/VTRC 17-R15). Richmond, VA: Virginia Department of Transportation and US Department of Transportation Federal Highway Administration.

Imhoff, C. M. (1998). Testing and modeling of Bridge R-289 (Master's thesis, University of Missouri). Available from University of Missouri.

Johnston, S. B., \& Mattock, A. H. (1967). Lateral distribution of load in composite box girder bridges. Highway Research Record: Bridges and Structures, 167, 25-33.

Kassimali, A. (2015). Structural Analysis (5th ed). Stamford, CT: Cengage Learning.
Kelly, L. T. (2014). Experimental evaluation of non-composite shallow press-brake-formed steel tub girders (Master's thesis, West Virginia University). Available from ProQuest.

Khaleel, M. A., \& Itani, R. Y. (1990). Live-load moments for continuous skew bridges. Journal of Structural Engineering, 116(9), 2361-2373.

Khaloo, A. R., \& Mirzabozorg, H. (2003). Load distribution factors in simply supported skew bridges. Journal of Bridge Engineering, 8(4), 241-244. doi: 10.1061/(ASCE)10840702(2003)8:4(241)

Kim, S., \& Nowak, A. S. (1997). Load distribution and impact factors for I-girder bridges. Journal of Bridge Engineering, 2 (3), 97-104.

Kozhokin, P. (2016). Evaluation of modular press-brake-formed tub girders with UHPC joints (Master's thesis, West Virginia University). Available from ProQuest.

Kreitman, K., Azad, A. R.G., Patel, H., Engelhardt, M. D., Helwig, T. A., Williamson, E. B., \& Klingner, R. (2016). Strengthening existing continuous non-composite steel girder bridges using post-installed shear connectors (Report No. FHWA/TX-16/0-6719-1). Austin, TX: Texas Department of Transportation.

Lay, M. G., Adams, P. F., \& Galambos, T. V. (1964). Experiments on high strength steel members (Report No. 297.8). Bethlehem, Pennsylvania: Fritz Engineering Laboratory, Lehigh University.

Li, J., \& Chen, G. (2011). Method to computer live-load distribution in bridge girders. Practice Periodical on Structural Design and Construction, 16(4), 191-198. doi: 10.1061/(ASCE)SC.1943-5576.0000091

Li, V. C., Fischer, G., Kim, Y., Lepech, M., Qian, S., Weimann, M., \& Wang, S. (2003). Durable link slabs for jointless bridge decks based on strain-hardening cementitious composites (Report No: RC-1438). Lansing, Michigan: Michigan Department of Transportation.

Li, V. C., Lepech, M., \& Li, M. (2005). Field demonstration of durable link slabs for jointless bridge decks based on strain-hardening cementitious composites (Report No. RC-1471). Lansing, MI: Michigan Department of Transportation.

Mabsout, M. E., Tarhini, K. M., Frederick, G. R., \& Kesserwan, A. (1998) Effect of continuity on wheel load distribution in steel girder bridges. Journal of Bridge Engineering, 3(3), 103110.

Mabsout, M. E., Tarhini, K. M., Frederick, G. R., \& Kobrosly, M. (1997). Influence of sidewalks and railings on wheel load distribution in steel girder bridges. Journal of Structural Engineering, 2(3), 88-96.

Mabsout, M. E., Tarhini, K. M., Frederick, G. R., \& Tayar, C. (1997). Finite-element analysis of steel girder highway bridges. Journal of Structural Engineering, 2(3), 83-87.

The Mathworks, Inc. (2021). MATLAB. Natick, MA, The Mathworks, Inc.

Mertz, D. (2007). Simplified live load distribution factor equations (NCHRP Report No. 592). Washington, DC: The National Academies Press.

Mertz, D. (2022). Chapter 10: Limit states. Steel Bridge Design Handbook (pp. 1-14). Chicago, IL: American Institute of Steel Construction.

Mertz, D. (2022). Chapter 12: Design for failure. Steel Bridge Design Handbook (pp. 1-36). Chicago, IL: American Institute of Steel Construction.

Michaelson, G. K. (2010). Live load distribution factors for exterior girders in steel I-girder bridges (Master's thesis, West Virginia University). Available from ProQuest.

Michaelson, G. K. (2014). Development and feasibility assessment of shallow press-brake-formed steel tub girders for short-span bridge applications (Doctoral dissertation, West Virginia University). Available from ProQuest.

Micro-Measurements, Inc. (2010). StrainSmart [Data Collection]. Raleigh, North Carolina: MicroMeasurements, Inc.

Nakamura, S. (2002). Bending behavior of composite girders with cold formed steel U section. ASCE Journal of Structural Engineering, 128(9), 1169-1176. doi: 10.1061-(ASCE)07339455(2002)128:9(1169)

Newmark, N. M. (1949). Design of I-beam bridges. Transportation ASC, 114, 997-1022.
Newmark, N. M., \& Siess, C. P. (1942). Moments in I-beam bridges. University of Illinois Bulletin, 39(44), 1-154.

Newmark, N. M., Siess, C. P., \& Peckman, W. M. (1948). Studies of slab and beam highway bridges Part II: Tests of simple-span skew I-beam bridges. University of Illinois Bulletin, 45(31), 1-70.

Nowak, A. S., Eom, J., \& Ferrand, D. (2003). Verification of girder distribution factors for continuous steel girder bridges (Report No. RC-1429). Lansing, Michigan: Michigan Department of Transportation.

Powell, G., \& Simons, J. (1981). Improved iteration strategy for nonlinear structures. International Journal for Numerical Methods in Engineering, 17(10), 1455-1467.

Ramm, E. (1981) Strategies for tracing the nonlinear response near limit points. In E. Wunderlich, E. Stein, \& K. J. Bathe (Eds), Nonlinear Finite Element Analysis in Structural Mechanics (pp. 63-89). Berlin: Springer-Verlag.

Razzaq, M. K. (2017). Load distribution factors for skewed composite steel I-girder bridges (Doctoral dissertation, University of Windsor). Available from ProQuest.

Righman, J. (2005). Rotation compatibility approach to moment redistribution for design and rating of steel I-girders (Doctoral dissertation, West Virginia University). Available from ProQuest.

Riks, E. (1979). An incremental approach to the solution of snapping and buckling problems. International Journal of Solids and Structures, 15, 529-551.

Roberts, N. (2005). Evaluation of the ductility of composite steel I-girders in positive bending (Master's thesis, West Virginia University). Available from ProQuest.

Roh, A. D. (2020). Field evaluation of a modular press-brake-formed steel tub girder in an application that includes skew and superelevation (Master's thesis, West Virginia University). Available from ProQuest.

Royce, M. (2016, July 18-20). Utilization of ultra-high performance concrete (UHPC) in New York. In B. Graybeal, S. Sritharan, \& K. Wille (Chairs), International Interactive Symposium on Ultra-High Performance Concrete [Symposium]. Des Moines, IA.

Sanchez, T. A. S. (2011). Influence of bracing systems on the behavior of curved and skewed steel I-girder bridges during construction (Doctoral dissertation, Georgia Institute of Technology). Available from ProQuest.

Sanders, W. W. (1984). Distribution of wheel loads on highway bridges. National Cooperative Highway Research Program Synthesis of Highway Practice, 111, 1-22.

Schilling, C. G. \& Morcos, S. S. (1988). Moment-rotation tests of steel girders with ultracompact flanges. Project 188 Autostress Design of Highway Bridges. American Iron and Steel Institute.

Short Span Steel Bridge Alliance. (2020). Steel press-brake formed tub girder system selected for 2021 AASHTO Innovation Initiative focus technology. SSSBA, Washington, DC.
Stallings, J. M., \& Yoo, C. H. (1993). Tests and ratings of short-span steel bridges. Journal of Structural Engineering, 119(7), 2150-2168.

Strang, G. (2016). Introduction to linear algebra (5th ed). Wellesley, MA: Cambridge Press.
Taly, N. B., \& Gangarao, H. V. S. (1976). Development and design of standardized short span bridge superstructural. (Report No. FHWA-WV-76-5).

Tarhini, K. M., \& Frederick, G. R. (1992). Wheel load distribution in I-girder highway bridges. Journal of Structural Engineering, 118(5), 1285-1294.

Tennant, R. M. (2018). Fatigue performance of uncoated and galvanized composite press-brakeformed tub girders (Master's thesis, West Virginia University). Available from ProQuest.

Texas Steel Quality Council. (2021). Preferred practices for steel bridge design, fabrication, and erection. Austin, TX: Texas Department of Transportation.

Timoshenko, S., \& Goodier, J. N. (1961). Theory of elasticity. New York, NY: McGraw-Hill Book Company.

Timoshenko, S., \& Woinowsky-Krieger, S. (1959). Theory of plates and shells (2nd ed). New York, NY: McGraw-Hill Book Company.

Underwood, N. M. H. (2019). Field performance and rating evaluation of a modular press-brakeformed steel tub girder with a steel sandwich plate deck (Master's thesis, West Virginia University). Available from ProQuest.

Valmont Structures. (2022). Product details. https://bridge.constructbridge.com/ProductDetails
Walker, W. H. (1987). Lateral load distribution in multi-girder bridges. Engineering Journal, 24(1), 21-28.

Wallace, M. R. (1976). Studies of skewed concrete box-girder bridges. Committee on Concrete Bridges, 50-55.

White, D. (2022). Chapter 4: Strength behavior and design of steel. Steel Bridge Design Handbook (pp. 1-291). Chicago, IL: American Institute of Steel Construction.

White, D. W., \& Kamath. A. (2020). Straight steel I-girder bridges with skew index approaching 0.3 (Report No. FDOT BE535). Tallahassee, Florida: Florida Department of Transportation.

White, D. W., Coletti, D., Chavel, B. W., Sanchez, A., Ozgur, G., Chong, J. M. J.,...Kowatch, G. T. (2012). Guidelines for analysis methods and construction engineering of curved and skewed steel girder bridges (NCHRP Report No. 725). Washington, DC: The National Academies Press. doi: 10.17226/22729

White, D. W., Grubb, M., King, C., \& Slein, R. (2019). Proposed AASHTO guidelines for bottom flange limits of steel box girders.

Wing, K. M., \& Kowalsky, M. J. (2005). Behavior, analysis, and design of an instrumented link slab bridge. Journal of Bridge Engineering, 10(3), 331-344. doi: 10.1061/(ASCE)10840702(2005)10:3(331)

Yang, L. (2004). Evaluation of moment redistribution for hybrid HPS 70W bridge girders (Master's thesis, West Virginia University). Available from ProQuest.

Yousif, Z., \& Hindi, R. (2007). AASHTO-LRFD live load distribution for beam-and-slab bridges: Limitations and applicability. Journal of Bridge Engineering, 12(6), 765-773. doi: 10.1061/(ASCE)1084-0702(2007)12:6(765)

Ziemian, R. D. (2010). Guide to stability design criteria for metal structures. Hoboken, NJ: John Wiley \& Sons, Inc.

Zokaie, T. (2000). AASHTO-LRFD live load distribution specifications. Journal of Bridge Engineering, 131-138.

Zokaie, T., Osterkamp, T. A., \& Imbsen, R. A. (1991). Distribution of wheel loads on highway bridges. National Cooperative Highway Research Program Transportation Research Board National Research Council, 12-26/1, 1-710.

## APPENDIX A: LLDF SENSITIVITY MATRIX RESULTS

The following appendix lists in graphical form the comparison of LLDFs calculated from the finite element models of the typical bridges analyzed during the sensitivity study discussed in Section 8.2. For the reader's convenience, this data has been organized such that each graph is focused on the influence of a single parameter on the distribution of moment to a single girder for each live load distribution methodology. Note, some graphs are not available as the typical bridge may not be applicable in certain situations. These situations will be labeled 'No Data Available' in place of the typical graph. Additionally, not every parameter will be present in every graph, as some parameters are not feasible with every standard bridge. The graphs were generated using MATLAB (Mathworks, 2021).


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#1, Stallings/Yoo Methodology, IG OLL, Variable = Overhang Ratio

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = PBFTG Size

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = Span Length (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable $=$ Girder Spacing (ft)

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = Degree of Skew (deg)

## NO DATA AVAILABLE

[^1]
## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#1, Stallings/Yoo Methodology, EG OLL, Variable $=$ Overhang Ratio

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = PBFTG Size

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Girder Spacing (ft)

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Degree of Skew (deg)

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Deck Thickness (in)

## NO DATA AVAILABLE

Typical Bridge \#1, Stallings/Yoo Methodology, EG 2LL, Variable = Overhang Ratio


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Ratio

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = PBFTG Size

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Span Length (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Degree of Skew (deg)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable $=$ Overhang Ratio

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = PBFTG Size

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Girder Spacing (ft)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Degree of Skew (deg)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)

## NO DATA AVAILABLE

Typical Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Overhang Ratio


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#2, Stallings/Yoo Methodology, IG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Span Length (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#2, Stallings/Yoo Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#2, Stallings/Yoo Methodology, EG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#2, Stallings/Yoo Methodology, EG 2LL, Variable $=$ Overhang Ratio


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable $=$ Span Length (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Overhang Ratio


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#3, Stallings/Yoo Methodology, IG OLL, Variable = Overhang Ratio


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable $=$ Span Length (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#3, Stallings/Yoo Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#3, Stallings/Yoo Methodology, EG OLL, Variable = Overhang Ratio


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#3, Stallings/Yoo Methodology, EG 2LL, Variable = Overhang Ratio


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Span Length (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Overhang Ratio


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Overhang Ratio


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable $=$ Span Length (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable = Deck Thickness (in)


Typical Bridge \#4, Stallings/Yoo Methodology, IG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable $=$ Span Length (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#4, Stallings/Yoo Methodology, IG 2LL, Variable = Overhang Ratio


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#4, Stallings/Yoo Methodology, EG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#4, Stallings/Yoo Methodology, EG 2LL, Variable = Overhang Ratio


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable = PBFTG Size


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable = Span Length (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Girders


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Deck Thickness (in)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable = PBFTG Size


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable $=$ Span Length (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Girders


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable $=$ Girder Spacing (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#4, Tarhini/Frederick Methodology, IG 2LL, Variable $=$ Overhang Ratio


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = PBFTG Size


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = Span Length (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Girders


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG OLL, Variable $=$ Overhang Ratio


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = PBFTG Size


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Girders


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = Girder Spacing (ft)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = Degree of Skew (deg)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)


Typical Bridge \#4, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Overhang Ratio

## APPENDIX B: LLDF PARAMETRIC VARIATION RESULTS

The following appendix lists in graphical form the comparison of LLDFs calculated from the finite element models of the standard bridges analyzed during the parametric study discussed in Section 8.2. For the reader's convenience, this data has been organized such that each graph is focused on the influence of a single parameter on the distribution of moment to a single girder. Note, some graphs are not available as the typical bridge may not be applicable in certain situations. These situations will be labeled 'No Data Available' in place of the typical graph. Additionally, not every parameter will be present in every graph, as some parameters are not feasible with every standard bridge. The graphs were generated using MATLAB (Mathworks, 2021).


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Beams


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Distance (in)


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Plate Size


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Standard Bridge \#1, Tarhini/Frederick Methodology, IG OLL, Variable = Span Length (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Beams


Standard Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Distance (in)

## NO DATA AVAILABLE

Standard Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Plate Size

## NO DATA AVAILABLE

## NO DATA AVAILABLE

Standard Bridge \#1, Tarhini/Frederick Methodology, IG 2LL, Variable = Span Length (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Beams


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Overhang Distance (in)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Plate Size


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG OLL, Variable $=$ Span Length (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Beams


Standard Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Girder Spacing (ft)


Standard Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Overhang Distance (in)

## NO DATA AVAILABLE

## Standard Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Plate Size

## NO DATA AVAILABLE

## NO DATA AVAILABLE

Standard Bridge \#1, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Beams


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Distance (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Plate Size


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Span Length (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Beams


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Distance (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Plate Size


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, IG 2LL, Variable = Span Length (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Beams


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Overhang Distance (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Plate Size


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG OLL, Variable = Span Length (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Beams


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Girder Spacing (ft)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Overhang Distance (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Plate Size


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)


Standard Bridge \#2, Tarhini/Frederick Methodology, EG 2LL, Variable $=$ Span Length (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Number of Beams


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Overhang Distance (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Plate Size


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable = Deck Thickness (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG OLL, Variable $=$ Span Length (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Number of Beams


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Girder Spacing (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Overhang Distance (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Plate Size


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Deck Thickness (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, IG 2LL, Variable = Span Length (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Number of Beams


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Girder Spacing (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Overhang Distance (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Plate Size


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable = Deck Thickness (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG OLL, Variable $=$ Span Length (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Number of Beams


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Girder Spacing (ft)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Overhang Distance (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Plate Size


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Deck Thickness (in)


Standard Bridge \#3, Tarhini/Frederick Methodology, EG 2LL, Variable = Span Length (ft)

## APPENDIX C: COMPACTNESS SENSITIVITY MATRIX RESULTS

This appendix documents the results from the analytical study performed in Chapter 9. The graphs are titled according to the degrees of skew discussed in Chapter 9. In addition, for each graph, the concentrated load at midspan corresponding to the yield moment, plastic moment, and nominal capacity calculated from AASHTO LRFD BDS Article 6.11.7, assuming the section is considered noncompact, is displayed.


## Load-Deflection Graph for the Analytical Test with $0^{\circ}$ Skew



## Load-Deflection Graph for the Analytical Test with $5^{\circ}$ Skew



Load-Deflection Graph for the Analytical Test with $10^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $15^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $20^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $\mathbf{2 5}^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $30^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $35^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $40^{\circ}$ Skew


Load-Deflection Graph for the Analytical Test with $45^{\circ}$ Skew

## APPENDIX D: LOADING CALCULATIONS

## D. 1 Overview

According to Caner and Zia (1998), "Each span of a bridge with a jointless bridge deck [link slab] may be designed independently as a simply-supported span using standard design procedures without considering the effect of the link slab because the stiffness of the link slab is much smaller when compared to that of the composite girder." Therefore, influence line analysis was performed to determine the moments at midspan of a simply supported beam. Equation D. 1 presents the set of functions for the moment at midspan:

$$
f(x)=\left\{\begin{array}{cc}
x / 2 & {[0, L / 2]} \\
(L-x) / 2 & {[L / 2, L]}
\end{array}\right.
$$

Eq. D. 1

## D. 2 Simulated Link Slab

The constructed link slab was designed as if it was placed between two 80 foot spans. The 80 foot spans were chosen because modern press-brakes are limited to bending PBFTGs at 60 feet in length. A PBFTG 60 feet in length can be cut into three equal 20 foot pieces, which can be spliced onto a full 60 foot long PBFTG. Due to physical constraints in the Major Units Laboratory, two 80 -foot spans could not be built, so the largest possible set up was chosen. A link slab, designed for two 80 -foot spans, was placed between two 24.5 foot long PBFTG modules.

## D. 3 Design Loading

The first step to testing the link slab was to determine the load to induce Fatigue I moment. The Fatigue I load combination is related to infinite load-induced fatigue life and utilizes the load factors found in AASHTO LRFD BDS Table 3.4.1-1.

Fatigue $I=1.75(L L+I M)$

AASHTO LRFD BDS define the terms found in Equation D. 2 as follows:

- $\mathrm{LL}=$ vehicular live load
- Vehicular live loading on the roadway of bridges is defined in AASHTO LRFD BDS Article 3.6.1.2 as: the combination of either the design truck and the design lane or the design tandem and the design lane, whichever yields the largest force effect.
- Note, for the fatigue limit state, the fatigue load consists of only one design truck with a fixed rear axle spacing of 30 feet (AASHTO LRFD BDS Article 3.6.1.4.1).
- $\quad \mathrm{IM}=$ vehicular dynamic load allowance
- The load allowance serves to amplify the vehicular components of the HL-93 live load (i.e., the truck and tandem)
- For all limit states regarding deck joints, $\mathrm{IM}=75 \%$ (AASHTO LRFD BDS Article 3.6.2).

Utilizing a load factor of 1.75 , for infinite life fatigue, and an impact factor of 1.75 , for limit states regarding deck joints, the Fatigue I moment was simplified from Equation D. 2 into Equation D.3:

$$
\left.\begin{array}{l}
M_{\text {Fatigue }_{I}}=1.75\left(1.75 M_{\text {Fatigue }}^{\text {Truck }}\right.
\end{array}\right)
$$

The weights and spacings of axles and wheels for the design truck are shown in Figure D.1. The fatigue load is to be taken as one design truck or axles thereof with a constant spacing of 30 feet between the 32 kip axles.


Figure D.1: Characteristics of the Design Truck

## D. 4 Moment Envelope

A commercially available live load generator was used to determine the maximum negative moment at the interior support. Figure D. 2 shows the fatigue moment envelope, which is based on the data presented in Table D.1. For loads applied to the composite section, the envelope shown is based on the composite section properties, assuming the concrete deck to be effective over the entire span length.


