The Role of Venture Capital in an Endogenously Growing Economy

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Abstract

This paper presents an endogenous growth model in which R&D improves product quality and venture capital supports these quality-enhancing activities both financially and nonfinancially. In the model, the venture capitalists' skill in evaluating entrepreneurs' innovative abilities plays a key role in achieving innovation and economic growth. When their skill is sufficiently low, neither innovation nor economic growth occurs even if entrepreneurs are abundant in the economy. Moreover, insufficient market size discourages entrepreneurs from engaging in R&D activities. Therefore, competent venture capitalists and a sufficiently large market are indispensable to the economy's long-run growth.

Keywords: Venture Capital, Innovation, Endogenous Growth.

1 Introduction

Over the last two decades, some economists have examined the links between financial development and economic growth. Theory and evidence imply that better functioning financial systems mitigate the effects of information and transaction costs and thus relax the external financing constraints that impede firm and industrial expansion (Levine, 2005). In the process of reducing market frictions, financial intermediaries produce information and allocate capital by evaluating prospective entrepreneurs and revealing the expected profits from engaging in innovation (King and Levine, 1993), which illuminates one mechanism through which financial development influences economic growth.

While all financial intermediaries provide these functions, how well they do so may differ across different types of financial intermediaries. Allen and Gale (1999) note that although banks may be effective at eliminating duplication in information gathering and processing, they may not be effective gatherers and processors of information in new, uncertain situations involving innovative products and processes. In contrast, venture capital funds have emerged as prominent financial intermediaries facilitating the entry of young innovative firms. Empirical studies have found strong linkages among venture capital activity, innovation, and economic growth. Kortum and Lerner (2001) examine the influence of venture capital on patented inventions in the United States across twenty industries over three decades and report that venture capital may have accounted for 8% of industrial innovations in that period. Pradhan, Arvin, Nair, Bennett, and Bahmani (2019) assess the causal relationship between venture capital investment and economic growth in European countries over the past three decades and reveal the existence of both unidirectional and bidirectional causality between the variables.

Despite the empirical relevance of venture capital activities to innovation and growth, the literature has not provided a unified theoretical framework that explains this relationship. This paper provides a theoretical attempt to feature venture capitalists identifying and supporting prospective entrepreneurs.[1] I consider an economy with asymmetric information and uncertainty in which the type of agents seeking venture capital is not observable to venture capitalists, and the former are exposed to high risks of failure when engaging in innovative activities.[2] Such conditions incentivize non-entrepreneurs who lack the industry expertise necessary for innovation to seek venture funds provided by venture capitalists. Hence, venture capi-

[1] Opp (2019) develops a model of venture capital intermediation that can explain central empirical facts about venture capital activity and uses it to evaluate the impact on the macroeconomy. Compared to Opp's study, this paper focuses more on the relationship between venture capitalists' competency and economic growth.

[2] See Hall and Woodward (2010) for empirical evidence.

talists screen and identify the genuine entrepreneurs capable of running an R&D project. [3] Venture capitalists' cost of evaluating proposals measures their competency.

Based on the works of Grossman and Helpman (1991) and King and Levine (1993), I present another endogenous growth model in which entrepreneurs are alternately involved in innovative and productive activities, and venture capitalists support them financially and nonfinancially. In that model, the venture capitalists' skill in evaluating entrepreneurs' innovative abilities plays a crucial role in achieving innovation and economic growth. There are two main findings of my model. First, when venture capitalists' skill is sufficiently low, neither innovation nor economic growth occurs even if entrepreneurs are abundant in the economy. This finding theoretically explains the practical importance of venture capitalists' efforts to evaluate and screen entrepreneurs documented in Kaplan and Stromberg (2001). Second, insufficient market size discourages entrepreneurs from engaging in R&D activities. Therefore, competent venture capitalists and a sufficiently large market are indispensable to the economy's long-run growth.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 derives the stationary equilibria of that model. Section 4 discusses some welfare properties of those equilibria. Section 5 concludes the paper.