Figure D.2: Fatigue Live Load Moments

Table D.1: Unfactored and Undistributed Live Load Moments (kip-ft)

| Span |  | Truck |  | Tandem |  | Lane |  | Double Truck |  | Double Tandem |  | Fatigue Truck |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x} / \mathrm{L}$ | $\mathrm{x}(\mathrm{ft})$ | $(+)$ | $(-)$ | $(+)$ | $(-)$ | $(+)$ | $(-)$ | $(+)$ | $(-)$ | $(+)$ | $(-)$ | $(+)$ | $(-)$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.1 | 2.45 | 87 | -10 | 95 | -11 | 15 | -2 | 0 | 0 | 0 | 0 | 73 | -8 |
| 0.2 | 4.9 | 142 | -20 | 160 | -23 | 26 | -5 | 0 | 0 | 0 | 0 | 124 | -15 |
| 0.3 | 7.35 | 166 | -30 | 198 | -34 | 33 | -7 | 0 | 0 | 0 | 0 | 153 | -23 |
| 0.4 | 9.8 | 166 | -40 | 210 | -46 | 36 | -10 | 0 | 0 | 0 | 0 | 163 | -31 |
| 0.5 | 12.25 | 159 | -49 | 205 | -57 | 36 | -12 | 0 | 0 | 0 | 0 | 159 | -39 |
| 0.6 | 14.7 | 149 | -59 | 186 | -69 | 32 | -14 | 0 | 0 | 0 | 0 | 144 | -46 |
| 0.7 | 17.15 | 129 | -69 | 148 | -80 | 24 | -17 | 0 | 0 | 0 | 0 | 119 | -54 |
| 0.8 | 19.6 | 82 | -79 | 94 | -91 | 12 | -19 | 0 | -79 | 0 | -91 | 81 | -62 |
| 0.9 | 22.05 | 40 | -93 | 32 | -103 | 2 | -28 | 0 | -93 | 0 | -136 | 40 | -90 |
| 1 | 24.5 | 0 | -144 | 0 | -114 | 0 | -48 | 0 | -144 | 0 | -197 | 0 | -129 |

The Fatigue I moment induced into the link slab can be calculated from the unfactored and undistributed moment caused by the Fatigue Truck, found from Table D. 1 and inserted into Equation D.3:

$$
\begin{aligned}
& M_{\text {Fatigue }_{I}}=3.0625 M_{\text {Fatigue }} \text { Truck } \\
& M_{\text {Fatigue }} \text { I }
\end{aligned}=3.0625(129 \text { kip }-f t)=395 \mathrm{kip}-f t
$$

## D. 5 LoAd Application by Actuators

To determine the point loads which need to be applied to the spans in the lab to generate the moments which would be created by the vehicles, the applied load was considered symmetrically placed. The span length in the lab was 24.5 feet. The equation for the moment at an interior support of a continuous beam with two equal concentrated loads symmetrically placed is described by Equation D.4:

$$
M=\frac{3 P L}{16}
$$

Equation D. 4 can be manipulated into Equation D. 5 to determine the load required to generate the same Fatigue I Moment generated by design trucks from the AASHTO LRFD BDS:

$$
P=\frac{16 M}{3 L}
$$

$$
\begin{aligned}
& P_{\text {Fatigue } I}=\frac{16 M_{\text {Fatigue } I}}{3 L} \\
& P_{\text {Fatigue } I}=\frac{16(395 \mathrm{kip}-\mathrm{ft})}{3(24.5 \mathrm{ft})}=86.0 \mathrm{kip}
\end{aligned}
$$

## APPENDIX E: EXPERIMENTAL DATA

The purpose of this appendix is to document the experimental data recorded from the experimental laboratory test discussed in Chapter 10. The title of each table corresponds to the number of cycles at which the static test was performed or the date of the test after retrofitting the specimens.

| N = 0,000,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.4095 | 14.1759 | 34.2747 | 64.7742 | 95.9287 | 120.3684 | 142.3373 | 164.1203 | 185.4382 | 207.4564 |
| D1B | 0 | 0.466 | 0.8388 | 1.7708 | 2.33 | 3.262 | 4.0078 | 5.1732 | 5.7328 | 6.6182 | 7.4104 |
| D2A | 0 | 3.3109 | 15.483 | 47.0102 | 93.046 | 141.8375 | 193.06 | 243.4477 | 293.5608 | 342.2789 | 390.6277 |
| D2B | 0 | 1.4394 | 3.2505 | 5.0147 | 6.872 | 8.5437 | 10.401 | 12.1191 | 13.7906 | 15.3694 | 17.0874 |
| D3A | 0 | 4.5536 | 19.7952 | 53.3942 | 99.3579 | 146.3017 | 193.4826 | 239.5056 | 284.9283 | 328.1231 | 372.2986 |
| D3B | 0 | 3.2708 | 5.9341 | 8.6446 | 11.822 | 14.6725 | 18.551 | 21.7757 | 25.1399 | 28.5048 | 32.29 |
| D4A | 0 | 6.1719 | 26.3708 | 61.4871 | 104.1347 | 146.1782 | 188.0848 | 229.1526 | 269.1953 | 307.0418 | 346.8095 |
| D4B | 0 | 50.7866 | 81.6478 | 92.2492 | 98.3467 | 104.1168 | 110.1206 | 114.2019 | 118.5646 | 120.7225 | 124.6627 |
| G04-01 | 0 | 6.1256 | 9.4666 | 10.673 | 11.1834 | 12.3434 | 15.0351 | 17.5412 | 20.6964 | 24.0846 | 28.2606 |
| G04-02 | 0 | 12.4779 | 23.4195 | 33.4302 | 42.6496 | 51.9157 | 62.7652 | 73.3354 | 84.0924 | 95.0353 | 106.5845 |
| G11-03 | 0 | 23.6282 | 47.1173 | 68.9272 | 90.4102 | 111.5673 | 134.033 | 155.7058 | 177.3329 | 199.3813 | 220.9637 |
| G04-04 | 0 | 34.7487 | 70.8508 | 107.5142 | 142.8298 | 177.5884 | 214.3062 | 249.7222 | 284.7208 | 319.3496 | 353.794 |
| G04-05 | 0 | 143.7844 | 211.3537 | 257.6949 | 300.4191 | 336.5808 | 377.0909 | 417.6046 | 457.3482 | 501.2968 | 532.7884 |
| G05-06 | 0 | 34.0515 | 69.0827 | 104.8625 | 140.8307 | 175.4039 | 212.263 | 247.3533 | 282.5398 | 317.2624 | 351.4747 |
| G05-07 | 0 | 22.691 | 44.867 | 67.8417 | 91.3796 | 114.0742 | 138.553 | 161.6248 | 184.5579 | 207.9131 | 230.5668 |
| G05-08 | 0 | 14.7582 | 28.1621 | 41.8939 | 55.9526 | 69.4976 | 83.8375 | 97.6637 | 111.4439 | 125.3645 | 139.2387 |
| G05-09 | 0 | 8.612 | 14.7569 | 21.4138 | 28.6296 | 35.473 | 43.1082 | 50.2311 | 57.5403 | 65.362 | 73.1834 |
| G05-10 | 0 | -6.0478 | -13.6773 | -19.5849 | -24.6091 | -29.0748 | -32.5173 | -36.7042 | -39.5418 | -42.1933 | -44.6586 |
| G06-11 | 0 | 1.2578 | 0.3728 | 1.3975 | 3.0285 | 5.6837 | 8.3862 | 11.9268 | 15.7944 | 19.335 | 23.994 |
| G06-12 | 0 | 16.8123 | 32.4707 | 46.5128 | 59.354 | 71.9651 | 84.8537 | 97.3726 | 109.1988 | 121.5342 | 133.1766 |
| G06-13 | 0 | 30.8812 | 59.8112 | 86.6963 | 111.3966 | 135.8653 | 160.9399 | 184.9461 | 208.6738 | 232.0772 | 255.1557 |
| G06-14 | 0 | 35.5816 | 70.8406 | 100.1009 | 126.5247 | 153.4156 | 180.7265 | 207.0155 | 232.7933 | 257.9681 | 282.9115 |
| G06-15 | 0 | 28.5563 | 54.9716 | 80.4096 | 104.2181 | 127.422 | 152.0248 | 175.6044 | 199.2306 | 222.5321 | 245.4618 |
| G07-16 | 0 | 14.5767 | 27.5234 | 40.5172 | 52.9989 | 65.5742 | 78.9418 | 91.7506 | 104.42 | 117.043 | 129.7129 |
| G07-17 | 0 | 1.4908 | 3.4008 | 5.6836 | 8.2923 | 11.0409 | 14.3018 | 17.3767 | 21.0106 | 24.7838 | 28.185 |
| G07-18 | 0 | -10.729 | -20.0644 | -28.7958 | -36.459 | -43.4253 | -49.3697 | -55.0355 | -60.6548 | -65.6238 | -70.6858 |
| G07-19 | 0 | 0.9315 | -2.7955 | -9.6918 | -14.8639 | -20.3151 | -21.3867 | -20.9209 | -19.9423 | -20.0821 | -18.2647 |
| G07-20 | 0 | 11.262 | 19.7434 | 26.6955 | 33.9258 | 43.6586 | 53.8091 | 64.2382 | 75.1307 | 86.8577 | 98.2144 |
| G07-21 | 0 | 20.9159 | 41.2734 | 60.9793 | 81.9909 | 103.1433 | 124.2029 | 145.0773 | 165.9521 | 187.0153 | 207.9387 |
| G08-22 | 0 | 34.6276 | 70.1396 | 107.0008 | 143.2608 | 178.595 | 212.8634 | 246.577 | 280.3393 | 313.3608 | 345.9664 |
| G08-23 | 0 | 33.4473 | 70.9914 | 109.4688 | 146.46 | 190.2015 | 225.8494 | 260.6617 | 295.1506 | 333.1334 | 367.022 |
| G08-24 | 0 | 33.8644 | 70.4726 | 107.8735 | 144.5335 | 180.267 | 215.2593 | 249.371 | 283.2063 | 316.5323 | 350.1863 |
| G08-25 | 0 | 22.336 | 45.3244 | 67.0107 | 88.1864 | 109.9677 | 131.7035 | 153.2073 | 174.5259 | 196.1711 | 217.5386 |
| G09-26 | 0 | 15.2345 | 27.5434 | 37.3906 | 46.913 | 57.9682 | 69.2563 | 81.0088 | 92.9478 | 105.2584 | 118.127 |
| G09-27 | 0 | 3.9636 | 4.5696 | 0.4197 | -2.7042 | -3.6369 | -2.8443 | -1.0261 | 0.8862 | 4.0101 | 6.7612 |
| G09-28 | 0 | 3.4091 | 4.4832 | 3.3157 | 0.4202 | -1.2147 | -1.4482 | -1.7751 | -2.0086 | -2.4289 | -2.569 |
| G09-29 | 0 | 0.5266 | 2.2038 | -0.0966 | -12.7946 | -10.6873 | -6.7588 | -11.7904 | -10.0181 | -10.21 | -13.3245 |
| G09-30 | 0 | 16.7566 | 33.2344 | 50.877 | 67.0304 | 84.4876 | 102.7834 | 119.5903 | 136.3044 | 153.1125 | 169.7342 |
| G10-31 | 0 | 25.8184 | 49.7262 | 72.5642 | 96.009 | 119.9674 | 142.8549 | 165.5569 | 187.4672 | 210.0781 | 231.8505 |
| G10-32 | 0 | 22.0077 | 45.1332 | 70.0743 | 95.0634 | 120.8914 | 145.6968 | 170.131 | 194.2871 | 218.6309 | 242.5098 |
| G10-33 | 0 | 18.8603 | 38.1417 | 57.6103 | 77.0802 | 96.5511 | 115.3215 | 133.9528 | 152.2582 | 170.0042 | 187.8436 |
| G10-34 | 0 | 14.871 | 30.3021 | 44.8476 | 58.8807 | 72.9137 | 87.8337 | 102.0546 | 116.4621 | 130.9171 | 145.1391 |
| G10-35 | 0 | 11.3896 | 22.5927 | 30.9481 | 38.3703 | 46.6324 | 55.2688 | 64.6056 | 73.1953 | 82.4852 | 90.9355 |
| G11-36 | 0 | 5.5645 | 10.3807 | 11.7367 | 10.2403 | 10.6614 | 11.8302 | 13.9343 | 15.5242 | 17.0674 | 19.3589 |


| N = 0,100,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 7.8819 | 16.7435 | 36.1461 | 77.0999 | 111.2923 | 136.3438 | 159.7163 | 179.8711 | 201.7531 | 226.4822 |
| D1B | 0 | 0.7922 | 1.398 | 1.9106 | 2.7028 | 4.2406 | 5.3124 | 5.6852 | 3.5416 | 4.893 | 7.689 |
| D2A | 0 | 2.0517 | 10.4915 | 34.2265 | 80.4872 | 137.3397 | 194.6649 | 247.0519 | 297.5777 | 346.7557 | 397.1052 |
| D2B | 0 | 4.4579 | 6.4085 | 8.6375 | 11.5164 | 15.3247 | 17.925 | 19.7826 | 19.1324 | 22.1513 | 26.7484 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 4.4856 | 7.2423 | 11.588 | 17.3822 | 23.7838 | 29.6247 | 33.9237 | 36.0266 | 40.6058 | 46.4004 |
| D4A | 0 | 4.4891 | 12.6255 | 38.8127 | 85.766 | 138.5237 | 187.591 | 230.4882 | 269.3184 | 307.3098 | 348.1114 |
| D4B | 0 | 139.2539 | 156.6074 | 166.8327 | 174.4314 | 181.3736 | 188.1282 | 191.7869 | 202.1534 | 204.2175 | 207.3607 |
| G04-01 | 0 | 4.5936 | 4.3152 | -0.3712 | -5.4288 | -9.3264 | -9.9296 | -13.1776 | -16.6112 | -12.76 | -3.1088 |
| G04-02 | 0 | 12.5254 | 22.9564 | 30.6395 | 37.252 | 44.7027 | 53.3175 | 60.3494 | 64.6336 | 92.2972 | 108.83 |
| G11-03 | 0 | 24.9338 | 49.4021 | 72.0039 | 93.2057 | 113.6146 | 135.3784 | 154.4808 | 170.4547 | 192.6415 | 219.3132 |
| G04-04 | 0 | 37.9621 | 78.629 | 119.8584 | 159.3206 | 197.0621 | 235.1794 | 270.5959 | 305.7353 | 352.7637 | 389.214 |
| G04-05 | 0 | -534.354 | -478.469 | -562.906 | -633.247 | -735.352 | -629.173 | -577.118 | -269.559 | 49383.85 | 49414.87 |
| G05-06 | 0 | 36.7096 | 75.8903 | 116.6588 | 156.1716 | 192.939 | 230.7338 | 265.5027 | 295.7985 | 332.3426 | 373.3181 |
| G05-07 | 0 | 22.8338 | 45.9964 | 70.4727 | 94.6222 | 118.1166 | 141.5651 | 161.1214 | 178.5218 | 205.4439 | 237.2456 |
| G05-08 | 0 | 13.312 | 24.2888 | 35.7792 | 47.8775 | 59.9293 | 72.0283 | 81.6981 | 89.4063 | 104.262 | 124.9585 |
| G05-09 | 0 | 5.2144 | 6.2389 | 7.4494 | 9.4979 | 12.6173 | 15.6902 | 17.5058 | 18.2973 | 26.2591 | 39.8548 |
| G05-10 | 0 | -6.8861 | -15.7724 | -23.7746 | -34.0562 | -41.9186 | -50.3856 | -58.6197 | -72.1571 | -59.4568 | -58.6197 |
| G06-11 | 0 | 4.9828 | 7.5909 | 9.3606 | 8.6153 | 8.1496 | 8.2428 | 8.8951 | 5.6815 | 8.8948 | 15.9272 |
| G06-12 | 0 | 15.8898 | 29.7017 | 41.4354 | 51.1826 | 59.7753 | 69.8459 | 78.0233 | 83.4749 | 96.5033 | 111.5183 |
| G06-13 | 0 | 30.8386 | 62.4697 | 91.4979 | 118.8063 | 142.0685 | 168.1234 | 191.0622 | 211.3495 | 238.4319 | 266.3528 |
| G06-14 | 0 | 31.264 | 62.9494 | 92.4028 | 119.9968 | 144.4275 | 170.5354 | 194.0847 | 217.1229 | 224.7095 | 258.6411 |
| G06-15 | 0 | 30.8304 | 61.9423 | 91.4727 | 118.8619 | 142.4791 | 168.5203 | 192.0002 | 212.3596 | 206.2104 | 232.6742 |
| G07-16 | 0 | 13.1768 | 27.099 | 40.9282 | 52.6627 | 63.4191 | 75.7594 | 86.7027 | 95.5977 | 109.7551 | 126.3808 |
| G07-17 | 0 | 1.9106 | 3.0751 | 4.0999 | 3.9602 | 4.9852 | 6.5695 | 8.1068 | 6.6623 | 10.7157 | 18.2632 |
| G07-18 | 0 | -10.6826 | -22.8518 | -34.1844 | -46.5383 | -55.7806 | -64.5583 | -72.4997 | -84.1563 | -87.3606 | -88.986 |
| G07-19 | 0 | 0.3262 | -4.7988 | -15.0964 | -22.7854 | -25.7207 | -28.2366 | -28.9351 | -32.616 | -30.3336 | -23.7169 |
| G07-20 | 0 | 9.9183 | 16.9172 | 21.8302 | 26.8831 | 33.6501 | 40.1395 | 48.0658 | 51.5885 | 60.1641 | 74.627 |
| G07-21 | 0 | 22.0338 | 43.0444 | 63.5899 | 82.4122 | 101.6076 | 120.2912 | 139.4884 | 153.6077 | 173.7845 | 197.0384 |
| G08-22 | 0 | 36.9484 | 75.3854 | 115.5427 | 151.6638 | 185.5127 | 219.5028 | 252.5199 | 279.1769 | 311.6406 | 345.7786 |
| G08-23 | 0 | 34.4363 | 74.0413 | 116.1161 | 154.6101 | 191.292 | 219.1771 | 253.4891 | 300.9335 | 335.2042 | 371.6194 |
| G08-24 | 0 | 37.3486 | 77.5338 | 119.3023 | 157.4494 | 193.4156 | 228.2689 | 263.2644 | 293.4747 | 327.266 | 362.8723 |
| G08-25 | 0 | 23.4526 | 46.4875 | 69.0115 | 90.885 | 111.9683 | 133.6573 | 154.5562 | 170.8476 | 193.7496 | 220.47 |
| G09-26 | 0 | 12.7737 | 22.2497 | 29.6356 | 38.4616 | 47.5668 | 58.4372 | 69.7723 | 78.7388 | 91.3752 | 107.2643 |
| G09-27 | 0 | 2.2379 | -1.9117 | -11.0044 | -15.4341 | -17.5323 | -17.6256 | -17.1127 | -19.4439 | -15.1542 | -5.9218 |
| G09-28 | 0 | 2.0554 | 1.8686 | -5.0917 | -11.6784 | -15.4154 | -18.8722 | -23.2168 | -32.7461 | -32.9796 | -26.1131 |
| G09-29 | 0 | 9.5079 | 18.7292 | 23.3636 | 29.0018 | 37.3156 | 44.5782 | 50.79 | 46.7285 | 471.8658 | 471.08 |
| G09-30 | 0 | 16.9441 | 33.8889 | 49.81 | 65.2194 | 80.583 | 96.9714 | 111.2649 | 119.2267 | 137.1527 | 159.1768 |
| G10-31 | 0 | 28.6104 | 55.4054 | 81.4558 | 106.4828 | 130.0656 | 153.4167 | 176.3494 | 191.545 | 214.1065 | 240.445 |
| G10-32 | 0 | 24.0561 | 49.7422 | 77.5237 | 105.1203 | 130.8107 | 156.6882 | 181.8223 | 200.1619 | 224.7401 | 252.2515 |
| G10-33 | 0 | 21.1478 | 42.4368 | 64.5672 | 85.8583 | 105.516 | 125.1275 | 143.1984 | 155.2932 | 173.8326 | 196.5298 |
| G10-34 | 0 | 15.8513 | 32.0302 | 47.323 | 62.29 | 77.2108 | 92.5053 | 106.1217 | 113.4894 | 129.4845 | 149.7705 |
| G10-35 | 0 | 11.1096 | 21.0527 | 27.9618 | 35.2907 | 43.647 | 51.8166 | 59.3795 | 62.0408 | 72.6852 | 87.0178 |
| G11-36 | 0 | 5.4716 | 6.7811 | 3.1802 | 0.8886 | 0.2338 | -0.7951 | -2.8994 | -11.5977 | -9.1191 | 0.7949 |


| N = 0,200,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 6.1109 | 17.8193 | 50.2409 | 92.3694 | 124.5624 | 151.5316 | 175.8428 | 200.6214 | 222.4613 | 242.7618 |
| D1B | 0 | 1.1184 | 2.3766 | 2.9358 | 4.3338 | 6.1512 | 8.0152 | 8.621 | 10.3918 | 11.3238 | 12.816 |
| D2A | 0 | 3.3575 | 14.8756 | 46.2141 | 98.3097 | 159.1332 | 219.311 | 275.2503 | 329.7026 | 382.5274 | 427.0033 |
| D2B | 0 | -3.9118 | -8.4755 | -2.5615 | -10.6176 | -11.9217 | -12.9928 | -14.1103 | -12.9462 | -14.2965 | -12.8997 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 8.3164 | 12.3813 | 19.6236 | 25.4172 | 32.7998 | 39.4349 | 44.9017 | 50.4154 | 55.3221 | 59.901 |
| D4A | 0 | -1.265 | 4.3516 | -72.0018 | -28.5208 | 21.1435 | 67.863 | 111.6834 | 154.7126 | 193.8583 | 229.5884 |
| D4B | 0 | 158.1744 | 170.7465 | 188.2918 | 194.1559 | 193.4521 | 196.2671 | 200.3489 | 205.2277 | 209.4971 | 213.2976 |
| G04-01 | 0 | 5.6613 | 2.6917 | -4.5478 | -12.5759 | -17.2628 | -19.9076 | -21.811 | -20.511 | -19.026 | -14.618 |
| G04-02 | 0 | 11.4576 | 19.9811 | 29.3899 | 34.2341 | 40.2892 | 48.3012 | 56.1265 | 66.5609 | 76.0633 | 87.4765 |
| G11-03 | 0 | 25.1723 | 49.2238 | 72.0631 | 93.1746 | 114.1945 | 135.5414 | 156.8893 | 179.1731 | 200.5698 | 222.1075 |
| G04-04 | 0 | 39.7412 | 81.4894 | 126.6433 | 168.2585 | 207.4071 | 245.6737 | 283.1503 | 320.9566 | 356.3875 | 390.6088 |
| G04-05 | 0 | 40.4773 | 112.6819 | 177.5135 | 45.7826 | 63.7908 | 106.9579 | 96.3183 | 106.8581 | 121.471 | 135.3185 |
| G05-06 | 0 | 38.7145 | 79.1088 | 121.9758 | 163.4488 | 202.2683 | 241.1374 | 278.9842 | 316.7878 | 352.6356 | 386.9478 |
| G05-07 | 0 | 24.8969 | 47.9189 | 73.8031 | 98.0938 | 121.2135 | 144.7563 | 167.878 | 192.9705 | 215.2969 | 238.61 |
| G05-08 | 0 | 13.8719 | 23.7266 | 33.5823 | 44.6522 | 55.6287 | 67.0263 | 78.7508 | 92.344 | 104.9102 | 119.2988 |
| G05-09 | 0 | 5.3077 | 4.4697 | 2.5609 | 2.4678 | 2.8868 | 5.1679 | 7.6356 | 13.1765 | 18.81 | 26.0271 |
| G05-10 | 0 | -8.5629 | -19.9173 | -29.0847 | -39.3218 | -50.583 | -60.5408 | -70.1262 | -77.3849 | -85.6205 | 90.2733 |
| G06-11 | 0 | 16.0459 | 20.9296 | 34.2794 | 35.3954 | 35.4881 | 36.837 | 40.6979 | 45.5358 | 47.7684 | 9 |
| G06-12 | 0 | 14.7358 | 27.2088 | 40.4672 | 50.6307 | 59.7321 | 68.8797 | 78.9051 | 89.9007 | 100.4815 | 111.4312 |
| G06-13 | 0 | 32.8865 | 65.7745 | 97.409 | 126.6726 | 154.1235 | 181.25 | 208.2847 | 236.0192 | 262.9174 | 287.2574 |
| G06-14 | 0 | 39.7361 | 75.3335 | 107.2573 | 138.7643 | 167.9463 | 196.8037 | 225.8957 | 254.6634 | 282.8277 | 307.8278 |
| G06-15 | 0 | 35.2562 | 69.0728 | 105.9154 | 137.6901 | 165.4188 | 193.6142 | 221.532 | 250.1493 | 271.974 | 299.3846 |
| G07-16 | 0 | 14.0154 | 27.4723 | 40.7431 | 52.6175 | 62.5829 | 73.1073 | 84.7029 | 97.1372 | 108.1749 | 120.9822 |
| G07-17 | 0 | 0.9315 | 0.1864 | -0.9782 | -3.0279 | -6.8941 | -9.316 | -10.8065 | -9.9216 | -10.1545 | -6.7077 |
| G07-18 | 0 | -12.4021 | -26.1507 | -40.2242 | -55.412 | -72.039 | -86.3898 | -99.3005 | -109.982 | -120.988 | -126.468 |
| G07-19 | 0 | 2.5585 | -5.5365 | -16.0965 | -24.005 | -29.0756 | -32.7509 | -34.1928 | -33.495 | -31.4946 | -27.9127 |
| G07-20 | 0 | 10.5685 | 17.5215 | 18.1705 | 23.2233 | 29.3418 | 36.3414 | 44.9639 | 54.8378 | 64.8975 | 75.8846 |
| G07-21 | 0 | 23.4353 | 45.2872 | 66.0222 | 85.733 | 104.139 | 123.3392 | 142.819 | 163.0444 | 183.3184 | 203.1264 |
| G08-22 | 0 | 37.6499 | 76.8345 | 116.8116 | 154.6092 | 188.3696 | 222.1321 | 255.4329 | 289.0143 | 320.9255 | 352.1883 |
| G08-23 | 0 | 46.0509 | 99.293 | 138.9596 | 177.789 | 212.9812 | 248.1288 | 282.8125 | 317.9186 | 351.0662 | 385.7098 |
| G08-24 | 0 | 38.1453 | 78.5235 | 119.9736 | 158.964 | 194.3322 | 229.6568 | 264.426 | 299.29 | 332.762 | 365.0745 |
| G08-25 | 0 | 23.826 | 46.8154 | 68.9219 | 91.2617 | 111.2289 | 132.3604 | 153.7721 | 176.6282 | 198.0416 | 219.5957 |
| G09-26 | 0 | 13.4224 | 22.3398 | 29.2605 | 37.2956 | 45.6563 | 55.6431 | 66.6052 | 78.9609 | 91.2706 | 104.324 |
| G09-27 | 0 | 2.5645 | -1.9584 | -11.7964 | -17.5779 | -20.5618 | -21.5876 | -21.2612 | -18.557 | -15.1066 | -10.1179 |
| G09-28 | 0 | 2.8493 | 2.5225 | -5.2318 | -12.1454 | -16.209 | -19.5722 | -23.5425 | -24.8508 | -26.8589 | -25.7848 |
| G09-29 | 0 | 36.5604 | 51.7424 | 65.8016 | 74.5892 | 79.7639 | 81.7169 | 93.922 | 103.3938 | 111.9376 | 130.3444 |
| G09-30 | 0 | 18.3352 | 34.8094 | 51.2377 | 67.3874 | 83.3048 | 99.6415 | 115.56 | 133.015 | 148.3294 | 164.6682 |
| G10-31 | 0 | 30.049 | 57.4438 | 82.93 | 108.79 | 132.8346 | 156.554 | 180.1812 | 204.089 | 225.6202 | 247.0136 |
| G10-32 | 0 | 24.8405 | 49.8677 | 78.3395 | 106.6265 | 132.2631 | 158.4595 | 183.7729 | 209.2272 | 232.8213 | 255.672 |
| G10-33 | 0 | 22.0808 | 43.1353 | 66.0118 | 88.0488 | 107.6587 | 127.3159 | 146.4609 | 166.3531 | 183.8646 | 201.5639 |
| G10-34 | 0 | 16.7335 | 32.4422 | 47.3121 | 62.4622 | 76.9136 | 91.6452 | 105.7247 | 121.0166 | 134.8166 | 149.2236 |
| G10-35 | 0 | 11.8578 | 21.475 | 28.3379 | 35.1541 | 42.4841 | 50.0943 | 57.7045 | 67.1826 | 75.4471 | 84.7852 |
| G11-36 | 0 | 6.5483 | 7.8113 | 4.2096 | 0.7482 | -0.8885 | -2.0578 | -3.7884 | -2.2451 | -2.0112 | 1.4499 |


| N = 0,300,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.469 | 17.155 | 48.9897 | 86.3899 | 118.1352 | 146.4694 | 173.4494 | 197.7649 | 221.7079 | 238.7772 |
| D1B | 0 | 0.8854 | 1.864 | 2.8892 | 3.5882 | 4.8464 | 7.0366 | 8.854 | 10.1588 | 11.184 | 12.7218 |
| D2A | 0 | 3.3109 | 15.1568 | 48.8303 | 94.6801 | 155.7884 | 217.9774 | 278.2143 | 333.4652 | 386.2949 | 435.0693 |
| D2B | 0 | -16.4967 | -14.3937 | -17.8985 | 2.7575 | -1.6356 | 13.2261 | 3.5052 | 3.879 | 3.178 | 2.1499 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 7.1483 | 10.8858 | 15.6517 | 20.5574 | 26.7251 | 32.7058 | 39.1539 | 44.1532 | 48.6858 | 52.9845 |
| D4A | 0 | 1.1712 | 8.338 | 38.4108 | 77.7144 | 125.3608 | 172.4963 | 218.4177 | 260.5946 | 300.6655 | 337.8334 |
| D4B | 0 | 157.0125 | 167.4677 | 174.2197 | 180.9248 | 186.8796 | 194.7099 | 199.0706 | 203.6658 | 207.1359 | 210.9338 |
| G04-01 | 0 | 0.3717 | -6.9615 | -20.6055 | -33.4609 | -44.3195 | -51.9765 | -56.7103 | -59.1695 | -58.8911 | -56.4319 |
| G04-02 | 0 | 10.2503 | 17.239 | 19.8952 | 23.0631 | 27.723 | 33.1276 | 39.6972 | 48.2241 | 58.1964 | 68.8202 |
| G11-03 | 0 | 24.4718 | 47.637 | 69.7758 | 91.5882 | 112.9813 | 134.002 | 154.8365 | 176.7926 | 198.4234 | 220.4286 |
| G04-04 | 0 | 40.4134 | 84.7934 | 131.974 | 177.341 | 221.1264 | 262.0244 | 301.4797 | 340.4717 | 377.5543 | 413.5667 |
| G04-05 | 0 | 71.4591 | 123.0366 | 179.9728 | 230.4249 | 274.391 | 321.3461 | 364.1356 | 406.1703 | 446.0283 | 483.4723 |
| G05-06 | 0 | 39.1742 | 82.7312 | 129.7396 | 175.4472 | 219.4356 | 260.5378 | 300.5712 | 340.0025 | 377.805 | 414.1188 |
| G05-07 | 0 | 23.4451 | 47.173 | 71.0894 | 96.4134 | 120.8014 | 143.971 | 167.3288 | 191.1099 | 214.8928 | 238.3008 |
| G05-08 | 0 | 12.2364 | 20.6901 | 26.4342 | 34.561 | 41.8939 | 49.975 | 59.3627 | 70.1525 | 81.737 | 94.7693 |
| G05-09 | 0 | 2.2347 | -2.1885 | -12.1521 | -18.2512 | -24.1176 | -28.82 | -30.7755 | -30.3564 | -27.1905 | -22.0693 |
| G05-10 | 0 | -9.7699 | -21.493 | -33.2162 | -43.5438 | -55.2199 | -66.9419 | -77.7803 | -86.8972 | -94.6184 | -100.758 |
| G06-11 | 0 | 2.002 | 2.3742 | 2.6071 | 2.7001 | 0.9775 | -1.7693 | -4.1437 | -4.0039 | -2.8403 | -0.6518 |
| G06-12 | 0 | 14.8729 | 28.3608 | 40.7862 | 52.6117 | 63.6521 | 72.6139 | 82.0839 | 92.9871 | 103.6121 | 114.9311 |
| G06-13 | 0 | 33.0698 | 66.1883 | 99.3088 | 130.3379 | 160.6247 | 189.0991 | 216.644 | 244.3768 | 270.808 | 295.984 |
| G06-14 | 0 | 35.253 | 69.531 | 104.3704 | 136.467 | 168.4729 | 198.201 | 226.9072 | 255.3827 | 282.8357 | 309.6394 |
| G06-15 | 0 | 32.4208 | 63.7264 | 96.7108 | 127.4142 | 155.7901 | 184.0743 | 210.6353 | 238.1769 | 263.6225 | 288.8367 |
| G07-16 | 0 | 13.6886 | 27.1448 | 39.577 | 52.5683 | 63.2783 | 73.0107 | 83.2552 | 94.804 | 106.7256 | 118.6474 |
| G07-17 | 0 | 0.3728 | 0.5126 | -2.7028 | -3.355 | -7.1754 | -11.5558 | -14.2576 | -14.8171 | -13.8385 | -11.8354 |
| G07-18 | 0 | -13.1904 | -26.2415 | -43.8899 | -56.8008 | -74.3551 | -91.259 | -106.676 | -118.935 | -128.501 | -135.28 |
| G07-19 | 0 | 5.8166 | -2.1399 | -12.6549 | -28.5208 | -40.0125 | -42.1991 | -44.5253 | -44.5254 | -43.083 | -38.2445 |
| G07-20 | 0 | 10.1966 | 16.9168 | 19.6978 | 26.3721 | 29.9876 | 37.3108 | 44.4491 | 54.8782 | 64.7048 | 75.4129 |
| G07-21 | 0 | 23.295 | 44.3081 | 63.7839 | 84.8451 | 104.5097 | 122.8231 | 141.1845 | 161.1769 | 181.4965 | 201.4443 |
| G08-22 | 0 | 37.5029 | 76.8659 | 117.0208 | 155.6936 | 191.6765 | 224.4112 | 257.0087 | 289.748 | 321.5138 | 352.7706 |
| G08-23 | 0 | 31.3768 | 71.6616 | 114.7005 | 156.904 | 192.4418 | 215.0619 | 249.7638 | 285.6343 | 319.0819 | 374.0862 |
| G08-24 | 0 | 38.2268 | 79.1045 | 121.1469 | 162.0776 | 199.3876 | 234.0978 | 268.5781 | 302.643 | 335.873 | 369.1059 |
| G08-25 | 0 | 23.7322 | 46.7673 | 68.9191 | 91.7237 | 112.9932 | 133.3795 | 154.5579 | 176.4352 | 197.7084 | 219.4946 |
| G09-26 | 0 | 12.5886 | 21.6468 | 27.918 | 35.9546 | 44.7812 | 53.7936 | 64.2465 | 76.1396 | 88.3121 | 101.4138 |
| G09-27 | 0 | 2.0052 | -3.4976 | -14.2225 | -21.0307 | -24.8076 | -26.813 | -26.9993 | -24.9011 | -21.4038 | -16.7874 |
| G09-28 | 0 | 1.6813 | 1.0741 | -7.8485 | -14.9489 | -19.7134 | -23.7307 | -27.3744 | -29.0097 | -29.7102 | -29.5234 |
| G09-29 | 0 | 2.1647 | 7.5309 | 9.508 | 11.9556 | 15.7214 | 18.828 | 24.1944 | 29.9373 | 36.3393 | 41.7061 |
| G09-30 | 0 | 17.3533 | 34.6145 | 50.713 | 67.2306 | 84.2143 | 100.3609 | 116.6474 | 132.6555 | 148.6642 | 164.9525 |
| G10-31 | 0 | 28.9281 | 55.9479 | 82.6429 | 109.5251 | 134.5458 | 158.1695 | 181.0491 | 204.2558 | 226.0647 | 247.2231 |
| G10-32 | 0 | 24.0951 | 49.5872 | 78.2439 | 106.3906 | 133.9336 | 159.0587 | 184.5111 | 209.1273 | 232.9997 | 255.9894 |
| G10-33 | 0 | 21.4701 | 43.2216 | 65.2539 | 87.8008 | 108.0612 | 127.2485 | 146.3431 | 165.5789 | 183.5077 | 200.5968 |
| G10-34 | 0 | 15.754 | 31.5086 | 46.2382 | 61.4342 | 75.9315 | 90.4291 | 104.881 | 119.333 | 133.319 | 147.7252 |
| G10-35 | 0 | 12.0456 | 21.1039 | 27.314 | 34.6444 | 41.6953 | 48.326 | 56.1239 | 63.5482 | 72.2801 | 81.7135 |
| G11-36 | 0 | 5.6603 | 7.625 | 2.2918 | -1.6375 | -3.7892 | -5.0524 | -6.83 | -6.3622 | -4.8187 | -2.6199 |


| N = 3/16 Link Slab Restart |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.4192 | 19.7163 | 45.5545 | 71.7679 | 100.2258 | 132.1435 | 160.8851 | 189.9556 | 215.4285 | 240.5292 |
| D1B | 0 | 2.5169 | 3.8226 | 4.8945 | 6.4326 | 7.4116 | 9.3226 | 10.7214 | 12.8655 | 14.4039 | 16.5014 |
| D2A | 0 | 8.3921 | 28.5797 | 59.3992 | 105.5153 | 157.6512 | 217.4423 | 272.7624 | 329.1613 | 379.3145 | 428.3999 |
| D2B | 0 | 16.1902 | 19.0293 | 104.7963 | 113.8255 | 121.924 | 122.0636 | 127.4632 | 138.4955 | 149.6212 | 233.6578 |
| D3A | 0 | -838.345 | -205.878 | 46433.07 | 46433.07 | 46433.07 | 46433.07 | 46433.07 | 46433.07 | 46433.07 | 46433.07 |
| D3B | 0 | 16.1586 | 26.5735 | 33.3919 | 50.813 | 56.371 | 57.0246 | 65.5253 | 82.2005 | 80.8453 | 90.8409 |
| D4A | 0 | -115.214 | -143.869 | -130.294 | -86.2397 | -38.857 | 7.4531 | 51.0042 | 95.0746 | 133.0595 | 174.5601 |
| D4B | 0 | 436.5933 | 60.2963 | 1197.459 | 443.1239 | 959.8226 | 697.4486 | 159.9857 | 60.9232 | 51.0196 | 1030.724 |
| G04-01 | 0 | 2.2282 | -4.873 | -10.5355 | -15.548 | -19.2141 | -23.8088 | -26.3153 | -27.3825 | -27.6145 | -30.2136 |
| G04-02 | 0 | 10.4763 | 18.8108 | 25.795 | 33.4313 | 41.2073 | 49.7286 | 58.7622 | 68.1684 | 78.3201 | 88.6115 |
| G11-03 | 0 | 24.7957 | 49.2198 | 72.5708 | 96.2954 | 119.555 | 142.6288 | 166.0772 | 188.7327 | 211.5293 | 234.1868 |
| G04-04 | 0 | 39.0515 | 81.5977 | 124.1008 | 166.6541 | 208.1868 | 249.2572 | 289.1663 | 328.753 | 366.7127 | 403.7896 |
| G04-05 | 0 | 78.4619 | 120.384 | 256.2339 | -523.929 | 140.4106 | 11419.39 | 3269.838 | 41989.73 | 50739.68 | 50653.11 |
| G05-06 | 0 | 38.4308 | 80.2185 | 122.429 | 164.3636 | 205.4628 | 245.7738 | 285.4348 | 324.2135 | 362.296 | 399.216 |
| G05-07 | 0 | 22.877 | 46.8341 | 71.2607 | 95.0325 | 118.4765 | 142.0627 | 165.5561 | 189.0036 | 212.4995 | 235.9495 |
| G05-08 | 0 | 12.7968 | 23.5855 | 33.627 | 44.0421 | 54.7851 | 65.2937 | 76.9248 | 88.4151 | 100.9804 | 113.032 |
| G05-09 | 0 | 3.9107 | 2.9797 | 1.2105 | 0.0001 | 0.233 | 0.4658 | 2.6073 | 4.7022 | 8.8459 | 13.0363 |
| G05-10 | 0 | -5.3024 | -12.3259 | -17.6279 | -23.3024 | -26.6044 | -31.2089 | -34.9299 | -38.8826 | -41.3014 | 45.2079 |
| G06-11 | 0 | -10.8386 | -12.4201 | -9.9086 | -11.8624 | -10.5598 | -9.5362 | -8.2808 | -7.5366 | -4.7456 | 1 |
| G06-12 | 0 | 17.5973 | 27.805 | 32.2391 | 46.1424 | 58.5215 | 71.4093 | 83.6041 | 95.6608 | 108.0876 | 120.191 |
| G06-13 | 0 | 30.416 | 61.2995 | 91.1149 | 120.3735 | 146.331 | 173.4993 | 199.3665 | 225.6073 | 250.2675 | 276.8367 |
| G06-14 | 0 | 57.7772 | 86.7198 | 60.769 | 80.5707 | 148.5971 | 201.3785 | 255.0449 | 314.7572 | 366.9602 | 420.8391 |
| G06-15 | 0 | 31.0736 | 63.3591 | 93.8319 | 123.2829 | 149.7111 | 177.1185 | 203.1307 | 229.889 | 255.0664 | 281.4088 |
| G07-16 | 0 | 12.7607 | 25.8943 | 38.7488 | 50.8121 | 63.5738 | 75.0785 | 86.9561 | 98.6944 | 110.9451 | 122.544 |
| G07-17 | 0 | 0.6524 | -0.3261 | 0.3262 | 0.0933 | 1.864 | 2.1434 | 3.5876 | 3.6809 | 5.9636 | 6.1031 |
| G07-18 | 0 | -11.1055 | -24.9987 | -35.732 | -47.348 | -54.5959 | -65.2821 | -72.2978 | -82.7977 | -89.488 | -99.012 |
| G07-19 | 0 | -14.8806 | -19.484 | -33.6195 | -45.4295 | -57.7505 | -66.3988 | -69.514 | -71.7455 | -73.2335 | -72.0245 |
| G07-20 | 0 | 6.9054 | 12.3281 | 5.7467 | 10.1034 | 16.1749 | 23.0346 | 31.1451 | 40.786 | 49.9632 | 62.6175 |
| G07-21 | 0 | 21.435 | 40.7284 | 63.0974 | 83.5102 | 103.2716 | 123.4068 | 143.4022 | 163.4454 | 184.2823 | 205.8655 |
| G08-22 | 0 | 36.0652 | 75.1505 | 116.8387 | 155.9301 | 193.3069 | 229.2002 | 264.0744 | 297.4187 | 330.8583 | 362.9526 |
| G08-23 | 0 | 36.8572 | 81.5044 | 123.2293 | 162.7863 | 202.1576 | 239.6901 | 274.2987 | 307.3983 | 341.7278 | 381.0657 |
| G08-24 | 0 | 21.3108 | -122.572 | -84.1365 | -44.1619 | -6.0458 | 30.3042 | 66.3776 | 100.9177 | 135.2736 | 168.6546 |
| G08-25 | 0 | 23.5408 | 46.7575 | 69.2307 | 91.5648 | 113.2488 | 135.0736 | 157.1315 | 178.3531 | 200.2735 | 223.1723 |
| G09-26 | 0 | 12.3124 | 21.1872 | 26.4842 | 33.8719 | 42.3754 | 51.901 | 62.7746 | 74.1593 | 86.6598 | 99.5325 |
| G09-27 | 0 | 1.9121 | -2.1918 | -14.4549 | -21.9157 | -27.2313 | -29.003 | -29.1428 | -28.024 | -24.9932 | -20.8897 |
| G09-28 | 0 | 3.3633 | 3.8773 | -3.0363 | -9.903 | -15.088 | -18.7783 | -21.6747 | -24.7102 | -25.7382 | -25.785 |
| G09-29 | 0 | -87.849 | -115.304 | -109.468 | -143.13 | -138.508 | 568.5618 | 931.6751 | 926.5476 | 1346.745 | 50000.99 |
| G09-30 | 0 | 22.4555 | 46.17 | -62.541 | -41.7595 | -22 | -1.9145 | 17.3354 | 38.771 | 59.1845 | 78.9945 |
| G10-31 | 0 | 27.3399 | 53.4236 | 79.6951 | 107.4587 | 132.4289 | 157.4928 | 182.6984 | 207.0196 | 230.3637 | 251.9846 |
| G10-32 | 0 | 27.621 | 55.2435 | 81.8915 | 113.517 | 142.633 | 171.146 | 198.265 | 224.7806 | 250.9723 | 275.4435 |
| G10-33 | 0 | 21.1879 | 42.7035 | 65.4802 | 88.7247 | 110.3364 | 131.5291 | 152.489 | 172.9366 | 192.8244 | 211.546 |
| G10-34 | 0 | 21.6173 | 41.9309 | 62.0589 | 77.7609 | 93.6039 | 109.3075 | 125.2438 | 140.7621 | 156.327 | 171.1002 |
| G10-35 | 0 | 11.2517 | 21.8039 | 29.0404 | 35.1571 | 41.9743 | 49.0708 | 56.7752 | 64.6665 | 74.0054 | 82.457 |
| G11-36 | 0 | 6.0296 | 8.6471 | 6.0763 | 1.3086 | -1.2154 | -2.8981 | -3.9731 | -4.9079 | -3.6926 | -1.1687 |


| N = 0,400,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 6.5907 | 20.0526 | 46.6042 | 72.5025 | 96.9526 | 119.0666 | 141.6489 | 163.1569 | 180.1769 | 196.5429 |
| D1B | 0 | 0.3264 | 0.6526 | 1.5384 | 3.4034 | 5.3151 | 6.807 | 8.1125 | 9.8378 | 10.91 | 12.6817 |
| D2A | 0 | 4.5228 | 21.4022 | 54.3696 | 103.8014 | 162.7531 | 220.4521 | 280.7704 | 337.1765 | 387.243 | 437.1275 |
| D2B | 0 | 1.8153 | 3.3512 | 5.7255 | 9.4493 | 16.5712 | 20.2949 | 23.5532 | 27.7893 | 30.7685 | 33.7477 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 2.9916 | 5.9363 | 12.1531 | 19.5857 | 28.701 | 33.9366 | 43.4261 | 49.4566 | 57.0298 | 62.4526 |
| D4A | 0 | 11.2243 | 34.2356 | 77.9688 | 141.4937 | 230.8547 | 311.7604 | 388.7484 | 453.67 | 506.3814 | 556.0083 |
| D4B | 0 | 5.8701 | 10.1434 | 17.329 | 26.1113 | 33.3438 | 39.1675 | 44.8505 | 50.4865 | 55.8407 | 61.4296 |
| G04-01 | 0 | 0.8326 | -9.2993 | -24.4279 | -27.8517 | -123.926 | -126.608 | -129.939 | -131.002 | -130.262 | -106.583 |
| G04-02 | 0 | 11.0372 | 16.4861 | 19.1875 | 23.6116 | 28.5949 | 35.7672 | 43.5912 | 53.046 | 63.6653 | 74.4709 |
| G11-03 | 0 | 26.5174 | 51.2621 | 74.001 | 95.8993 | 117.0057 | 139.3263 | 161.6489 | 184.2517 | 206.5753 | 228.4339 |
| G04-04 | 0 | 41.2448 | 88.6384 | 136.9212 | 181.111 | 222.9759 | 264.5652 | 304.6208 | 343.7011 | 381.5268 | 417.4454 |
| G04-05 | 0 | 44.1124 | 89.413 | 138.6039 | 188.0365 | 234.5349 | 275.6337 | 318.2526 | 356.3712 | 395.4406 | 428.9652 |
| G05-06 | 0 | 39.839 | 86.8584 | 134.3478 | 178.8122 | 220.67 | 262.1119 | 302.5316 | 341.742 | 379.3234 | 415.229 |
| G05-07 | 0 | 24.4715 | 49.5999 | 75.0584 | 99.7205 | 123.3057 | 147.548 | 171.6516 | 195.6147 | 219.1573 | 242.0445 |
| G05-08 | 0 | 12.3786 | 20.0861 | 26.9998 | 35.6421 | 42.6026 | 52.6931 | 63.1578 | 74.7902 | 86.8905 | 99.8316 |
| G05-09 | 0 | 2.1421 | -3.8647 | -12.0603 | -16.8559 | -20.0224 | -21.4658 | -21.3261 | -19.5566 | -15.8781 | -10.2907 |
| G05-10 | 0 | -8.7007 | -18.8908 | -29.685 | -38.6646 | -47.5976 | -54.9019 | -62.2525 | -68.068 | -72.1154 | -77.3726 |
| G06-11 | 0 | 2.0945 | 2.7464 | 1.2102 | -0.0001 | -0.9776 | -0.0933 | 0.9309 | 3.0255 | 6.6099 | 8.984 |
| G06-12 | 0 | 15.0602 | 28.7354 | 40.1468 | 51.513 | 60.9383 | 72.2121 | 83.1625 | 94.5754 | 105.9883 | 117.0783 |
| G06-13 | 0 | 32.9385 | 66.8098 | 100.218 | 129.9525 | 157.1287 | 184.6329 | 212.9758 | 239.7843 | 264.7794 | 290.8926 |
| G06-14 | 0 | 31.9196 | 65.284 | 98.6972 | 128.2961 | 156.3143 | 185.4976 | 215.1016 | 242.8455 | 268.589 | 296.522 |
| G06-15 | 0 | 32.0567 | 65.8838 | 99.2011 | 128.5188 | 155.7437 | 183.1093 | 211.0821 | 238.0321 | 262.7958 | 288.864 |
| G07-16 | 0 | 14.1108 | 27.9893 | 40.8432 | 52.4867 | 63.6182 | 75.6349 | 87.6517 | 99.6688 | 111.5934 | 123.518 |
| G07-17 | 0 | 0.8387 | -0.2793 | -3.1679 | -5.3571 | -6.1959 | -6.4288 | -7.4072 | -7.0813 | -5.4039 | -4.7052 |
| G07-18 | 0 | -12.2241 | -27.1896 | -43.689 | -57.3055 | -70.0395 | -80.914 | -93.2288 | -103.638 | -110.887 | -119.81 |
| G07-19 | 0 | -0.233 | -7.3613 | -20.1271 | -35.5017 | -43.0954 | -45.6111 | -48.4064 | -49.4312 | -52.1334 | -48.8258 |
| G07-20 | 0 | 10.2869 | 16.9599 | 19.5088 | 19.3232 | 24.4208 | 31.9745 | 40.3157 | 48.9818 | 59.0848 | 69.3268 |
| G07-21 | 0 | 22.0418 | 43.1991 | 63.5183 | 84.9106 | 103.4139 | 125.2273 | 144.9906 | 165.221 | 186.1511 | 205.8236 |
| G08-22 | 0 | 37.3673 | 77.8946 | 119.075 | 158.1228 | 195.1771 | 230.5621 | 265.1604 | 299.0641 | 331.9486 | 363.3947 |
| G08-23 | 0 | 36.6472 | 77.7543 | 121.9468 | 163.8669 | 203.514 | 245.915 | 274.564 | 201.6285 | 201.8287 | 202.3135 |
| G08-24 | 0 | 37.5407 | 78.5693 | 122.3895 | 163.0066 | 200.4669 | 237.9761 | 273.5825 | 308.7265 | 342.6176 | 375.3021 |
| G08-25 | 0 | 23.1775 | 46.7282 | 69.3496 | 90.8081 | 111.4293 | 132.8897 | 154.6302 | 176.6043 | 198.3466 | 220.1831 |
| G09-26 | 0 | 12.402 | 21.7386 | 27.0805 | 33.073 | 40.041 | 48.8671 | 59.7374 | 71.2584 | 82.733 | 95.6947 |
| G09-27 | 0 | 2.4244 | -1.3988 | -14.0344 | -23.3592 | -29.3273 | -33.0104 | -32.9638 | -31.3787 | -28.9078 | -24.8047 |
| G09-28 | 0 | 2.0091 | 0.0937 | -9.2976 | -17.4268 | -23.4537 | -28.92 | -32.3768 | -35.6938 | -36.9554 | -36.3483 |
| G09-29 | 0 | 11.8424 | 21.2602 | 26.4355 | 31.984 | 37.8589 | 44.8531 | 52.9195 | 60.8931 | 69.7989 | 79.1715 |
| G09-30 | 0 | 17.3476 | 33.952 | 49.9519 | 65.6731 | 81.9533 | 98.606 | 115.3991 | 131.9601 | 148.8469 | 164.8506 |
| G10-31 | 0 | 28.418 | 56.1385 | 83.0222 | 109.1152 | 134.2311 | 159.6737 | 184.9319 | 209.2596 | 233.1684 | 254.9345 |
| G10-32 | 0 | 25.0231 | 50.513 | 79.539 | 108.0552 | 135.2703 | 163.0453 | 189.4261 | 215.8546 | 240.563 | 263.7366 |
| G10-33 | 0 | 21.3329 | 43.1805 | 66.1025 | 89.6795 | 110.3161 | 131.42 | 152.1518 | 172.3237 | 191.0489 | 209.0746 |
| G10-34 | 0 | 15.8906 | 31.8744 | 47.3922 | 62.4451 | 77.2192 | 92.4128 | 107.3739 | 122.4282 | 137.2039 | 151.6066 |
| G10-35 | 0 | 11.0154 | 21.1908 | 28.0522 | 34.3538 | 40.7489 | 48.6843 | 56.4798 | 64.2754 | 74.312 | 83.4616 |
| G11-36 | 0 | 5.0006 | 7.4777 | 3.4585 | -1.4489 | -5.0003 | -7.6175 | -9.4403 | -10.0009 | -9.4868 | -7.4306 |