2 The Model

2.1 Numeraire and Indexed Goods

Consider an economy in which all goods are produced from labor one-to-one. They are categorized into *numeraire* and *indexed goods*. The numeraire is produced competitively, and thus the wage rate in terms of that good, denoted by w, is determined as $w = 1$. Indexed goods constitute a continuum of measure one, each of which is identified by a real number in $[0, 1]$ and can be potentially supplied in a countable number of qualities. The quality j of the good indexed by ω ($\in [0, 1]$) is expressed as $q_j(\omega) = \lambda^j$, where the quality step λ (> 1) is common to all indexed goods. Stepping up the quality ladder requires R&D activities, while how to produce the lowest quality (i.e., $q_0(\omega) = 1$) is publicly known.

2.2 Economic Agents

There are three types of agents in the economy: *entrepreneurs*, *non-entrepreneurs* and *venture capital*. The first two are utility-maximizing individuals, while the last is profit-maximizing organizations. Assume that the populations of

[3] As shown in Kortum and Lerner (2000), venture capital backed firms are more innovative and produce more valuable patents. See also Samila and Sorenson (2011).

entrepreneurs and non-entrepreneurs are M (> 1) and N in measure, respectively, and that they are endowed with one unit of labor in each period.

2.2.1 Entrepreneurs

Entrepreneurs share a common utility function:

$$U_e = \sum_{t=0}^{\infty} \beta^t c_t. \tag{1}$$

where β (< 1) is the subjective discount factor. They consume only the numeraire. In each period, depending on their current status, they expend their endowed labor to engage in one of the following three activities: producing an indexed good, R&D, or working for others.

2.2.2 Non-entrepreneurs

Non-entrepreneurs can only work for others. Specifically, they inelastically supply their endowed labor to the labor market and consume all indexed goods. They share an intertemporal utility function:

$$U_n = \sum_{t=0}^{\infty} \beta^t \log X_t \tag{2}$$

Note that the entrepreneurs and the non-entrepreneurs have a common discount factor (i.e., β). The contents of $\log X_t$ are given by

$$\log X_t = \int_0^1 \log \left(\sum_{j=0}^{J_t(\omega)} q_{jt}(\omega) d_{jt}(\omega) \right) d\omega \tag{3}$$

where $d_{jt}(\omega)$ and $J_t(\omega)$ denote the demand for indexed good ω of quality j and the highest quality of that good available in period t, respectively. The non-entrepreneurs maximize utility in two stages. The first step is to maximize (3) subject to

$$E_t = \int_0^1 \left(\sum_{j=0}^{J_t(\omega)} p_{jt}(\omega) d_{jt}(\omega) \right) d\omega \tag{4}$$

where E_t is the total expenditure on indexed goods in period t, measured by the numeraire. The solution to this problem is derived as

$$d_{jt}(\omega) = \begin{cases} E_t / p_{jt}(\omega) & \text{if } j = J_t^*(\omega) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $J_t^*(\omega)$ is the single quality that carries the lowest quality-adjusted price $p_{jt}(\omega)/q_{jt}(\omega)$. Using (5), one can rewrite (2) as

$$U_n = \sum_{t=0}^{\infty} \beta^t \left[\log E_t + \int_0^1 (\log q_t(\omega) - \log p_t(\omega)) d\omega \right] \tag{6}$$

where $q_t(\omega)$ and $p_t(\omega)$ represent the quality and price of $J_t^*(\omega)$, respectively. The second step of non-entrepreneurs' optimization is to maximize (6) subject to their flow budget constraint:

$$A_{t+1} = (1 + r_t)A_t + 1 - E_t \tag{7}$$

where A_t and r_t denote the asset of a non-entrepreneur in period t and the one-period interest rate from period $t-1$ to t, and 1 in the RHS is the labor income earned in period t, respectively. From (6) and (7), the Euler equation is derived as

$$E_{t+1}/E_t = \beta(1 + r_{t+1}). \tag{8}$$

2.2.3 Venture Capital

Entrepreneurs seek venture capital to start their innovative activities because they are assumed to be incapable of raising capital in public markets or securing a bank loan due to their limited operating history or lack of initial wealth. The venture capitalists invest in startup entrepreneurs who are expected to succeed at R&D activity and become the monopolistic supplier of an indexed good. The startups face high uncertainty, and the venture capital firms take on their risk hoping that some of them will become successful. Specifically, the venture capitalists evaluate R&D projects proposed by the startups and acquire a sufficient percentage, say α, of the equities issued by those whose proposals are promising in return for paying in advance all of the expenses necessary to start their projects, say C_I. After having taken significant control over promising projects, the venture capitalists monitor the R&D activities of their proposers, provide them with advice on how to proceed with their projects, and receive α percentage of the profits if they succeed at innovation.

Nonetheless, non-entrepreneurs can make proposals to the venture fund C_I. They do not endeavor to innovate and work for others. Since entrepreneurs face a high risk of failure, non-entrepreneurs may be incentivized to disguise their type, given that venture capitalists do not evaluate proposals. Investors gain no return from non-entrepreneurs they support if they do not clarify the type of agents. Therefore, venture capitalists choose to evaluate, and non-entrepreneurs do not make proposals. C_E denotes the cost of evaluation. [4] Total costs arising from the venture capitalists' support activities are defined as $C = C_I + C_E$. C_E's reciprocal (i.e., $1/C_E$) can be interpreted as a measure of their skill, as a more competent venture capitalist can verify entrepreneurs at a lower cost. In this model, free entry to venture capital investment is assumed, and thus the operating ventures earn zero profit.

3 Stationary Equilibria

In what follows, the analysis will be made under the assumption that the economy is in a stationary equilibrium, where all variables are time-invariant in value. Thus the time subscript t can be omitted. [5] In any stationary equilibrium, the interest rate is determined by (8) as

$$r = (1/\beta) - 1. \tag{9}$$

3.1 Venture Funding and Innovation

In any period, every entrepreneur stays in one of the following three statuses: *quality leaders*, the population of which is equal to 1; *innovators*, the population of which is equal to I; and *workers*, the population of which is equal to $\min[0, M - 1 - I]$.

3.1.1 Quality Leaders

A quality leader can produce the highest quality of an indexed good. He can monopolize all of the demand for that good by setting his price at a level that carries a lower quality-adjusted price against his rival, who stands one step near him. Specifically, he sets his price as $p = \lambda$, which yields NE/λ units of the demand for his good. In doing so, he earns a profit that amounts to

$$\delta = (1 - 1/\lambda)NE. \tag{10}$$

However, the status of a quality leader is not permanent. If an innovator succeeds in producing a higher quality of that good, the incumbent loses the status of a quality leader. Such a replacement occurs with probability Π per period. Hence, in any period, Π of quality leaders lose their status. They choose to become either innovators or workers in the next period.

[4] Sunaga (2017) analyzes the effects of financial intermediaries' monitoring costs on economic growth. In her context, it is costly to monitor entrepreneurs who can conceal the result of successful innovation and avoid repaying financial intermediaries. Differing from her study, the cost C_E here arises from adverse selection.

[5] It can be shown that any stationary equilibria of this economy are locally unstable, meaning that the growth path that converges to the stationary equilibria (except the trivial case) does not exist. See Appendix A.

3.1.2 Innovators

An innovator succeeds in his innovation with probability π ($< 1/(M-1)$) per period. Since there are I innovators in any period, the number of successes per period is πI ($= \Pi$). Furthermore, since the profit flows are the same for all indexed goods, and since the prospective leadership position is expected to last equally long for any good, innovators target all goods to the same extent. Let Q and V_I denote the total value of an innovator and the net worth of being an innovator, respectively. Since α percentage of the profit in each period goes to venture capital, Q and V_I satisfy

$$V_I = (1-\alpha)Q.$$

As long as there is positive innovation, competition among venture capitalists reduces their expected profits to zero, i.e.,

$$\alpha Q = [\alpha/(1-\alpha)]V_I = C. \tag{11}$$

If $[\alpha/(1-\alpha)]V_I < C$, venture capitalists reject all R&D proposals, and thus $I = 0$. Let V_P denote the net worth of being a quality leader. Then, the argument thus far implies that V_I and V_P satisfy

$$V_I = \beta[\pi V_P + (1-\pi)V_I] \tag{12}$$

$$V_P = (1-\alpha)\delta + \beta[\Pi V_I + (1-\Pi)V_P]. \tag{13}$$

By solving (12) and (13) with respect to V_I and V_P, one can obtain

$$V_I = \frac{(1-\alpha)\pi\beta\delta}{(1-\beta)[1-\beta+\beta(\Pi+\pi)]}. \tag{14}$$