| N = 0,500,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 6.2626 | 20.0024 | 47.717 | 70.5252 | 97.2147 | 118.1559 | 140.5 | 162.3311 | 187.5294 | 207.7258 |
| D1B | 0 | 0.1397 | 0.4197 | 1.8654 | 2.8449 | 3.9646 | 6.4363 | 5.83 | 6.5297 | 7.6024 | 8.6285 |
| D2A | 0 | 4.429 | 19.9543 | 54.7374 | 96.2839 | 157.3748 | 214.3216 | 270.2018 | 322.543 | 377.2224 | 422.4819 |
| D2B | 0 | -6.5813 | -8.4477 | -9.7074 | 157.8622 | 77.0982 | 129.226 | 108.5452 | 134.1226 | 130.4338 | 191.171 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | -0.4672 | 4.7658 | 11.8682 | 19.8117 | 28.082 | 35.5586 | 41.1658 | 47.6142 | 54.3433 | 59.8573 |
| D4A | 0 | 10.804 | 34.4233 | 78.0642 | 133.1707 | 224.2632 | 307.089 | 385.8556 | 450.2642 | 518.5667 | 568.1491 |
| D4B | 0 | 10.7051 | 20.8948 | 27.281 | 38.222 | 44.9374 | 51.7935 | 55.2214 | 59.2131 | 63.064 | 66.7266 |
| G04-01 | 0 | 0.0927 | -12.0009 | -26.225 | -58.9349 | -70.3788 | -76.2162 | -83.4898 | -85.065 | -85.8526 | -82.9338 |
| G04-02 | 0 | 9.0783 | 12.7099 | 15.5966 | 18.7623 | 21.8815 | 35.5234 | 37.6649 | 45.9529 | 54.3338 | 64.7636 |
| G11-03 | 0 | 22.8413 | 44.6552 | 69.3667 | 91.4168 | 114.729 | 139.3973 | 156.5438 | 177.1025 | 197.5684 | 218.5023 |
| G04-04 | 0 | 41.3025 | 88.9889 | 139.5675 | 183.6768 | 226.5784 | 277.637 | 313.6508 | 353.0223 | 389.8339 | 424.8779 |
| G04-05 | 0 | 17.6995 | 47.3376 | 83.1764 | 2772.588 | 1829.752 | 1472.185 | 1302.894 | 1189.475 | 1138.198 | 1105.571 |
| G05-06 | 0 | 39.6952 | 88.7595 | 141.6499 | 190.1181 | 231.5995 | 274.7624 | 316.8098 | 354.572 | 394.2952 | 434.5809 |
| G05-07 | 0 | 24.0512 | 48.7596 | 76.2364 | 104.4173 | 126.6913 | 159.4245 | 177.5274 | 201.0702 | 224.0522 | 247.5045 |
| G05-08 | 0 | 11.1648 | 17.6581 | 25.8331 | 32.5134 | 38.4933 | 48.5376 | 54.7977 | 65.9634 | 77.3165 | 90.8187 |
| G05-09 | 0 | -0.419 | -9.4536 | -17.1377 | -23.4246 | -29.6646 | -30.8286 | -33.8557 | -31.6669 | -28.64 | -22.3069 |
| G05-10 | 0 | -10.6094 | -22.4746 | -34.6654 | -43.5987 | -56.0218 | -64.8151 | -78.5404 | -85.6121 | -94.0328 | -99.2432 |
| G06-11 | 0 | 1.0227 | 1.2082 | -0.2326 | -1.7662 | -5.6704 | -7.9942 | -12.4561 | -11.1546 | -11.666 | -8.9238 |
| G06-12 | 0 | 16.3074 | 30.4038 | 44.2707 | 57.7691 | 67.0753 | 75.829 | 83.5233 | 95.4102 | 105.3624 | 116.1901 |
| G06-13 | 0 | 32.755 | 66.3967 | 102.4138 | 133.2209 | 161.1912 | 184.2756 | 208.6182 | 234.4048 | 260.5187 | 284.1203 |
| G06-14 | 0 | 32.6785 | 66.942 | 103.6289 | 136.0818 | 165.4166 | 195.1723 | 221.3902 | 249.1465 | 277.51 | 302.7543 |
| G06-15 | 0 | 29.7856 | 59.0577 | 93.0158 | 121.6824 | 148.0556 | 169.7454 | 192.7009 | 217.3442 | 243.9568 | 266.5409 |
| G07-16 | 0 | 13.1417 | 26.7965 | 40.5916 | 52.9885 | 63.1022 | 75.1736 | 84.1694 | 96.5211 | 107.661 | 119.9666 |
| G07-17 | 0 | -0.7467 | -1.5402 | -6.6736 | -11.2937 | -20.4871 | -24.6403 | -31.36 | -35.5133 | -39.1066 | -38.1735 |
| G07-18 | 0 | -14.965 | -30.0689 | -50.191 | -60.647 | -79.1417 | -91.9205 | -111.157 | -124.214 | -139.129 | -148.096 |
| G07-19 | 0 | -1.6788 | -9.3264 | -22.5229 | -32.8749 | -43.2734 | -51.0605 | -58.8473 | -60.7589 | -60.2925 | -58.008 |
| G07-20 | 0 | 9.8755 | 14.0479 | 19.5656 | 28.3288 | 36.7673 | 53.1815 | 57.9572 | 61.8522 | 70.9869 | 81.374 |
| G07-21 | 0 | 21.4465 | 42.8471 | 63.8291 | 84.0658 | 101.599 | 121.4644 | 138.2057 | 157.4662 | 177.6135 | 197.1087 |
| G08-22 | 0 | 38.4895 | 78.468 | 121.6538 | 162.7536 | 197.7256 | 232.8394 | 264.6116 | 297.268 | 329.4623 | 360.2646 |
| G08-23 | 0 | 37.323 | 78.2583 | 123.709 | 168.0705 | 203.4585 | 241.129 | 270.6309 | 304.4107 | 338.9525 | 371.0262 |
| G08-24 | 0 | 37.9184 | 79.5109 | 124.6859 | 167.3081 | 203.9379 | 240.7559 | 273.9035 | 308.4019 | 341.6004 | 373.5459 |
| G08-25 | 0 | 23.5451 | 46.858 | 70.8242 | 92.7899 | 112.1503 | 133.4199 | 151.3856 | 172.7965 | 194.115 | 215.4814 |
| G09-26 | 0 | 11.6595 | 20.2066 | 25.688 | 31.1695 | 36.8371 | 45.3379 | 53.3284 | 63.874 | 75.5812 | 88.0321 |
| G09-27 | 0 | 0.3264 | -6.0595 | -18.5053 | -29.5524 | -38.0355 | -41.2511 | -45.0737 | -44.561 | -41.4845 | -38.1749 |
| G09-28 | 0 | 2.3359 | 0.6071 | -8.1755 | -17.9394 | -25.8811 | -28.5904 | -36.9992 | -39.5217 | -40.5962 | -39.5684 |
| G09-29 | 0 | 19.9639 | 44.3149 | 49.7477 | 58.8967 | 64.4688 | -189.89 | -182.033 | -169.7 | -164.319 | -145.366 |
| G09-30 | 0 | 12.0355 | 9.5175 | 12.878 | 21.4157 | 32.6586 | 139.2621 | 149.3408 | 163.9461 | 178.3651 | 192.1779 |
| G10-31 | 0 | 28.2849 | 54.7551 | 82.7637 | 110.4012 | 133.7055 | 158.829 | 179.5717 | 202.0868 | 223.7171 | 244.0431 |
| G10-32 | 0 | 23.877 | 50.8272 | 80.4321 | 110.7839 | 137.2267 | 163.9503 | 187.4159 | 213.0246 | 237.8896 | 260.0549 |
| G10-33 | 0 | 20.0333 | 41.095 | 63.979 | 86.8176 | 105.7336 | 125.8178 | 141.5592 | 159.9168 | 177.5277 | 194.1112 |
| G10-34 | 0 | 15.5255 | 31.3311 | 48.1629 | 63.2701 | 76.2796 | 88.2638 | 99.1754 | 113.8179 | 128.554 | 142.3577 |
| G10-35 | 0 | 10.8299 | 20.4456 | 27.3547 | 32.676 | 37.5313 | 45.2337 | 49.108 | 56.5779 | 65.3077 | 73.8976 |
| G11-36 | 0 | 4.16 | 5.3754 | 2.1037 | -5.1884 | -11.4055 | -13.7893 | -21.1746 | -22.9509 | -22.6703 | -21.7355 |


| N = 0,600,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.605 | 19.2429 | 43.7649 | 69.9232 | 101.0813 | 134.1093 | 162.2813 | 193.4449 | 219.5639 | 247.1328 |
| D1B | 0 | 0.2799 | 0.7928 | 1.306 | 2.5656 | 4.5244 | 6.6236 | 7.6498 | 9.0025 | 10.4485 | 11.5213 |
| D2A | 0 | 4.8961 | 21.5433 | 53.9073 | 100.6846 | 159.9202 | 224.0144 | 278.9721 | 337.5757 | 388.1126 | 438.4213 |
| D2B | 0 | -7.2789 | -12.9244 | -14.6509 | -22.8155 | -23.5621 | -24.402 | -24.2155 | -21.323 | -24.3555 | -25.2419 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 4.2055 | 8.6908 | 16.2598 | 19.811 | 29.1558 | 38.3608 | 46.8186 | 55.65 | 60.4162 | 66.6313 |
| D4A | 0 | 10.9886 | 36.1464 | 83.4728 | 152.7866 | 250.2258 | 360.458 | 447.7813 | 538.2096 | 604.4025 | 670.4171 |
| D4B | 0 | 6.0574 | 10.7531 | 17.0454 | 18.7357 | 25.3101 | 31.227 | 37.6133 | 44.2347 | 46.8176 | 51.7015 |
| G04-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 55650.23 |
| G04-02 | 0 | 9.6876 | 12.8544 | 13.8327 | 15.0436 | 17.3258 | 21.6111 | 27.8983 | 35.7704 | 45.2254 | 55.7986 |
| G11-03 | 0 | 25.0262 | 49.26 | 72.2339 | 93.621 | 113.8421 | 135.0444 | 157.1817 | 178.8995 | 200.6187 | 222.3382 |
| G04-04 | 0 | 43.4067 | 93.5246 | 144.1134 | 191.5855 | 235.4745 | 278.2493 | 319.7689 | 359.7547 | 397.5062 | 434.468 |
| G04-05 | 0 | 42.9189 | 94.4739 | 146.223 | 194.2027 | 238.7424 | 282.1064 | 324.8139 | 365.9196 | 404.7165 | 441.1558 |
| G05-06 | 0 | 41.8844 | 91.4138 | 141.2276 | 187.551 | 231.5018 | 273.6848 | 314.0534 | 354.7518 | 392.1897 | 428.4646 |
| G05-07 | 0 | 25.1291 | 50.5878 | 76.3758 | 101.4154 | 125.0959 | 148.7774 | 172.7413 | 196.3789 | 219.8758 | 243.1402 |
| G05-08 | 0 | 11.4465 | 17.8939 | 23.9681 | 31.2569 | 38.7791 | 46.8628 | 57.3756 | 67.7021 | 80.0847 | 93.1688 |
| G05-09 | 0 | 0.3726 | -9.0815 | -19.2809 | -26.6856 | -32.3208 | -36.9776 | -37.6296 | -36.9776 | -33.1122 | -27.617 |
| G05-10 | 0 | -10.3796 | -21.9695 | -34.5827 | -46.2183 | -57.2021 | -68.7904 | -75.5854 | -84.0087 | -88.8951 | -93.5022 |
| G06-11 | 0 | 1.7193 | 0.3249 | -0.372 | -3.2063 | -8.3177 | -12.7779 | -10.1294 | -8.6427 | -4.7862 | 1.8125 |
| G06-12 | 0 | 15.041 | 28.7025 | 40.9845 | 52.3005 | 61.915 | 71.1155 | 82.248 | 92.921 | 104.469 | 115.326 |
| G06-13 | 0 | 33.5444 | 67.6033 | 102.0368 | 133.4941 | 161.7425 | 190.6436 | 217.8711 | 246.4965 | 272.0974 | 298.8175 |
| G06-14 | 0 | 34.7236 | 69.2164 | 105.2479 | 139.2334 | 169.3102 | 200.9722 | 230.075 | 260.2038 | 287.3075 | 315.3905 |
| G06-15 | 0 | -10.9418 | -0.863 | 14.8019 | 37.2369 | 53.7087 | 81.0215 | 102.276 | 129.8285 | 144.4574 | 156.2463 |
| G07-16 | 0 | 14.5821 | 28.3261 | 41.4644 | 54.1375 | 65.1331 | 75.8967 | 88.151 | 99.4271 | 111.2165 | 123.0985 |
| G07-17 | 0 | -0.7939 | -2.7553 | -6.9583 | -11.1613 | -15.5511 | -21.6211 | -23.0221 | -26.9449 | -27.3185 | -28.5794 |
| G07-18 | 0 | -14.8692 | -31.3177 | -51.3895 | -70.4385 | -88.0463 | -109.138 | -123.261 | -141.053 | -152.387 | -165.022 |
| G07-19 | 0 | -0.9865 | -10.0054 | -24.755 | -35.8404 | -45.1875 | -52.1392 | -55.5679 | -56.8358 | -56.1785 | -48.6162 |
| G07-20 | 0 | 8.9088 | 13.3168 | 14.5696 | 16.2864 | 19.1178 | 23.897 | 31.1818 | 39.674 | 49.0478 | 84.4567 |
| G07-21 | 0 | 21.7755 | 42.0128 | 61.5984 | 80.6252 | 98.5336 | 116.9084 | 136.5437 | 155.8066 | 175.8161 | 196.6197 |
| G08-22 | 0 | 37.7509 | 78.1985 | 121.2959 | 161.9358 | 198.9094 | 234.0276 | 269.1482 | 302.3197 | 334.936 | 366.1142 |
| G08-23 | 0 | 43.1403 | 91.1457 | 143.4051 | 197.6524 | 244.4451 | 287.2755 | 330.5818 | 368.6963 | 409.1281 | 446.1622 |
| G08-24 | 0 | 38.405 | 80.0215 | 125.6409 | 168.4735 | 206.8448 | 243.1259 | 279.8281 | 314.393 | 347.983 | 380.5987 |
| G08-25 | 0 | 24.2422 | 48.0663 | 70.9144 | 93.019 | 112.9835 | 133.6007 | 155.3822 | 177.1183 | 199.0879 | 199.0418 |
| G09-26 | 0 | 11.9839 | 20.2514 | 25.0822 | 29.2164 | 34.7908 | 42.3622 | 52.3494 | 63.3125 | 75.2047 | 87.283 |
| G09-27 | 0 | 0.4662 | -6.2002 | -20.325 | -33.3773 | -42.6071 | -47.6414 | -48.9467 | -48.201 | -45.9167 | -42.2809 |
| G09-28 | 0 | 1.4013 | -1.5417 | -12.4728 | -24.1046 | -32.6527 | -39.1926 | -44.3774 | -48.0208 | -49.5621 | -62.3604 |
| G09-29 | 0 | 39.9448 | 46.2222 | 53.8311 | 60.1563 | 61.8683 | 61.9635 | 68.2888 | 77.8481 | 86.171 | -47.1968 |
| G09-30 | 0 | 32.0555 | 68.5668 | 100.256 | 125.5906 | 152.1332 | 176.357 | 202.8559 | 228.7998 | 255.1622 | 925.4723 |
| G10-31 | 0 | 28.896 | 55.4168 | 83.3839 | 111.2126 | 136.1992 | 160.3479 | 185.8963 | 209.4412 | 232.1948 | 252.7576 |
| G10-32 | 0 | 25.2779 | 52.2327 | 81.8895 | 112.0134 | 139.3921 | 166.912 | 193.8278 | 219.7206 | 244.2177 | 265.4088 |
| G10-33 | 0 | 21.5764 | 43.1538 | 65.5253 | 88.7864 | 109.4788 | 129.5181 | 150.4461 | 169.8799 | 188.1464 | 207.7219 |
| G10-34 | 0 | 16.8762 | 32.8666 | 48.1583 | 63.031 | 77.345 | 92.2181 | 107.1388 | 122.2931 | 136.7949 | 6.9201 |
| G10-35 | 0 | 11.0629 | 20.5387 | 27.2144 | 31.4156 | 36.5975 | 43.0858 | 50.555 | 58.1649 | 66.6143 | 75.7181 |
| G11-36 | 0 | 4.2075 | 4.8151 | -0.5139 | -9.0223 | -15.0529 | -19.3068 | -22.2988 | -24.1689 | -23.7481 | -22.1586 |


| N = 0,700,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 7.3319 | 19.2871 | 43.7122 | 67.4845 | 98.6374 | 131.3334 | 165.0132 | 194.6309 | 226.9131 | 252.236 |
| D1B | 0 | 0.6064 | 0.8396 | 1.1662 | 2.1458 | 3.8719 | 6.064 | 7.5568 | 9.0496 | 10.449 | 11.8948 |
| D2A | 0 | 4.8961 | 20.5178 | 54.0942 | 97.1874 | 155.7233 | 218.0448 | 280.3742 | 335.1984 | 391.9889 | 438.1914 |
| D2B | 0 | 5.8805 | 11.4344 | -1.4928 | 7.1406 | 13.6277 | 20.582 | 21.2354 | 29.9631 | 32.3902 | 35.9374 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 8.7838 | 16.0726 | 17.2874 | 28.0337 | 37.3788 | 47.7518 | 53.3591 | 62.1906 | 71.1622 | 79.4332 |
| D4A | 0 | 14.5837 | 37.956 | 84.7034 | 146.5103 | 247.9329 | 343.0147 | 444.9915 | 527.2431 | 606.3722 | 665.7162 |
| D4B | 0 | 15.2323 | 23.6687 | 20.2475 | 27.7464 | 35.9491 | 45.136 | 46.8233 | 56.4791 | 59.1977 | 65.1977 |
| G04-01 | 0 | -31.9305 | -67.2955 | -2160.58 | -2369.32 | -2418.94 | -2584.6 | -2673.3 | -2661.35 | -2746.94 | -2807.4 |
| G04-02 | 0 | 9.4082 | 13.3672 | 14.0191 | 14.9972 | 16.6742 | 20.3069 | 26.2223 | 34.1873 | 43.2235 | 54.03 |
| G11-03 | 0 | 25.915 | 51.0376 | 74.1998 | 96.3823 | 117.3983 | 138.2752 | 160.1795 | 182.366 | 204.5068 | 226.4618 |
| G04-04 | 0 | 43.2734 | 92.8396 | 143.902 | 191.8471 | 237.1879 | 279.7823 | 322.1477 | 362.2793 | 400.3162 | 436.4913 |
| G04-05 | 0 | 43.5713 | 92.2395 | 144.3556 | 191.7128 | 238.5556 | 281.2038 | 323.7611 | 363.8681 | 401.9966 | 438.0982 |
| G05-06 | 0 | 42.1659 | 90.9521 | 141.0937 | 188.5375 | 233.4223 | 275.4677 | 317.4701 | 357.0515 | 394.7241 | 430.4875 |
| G05-07 | 0 | 24.6602 | 50.7745 | 76.2342 | 101.9763 | 126.032 | 149.479 | 173.1146 | 196.9388 | 220.4367 | 243.6063 |
| G05-08 | 0 | 11.6331 | 18.9216 | 24.3411 | 31.7234 | 38.5449 | 46.5345 | 55.5991 | 66.7198 | 78.1682 | 91.5321 |
| G05-09 | 0 | -0.0002 | -8.4302 | -20.0739 | -27.06 | -34.9312 | -38.9362 | -41.6842 | -40.7061 | -38.2844 | -32.5557 |
| G05-10 | 0 | -10.3324 | -21.2225 | -34.9518 | -45.9351 | -58.5467 | -67.9004 | -77.8123 | -83.8154 | -90.8421 | 5 |
| G06-11 | 0 | 1.2078 | 1.115 | -2.1369 | -3.9487 | -8.4548 | -9.8947 | -11.4279 | -9.7555 | -9.2909 | -6.039 |
| G06-12 | 0 | 14.3503 | 27.4132 | 38.545 | 48.895 | 57.956 | 66.282 | 75.989 | 86.707 | 97.15 | 108.007 |
| G06-13 | 0 | 33.3096 | 67.1337 | 102.3089 | 133.5781 | 164.3837 | 190.9098 | 220.2763 | 246.5261 | 274.034 | 298.0058 |
| G06-14 | 0 | 33.6536 | 67.8218 | 104.1803 | 136.5837 | 168.6636 | 197.1137 | 228.1728 | 256.3465 | 285.174 | 309.9512 |
| G06-15 | 0 | 33.9649 | 67.1772 | 103.1279 | 135.3065 | 166.9215 | 195.3759 | 225.5314 | 253.8003 | 283.0627 | 306.6145 |
| G07-16 | 0 | 13.6983 | 27.8151 | 40.4426 | 53.3955 | 64.3455 | 75.1557 | 86.1533 | 98.6421 | 109.8261 | 121.4761 |
| G07-17 | 0 | -0.6071 | -0.9807 | -6.8649 | -9.2466 | -15.3643 | -19.3338 | -24.5642 | -26.152 | -28.4403 | -27.3195 |
| G07-18 | 0 | -14.8216 | -29.8286 | -52.1757 | -68.3892 | -89.9914 | -107.505 | -127.619 | -140.996 | -156.557 | -165.243 |
| G07-19 | 0 | 0.1408 | -11.1133 | -26.0245 | -40.0441 | -50.8285 | -60.0184 | -64.285 | -65.7856 | -65.3166 | -63.2536 |
| G07-20 | 0 | 10.0209 | 14.15 | 14.6604 | 15.7276 | 19.1602 | 23.0578 | 29.7848 | 37.7182 | 46.9508 | 56.7875 |
| G07-21 | 0 | 22.3789 | 43.0338 | 62.0107 | 81.0817 | 99.9202 | 117.7802 | 137.0399 | 156.3935 | 176.2609 | 195.4762 |
| G08-22 | 0 | 37.8376 | 78.557 | 121.4153 | 163.674 | 201.7559 | 237.0078 | 271.4721 | 305.3817 | 337.8536 | 370.1415 |
| G08-23 | 0 | 38.7485 | 81.2947 | 127.1175 | 171.901 | 212.4656 | 249.7117 | 286.6762 | 322.4567 | 357.2434 | 390.2763 |
| G08-24 | 0 | 38.7235 | 81.169 | 126.7805 | 171.094 | 211.0395 | 247.7795 | 283.9177 | 319.3141 | 352.6661 | 385.5087 |
| G08-25 | 0 | 23.6839 | 47.0895 | 70.3103 | 91.8103 | 112.0081 | 132.2998 | 153.2903 | 174.7939 | 196.3449 | 217.6177 |
| G09-26 | 0 | 12.0775 | 20.7169 | 24.8048 | 28.0101 | 32.8411 | 39.6233 | 49.1464 | 59.6452 | 71.445 | 83.2913 |
| G09-27 | 0 | 0.2331 | -6.2473 | -21.2585 | -36.549 | -46.7113 | -53.3306 | -55.5215 | -55.8011 | -53.7968 | -51.0465 |
| G09-28 | 0 | 0.8876 | -2.0555 | -14.5749 | -27.7478 | -36.9974 | -44.0974 | -49.7028 | -53.9535 | -55.9616 | -56.0083 |
| G09-29 | 0 | 13.0424 | 18.6388 | 22.3855 | 26.1321 | 30.3533 | 34.9065 | 42.4477 | 47.0959 | 55.6809 | 65.7362 |
| G09-30 | 0 | 17.7876 | 34.7843 | 50.0118 | 65.2863 | 80.8408 | 95.9767 | 112.2771 | 128.6252 | 144.8335 | 160.3903 |
| G10-31 | 0 | 28.3338 | 55.8764 | 83.5142 | 111.8996 | 137.6291 | 162.2416 | 187.0877 | 210.8634 | 233.0082 | 254.408 |
| G10-32 | 0 | 24.5289 | 51.6193 | 81.0853 | 111.7168 | 139.6964 | 166.6535 | 193.5189 | 219.5007 | 243.9475 | 267.0915 |
| G10-33 | 0 | 20.6404 | 42.3569 | 64.9607 | 88.0793 | 109.3779 | 129.556 | 149.7351 | 169.4945 | 187.6194 | 205.0445 |
| G10-34 | 0 | 15.7491 | 31.6848 | 46.7822 | 61.3214 | 75.4412 | 90.2139 | 105.1734 | 119.7608 | 134.3019 | 148.4704 |
| G10-35 | 0 | 11.0172 | 20.7743 | 26.6097 | 30.4379 | 35.62 | 41.8295 | 48.6924 | 56.2558 | 64.9395 | 73.9036 |
| G11-36 | 0 | 4.3018 | 5.3303 | -0.7008 | -10.3777 | -16.9695 | -21.8313 | -25.0099 | -27.3475 | -27.0203 | -25.2438 |


| N = 0,800,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | -29.131 | -7.6238 | 17.6251 | 36.7964 | 62.7955 | 86.8323 | 111.5243 | 132.5229 | 156.8903 | 177.4701 |
| D1B | 0 | 0.9332 | 2.2393 | 3.5923 | 4.6186 | 5.4116 | 6.9511 | 7.8374 | 9.3302 | 10.4497 | 11.9425 |
| D2A | 0 | 10.0743 | 33.9552 | 74.2105 | 116.2416 | 174.4191 | 232.5571 | 292.6147 | 347.0791 | 401.5493 | 447.1554 |
| D2B | 0 | 16.1086 | 19.5473 | 26.943 | 35.9397 | 40.0381 | 44.419 | 42.3463 | 55.6304 | 57.3734 | 63.8744 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 10.7624 | 17.7818 | 22.5547 | 32.7097 | 42.5836 | 50.5863 | 52.7386 | 55.7807 | 68.7906 | 77.1211 |
| D4A | 0 | 16.9879 | 47.6424 | 100.719 | 162.6022 | 259.4673 | 344.5967 | 428.1949 | 504.1245 | 577.4885 | 634.1355 |
| D4B | 0 | 46.4838 | 58.903 | 69.9165 | 76.9932 | 83.0391 | 90.8191 | 96.6781 | 104.6926 | 107.1766 | 112.6605 |
| G04-01 | 0 | -3.7597 | -19.3538 | -39.8671 | -56.249 | -71.7494 | -82.0976 | -90.3114 | -94.1629 | -95.8335 | -93.745 |
| G04-02 | 0 | 8.7112 | 10.9942 | 10.6218 | 11.8791 | 13.8826 | 18.2149 | 23.8522 | 31.9116 | 41.0892 | 51.712 |
| G11-03 | 0 | 24.6966 | 51.0289 | 75.4942 | 98.5605 | 120.8339 | 143.2012 | 165.337 | 188.3134 | 211.1984 | 233.3362 |
| G04-04 | 0 | 43.3288 | 93.2785 | 145.2365 | 193.2385 | 240.0795 | 284.7346 | 327.7616 | 368.9274 | 408.4648 | 445.6272 |
| G04-05 | 0 | 39.8527 | 86.116 | 133.8913 | 178.0429 | 221.8213 | 265.509 | 306.3255 | 344.7414 | 383.3959 | 418.8 |
| G05-06 | 0 | 42.1174 | 90.854 | 140.249 | 186.9918 | 232.7134 | 275.9684 | 318.4816 | 358.6671 | 397.4107 | 434.0127 |
| G05-07 | 0 | 24.3361 | 49.6591 | 74.3724 | 99.3691 | 124.4137 | 148.663 | 172.8654 | 196.8353 | 220.6647 | 244.0736 |
| G05-08 | 0 | 10.8862 | 16.2596 | 20.9318 | 28.5479 | 36.0237 | 44.855 | 53.9667 | 64.5735 | 75.9752 | 89.0594 |
| G05-09 | 0 | -1.3041 | -11.5979 | -23.6614 | -30.6481 | -37.3085 | -41.1276 | -43.4098 | -43.0373 | -41.0346 | -35.7716 |
| G05-10 | 0 | -10.6591 | -22.2024 | -35.9796 | -46.9175 | -58.3204 | -66.698 | -75.8198 | -81.963 | -88.8044 | -92.8068 |
| G06-11 | 0 | 0.7898 | -0.0467 | -4.0434 | -5.8093 | -8.6908 | -9.1555 | -9.8526 | -7.8544 | -10.4102 | 6.6459 |
| G06-12 | 0 | 13.9933 | 25.8697 | 36.7334 | 47.091 | 57.4027 | 67.3467 | 77.2905 | 88.5237 | 99.8952 | 110.8985 |
| G06-13 | 0 | 33.8851 | 66.562 | 100.6381 | 131.7834 | 163.303 | 192.2171 | 221.6916 | 249.2588 | 276.8271 | 302.1617 |
| G06-14 | 0 | 33.7057 | 67.4604 | 101.7762 | 134.0453 | 167.2013 | 197.9378 | 228.9556 | 257.7399 | 286.805 | 313.4027 |
| G06-15 | 0 | 33.0412 | 65.6192 | 99.8282 | 131.9915 | 165.134 | 195.6258 | 226.9573 | 255.6365 | 284.4109 | 310.0669 |
| G07-16 | 0 | 12.3903 | 23.1507 | 33.7249 | 46.7219 | 59.2064 | 72.0645 | 83.4316 | 95.3116 | 107.0984 | 119.3983 |
| G07-17 | 0 | -1.4477 | -1.9609 | -6.4907 | -8.6852 | -13.3076 | -15.5958 | -15.7824 | -17.0897 | -19.9384 | -18.911 |
| G07-18 | 0 | -17.0537 | -34.4317 | -57.7103 | -74.3903 | -94.5539 | -110.489 | -129.769 | -143.52 | -159.129 | -168.281 |
| G07-19 | 0 | -1.0732 | -11.3374 | -28.2733 | -44.5085 | -55.2851 | -63.1692 | -67.8807 | -71.2864 | -71.053 | -70.1667 |
| G07-20 | 0 | 42.3736 | -164.754 | -271.161 | -227.942 | -364.191 | -384.961 | -1046.32 | -701.556 | -1319.76 | -858.725 |
| G07-21 | 0 | 21.907 | 40.925 | 59.1514 | 77.9377 | 97.191 | 116.5846 | 136.2124 | 155.934 | 176.0297 | 196.0329 |
| G08-22 | 0 | 37.3705 | 77.8544 | 119.7809 | 161.7112 | 200.9046 | 238.8473 | 274.5172 | 309.4924 | 344.9816 | 375.8274 |
| G08-23 | 0 | 38.5106 | 79.9428 | 125.3342 | 170.5883 | 212.361 | 253.0544 | 290.7835 | 327.8557 | 363.6584 | 398.5218 |
| G08-24 | 0 | 38.1486 | 79.1355 | 124.4941 | 169.1596 | 210.5757 | 249.8099 | 286.5831 | 323.0334 | 357.5341 | 391.9896 |
| G08-25 | 0 | 23.2654 | 45.2747 | 66.9601 | 88.4133 | 109.4953 | 130.7179 | 151.8015 | 173.5841 | 195.2284 | 216.9197 |
| G09-26 | 0 | 11.567 | 18.9532 | 21.4154 | 23.413 | 28.5698 | 35.306 | 44.2258 | 53.982 | 65.1789 | 76.608 |
| G09-27 | 0 | 0.466 | -6.9464 | -24.474 | -41.1628 | -52.3032 | -59.7612 | -63.397 | -64.609 | -64.0961 | -61.999 |
| G09-28 | 0 | 0.7008 | -3.6438 | -16.2105 | -30.1316 | -40.2215 | -48.0694 | -53.6749 | -58.4393 | -61.2421 | -62.0362 |
| G09-29 | 0 | 10.6882 | 18.5642 | 21.3299 | 23.6739 | 28.4089 | 34.2689 | 41.3948 | 48.3335 | 56.163 | 64.602 |
| G09-30 | 0 | 15.2004 | 29.2823 | 42.0125 | 55.2088 | 69.9915 | 85.2407 | 100.7235 | 114.8544 | 129.9186 | 145.3564 |
| G10-31 | 0 | 27.5421 | 54.7129 | 81.8386 | 110.8769 | 137.6323 | 163.2709 | 189.1437 | 213.3395 | 236.837 | 258.4707 |
| G10-32 | 0 | 24.6674 | 50.4531 | 79.2656 | 110.0352 | 139.4101 | 168.088 | 195.0919 | 221.8176 | 246.6821 | 271.0359 |
| G10-33 | 0 | 20.6464 | 42.0407 | 64.3704 | 87.9158 | 110.0139 | 131.5525 | 152.4376 | 172.5762 | 191.407 | 209.164 |
| G10-34 | 0 | 15.5712 | 31.236 | 46.1087 | 60.7483 | 75.4823 | 90.4496 | 105.4639 | 120.8983 | 135.4939 | 149.6702 |
| G10-35 | 0 | 10.5979 | 19.5622 | 25.4445 | 28.7125 | 33.615 | 39.7784 | 46.6884 | 54.2991 | 62.61 | 71.6218 |
| G11-36 | 0 | 3.9746 | 4.3487 | -1.6831 | -11.5967 | -18.7044 | -24.596 | -27.5419 | -30.3941 | -30.2541 | -29.5992 |


| N = 0,900,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 2.9909 | 16.2171 | 37.9967 | 57.0664 | 72.6778 | 90.4868 | 108.0157 | 128.5372 | 146.4884 | 165.3749 |
| D1B | 0 | 3.4023 | 6.1529 | 8.2038 | 11.4204 | 14.5435 | 17.3868 | 19.6246 | 22.2817 | 23.9133 | 26.337 |
| D2A | 0 | 5.2679 | 25.0344 | 59.1158 | 110.2651 | 161.6525 | 221.5801 | 277.2236 | 336.8383 | 386.8495 | 436.1659 |
| D2B | 0 | 1.6756 | 2.2804 | 3.677 | 9.169 | 13.4977 | 18.8503 | 21.5966 | 27.0424 | 29.8351 | 33.2796 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | -0.9809 | 0.2803 | 2.663 | 4.8122 | 25.3687 | 36.7222 | 45.8327 | 57.7474 | 61.0179 | 66.3912 |
| D4A | 0 | 7.2541 | 29.9075 | 65.0127 | 139.7712 | 210.186 | 296.6738 | 371.4196 | 450.7678 | 510.3576 | 566.7208 |
| D4B | 0 | 11.0391 | 16.5825 | 22.9715 | 28.8907 | 37.2529 | 43.5955 | 46.2734 | 53.3674 | 58.0655 | 65.2068 |
| G04-01 | 0 | -5.0609 | -21.637 | -41.044 | -59.9865 | -74.6572 | -87.0985 | -94.8977 | -99.9582 | -102.325 | -101.676 |
| G04-02 | 0 | 5.3218 | 5.3688 | 3.5477 | 1.587 | 2.1475 | 1.4471 | 3.8746 | 8.6832 | 13.9583 | 19.5143 |
| G11-03 | 0 | 19.8475 | 39.9762 | 59.7318 | 79.3488 | 98.779 | 119.4711 | 140.8189 | 162.774 | 184.0763 | 204.9125 |
| G04-04 | 0 | 43.866 | 94.1164 | 143.6732 | 190.9989 | 236.2793 | 280.6318 | 322.7051 | 363.7101 | 401.0834 | 436.5489 |
| G04-05 | 0 | 38.7547 | 89.9058 | 132.7834 | 174.42 | 214.8636 | 277.6176 | 323.623 | 365.6587 | 392.4239 | 420.819 |
| G05-06 | 0 | 44.9987 | 96.1982 | 147.822 | 196.4226 | 242.977 | 287.9976 | 329.7131 | 371.4783 | 408.9586 | 444.8097 |
| G05-07 | 0 | 26.1647 | 52.284 | 77.9353 | 102.5089 | 127.6473 | 151.3328 | 175.3943 | 199.4572 | 221.7855 | 243.8331 |
| G05-08 | 0 | 11.6342 | 17.5211 | 23.175 | 29.4831 | 37.8467 | 45.3229 | 55.0421 | 65.3692 | 76.5376 | 87.8927 |
| G05-09 | 0 | 1.0247 | -10.1548 | -20.4489 | -28.5536 | -34.1894 | -39.4064 | -41.3627 | -41.8749 | -39.7789 | -36.0061 |
| G05-10 | 0 | -10.9897 | -24.4936 | -37.1591 | -50.3365 | -60.021 | -71.1491 | -78.4124 | -86.2809 | -92.0078 | -98.5257 |
| G06-11 | 0 | 1.254 | -1.1148 | -3.5765 | -7.3852 | -9.0572 | -11.4725 | -10.59 | -10.8687 | -8.4532 | 8.0819 |
| G06-12 | 0 | 19.1958 | 35.5314 | 48.3143 | 60.5436 | 71.4351 | 82.7881 | 94.2799 | 105.5875 | 116.8028 | 127.0492 |
| G06-13 | 0 | 33.0269 | 67.3609 | 99.8799 | 132.075 | 160.1252 | 190.1802 | 217.5344 | 246.5216 | 271.0828 | 296.5304 |
| G06-14 | 0 | 35.0882 | 70.9702 | 105.412 | 138.3663 | 168.9486 | 202.0471 | 231.097 | 261.6387 | 287.9449 | 314.6255 |
| G06-15 | 0 | 32.8979 | 67.5201 | 101.3535 | 134.3049 | 165.1171 | 196.9086 | 226.5612 | 257.2395 | 283.0312 | 309.2897 |
| G07-16 | 0 | 13.8943 | 27.7888 | 40.0053 | 50.4034 | 62.0605 | 73.8117 | 85.6096 | 97.7337 | 109.2525 | 120.3983 |
| G07-17 | 0 | -0.3262 | -2.4228 | -6.6624 | -13.045 | -16.4929 | -22.0834 | -24.3197 | -27.8138 | -28.7453 | -30.3296 |
| G07-18 | 0 | -14.3638 | -31.6557 | -50.62 | -72.4657 | -89.802 | -110.112 | -125.309 | -143.201 | -154.68 | -167.18 |
| G07-19 | 0 | -4.0695 | -15.4819 | -34.1439 | -52.1031 | -65.3853 | -72.447 | -77.6849 | -80.9116 | -81.2858 | -80.6777 |
| G07-20 | 0 | 49.6323 | -7.0165 | -20.7703 | -75.6397 | -128.701 | -119.737 | -68.3975 | -93.0181 | -81.5434 | -48.4634 |
| G07-21 | 0 | 25.5968 | 46.8122 | 65.8367 | 84.0226 | 102.3023 | 121.7491 | 141.2425 | 161.2033 | 180.6521 | 199.9149 |
| G08-22 | 0 | 38.8315 | 79.9887 | 124.4948 | 166.8671 | 205.8975 | 243.3509 | 279.5524 | 314.0366 | 347.0826 | 378.6432 |
| G08-23 | 0 | 43.0429 | 84.5784 | 131.9239 | 176.6772 | 218.1768 | 258.5938 | 296.6528 | 333.8176 | 368.8597 | 401.0232 |
| G08-24 | 0 | 38.889 | 80.8985 | 127.4244 | 171.4885 | 212.718 | 251.1583 | 288.1121 | 323.4395 | 357.6527 | 389.8656 |
| G08-25 | 0 | 23.9686 | 47.5661 | 69.8612 | 90.2491 | 110.5909 | 131.6786 | 152.2086 | 173.9496 | 194.9004 | 215.3861 |
| G09-26 | 0 | 11.9363 | 19.7393 | 21.7831 | 22.6188 | 26.8458 | 33.0232 | 41.1517 | 51.0452 | 61.2641 | 72.5052 |
| G09-27 | 0 | 2.3286 | -2.7942 | -19.1404 | -35.5324 | -46.2428 | -51.1789 | -53.8333 | -53.8798 | -52.6226 | -49.3163 |
| G09-28 | 0 | 0.7477 | -3.7381 | -17.3824 | -31.3069 | -41.4929 | -49.5293 | -55.9303 | -60.6024 | -62.9384 | -63.4525 |
| G09-29 | 0 | -21.628 | -30.896 | -41.1467 | -48.6352 | -55.235 | -58.4174 | -58.0898 | -55.8907 | -53.4572 | -52.5684 |
| G09-30 | 0 | 21.1393 | 39.0664 | 54.8524 | 71.1045 | 87.1705 | 104.3553 | 121.4472 | 138.6791 | 155.0736 | 171.3291 |
| G10-31 | 0 | 30.1665 | 58.8895 | 87.7073 | 115.6409 | 142.3166 | 168.201 | 193.9002 | 217.9214 | 240.4046 | 261.3493 |
| G10-32 | 0 | 25.8382 | 53.121 | 83.8047 | 114.1637 | 142.4295 | 170.5572 | 197.848 | 224.302 | 248.2886 | 270.7392 |
| G10-33 | 0 | 20.5651 | 41.7854 | 64.7363 | 86.893 | 108.1631 | 129.2002 | 149.2558 | 168.6585 | 186.3315 | 202.2752 |
| G10-34 | 0 | 16.6012 | 32.8762 | 47.986 | 61.837 | 76.5282 | 91.406 | 106.5177 | 121.4899 | 135.3896 | 148.8232 |
| G10-35 | 0 | 11.6667 | 20.8133 | 25.4802 | 28.7938 | 33.3671 | 39.7141 | 46.5277 | 54.6485 | 62.8627 | 71.5435 |
| G11-36 | 0 | 4.5378 | 4.5381 | -3.6958 | -14.0347 | -20.5845 | -25.6369 | -29.1454 | -30.6424 | -30.2679 | -29.0518 |


| N = 1,000,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.3776 | 15.7588 | 38.2063 | 56.9596 | 76.2281 | 95.1235 | 117.2932 | 137.4061 | 159.6718 | 178.8506 |
| D1B | 0 | 0.0932 | 0.6062 | 1.5386 | 2.4711 | 3.9168 | 5.7817 | 7.1807 | 8.7195 | 10.1185 | 11.6573 |
| D2A | 0 | 5.688 | 21.7266 | 56.1838 | 100.8092 | 153.7398 | 210.9203 | 270.0202 | 323.5749 | 380.5885 | 427.3423 |
| D2B | 0 | 5.1837 | 1.1675 | 11.8151 | 11.3481 | 13.7298 | 19.4739 | 24.9851 | 27.0869 | 31.477 | 35.3064 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 7.5224 | 14.6244 | 19.624 | 26.3991 | 36.0715 | 45.0432 | 54.3418 | 58.7812 | 67.3328 | 74.9029 |
| D4A | 0 | 10.7157 | 30.0889 | 72.6281 | 136.2801 | 223.019 | 302.0006 | 392.9828 | 465.2462 | 537.8482 | 591.4396 |
| D4B | 0 | 8.6029 | 12.7399 | 22.7537 | 27.3608 | 32.7674 | 43.3927 | 48.8466 | 58.9549 | 66.5246 | 71.5556 |
| G04-01 | 0 | -4.9204 | -21.3524 | -42.1469 | -61.5946 | -78.8601 | -91.0199 | -100.256 | -105.175 | -106.474 | -104.804 |
| G04-02 | 0 | 8.3451 | 11.3752 | 11.6083 | 11.282 | 12.0743 | 15.3383 | 20.5129 | 27.6929 | 36.9241 | 47.5078 |
| G11-03 | 0 | 15.5464 | 32.2171 | 58.7227 | 81.9047 | 102.7461 | 118.7641 | 139.4196 | 159.8419 | 180.499 | 199.8921 |
| G04-04 | 0 | 42.85 | 92.0854 | 143.3294 | 191.4096 | 236.6527 | 280.1751 | 323.0957 | 363.3165 | 401.4433 | 437.7547 |
| G04-05 | 0 | 398.7473 | 1422.122 | 2950.122 | 5497.938 | 48504.51 | 48504.51 | 48504.51 | 48504.51 | 48504.51 | 48504.51 |
| G05-06 | 0 | 43.2413 | 93.4771 | 144.9761 | 193.8234 | 239.9253 | 283.5138 | 326.8727 | 367.438 | 405.8615 | 442.1427 |
| G05-07 | 0 | 24.7555 | 50.5907 | 76.8487 | 102.2176 | 126.5567 | 151.3179 | 176.0811 | 199.7666 | 223.1717 | 246.7654 |
| G05-08 | 0 | 11.0738 | 17.7085 | 22.8483 | 29.1563 | 36.1189 | 44.8569 | 53.9222 | 64.3896 | 76.3059 | 89.0167 |
| G05-09 | 0 | -0.233 | -9.7827 | -21.661 | -31.0241 | -38.3369 | -43.088 | -45.8364 | -46.0228 | -43.3676 | -37.6852 |
| G05-10 | 0 | -11.3127 | -22.8577 | -36.2648 | -48.7869 | -62.2399 | -72.7133 | -84.3498 | -92.03 | -100.594 | -104.923 |
| G06-11 | 0 | -0.0932 | 1.2574 | -3.8199 | -7.5 | -12.9035 | -16.8628 | -19.9372 | -18.1206 | -17.7481 | 13.5554 |
| G06-12 | 0 | 15.706 | 28.9637 | 41.3445 | 52.0157 | 61.2095 | 70.8191 | 80.9831 | 91.6091 | 102.0041 | 113.4627 |
| G06-13 | 0 | 32.5992 | 66.3182 | 100.878 | 133.0178 | 162.6912 | 191.7602 | 222.3683 | 249.9503 | 277.3473 | 301.9957 |
| G06-14 | 0 | 35.7363 | 71.2892 | 108.3805 | 142.2164 | 174.4722 | 205.1939 | 237.919 | 266.9685 | 296.7181 | 322.1857 |
| G06-15 | 0 | 32.618 | 65.9357 | 101.8155 | 134.6261 | 166.2752 | 196.2504 | 228.4619 | 256.7189 | 285.6288 | 310.5828 |
| G07-16 | 0 | 12.5374 | 26.1937 | 38.3588 | 49.8253 | 60.4061 | 71.5931 | 83.0604 | 94.1548 | 104.7366 | 116.2043 |
| G07-17 | 0 | -1.9114 | -0.5595 | -6.5734 | -14.2185 | -20.6517 | -25.2667 | -29.5553 | -32.5852 | -35.1026 | -33.6575 |
| G07-18 | 0 | -15.2451 | -30.3041 | -53.2173 | -73.1083 | -93.5564 | -110.518 | -130.082 | -144.533 | -160.239 | -168.835 |
| G07-19 | 0 | 2.8481 | -9.5243 | -28.2925 | -46.1733 | -60.7854 | -70.4021 | -76.4711 | -79.9251 | -80.7189 | -79.5053 |
| G07-20 | 0 | 7.2797 | 19.707 | 23.6024 | 29.2595 | 33.155 | 40.5746 | 49.8032 | 61.7218 | 66.4986 | 79.5305 |
| G07-21 | 0 | 22.565 | 43.5461 | 63.0822 | 82.2464 | 100.1989 | 119.3179 | 139.0435 | 158.8636 | 178.6841 | 199.0652 |
| G08-22 | 0 | 38.4867 | 80.273 | 125.2201 | 168.4536 | 207.3246 | 244.8983 | 281.4529 | 316.1522 | 349.4132 | 381.5623 |
| G08-23 | 0 | 40.9048 | 84.9742 | 134.2387 | 180.0624 | 221.1234 | 261.3378 | 299.7154 | 337.4338 | 373.7863 | 407.7336 |
| G08-24 | 0 | 39.0431 | 82.459 | 129.6445 | 175.0215 | 216.1245 | 255.045 | 293.1315 | 328.895 | 363.5455 | 397.361 |
| G08-25 | 0 | 23.9678 | 47.4249 | 69.6733 | 90.9912 | 110.2624 | 131.5352 | 152.67 | 173.8519 | 195.7797 | 217.1497 |
| G09-26 | 0 | 11.2422 | 18.6289 | 20.3014 | 21.2773 | 23.7394 | 29.5467 | 37.3054 | 46.3651 | 57.1906 | 68.4809 |
| G09-27 | 0 | 0.0932 | -8.3424 | -26.9839 | -46.4171 | -61.0029 | -69.7171 | -74.8889 | -77.4517 | -77.2187 | -75.3081 |
| G09-28 | 0 | -0.234 | -4.8592 | -19.0615 | -33.1704 | -44.6159 | -51.9503 | -58.5837 | -63.2552 | -65.077 | -64.7501 |
| G09-29 | 0 | 7.6111 | 13.7278 | 14.942 | 16.2029 | 18.7245 | 23.6744 | 29.5576 | 35.9088 | 44.1741 | 52.7665 |
| G09-30 | 0 | 17.7356 | 34.8663 | 50.9737 | 67.1747 | 82.5377 | 100.183 | 117.3632 | 134.2644 | 151.3528 | 167.6498 |
| G10-31 | 0 | 29.5563 | 57.5755 | 86.9485 | 115.9032 | 141.7824 | 168.3625 | 193.4049 | 217.1891 | 240.1352 | 261.7762 |
| G10-32 | 0 | 25.2303 | 52.6503 | 84.215 | 115.2697 | 143.7188 | 172.3092 | 199.8768 | 226.0951 | 251.2905 | 274.2978 |
| G10-33 | 0 | 20.603 | 42.2342 | 65.5953 | 88.9109 | 109.5171 | 130.7785 | 151.1528 | 169.9393 | 188.1186 | 205.1306 |
| G10-34 | 0 | 16.2268 | 32.4077 | 47.703 | 62.2992 | 76.0567 | 91.3065 | 106.3704 | 121.0618 | 136.08 | 150.2592 |
| G10-35 | 0 | 10.784 | 20.0273 | 24.6022 | 27.8235 | 31.6986 | 38.4681 | 45.5645 | 52.801 | 62.138 | 71.3827 |
| G11-36 | 0 | 4.6778 | 3.6954 | -4.7245 | -15.2023 | -23.6688 | -28.1592 | -31.0124 | -33.0705 | -31.1527 | -28.8607 |