Eqs.(10)(11)(14) and $\Pi = \pi I$ jointly produce

$$\frac{\pi\alpha\beta(1-1/\lambda)NE}{(1-\beta)[1-\beta+\pi\beta(1+I)]} = C. \tag{15}$$

If $I > 0$, then α, E, and I must satisfy this condition.

3.1.3 Workers

Entrepreneurs can work for others, and the net worth of being a worker is given by $1/(1-\beta)$, as the wage rate is determined as $w = 1$. When $V_I \leq 1/(1-\beta)$, they may find it rational to become workers. Specifically, when $V_I < 1/(1-\beta)$, being a worker is more attractive than being an innovator, and thus all of the entrepreneurs choose to become a worker, resulting in $I = 0$. When $V_I = 1/(1-\beta)$, being a worker and being an innovator is equally attractive, and thus it is likely that some of the entrepreneurs become a worker and others become an innovator. In the second case, eq.(11) and $V_I = 1/(1-\beta)$ jointly determine α as

$$\alpha = \frac{(1-\beta)C}{1+(1-\beta)C}. \tag{16}$$

3.2 Clearance of the Labor Market

The labor market clearing condition can be written as

$$\begin{aligned}\pi CI + (1-\alpha)(1-1/\lambda)NE + \min[0, M-1-I] + NE/\lambda \\ = N + \min[0, M-1-I].\end{aligned} \tag{17}$$

The LHS and RHS of this equation are the aggregate demand for labor and its aggregate supply, respectively. More specifically, $\pi CI + (1-\alpha)(1-1/\lambda)NE + \min[0, M-1-I]$ is the labor demand arising from numeraire production, NE/λ is the labor demand arising from indexed goods production, and $N + \min[0, M-1-I]$ is the labor supplied by entrepreneurs and non-entrepreneurs. Eq.(17) can be simplified as

$$(1-E)N + \alpha(1-1/\lambda)NE = \pi CI. \tag{18}$$

This equation means that in any stationary equilibrium, the net income flow from the financial sector is always zero, since $(1-E)N$, $\alpha(1-1/\lambda)NE$, and πCI are the additional deposits by the non-entrepreneurs, the dividends from the quality leaders, and the venture capitalists' expenditure, respectively.

3.3 Innovation Equilibrium

Two types of stationary equilibria are conceivable in this model. The first type is called the *innovation equilibrium*, in which some entrepreneurs are engaging in R&D activities. Possible innovation equilibria can be further divided into *full* and *partial* innovation cases. In the full innovation case, no entrepreneurs choose to work for others, and thus the equilibrium value of I is determined as

$$I = M - 1. \tag{19}$$

Then, conditions (15)(18) and (19) jointly determine the equilibrium values of E and α as

$$E = 1 + \left[(1-\pi-\beta)M + \pi + \tfrac{(1-\beta)^2}{\pi\beta}\right]\tfrac{C}{N} \tag{20}$$

$$\alpha = \tfrac{(1-\beta)(1-\beta+\pi\beta M)C}{\pi\beta(1-1/\lambda)N}\left\{1 + \left[(1-\pi-\beta)M + \pi + \tfrac{(1-\beta)^2}{\pi\beta}\right]\tfrac{C}{N}\right\}^{-1} \tag{21}$$

These equilibrium values satisfy $V_I \geq 1/(1-\beta)$ if and only if $C(= C_I + C_E) \in (0, C_1]$, where C_1 is defined as

$$C_1 \equiv \frac{\pi\beta(\lambda-1)N - \lambda(1-\beta+\pi\beta M)}{(1-\beta)(1-\beta+\pi\beta M) + \pi^2\beta(\lambda-1)(M-1)}. \tag{22}$$