| N = 1,200,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 3.1793 | 12.2965 | 35.675 | 50.9651 | 68.2193 | 84.4925 | 102.4961 | 119.3316 | 140.3297 | 158.7098 |
| D1B | 0 | 0.1399 | 1.0734 | 2.1 | 3.0332 | 4.8999 | 6.253 | 7.9799 | 9.5198 | 11.153 | 12.7862 |
| D2A | 0 | 5.0804 | 21.208 | 57.2874 | 97.332 | 150.0148 | 206.7603 | 265.0972 | 319.4299 | 376.4741 | 423.5884 |
| D2B | 0 | 1.7309 | 9.2149 | 10.4779 | 13.8458 | 17.4474 | 25.3058 | 31.1533 | 35.0825 | 39.5734 | 43.5961 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 2.8503 | 3.6446 | 11.3077 | 14.8589 | 20.8868 | 29.8121 | 37.1482 | 41.0735 | 50.3729 | 55.6068 |
| D4A | 0 | 8.3303 | 27.6578 | 68.0482 | 122.4841 | 205.904 | 280.3009 | 361.2185 | 430.6739 | 499.5766 | 550.3576 |
| D4B | 0 | 16.2666 | 19.9342 | -0.3764 | -14.4799 | 5.5469 | 15.9834 | 15.137 | 17.2527 | 14.1498 | 18.193 |
| G04-01 | 0 | -5.1542 | -20.3382 | -40.8147 | -60.4083 | -78.5623 | -92.3045 | -101.125 | -106.465 | -108.972 | -107.579 |
| G04-02 | 0 | 8.8151 | 14.7853 | 13.7125 | 13.899 | 12.1735 | 13.1994 | 18.9832 | 26.5859 | 35.6817 | 45.9902 |
| G11-03 | 0 | 26.7822 | 54.2214 | 79.9767 | 104.047 | 126.3857 | 148.3976 | 171.0664 | 193.7357 | 216.0316 | 238.0942 |
| G04-04 | 0 | 44.6243 | 94.9829 | 147.3498 | 195.9476 | 243.4786 | 287.938 | 330.8165 | 372.7664 | 411.4104 | 447.7725 |
| G04-05 | 0 | 40.8603 | 87.7879 | 135.4905 | 179.4422 | 224.1679 | 266.2012 | 304.3855 | 338.9128 | 374.0686 | 406.6261 |
| G05-06 | 0 | 41.9826 | 91.9368 | 144.0406 | 192.8398 | 240.1991 | 284.2527 | 326.9578 | 367.9883 | 407.0636 | 443.7633 |
| G05-07 | 0 | 24.4309 | 51.0671 | 77.4708 | 103.64 | 128.7797 | 152.9359 | 177.28 | 201.5318 | 225.0342 | 248.5845 |
| G05-08 | 0 | 10.608 | 17.6647 | 23.0858 | 30.0024 | 36.4517 | 44.4901 | 53.5572 | 64.2598 | 75.2899 | 87.9563 |
| G05-09 | 0 | -0.8388 | -9.6908 | -24.2254 | -31.5392 | -39.6912 | -45.2814 | -48.7754 | -48.915 | -48.7286 | -43.7443 |
| G05-10 | 0 | -11.2245 | -21.7968 | -34.6509 | -45.7816 | -58.3559 | -68.8807 | -79.638 | -86.9958 | -95.9833 | -100.966 |
| G06-11 | 0 | 1.3506 | 1.4436 | -0.7455 | -2.3756 | -5.4965 | -7.7322 | -10.0609 | -8.6173 | -10.1542 | -7.2199 |
| G06-12 | 0 | 14.2798 | 27.6358 | 39.3746 | 50.3279 | 60.4036 | 69.3697 | 79.3072 | 89.7066 | 99.7829 | 110.3216 |
| G06-13 | 0 | 33.0374 | 67.4758 | 103.5938 | 136.2192 | 168.1939 | 197.2803 | 228.0471 | 256.1581 | 284.6904 | 309.914 |
| G06-14 | 0 | 34.4768 | 69.8399 | 107.2067 | 141.0393 | 175.3394 | 205.8718 | 238.594 | 268.1529 | 297.8995 | 323.7373 |
| G06-15 | 0 | 32.7567 | 66.5858 | 103.0235 | 136.3451 | 169.809 | 200.3894 | 232.1815 | 260.8567 | 289.906 | 314.7202 |
| G07-16 | 0 | 13.1137 | 29.5877 | 42.2818 | 54.6968 | 65.8513 | 76.8661 | 87.6946 | 98.9435 | 110.0059 | 121.9087 |
| G07-17 | 0 | -1.0718 | -1.0718 | -7.5036 | -9.8346 | -16.9188 | -21.0672 | -26.5667 | -28.7106 | -31.5998 | -30.7144 |
| G07-18 | 0 | -15.527 | -28.8687 | -51.414 | -69.8213 | -91.6677 | -109.748 | -129.129 | -143.212 | -159.245 | -167.889 |
| G07-19 | 0 | -2.4762 | -13.2694 | -33.5001 | -50.1329 | -63.2613 | -72.0907 | -75.5013 | -78.9119 | -77.6505 | -75.081 |
| G07-20 | 0 | 8.2938 | 19.6631 | 18.2186 | 24.1833 | 24.0896 | 28.237 | 35.6459 | 43.3344 | 52.6544 | 62.0205 |
| G07-21 | 0 | 21.3662 | 42.5468 | 61.9552 | 81.3647 | 99.8881 | 118.7857 | 138.2436 | 158.1693 | 178.2358 | 197.8363 |
| G08-22 | 0 | 39.0068 | 81.9646 | 127.3878 | 172.0253 | 212.4391 | 249.5115 | 285.7037 | 320.737 | 353.0776 | 386.3961 |
| G08-23 | 0 | 39.8348 | 84.5352 | 133.4414 | 181.7378 | 225.7421 | 265.17 | 304.0817 | 341.8626 | 378.0405 | 412.5205 |
| G08-24 | 0 | 39.5407 | 83.458 | 131.2879 | 178.005 | 220.8642 | 259.6315 | 297.843 | 334.8007 | 369.4804 | 403.045 |
| G08-25 | 0 | 22.8505 | 46.1212 | 68.1826 | 89.2673 | 109.2828 | 129.2521 | 150.5257 | 171.754 | 193.0758 | 214.399 |
| G09-26 | 0 | 10.9619 | 17.4651 | 18.812 | 18.7656 | 21.2741 | 26.3837 | 33.7229 | 42.4096 | 52.2115 | 63.5 |
| G09-27 | 0 | -1.2119 | -10.861 | -30.3444 | -51.3188 | -66.4658 | -76.1606 | -81.7528 | -86.5068 | -86.5534 | -85.3418 |
| G09-28 | 0 | -0.5607 | -6.0736 | -20.3236 | -36.5352 | -48.7752 | -57.2309 | -64.0046 | -70.358 | -72.18 | -72.9272 |
| G09-29 | 0 | 10.7243 | 18.4181 | 20.6562 | 22.1948 | 25.7855 | 30.6815 | 37.0701 | 43.9248 | 52.738 | 61.6447 |
| G09-30 | 0 | 18.0655 | 34.5486 | 48.7043 | 63.7447 | 79.3446 | 95.5503 | 112.4087 | 128.9882 | 145.9874 | 162.0088 |
| G10-31 | 0 | 28.9628 | 57.3676 | 86.5201 | 116.7008 | 142.8247 | 169.4164 | 195.6827 | 217.9381 | 241.7343 | 263.5717 |
| G10-32 | 0 | 24.96 | 52.3897 | 83.733 | 115.8701 | 145.7267 | 173.5357 | 201.3926 | 228.5991 | 253.1047 | 276.4463 |
| G10-33 | 0 | 20.2902 | 41.7971 | 65.4556 | 89.1619 | 110.8118 | 131.294 | 151.7764 | 170.7633 | 188.9095 | 206.2608 |
| G10-34 | 0 | 15.5312 | 31.6698 | 46.6426 | 61.4294 | 75.0034 | 89.7909 | 104.4857 | 119.3675 | 133.8296 | 148.2459 |
| G10-35 | 0 | 10.92 | 19.6468 | 23.2866 | 25.8533 | 29.6802 | 35.5138 | 42.421 | 50.028 | 59.2222 | 68.9298 |
| G11-36 | 0 | 3.929 | 3.1805 | -6.8287 | -19.2701 | -27.7824 | -33.4883 | -37.2298 | -39.9426 | -38.7731 | -36.014 |


| N = 10/18 Link Slab Restart |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | -1.2653 | 5.8089 | 16.444 | 25.6737 | 35.6068 | 46.3362 | 63.7197 | 80.8691 | 102.4244 | 120.419 |
| D1B | 0 | 3.8786 | 9.5328 | 14.8599 | 19.9067 | 24.4859 | 28.879 | 34.3463 | 39.8138 | 45.7486 | 51.45 |
| D2A | 0 | 11.0027 | 29.0923 | 55.9488 | 84.5782 | 113.5362 | 147.3915 | 187.9662 | 226.5843 | 271.4565 | 311.9474 |
| D2B | 0 | 16.511 | 41.14 | 68.1497 | 96.2336 | 124.5062 | 156.5593 | 194.2608 | 235.2785 | 275.4127 | 310.7894 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 2.0577 | 16.4167 | 31.6646 | 54.2104 | 72.5468 | 91.9131 | 123.4435 | 154.508 | 191.2357 | 228.5275 |
| D4A | 0 | 1.3131 | 21.4321 | 54.543 | 98.5378 | 152.9504 | 215.0168 | 297.5483 | 370.2853 | 458.8035 | 530.6251 |
| D4B | 0 | 8.0914 | 17.3123 | 27.5686 | 41.5418 | 54.1509 | 71.5594 | 91.1331 | 111.5074 | 132.2591 | 153.4821 |
| G04-01 | 0 | -3.2957 | -20.7965 | -33.3295 | -51.0612 | -65.9143 | -78.6782 | -88.8429 | -95.7121 | -102.256 | -105.691 |
| G04-02 | 0 | 5.6843 | 10.5698 | 14.2806 | 19.1191 | 22.1249 | 29.1697 | 37.3423 | 57.6781 | 70.9232 | 94.6447 |
| G11-03 | 0 | 15.4668 | 27.7934 | 28.5893 | 29.8548 | 44.573 | 62.573 | 83.0107 | 103.8249 | 123.8429 | 141.236 |
| G04-04 | 0 | 37.05 | 77.3655 | 120.8075 | 159.9181 | 197.4928 | 235.0704 | 272.3713 | 308.509 | 344.9757 | 380.0461 |
| G04-05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-06 | 0 | 31.5321 | 160.1252 | 258.084 | 535.8212 | 623.1384 | 403.4362 | 682.1315 | 2044.369 | 1772.523 | 903.3965 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 9.0215 | 10.9849 | 15.1917 | 18.4638 | 21.8296 | 28.0468 | 35.947 | 45.2031 | 53.3838 | 62.7802 |
| G05-09 | 0 | -2.4222 | -14.5818 | -23.76 | -32.8444 | -41.4625 | -46.1676 | -48.9627 | -49.1959 | -49.7548 | -48.4039 |
| G05-10 | 0 | -9.1263 | -18.6717 | -27.0529 | -35.8064 | -44.8392 | -52.8474 | -60.2035 | -64.8594 | -72.1225 | -76.7781 |
| G06-11 | 0 | 1.1146 | 0.4644 | 0.0463 | -1.2075 | -3.1117 | -4.5051 | -4.4121 | -2.7867 | -3.5298 | 2.3686 |
| G06-12 | 0 | 19.0502 | 35.2883 | 50.5113 | 64.6738 | 76.4844 | 89.633 | 101.3054 | 113.1167 | 123.7283 | 134.848 |
| G06-13 | 0 | 26.9326 | 53.867 | 81.6865 | 108.3911 | 133.8872 | 159.385 | 185.4422 | 209.872 | 233.6518 | 256.2688 |
| G06-14 | 0 | -7.3493 | -71.2062 | -43.943 | 24.5691 | 21.5063 | 23.5279 | 38.8346 | 43.1174 | 45.0895 | 21.8147 |
| G06-15 | 0 | -29.7397 | -46.8881 | -50.1325 | -42.7159 | -31.0246 | -20.7895 | -8.6283 | 2.5002 | 14.7094 | 25.3216 |
| G07-16 | 0 | 12.3608 | 23.1357 | 34.3774 | 44.4997 | 54.4359 | 64.9783 | 75.5211 | 87.2774 | 97.0742 | 107.8508 |
| G07-17 | 0 | -0.7918 | -3.0735 | -8.7082 | -14.4355 | -19.7902 | -24.5864 | -29.5224 | -30.8726 | -34.2719 | -34.1785 |
| G07-18 | 0 | -12.4529 | -26.1135 | -44.931 | -64.4449 | -82.9821 | -100.032 | -117.965 | -130.043 | -145.001 | -154.617 |
| G07-19 | 0 | -8.0126 | -20.4969 | -33.7729 | -43.229 | -51.7997 | -61.6744 | -69.7325 | -76.9054 | -83.7057 | -88.4563 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 14.9867 | 30.348 | 44.7753 | 59.8573 | 73.5852 | 87.5937 | 101.9762 | 116.733 | 131.4435 | 147.1814 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1-1 |
| G08-23 | 0 | 31.6794 | 65.0409 | 99.3378 | 134.1502 | 167.565 | 200.9353 | 232.6277 | 264.8823 | 296.4851 | 327.8101 |
| G08-24 | 0 | 32.55 | 66.3128 | 100.5904 | 134.0783 | 167.3357 | 199.6168 | 231.2479 | 263.5334 | 294.8424 | 325.4546 |
| G08-25 | 0 | 15.1984 | 30.1175 | 44.6176 | 59.3046 | 73.9923 | 87.5611 | 102.0159 | 116.9845 | 131.9535 | 147.0627 |
| G09-26 | 0 | 5.0662 | 7.762 | 9.6211 | 11.3872 | 13.3393 | 15.7562 | 18.638 | 22.4029 | 25.517 | 29.9792 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -14.4893 | -30.4738 | -48.0937 | -66.0398 | -84.3125 | -102.024 | -119.174 | -135.622 | -152.538 | -167.583 |
| G09-29 | 0 | 30.8304 | 81.211 | 47.057 | 37.4006 | 24.0837 | 26.5076 | -9.9592 | -5.7073 | -46.9917 | -89.286 |
| G09-30 | 0 | 11.2261 | 21.3348 | 31.2101 | 42.9497 | 53.9438 | 66.4757 | 77.2839 | 90.0027 | 100.7187 | 111.5744 |
| G10-31 | 0 | 27.0223 | 51.9422 | 76.3486 | 101.6454 | 125.5869 | 50.2027 | 73.3001 | 97.6613 | 120.9012 | 144.9841 |
| G10-32 | 0 | 26.8167 | 53.9606 | 81.5722 | 109.8835 | 137.3115 | 165.4396 | 192.0327 | 219.4186 | 246.0144 | 272.7511 |
| G10-33 | 0 | 20.9303 | 42.1885 | 62.9804 | 83.7268 | 104.5205 | 124.4739 | 144.3344 | 165.0374 | 184.9929 | 204.9961 |
| G10-34 | 0 | 12.5965 | 24.773 | 36.1101 | 47.8673 | 59.2983 | 71.1494 | 82.2076 | 94.6656 | 105.6776 | 117.9964 |
| G10-35 | 0 | 0.2806 | -0.4204 | -1.6357 | -2.9909 | -4.1592 | -5.1873 | -6.6831 | -6.7298 | -7.8516 | -6.8701 |
| G11-36 | 0 | -13.1398 | -28.103 | -43.3459 | -59.2434 | -75.5143 | -91.1769 | -106.559 | -120.631 | -135.824 | -148.727 |


| N = 1,300,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 5.5234 | 14.744 | 26.2588 | 40.0204 | 54.3911 | 72.5539 | 93.7131 | 117.4019 | 139.4056 | 163.4711 |
| D1B | 0 | 1.3086 | 5.093 | 8.504 | 12.8496 | 18.5035 | 24.0173 | 30.0451 | 37.1013 | 42.8956 | 48.9703 |
| D2A | 0 | 6.0613 | 20.3296 | 42.2916 | 70.4569 | 104.6398 | 145.7746 | 190.4575 | 236.3107 | 279.2288 | 324.67 |
| D2B | 0 | 6.0671 | 19.7416 | 38.7836 | 67.5349 | 102.7295 | 146.9826 | 192.8274 | 238.6293 | 275.0501 | 313.2946 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 4.7741 | 15.4926 | 25.9301 | 43.4828 | 69.6958 | 105.8346 | 144.9726 | 184.4878 | 219.1364 | 264.9796 |
| D4A | 0 | 18.288 | 71.5803 | 61.5971 | 146.5597 | 253.3573 | 443.4209 | 807.5434 | 1708.935 | 2468.185 | 1687.001 |
| D4B | 0 | 2.1175 | 7.9993 | 13.3635 | 24.5628 | 41.6915 | 65.3623 | 87.3869 | 107.3418 | 123.579 | 142.8294 |
| G04-01 | 0 | -47577.3 | -50169.9 | -48349.8 | -51028.7 | -97435.7 | -97435.7 | -97435.7 | -97435.7 | -48316.2 | -48017.1 |
| G04-02 | 0 | 138.7213 | -146.967 | -186.317 | -196.75 | -259.134 | -317.361 | -333.059 | -364.974 | -387.311 | -384.44 |
| G11-03 | 0 | 19.1402 | 40.1066 | 56.9551 | 74.7402 | 92.0108 | 108.7677 | 125.899 | 142.9845 | 159.8365 | 175.9866 |
| G04-04 | 0 | 34.3656 | 70.2704 | 109.5308 | 151.7289 | 193.3714 | 230.4988 | 264.1345 | 301.3138 | 334.1152 | 364.8692 |
| G04-05 | 0 | -73.338 | -142.957 | -190.6 | -280.149 | -293.672 | -435.95 | -530.377 | -1169.63 | -1155.58 | -1223.3 |
| G05-06 | 0 | 34.4488 | 71.9258 | 112.6185 | 152.6161 | 191.406 | 228.5689 | 263.5457 | 297.6399 | 329.5933 | 361.1762 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 7.9428 | 11.4004 | 11.6807 | 15.1384 | 19.7175 | 25.3248 | 30.698 | 37.4735 | 44.4355 | 52.9869 |
| G05-09 | 0 | -1.7229 | -11.6881 | -25.1927 | -35.3905 | -43.4462 | -49.4996 | -54.8077 | -57.9741 | -60.1623 | -59.5571 |
| G05-10 | 0 | -9.4979 | -9.032 | -22.3006 | -30.4011 | -39.5259 | -49.9072 | -61.6854 | -69.8322 | -80.213 | -88.2659 |
| G06-11 | 0 | 0.8363 | 0.5111 | -1.208 | -2.8338 | -4.6917 | -6.6426 | -8.8722 | -10.6374 | -12.8672 | -13.982 |
| G06-12 | 0 | 12.4197 | 23.085 | 31.3958 | 40.1685 | 48.2949 | 56.1447 | 63.1169 | 70.9204 | 77.4311 | 85.3275 |
| G06-13 | 0 | 24.6724 | 49.5322 | 72.0655 | 95.4845 | 120.1617 | 145.4455 | 167.3776 | 202.909 | 220.466 | 241.4699 |
| G06-14 | 0 | 24.165 | 53.3425 | 79.2899 | 98.915 | 120.6959 | 150.7691 | 170.6311 | 202.7688 | 207.7345 | 226.0531 |
| G06-15 | 0 | 22.4423 | 48.2853 | 74.0357 | 100.5794 | 126.8452 | 152.1809 | 176.6329 | 201.1792 | 223.3512 | 245.3844 |
| G07-16 | 0 | 11.1713 | 20.9465 | 29.0925 | 39.1477 | 49.0165 | 58.0941 | 66.3805 | 75.4586 | 83.4195 | 92.2183 |
| G07-17 | 0 | -1.5834 | -4.0055 | -10.2003 | -15.4171 | -20.4473 | -25.5704 | -31.3923 | -35.9102 | -39.4497 | -41.3595 |
| G07-18 | 0 | -13.5193 | -27.3166 | -46.3164 | -64.7117 | -82.8279 | -100.293 | -117.989 | -134.663 | -148.69 | -161.229 |
| G07-19 | 0 | -13.4106 | -31.0116 | -51.871 | -71.8451 | -90.5147 | -107.694 | -121.846 | -133.438 | -142.609 | -148.195 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 14.0046 | 26.5159 | 37.7202 | 50.6988 | 62.8842 | 75.1631 | 87.1156 | 99.8153 | 111.4415 | 125.1226 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 37.8707 | 76.3048 | 115.349 | 155.6571 | 195.6418 | 233.2001 | 268.2853 | 302.6252 | 329.725 | 358.6958 |
| G08-24 | 0 | 37.8457 | 77.3222 | 114.5222 | 154.7022 | 194.2342 | 232.6531 | 267.7719 | 304.5212 | 337.8772 | 368.1647 |
| G08-25 | 0 | 15.6577 | 30.0575 | 42.6404 | 58.2062 | 72.9335 | 86.6359 | 100.8514 | 114.6475 | 160.5603 | 170.2097 |
| G09-26 | 0 | -0.2321 | -3.2502 | -6.7792 | -9.797 | -12.6295 | -16.0189 | -17.1333 | -17.5048 | -16.3441 | -13.001 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -16.3551 | -36.2142 | -60.7451 | -84.4335 | -106.159 | -128.444 | -149.233 | -168.807 | -188.566 | -204.869 |
| G09-29 | 0 | -6.7644 | -13.8225 | -31.2722 | -42.4965 | -53.3775 | -61.317 | -70.5803 | -77.0984 | -85.8713 | -90.8209 |
| G09-30 | 0 | 11.8263 | 22.0697 | 30.1246 | 40.1823 | 51.3108 | 61.3689 | 71.1943 | 41.5312 | 54.4293 | 64.627 |
| G10-31 | 0 | 29.2446 | 55.086 | 78.6427 | 103.8803 | 129.6786 | 153.9387 | 177.8733 | 200.7828 | 222.1065 | 244.6443 |
| G10-32 | 0 | 26.5766 | 54.3181 | 80.9907 | 109.9919 | 139.0885 | 166.6503 | 193.1425 | 219.4031 | 244.6876 | 269.5074 |
| G10-33 | 0 | 20.3687 | 41.112 | 59.567 | 79.7043 | 100.3568 | 119.3745 | 137.9726 | 156.1504 | 173.4414 | 191.013 |
| G10-34 | 0 | 10.8194 | 20.753 | 29.1944 | 39.4545 | 49.2954 | 57.4574 | 67.019 | 76.5338 | 86.3288 | 96.9636 |
| G10-35 | 0 | -0.6538 | -3.3157 | -8.1258 | -11.3486 | -13.1699 | -16.0653 | -18.5871 | -19.8947 | -21.8094 | -21.6693 |
| G11-36 | 0 | -15.7503 | -34.2573 | -55.5674 | -74.7273 | -92.344 | -111.035 | -127.763 | -144.304 | -160.423 | -174.206 |


| N = 1,400,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 7.106 | 16.7834 | 31.5568 | 43.8062 | 62.9754 | 81.1169 | 106.4599 | 129.9333 | 156.8688 | 182.9638 |
| D1B | 0 | 1.9619 | 5.5589 | 11.5381 | 15.6024 | 23.2172 | 29.6175 | 38.4472 | 46.1555 | 55.1253 | 62.6002 |
| D2A | 0 | 6.1067 | 17.8073 | 42.0959 | 64.0545 | 101.4001 | 140.4269 | 189.1568 | 232.9474 | 281.2196 | 325.8581 |
| D2B | 0 | 6.7679 | 15.4025 | 41.5416 | 64.7409 | 104.6999 | 146.6234 | 200.1763 | 245.4236 | 289.3663 | 324.2065 |
| D3A | 0 | -698.653 | 45831.62 | 45831.62 | 45831.62 | 45831.62 | 45831.62 | 45831.62 | 45831.62 | 45831.62 | 45831.62 |
| D3B | 0 | 9.3937 | 12.118 | 34.805 | 52.5609 | 80.5582 | 112.973 | 154.7866 | 193.4554 | 234.9467 | 270.4261 |
| D4A | 0 | 13.1128 | 21.667 | 58.0929 | 87.3772 | 148.3007 | 212.5229 | 305.0625 | 387.0363 | 469.0232 | 536.3917 |
| D4B | 0 | 5.7411 | 5.506 | 19.8116 | 32.6587 | 49.4595 | 71.8142 | 96.2408 | 117.5623 | 137.3784 | 155.6418 |
| G04-01 | 0 | -4465.07 | -325.076 | 787.6492 | -1443.28 | 41.1463 | -1846.73 | -3322.9 | -4510.9 | -4524.87 | -5650.65 |
| G04-02 | 0 | 6.9402 | 11.8647 | 14.3504 | 15.945 | 16.6015 | 17.586 | 19.0863 | 22.7445 | 27.1531 | 33.3437 |
| G11-03 | 0 | 20.7792 | 41.9342 | 62.903 | 82.4676 | 101.0975 | 118.5574 | 135.6904 | 152.7772 | 170.8012 | 187.702 |
| G04-04 | 0 | 36.0917 | 76.4701 | 118.8551 | 160.0329 | 200.5615 | 237.7856 | 274.3599 | 309.4457 | 342.9967 | 374.0799 |
| G04-05 | 0 | 29.0324 | 61.2973 | 96.8933 | 138.2798 | 176.9204 | 211.7525 | 243.9329 | 274.5231 | 306.2737 | 335.4681 |
| G05-06 | 0 | 35.1782 | 74.5937 | 115.9669 | 156.2729 | 196.3498 | 232.6588 | 268.6446 | 302.9103 | 336.1078 | 367.5374 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 8.4106 | 12.85 | 15.2329 | 19.9994 | 23.9248 | 30.0462 | 36.2146 | 43.2243 | 50.935 | 60.4682 |
| G05-09 | 0 | -1.1642 | -10.1987 | -22.5391 | -31.9455 | -41.0726 | -46.4275 | -51.4096 | -54.1571 | -55.7868 | -54.39 |
| G05-10 | 0 | -7.68 | -16.3374 | -25.3206 | -33.326 | -43.3794 | -51.6174 | -60.646 | -68.6046 | -76.7956 | -83.0318 |
| G06-11 | 0 | 1.5326 | 2.3686 | 1.8577 | 1.7183 | -0.4644 | -1.9041 | -4.1803 | -5.1555 | -7.1066 | 7.0138 |
| G06-12 | 0 | 11.5892 | 22.2557 | 32.5528 | 42.1573 | 50.6079 | 58.4582 | 66.124 | 73.651 | 81.5477 | 89.2139 |
| G06-13 | 0 | 25.7507 | 51.6893 | 79.5851 | 105.107 | 132.214 | 154.7114 | 179.0258 | 201.8045 | 225.4229 | 246.154 |
| G06-14 | 0 | 27.0601 | 54.0748 | 83.1983 | 110.8247 | 140.373 | 164.6307 | 190.7632 | 214.8361 | 240.0811 | 262.5639 |
| G06-15 | 0 | 26.4935 | 51.545 | 80.4161 | 107.9387 | 137.1392 | 161.4975 | 187.3475 | 211.8478 | 237.1415 | 258.6165 |
| G07-16 | 0 | 11.2649 | 22.3898 | 32.1652 | 42.9653 | 53.1604 | 62.2847 | 70.2917 | 78.9977 | 87.8897 | 97.5267 |
| G07-17 | 0 | -0.8848 | -1.2108 | -7.1716 | -9.1275 | -16.2522 | -19.2323 | -25.7053 | -29.0578 | -33.6213 | -33.9006 |
| G07-18 | 0 | -13.2395 | -24.5739 | -44.0373 | -58.7149 | -80.7318 | -94.2014 | -113.151 | -128.338 | -144.36 | -154.02 |
| G07-19 | 0 | -9.3093 | -22.389 | -38.9124 | -55.063 | -73.6799 | -90.2014 | -104.303 | -115.984 | -125.198 | -131.667 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 16.0544 | 31.3624 | 46.4377 | 61.8865 | 75.8425 | 89.0519 | 102.7285 | 116.6854 | 130.783 | 144.8806 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 36.6533 | 74.4296 | 114.0759 | 154.2405 | 193.8007 | 231.3083 | 267.2772 | 301.9876 | 335.392 | 366.8828 |
| G08-24 | 0 | 36.398 | 73.8211 | 112.596 | 152.1635 | 191.456 | 228.565 | 264.096 | 298.8385 | 331.4905 | 362.7028 |
| G08-25 | 0 | 17.9965 | 35.1072 | 51.9394 | 68.9584 | 85.2318 | 100.713 | 115.1224 | 130.3249 | 145.5276 | 160.6843 |
| G09-26 | 0 | 3.9473 | 6.3153 | 5.6187 | 5.2011 | 3.5757 | 1.3467 | -0.4178 | -0.4178 | 0.6035 | 3.2507 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -15.7475 | -33.0367 | -54.4367 | -75.6959 | -98.7764 | -120.127 | -139.701 | -158.855 | -176.606 | -192.628 |
| G09-29 | 0 | 0.9798 | -3.038 | -7.3986 | -12.445 | -18.7652 | -25.5261 | -29.348 | -32.7281 | -37.2354 | -38.2645 |
| G09-30 | 0 | 14.4602 | 27.2467 | 39.4292 | 51.7512 | 63.8417 | 75.9787 | 87.1857 | 98.7187 | 109.9732 | 121.1812 |
| G10-31 | 0 | 27.9371 | 54.5697 | 80.3641 | 107.4193 | 133.7763 | 158.9683 | 183.6951 | 208.0028 | 231.099 | 253.8694 |
| G10-32 | 0 | 26.853 | 54.4991 | 82.9842 | 112.5415 | 141.8213 | 169.6599 | 196.7549 | 223.3858 | 249.4596 | 274.8361 |
| G10-33 | 0 | 21.8134 | 43.7212 | 65.6301 | 87.5863 | 109.2638 | 129.4931 | 149.6302 | 169.2075 | 188.1313 | 207.1027 |
| G10-34 | 0 | 12.5447 | 24.7632 | 36.3761 | 47.4762 | 58.0166 | 67.9044 | 78.5387 | 89.0797 | 99.9475 | 111.0022 |
| G10-35 | 0 | 0.7939 | -0.3269 | -2.7086 | -4.6233 | -7.5187 | -10.274 | -11.7684 | -12.8892 | -13.2161 | -12.7024 |
| G11-36 | 0 | -13.8797 | -30.4229 | -48.9749 | -66.8254 | -86.4973 | -104.627 | -120.373 | -135.791 | -150.134 | -163.403 |


| N = 1,500,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 4.5838 | 14.1722 | 24.5091 | 42.1435 | 56.3638 | 79.7059 | 101.5995 | 128.8734 | 154.6993 | 183.5671 |
| D1B | 0 | 1.2147 | 6.0268 | 9.7641 | 17.2863 | 23.36 | 33.3115 | 40.9271 | 49.9446 | 58.9153 | 67.9334 |
| D2A | 0 | 4.8481 | 19.5329 | 36.1293 | 68.1585 | 99.1635 | 147.5637 | 190.8854 | 237.9883 | 281.3644 | 329.3624 |
| D2B | 0 | 4.8595 | 16.4471 | 34.483 | 68.2207 | 100.9321 | 155.5656 | 214.5989 | 282.2871 | 303.2308 | 340.9605 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 5.2562 | 11.8734 | 24.8736 | 45.0549 | 68.5694 | 106.7304 | 60.1269 | 77.8856 | 242.6873 | 275.79 |
| D4A | 0 | 10.7519 | 25.6772 | 51.3061 | 98.3954 | 146.5201 | 240.8185 | 240.873 | 249.7984 | 508.4859 | 582.5521 |
| D4B | 0 | 4.7505 | 7.7607 | 16.8857 | 29.4444 | 46.2366 | 73.6609 | 55.5984 | 66.5169 | 147.9442 | 162.5766 |
| G04-01 | 0 | -4.4575 | -14.3946 | -27.4417 | -43.2282 | -58.3641 | -73.1278 | -83.6667 | -93.1839 | -98.4302 | -101.819 |
| G04-02 | 0 | 8.0152 | 13.8868 | 15.9838 | 16.5896 | 19.4788 | 19.1992 | 21.1564 | 23.8126 | 29.1255 | 35.0908 |
| G11-03 | 0 | 20.3224 | 41.5821 | 62.2337 | 81.4346 | 99.4187 | 117.0753 | 134.2172 | 151.5945 | 169.0654 | 186.7716 |
| G04-04 | 0 | 36.3186 | 78.0488 | 120.0158 | 161.0067 | 200.602 | 239.2207 | 274.9038 | 309.9362 | 342.9651 | 374.5031 |
| G04-05 | 0 | 25.9674 | 29.9786 | 48.4979 | 26.2392 | 31.0831 | 2.1048 | 21.9565 | 49.098 | 44.3246 | 54.8233 |
| G05-06 | 0 | 35.7195 | 75.5978 | 117.3009 | 157.7924 | 197.3537 | 235.7963 | 272.187 | 306.7581 | 340.4904 | 372.6363 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 8.7855 | 12.2435 | 15.9355 | 19.0197 | 24.394 | 29.3946 | 36.685 | 43.2747 | 51.8272 | 60.8474 |
| G05-09 | 0 | -1.1184 | -11.4606 | -22.1751 | -33.1692 | -40.4359 | -48.0291 | -51.709 | -55.1098 | -55.6688 | -54.5972 |
| G05-10 | 0 | -5.3988 | -13.171 | -19.7796 | -27.7842 | -35.37 | -43.3745 | -49.4707 | -57.5213 | -63.4784 | -69.9464 |
| G06-11 | 0 | 4.2173 | 6.581 | 6.6273 | -1.1129 | 0.6484 | -1.8543 | -1.5301 | -1.5301 | -0.9274 | -0.2323 |
| G06-12 | 0 | 88.4542 | 165.725 | 259.305 | 309.4935 | 349.9624 | 429.2066 | 437.5293 | 460.7807 | 423.3866 | 472.1278 |
| G06-13 | 0 | 25.9316 | 53.2613 | 79.8477 | 106.7613 | 131.4412 | 156.7743 | 179.8732 | 203.5788 | 225.9345 | 248.99 |
| G06-14 | 0 | 27.0425 | 54.6531 | 79.7125 | 101.8885 | 125.0112 | 145.3451 | 166.7205 | 190.3188 | 210.6552 | 233.5469 |
| G06-15 | 0 | 19.4956 | 41.3709 | 64.8797 | 89.8825 | 111.1077 | 134.7132 | 156.0335 | 178.148 | 198.9104 | 220.8403 |
| G07-16 | 0 | 11.0378 | 20.958 | 30.832 | 40.1009 | 50.1615 | 58.1263 | 67.1628 | 74.988 | 84.6768 | 93.7603 |
| G07-17 | 0 | -0.9319 | -3.8199 | -6.8474 | -13.4155 | -16.3035 | -23.9429 | -27.8553 | -33.3982 | -35.1219 | -38.1491 |
| G07-18 | 0 | -12.7313 | -27.8321 | -42.8392 | -63.1884 | -77.5908 | -98.8217 | -113.176 | -131.386 | -143.092 | -157.074 |
| G07-19 | 0 | -2.4299 | -11.4471 | -21.1185 | -34.7138 | -49.1033 | -64.6603 | -78.6752 | -90.2138 | -100.164 | -106.984 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 16.6194 | 33.0056 | 49.7195 | 65.0336 | 79.9274 | 93.1876 | 107.1485 | 121.2499 | 135.0249 | 149.6876 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 31.4763 | 64.6363 | 99.7134 | 136.8952 | 172.7247 | 207.1087 | 241.4952 | 274.4824 | 307.3315 | 337.4721 |
| G08-24 | 0 | 35.4745 | 70.9985 | 107.0825 | 144.658 | 180.5615 | 216.2815 | 250.5625 | 283.6825 | 316.107 | 346.4399 |
| G08-25 | 0 | 17.6664 | 34.3071 | 51.4153 | 68.1039 | 84.2337 | 99.5717 | 114.9568 | 129.6431 | 145.1222 | 160.229 |
| G09-26 | 0 | 5.1093 | 8.3139 | 10.9151 | 11.4723 | 12.2156 | 11.3796 | 11.3794 | 12.587 | 14.3056 | 17.7427 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -14.9537 | -32.3364 | -50.4664 | -72.1001 | -92.4247 | -113.029 | -132.37 | -150.403 | -167.875 | -183.478 |
| G09-29 | 0 | -1.0887 | -3.7396 | -6.1538 | -10.7456 | -14.4378 | -18.5562 | -21.7278 | -25.2306 | -27.8815 | -29.1122 |
| G09-30 | 0 | 12.6588 | 24.247 | 35.743 | 47.7971 | 59.4796 | 70.8831 | 81.8214 | 92.5736 | 103.4652 | 114.7768 |
| G10-31 | 0 | 29.3892 | 54.0676 | 80.5673 | 106.6484 | 132.5439 | 157.5544 | 182.0996 | 206.0394 | 229.3267 | 252.1017 |
| G10-32 | 0 | 26.6818 | 54.0635 | 82.2387 | 111.5331 | 139.3852 | 167.6115 | 194.8147 | 221.2271 | 247.1287 | 272.3799 |
| G10-33 | 0 | 21.7796 | 43.1395 | 65.1083 | 86.7506 | 107.2249 | 127.6535 | 147.615 | 166.8767 | 185.9989 | 204.7473 |
| G10-34 | 0 | 11.7029 | 20.4686 | 32.9642 | 44.4813 | 55.2058 | 65.8373 | 76.5157 | 86.8678 | 98.1997 | 109.3918 |
| G10-35 | 0 | 0.6538 | -1.5878 | -2.9421 | -5.6507 | -8.0791 | -11.1613 | -12.6557 | -14.01 | -14.1501 | -13.7298 |
| G11-36 | 0 | -14.0204 | -33.0416 | -49.8187 | -69.7262 | -87.3434 | -105.427 | -121.688 | -137.34 | -151.778 | -164.44 |


| N = 1,700,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 7.3466 | 55.8742 | 89.5264 | 106.5168 | 115.5037 | 140.3596 | 170.6931 | 198.7819 | 226.17 | 255.1044 |
| D1B | 0 | -0.5604 | 2.803 | 8.5491 | 13.1734 | 18.0789 | 29.5709 | 40.2689 | 49.052 | 57.5549 | 68.1136 |
| D2A | 0 | 5.1732 | 15.8461 | 37.5187 | 57.7009 | 93.1266 | 138.5308 | 187.4827 | 230.9374 | 276.6338 | 323.5007 |
| D2B | 0 | 5.5151 | 15.8447 | 37.1584 | 60.0624 | 97.5059 | 153.1852 | 218.6435 | 272.2318 | 320.9621 | 362.2602 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 2.9572 | 9.2481 | 22.4396 | 37.932 | 60.7964 | 98.3106 | 143.012 | 182.3641 | 222.9872 | 260.9362 |
| D4A | 0 | 6.4564 | 16.7961 | 43.0912 | 71.8208 | 131.7183 | 214.6977 | 319.2724 | 402.9376 | 487.7882 | 555.9785 |
| D4B | 0 | 6.8649 | 11.1916 | 19.4678 | 31.7883 | 46.2728 | 74.4431 | 102.5678 | 128.1073 | 148.568 | 169.9232 |
| G04-01 | 0 | -3.5746 | -12.5802 | -26.8772 | -41.2202 | -63.6391 | -73.8504 | -84.9898 | -93.0196 | -105.643 | -103.508 |
| G04-02 | 0 | 12.6252 | 18.2826 | 28.8477 | 24.2664 | -101.553 | -79.2025 | -142.83 | -141.941 | -142.314 | -133.105 |
| G11-03 | 0 | 20.7951 | 43.2306 | 63.981 | 84.4977 | 98.7385 | 120.3341 | 139.1666 | 157.6253 | 169.5254 | 191.5456 |
| G04-04 | 0 | 35.909 | 76.5312 | 118.2295 | 156.3395 | 182.043 | 224.8231 | 259.1151 | 293.7361 | 320.3801 | 356.2187 |
| G04-05 | 0 | 25.6202 | 46.7322 | 68.5658 | 89.1057 | 3454.169 | 49997.49 | 49997.49 | 49997.49 | 49997.49 | 49997.49 |
| G05-06 | 0 | 33.796 | 73.2665 | 113.7163 | 154.0759 | 1207.486 | 759.5659 | 769.8362 | 802.2388 | 1809.825 | 1779.342 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 9.2549 | 14.0697 | 16.3601 | 21.3149 | 20.5205 | 30.6172 | 37.3487 | 45.2021 | 44.4074 | 60.3019 |
| G05-09 | 0 | -1.3048 | -9.9248 | -22.9713 | -31.5911 | -45.9882 | -47.4794 | -51.952 | -54.2354 | -63.6462 | -56.611 |
| G05-10 | 0 | -7.5905 | -16.345 | -25.7049 | -33.0619 | -46.6123 | -49.7787 | -57.7413 | -65.3306 | -80.9289 | -81.4409 |
| G06-11 | 0 | 0.696 | -0.0464 | -2.5056 | -1.7632 | -11.5994 | -7.1451 | -6.6347 | -4.8254 | -8.9083 | -4.82 |
| G06-12 | 0 | 14.3 | 27.5394 | 39.6258 | 50.7901 | 57.9869 | 71.1811 | 82.1614 | 91.9885 | 97.0176 | 109.7515 |
| G06-13 | 0 | 26.2145 | 52.384 | 80.7435 | 106.8695 | 128.1528 | 155.8644 | 180.3641 | 204.0261 | 222.4722 | 247.2078 |
| G06-14 | 0 | 25.0719 | 51.7006 | 79.8862 | 109.9588 | 132.6322 | 162.5663 | 189.2966 | 214.7554 | 234.8405 | 260.8202 |
| G06-15 | 0 | 26.1767 | 51.6069 | 79.2355 | 102.9853 | 198.0911 | 202.3465 | 212.9142 | 229.0471 | 235.1721 | 257.3394 |
| G07-16 | 0 | 11.3204 | 21.9432 | 31.4476 | 42.7223 | 47.4281 | 60.5671 | 69.2803 | 78.4595 | 80.8822 | 94.4879 |
| G07-17 | 0 | -1.3519 | -2.0043 | -7.9691 | -9.554 | -21.903 | -22.835 | -28.3804 | -31.6424 | -41.0556 | -38.9586 |
| G07-18 | 0 | -13.0593 | -24.9568 | -44.7537 | -58.0444 | -82.8123 | -94.6617 | -112.597 | -126.769 | -147.91 | -154.972 |
| G07-19 | 0 | -6.4877 | -13.7218 | -24.3155 | -37.896 | -54.183 | -62.7698 | -77.1891 | -87.0352 | -101.314 | -109.153 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 17.459 | 35.0582 | 52.3314 | 68.9521 | 76.4219 | 96.7315 | 112.0927 | 126.9408 | 137.0265 | 154.2565 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 35.3339 | 71.7441 | 108.8104 | 147.42 | 197.8924 | 241.3652 | 277.7907 | 303.429 | 330.0038 | 362.979 |
| G08-24 | 0 | 35.6119 | 72.1098 | 108.6568 | 150.1353 | 184.1768 | 223.1036 | 258.5453 | 292.0358 | 320.7837 | 352.8825 |
| G08-25 | 0 | 17.6999 | 34.9345 | 51.5173 | 69.9642 | 79.8407 | 101.2233 | 116.877 | 133.2766 | 142.5946 | 162.3961 |
| G09-26 | 0 | 4.5978 | 7.106 | 8.6852 | 10.7286 | 7.431 | 10.7288 | 10.1714 | 12.5866 | 12.0758 | 17.8813 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -15.7027 | -32.9004 | -51.9668 | -71.1258 | -99.3021 | -114.862 | -133.972 | -150.511 | -173.964 | -183.589 |
| G09-29 | 0 | 2.441 | 2.825 | -0.9544 | -15.5982 | -35.84 | -29.7601 | -39.2329 | -53.3008 | -70.4313 | -69.7611 |
| G09-30 | 0 | 10.9285 | 21.1585 | 31.7616 | 42.5975 | 39.3425 | 55.712 | 66.7807 | 77.2451 | 81.338 | 96.0352 |
| G10-31 | 0 | 28.6409 | 54.4376 | 80.3757 | 107.0616 | 139.4875 | 167.1098 | 191.2335 | 215.9655 | 233.6519 | 260.2991 |
| G10-32 | 0 | 26.4923 | 54.0569 | 81.9491 | 111.2397 | 133.3141 | 167.1719 | 195.6759 | 222.3648 | 243.2326 | 271.5547 |
| G10-33 | 0 | 21.254 | 42.6024 | 63.7182 | 86.3763 | 98.6168 | 125.9956 | 147.4421 | 167.6275 | 180.5709 | 203.7953 |
| G10-34 | 0 | 12.0329 | 20.1487 | 31.669 | 44.5421 | 48.787 | 66.2783 | 77.7532 | 89.5082 | 92.9135 | 108.7274 |
| G10-35 | 0 | 0.0463 | -0.9346 | -2.7571 | -4.9066 | -14.5794 | -10.8879 | -12.57 | -13.0839 | -19.0186 | -14.1589 |
| G11-36 | 0 | -13.3649 | -29.9537 | -47.8499 | -65.4657 | -93.2661 | -105.087 | -121.86 | -135.782 | -157.645 | -164.139 |


| N = 1,900,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 3.748 | 10.4943 | 22.1131 | 33.1698 | 53.457 | 74.6821 | 101.0152 | 126.7874 | 154.5294 | 181.4762 |
| D1B | 0 | 2.1021 | 6.3999 | 12.4263 | 16.7708 | 26.5809 | 35.9246 | 45.5482 | 54.425 | 63.9091 | 72.412 |
| D2A | 0 | 6.0605 | 16.55 | 37.9497 | 58.6042 | 99.6828 | 142.8631 | 190.9439 | 234.878 | 281.8948 | 326.8633 |
| D2B | 0 | 4.0673 | 15.6618 | 37.8228 | 61.4812 | 107.3516 | 162.3926 | 221.7421 | 277.1697 | 299.9512 | 414.9044 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 3.3789 | 9.1048 | 22.1992 | 37.9697 | 66.836 | 103.0276 | 141.6153 | 186.122 | 124.3524 | 111.4097 |
| D4A | 0 | 7.3927 | 16.1894 | 42.3931 | 71.6866 | 138.6099 | 223.6575 | 317.802 | 408.9204 | 409.8158 | 397.4247 |
| D4B | 0 | 1.364 | 3.7157 | 10.3942 | 21.4466 | 41.7655 | 67.4468 | 90.7308 | 118.2965 | 78.2221 | 77.7072 |
| G04-01 | 0 | -5.0143 | -13.696 | -27.9952 | -43.2689 | -59.192 | -73.0721 | -85.0952 | -94.6577 | -100.739 | -103.802 |
| G04-02 | 0 | 7.1858 | 12.785 | 16.0979 | 17.3579 | 30.9831 | 31.2632 | 34.7159 | 37.7958 | 42.5555 | 48.3883 |
| G11-03 | 0 | 20.1823 | 42.0978 | 63.0778 | 81.9979 | 101.3404 | 118.4819 | 136.0931 | 153.751 | 171.4097 | 188.6479 |
| G04-04 | 0 | 35.8311 | 76.9774 | 119.2447 | 160.1181 | 201.6939 | 239.2175 | 276.1845 | 311.6158 | 345.0911 | 376.4238 |
| G04-05 | 0 | 41.4997 | 55.2706 | 91.8646 | 129.3766 | 165.2068 | 179.605 | 206.2384 | 245.493 | 266.7351 | 282.6311 |
| G05-06 | 0 | 33.9705 | 73.7928 | 115.6778 | 155.6002 | 195.9003 | 233.3479 | 270.2837 | 306.0049 | 339.0126 | 370.3372 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 9.0662 | 14.1599 | 16.6835 | 21.497 | 25.9368 | 31.4512 | 37.7139 | 44.8642 | 52.529 | 61.7362 |
| G05-09 | 0 | -0.9783 | -9.5489 | -21.7057 | -30.555 | -39.9169 | -46.2511 | -51.2347 | -54.2619 | -55.8453 | -54.6808 |
| G05-10 | 0 | -6.9364 | -15.4084 | -24.9978 | -33.3763 | -42.686 | -51.7625 | -61.0721 | -69.1709 | -77.3626 | -83.553 |
| G06-11 | 0 | 1.5312 | 2.1346 | 1.6241 | 0.8352 | -1.4848 | -3.7121 | -6.2186 | -7.4714 | -9.049 | -9.3274 |
| G06-12 | 0 | 12.2766 | 23.5418 | 33.6119 | 43.4981 | 52.5568 | 60.0522 | 68.1452 | 76.8369 | 84.9301 | 91.9202 |
| G06-13 | 0 | 32.5213 | 58.9876 | 86.899 | 112.2028 | 163.8386 | 186.1627 | 210.8656 | 233.705 | 256.5921 | 277.6623 |
| G06-14 | 0 | 25.3668 | 52.0549 | 81.5267 | 111.6606 | 143.6827 | 170.8016 | 197.5918 | 226.4117 | 256.9316 | 282.7356 |
| G06-15 | 0 | 26.0538 | 50.7994 | 78.6333 | 104.0365 | 101.4145 | 113.9523 | 137.7195 | 161.7214 | 184.6489 | 205.3309 |
| G07-16 | 0 | 11.4602 | 22.6876 | 32.378 | 43.5595 | 53.2967 | 62.0096 | 70.0228 | 79.435 | 87.7754 | 97.0478 |
| G07-17 | 0 | -1.3048 | -2.0504 | -7.8298 | -10.3928 | -15.9858 | -20.2733 | -26.6582 | -30.1998 | -34.3015 | -35.42 |
| G07-18 | 0 | -13.1978 | -25.4192 | -44.6102 | -58.3639 | -78.7618 | -94.2804 | -113.144 | -128.057 | -143.574 | -153.516 |
| G07-19 | 0 | -5.4172 | -12.7491 | -22.9764 | -37.2656 | -50.2472 | -65.8907 | -79.9921 | -93.1595 | -103.852 | -111.743 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 16.9419 | 34.2102 | 51.1528 | 67.6289 | 82.6123 | 96.2424 | 110.0125 | 124.1098 | 138.1613 | 152.1197 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 36.0402 | 73.7169 | 111.5833 | 150.8535 | 188.1186 | 224.7791 | 259.4812 | 294.3254 | 327.5838 | 359.0229 |
| G08-24 | 0 | 35.7394 | 72.6429 | 109.8744 | 148.0853 | 185.2763 | 221.3543 | 255.807 | 290.3088 | 322.6739 | 353.4601 |
| G08-25 | 0 | 18.068 | 35.0659 | 51.8782 | 69.2494 | 86.0163 | 101.3863 | 116.6633 | 132.8263 | 147.9648 | 163.3365 |
| G09-26 | 0 | 3.2964 | 4.9677 | 5.9427 | 6.6854 | 6.9642 | 6.0355 | 6.082 | 7.0572 | 8.7282 | 11.8389 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -15.2348 | -32.7117 | -52.0573 | -73.2718 | -93.4096 | -114.062 | -132.143 | -150.596 | -167.742 | -183.485 |
| G09-29 | 0 | -1.8651 | -5.2119 | -9.3723 | -15.3015 | -20.179 | -23.2869 | -22.7609 | -25.9169 | -29.5986 | -31.7981 |
| G09-30 | 0 | 10.8907 | 21.7351 | 32.3467 | 42.959 | 52.9196 | 63.2061 | 73.8654 | 84.1059 | 93.8813 | 103.9826 |
| G10-31 | 0 | 28.7318 | 55.4594 | 82.2816 | 109.0121 | 136.0239 | 161.4039 | 186.3657 | 210.722 | 234.4262 | 256.8716 |
| G10-32 | 0 | 27.2349 | 54.7971 | 83.1064 | 112.3015 | 141.545 | 169.8122 | 196.9169 | 224.2559 | 250.2455 | 275.3515 |
| G10-33 | 0 | 22.0951 | 44.0514 | 66.1019 | 88.2467 | 110.299 | 131.4176 | 152.1169 | 172.6299 | 191.6488 | 210.9486 |
| G10-34 | 0 | 12.3577 | 24.6226 | 35.9081 | 47.0074 | 55.169 | 67.2012 | 77.8816 | 89.5882 | 100.6421 | 111.7428 |
| G10-35 | 0 | -0.0934 | -1.2142 | -3.2223 | -5.8375 | -8.4061 | -11.3958 | -13.1704 | -14.1044 | -14.945 | -14.8983 |
| G11-36 | 0 | -15.0423 | -31.9994 | -49.6565 | -69.5547 | -88.4722 | -107.435 | -123.688 | -140.314 | -155.213 | -168.989 |


| N = 2,100,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 4.2035 | 10.7892 | 22.6997 | 33.2556 | 52.8272 | 72.9132 | 100.0539 | 124.72 | 154.2928 | 183.0734 |
| D1B | 0 | 1.6819 | 6.0268 | 12.3805 | 16.585 | 26.5361 | 34.5258 | 44.7571 | 53.1672 | 63.2128 | 73.3984 |
| D2A | 0 | 5.2669 | 15.7087 | 36.3126 | 55.7992 | 95.0067 | 135.3364 | 183.7829 | 227.1975 | 274.1603 | 320.5214 |
| D2B | 0 | 5.0908 | 14.5255 | 36.1516 | 60.7684 | 104.5392 | 154.8548 | 212.371 | 396.8966 | 404.3673 | 359.4863 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 0.422 | 5.5351 | 16.7931 | 32.8354 | 58.5425 | 92.977 | 121.9715 | 65.0755 | 115.7415 | 180.7227 |
| D4A | 0 | 7.5746 | 14.3547 | 41.4752 | 69.9067 | 132.527 | 211.9475 | 303.1706 | 287.5566 | 373.6001 | 511.5031 |
| D4B | 0 | -1.6447 | -0.3758 | 7.1427 | 19.5017 | 38.111 | 62.5487 | 77.3532 | 65.0421 | 86.1443 | 132.3912 |
| G04-01 | 0 | -4.4569 | -13.2311 | -27.2507 | -43.3594 | -59.2812 | -73.8107 | -86.1573 | -95.0693 | -101.429 | -104.863 |
| G04-02 | 0 | 6.9444 | 12.9092 | 16.1256 | 17.1042 | 18.0828 | 19.1546 | 21.0652 | 24.8408 | 29.6414 | 35.8865 |
| G11-03 | 0 | 20.2618 | 41.8789 | 63.2635 | 81.8004 | 100.8986 | 118.2224 | 135.6872 | 153.4332 | 171.0858 | 188.4592 |
| G04-04 | 0 | 35.1013 | 75.6056 | 118.3015 | 159.6509 | 200.2117 | 237.7487 | 274.4501 | 309.617 | 343.2959 | 375.1595 |
| G04-05 | 0 | 22.2631 | 60.4917 | 98.7234 | 134.5889 | 173.442 | 207.3697 | 240.4469 | 267.8869 | 299.8302 | 329.0744 |
| G05-06 | 0 | 34.5489 | 74.7424 | 116.9445 | 157.7974 | 198.3272 | 235.9682 | 272.6328 | 307.2936 | 341.024 | 372.937 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 8.4574 | 13.8311 | 16.9616 | 21.6345 | 25.8399 | 31.4007 | 37.4283 | 45.092 | 52.5221 | 61.588 |
| G05-09 | 0 | -1.3508 | -9.5479 | -21.2847 | -30.8317 | -39.8205 | -46.5735 | -51.6963 | -54.4904 | -56.2137 | -55.2356 |
| G05-10 | 0 | -7.2974 | -15.3849 | -23.937 | -32.8142 | -43.0389 | -51.2653 | -59.6775 | -66.9739 | -75.5715 | -81.1484 |
| G06-11 | 0 | 1.6246 | 2.2744 | 2.3206 | 1.0214 | -1.5316 | -3.899 | -6.3592 | -7.5662 | -9.5621 | 9.9335 |
| G06-12 | 0 | 11.5027 | 22.5898 | 33.8616 | 43.0544 | 51.4628 | 59.4554 | 67.217 | 75.302 | 83.0638 | 91.1958 |
| G06-13 | 0 | 26.1246 | 53.044 | 81.1311 | 107.3531 | 134.277 | 157.7953 | 182.2016 | 205.4424 | 228.451 | 250.4804 |
| G06-14 | 0 | 26.9465 | 54.3595 | 84.9855 | 111.8432 | 140.9829 | 166.1681 | 192.2851 | 216.7738 | 241.4971 | 264.4981 |
| G06-15 | 0 | 26.3777 | 52.8502 | 82.255 | 109.1956 | 137.8595 | 162.7554 | 188.4901 | 212.9698 | 237.4507 | 259.7451 |
| G07-16 | 0 | 11.0847 | 22.1233 | 32.7429 | 43.5024 | 52.8648 | 62.1808 | 70.1927 | 78.9965 | 87.2416 | 96.6052 |
| G07-17 | 0 | -1.6303 | -2.282 | -7.5916 | -9.9667 | -16.0681 | -19.8404 | -25.8483 | -29.2015 | -33.9517 | -35.1626 |
| G07-18 | 0 | -13.5198 | -25.9241 | -44.9717 | -59.0014 | -79.2551 | -94.3989 | -113.351 | -127.332 | -143.264 | -154.69 |
| G07-19 | 0 | -5.4568 | -13.6186 | -26.3974 | -42.627 | -58.0631 | -74.4785 | -90.8459 | -103.343 | -114.721 | -122.741 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 16.6222 | 34.4596 | 51.55 | 67.8004 | 83.118 | 97.1747 | 110.9045 | 124.8221 | 138.5529 | 153.0319 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 35.2193 | 73.1004 | 112.2906 | 152.3702 | 189.9802 | 226.9394 | 263.1547 | 297.9722 | 330.9722 | 362.2467 |
| G08-24 | 0 | 36.6741 | 74.7913 | 113.3771 | 152.6163 | 190.4168 | 226.8722 | 262.4928 | 297.0468 | 329.3705 | 360.7194 |
| G08-25 | 0 | 17.8297 | 34.9155 | 52.2813 | 69.1354 | 85.7571 | 101.1686 | 116.1154 | 131.6212 | 146.2893 | 161.5631 |
| G09-26 | 0 | 2.9259 | 3.8549 | 4.18 | 3.9943 | 4.273 | 2.9261 | 1.6719 | 2.7866 | 3.9945 | 7.3847 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -15.8438 | -33.2291 | -53.1375 | -74.9615 | -95.6626 | -117.671 | -137.156 | -155.332 | -173.226 | -188.831 |
| G09-29 | 0 | -1.4929 | -3.9649 | -7.1833 | -11.6611 | -15.7659 | -20.3838 | -23.882 | -26.9605 | -29.9921 | -31.1117 |
| G09-30 | 0 | 11.314 | 22.5816 | 33.9429 | 44.6991 | 55.7347 | 65.8858 | 76.5029 | 86.9341 | 96.527 | 107.3313 |
| G10-31 | 0 | 29.6246 | 56.8717 | 83.3266 | 110.3901 | 137.4081 | 163.1673 | 187.7151 | 212.4968 | 235.7398 | 258.4703 |
| G10-32 | 0 | 26.7664 | 55.0708 | 84.3084 | 113.8265 | 142.2755 | 169.9344 | 197.9676 | 224.931 | 250.6384 | 276.2075 |
| G10-33 | 0 | 21.95 | 44.4147 | 66.8337 | 88.7402 | 110.4605 | 131.715 | 152.363 | 172.4042 | 192.0726 | 211.2746 |
| G10-34 | 0 | 12.218 | 24.1561 | 35.535 | 46.7742 | 57.7807 | 67.9012 | 78.3017 | 89.7752 | 100.1761 | 111.7897 |
| G10-35 | 0 | 0.3736 | -0.7472 | -3.269 | -6.2116 | -9.2009 | -12.3765 | -14.8516 | -16.0191 | -16.6729 | -16.2526 |
| G11-36 | 0 | -13.9764 | -31.5052 | -50.1083 | -69.8793 | -88.9016 | -108.344 | -126.383 | -142.132 | -157.647 | -170.591 |


| N = 2,300,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P (kip) | 0 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 | 87 |
| D1A | 0 | 3.1296 | 11.0249 | 19.5273 | 34.6175 | 49.0069 | 71.4799 | 95.2621 | 122.5496 | 150.1663 | 181.2891 |
| D1B | 0 | 0.0465 | 4.0635 | 8.267 | 15.1801 | 21.1125 | 30.0809 | 39.563 | 49.0451 | 58.6683 | 69.2722 |
| D2A | 0 | 4.9406 | 16.4542 | 31.3703 | 59.0132 | 88.476 | 133.2331 | 177.9472 | 224.3906 | 269.393 | 318.7833 |
| D2B | 0 | 4.1565 | 13.8717 | 33.1152 | 64.7381 | 98.8855 | 153.7788 | 219.9855 | 276.8077 | 331.8137 | 375.6547 |
| D3A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D3B | 0 | 6.1079 | 11.0882 | 24.3851 | 42.334 | 65.3584 | 100.9308 | 143.3207 | 182.8467 | 229.9435 | 268.3011 |
| D4A | 0 | 9.9198 | 19.8865 | 41.4113 | 77.2567 | 124.4774 | 208.681 | 299.6869 | 389.3513 | 475.004 | 549.2421 |
| D4B | 0 | 12.3656 | 16.7854 | 26.1894 | 38.4619 | 57.553 | 84.4982 | 114.5958 | 135.3828 | 162.8024 | 181.428 |
| G04-01 | 0 | -3.3423 | -11.9771 | -25.8577 | -42.7084 | -57.7946 | -71.906 | -84.206 | -93.3498 | -99.291 | -103.144 |
| G04-02 | 0 | 7.3181 | 12.8646 | 15.5684 | 16.4538 | 17.7121 | 18.9246 | 20.6954 | 24.3778 | 29.7388 | 36.0318 |
| G11-03 | 0 | 20.8282 | 42.8239 | 62.8602 | 82.5229 | 101.0658 | 118.9549 | 135.911 | 153.8015 | 171.506 | 188.6971 |
| G04-04 | 0 | 35.4356 | 76.369 | 118.0976 | 160.3418 | 200.3066 | 238.9241 | 274.796 | 310.5302 | 344.1239 | 375.5302 |
| G04-05 | 0 | 35.2655 | 75.9056 | 117.2033 | 159.5792 | 199.6221 | 238.313 | 273.829 | 309.628 | 343.2328 | 374.2695 |
| G05-06 | 0 | 34.9378 | 75.1493 | 116.3902 | 157.9617 | 198.183 | 237.241 | 272.7552 | 307.5251 | 341.1769 | 372.591 |
| G05-07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G05-08 | 0 | 9.3951 | 14.4902 | 18.4637 | 21.7824 | 27.298 | 32.7205 | 38.891 | 45.6692 | 53.943 | 61.703 |
| G05-09 | 0 | -1.0248 | -9.689 | -20.822 | -32.141 | -39.7333 | -47.0927 | -51.7506 | -55.2904 | -56.548 | -56.6877 |
| G05-10 | 0 | -8.089 | -17.2937 | -26.173 | -37.3297 | -46.348 | -56.7141 | -66.5687 | -75.5866 | -82.8844 | -91.2972 |
| G06-11 | 0 | 1.6242 | 1.671 | 1.9494 | -0.7427 | -1.9959 | -4.827 | -7.3795 | -9.7004 | -10.4892 | -12.2993 |
| G06-12 | 0 | 12.1034 | 23.329 | 33.955 | 42.9178 | 51.6034 | 59.6422 | 66.9418 | 74.842 | 83.0666 | 90.5048 |
| G06-13 | 0 | 25.9277 | 53.5855 | 80.5905 | 108.391 | 134.091 | 159.4177 | 182.5031 | 207.3651 | 230.1255 | 252.3728 |
| G06-14 | 0 | 27.4626 | 55.5319 | 83.9287 | 113.6772 | 140.6343 | 167.732 | 191.665 | 217.7413 | 241.9563 | 265.1009 |
| G06-15 | 0 | 26.9851 | 54.3905 | 82.3094 | 111.6723 | 138.1051 | 165.284 | 188.9269 | 215.0844 | 238.6831 | 261.5387 |
| G07-16 | 0 | 10.8062 | 21.6591 | 31.9068 | 41.3628 | 52.5428 | 61.2073 | 69.2198 | 77.791 | 86.8293 | 94.9348 |
| G07-17 | 0 | -1.3972 | -3.2139 | -7.0332 | -13.1349 | -15.1381 | -20.541 | -25.4781 | -30.7414 | -33.1634 | -36.8428 |
| G07-18 | 0 | -13.9877 | -28.5328 | -44.5643 | -65.4744 | -79.2743 | -97.8131 | -113.842 | -131.404 | -144.041 | -158.536 |
| G07-19 | 0 | -6.3845 | -17.01 | -32.8074 | -47.8592 | -65.8458 | -82.0611 | -97.5769 | -111.741 | -123.669 | -133.593 |
| G07-20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G07-21 | 0 | 17.8496 | 36.0739 | 53.3639 | 69.9069 | 85.4692 | 100.7978 | 114.5383 | 127.9988 | 142.6746 | 156.5096 |
| G08-22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G08-23 | 0 | 36.6977 | 75.4523 | 116.8721 | 156.3803 | 196.2184 | 234.051 | 269.6445 | 304.8668 | 338.7366 | 369.9926 |
| G08-24 | 0 | 37.6535 | 76.891 | 117.247 | 156.211 | 195.4105 | 232.5208 | 268.285 | 303.2145 | 336.5185 | 367.173 |
| G08-25 | 0 | 16.813 | 33.1611 | 50.0685 | 66.8831 | 83.8382 | 99.21 | 113.8374 | 129.024 | 144.2572 | 159.0254 |
| G09-26 | 0 | 1.1617 | 0.2323 | -1.3005 | -1.3936 | -2.1836 | -4.2743 | -5.668 | -5.7146 | -4.6461 | -1.8118 |
| G09-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G09-28 | 0 | -14.3036 | -32.5804 | -55.0164 | -76.1429 | -98.6235 | -120.449 | -141.245 | -160.499 | -178.49 | -194.424 |
| G09-29 | 0 | -0.5133 | -2.5658 | -7.3709 | -11.2429 | -15.6745 | -20.1997 | -24.1648 | -27.8499 | -30.0422 | -32.0015 |
| G09-30 | 0 | 11.5929 | 22.7207 | 33.3828 | 44.3248 | 55.0341 | 65.697 | 75.662 | 85.5336 | 95.8711 | 106.1159 |
| G10-31 | 0 | 29.3004 | 56.5959 | 83.4268 | 110.7721 | 138.3522 | 164.2069 | 189.083 | 213.4468 | 237.0182 | 260.2173 |
| G10-32 | 0 | 26.1627 | 54.2356 | 84.1259 | 112.6209 | 142.7011 | 171.8049 | 199.0939 | 226.1051 | 252.373 | 277.6172 |
| G10-33 | 0 | 21.485 | 43.0167 | 64.9702 | 86.1771 | 108.1329 | 129.2481 | 149.4766 | 168.9118 | 188.8618 | 207.551 |
| G10-34 | 0 | 12.2668 | 23.368 | 34.3292 | 44.7314 | 55.6466 | 65.9557 | 76.2182 | 86.5278 | 97.9571 | 108.7336 |
| G10-35 | 0 | 0.8875 | -1.0747 | -4.4388 | -7.3353 | -10.5123 | -13.4559 | -15.8388 | -17.6139 | -18.2213 | -17.8476 |
| G11-36 | 0 | -12.1093 | -29.6416 | -50.1181 | -68.7242 | -88.4049 | -107.243 | -125.333 | -141.459 | -156.977 | -170.438 |


[^0]:    ${ }^{\text {a }}$ Dimensions are given in inches except for $A$, the overall width of the bridge.

[^1]:    Typical Bridge \#1, Stallings/Yoo Methodology, IG 2LL, Variable = Deck Thickness (in)