In the partial innovation case, some entrepreneurs choose to work for others, and hence

$$0 < I < M - 1. \tag{23}$$

In this case, the equilibrium value of α is given by (16), as already seen. Then, conditions (15)(16) and (18) jointly determine the equilibrium values of I and E as

$$I = \frac{(\lambda-1)N - [(1-\beta)/\pi\beta + 1][\lambda + C(1-\beta)]}{[\pi(\lambda-1)+1-\beta]C + \lambda} \tag{24}$$

$$E = \frac{\lambda[1+(1-\beta)C][(\pi+1/\beta-1)C+N]}{\{[\pi(\lambda-1)+1-\beta]C+\lambda\}N}. \tag{25}$$

These equilibrium values satisfy (23) if and only if $C \in (C_1, C_2)$, where C_2 is defined as

$$C_2 \equiv \frac{\pi\beta(\lambda-1)N - \lambda(1-\beta+\pi\beta)}{(1-\beta)(1-\beta+\pi\beta)}. \tag{26}$$

The next proposition summarizes the argument thus far.

Proposition 1. *Let C_1 and C_2 be as defined in (22) and (26), respectively. (a) When the parameters satisfy*

$$N \leq \frac{[(1-\beta)C_I+\lambda](1-\beta+\pi\beta)}{\pi\beta(\lambda-1)}, \tag{27}$$

there is no innovation equilibrium in this model. (b) When the parameters satisfy

$$N > \frac{[(1-\beta)C_I+\lambda](1-\beta+\pi\beta)}{\pi\beta(\lambda-1)} \tag{28}$$

and

$$N \leq \frac{(1-\beta)C_I+\lambda)(1-\beta+\pi\beta M)}{\pi\beta(\lambda-1)} + \pi(M-1), \tag{29}$$

there is a unique innovation equilibrium in this model if and only if C_E satisfies $C_E \in (0, C_2 - C_I)$. This equilibrium is of the partial innovation type. (c) When the parameters satisfy

$$N > \frac{(1-\beta)C_I+\lambda)(1-\beta+\pi\beta M)}{\pi\beta(\lambda-1)} + \pi(M-1), \tag{30}$$

there is a unique innovation equilibrium in this model if and only if C_E satisfies $C_E \in (0, C_2 - C_I)$. This is a full innovation equilibrium when $C_E \in [0, C_1 - C_I]$ and a partial innovation equilibrium when $C_E \in (C_1 - C_I, C_2 - C_I)$.

Proof. When (27) is valid, it is straightforward to verify that $C_I \geq C_2$, and thus $C \geq C_2$, making it impossible for this model to have any innovation equilibrium. When (28) and (29) are true, $C_1 \leq C_I < C_2$, meaning that the model has no full innovation equilibrium and that it has a unique partial innovation equilibrium if $C_E \in (0, C_2 - C_I)$. When (30) is true, $C_I < C_1$, meaning that the model has a unique full innovation equilibrium when $C_E \in [0, C_1 - C_I]$ and that it has a unique partial innovation equilibrium when $C_E \in (C_1 - C_I, C_2 - C_I)$. □

3.4 Zero-innovation Equilibrium

The second type of stationary equilibrium is called the *zero-innovation equilibrium*, in which no entrepreneurs are engaging in R&D activities. In a zero-innovation equilibrium, both the numeraire and indexed goods of the lowest quality are competitively produced, and thus the equilibrium values of E and I are determined as $E = 1$ and $I = 0$. [6] Although venture capital provides no support for startups in this type of equilibrium, it is possible to estimate the value of α that they would offer to startups. Substituting the equilibrium values of E and I into (15) yields this value:

$$\tilde{\alpha} = \frac{(1-\beta)(1-\beta+\beta\pi)}{\pi\beta(1-1/\lambda)N}C. \tag{31}$$

When $\tilde{\alpha} > 1$, the future profits of a new project cannot cover its necessary expenses (i.e., $Q < C$). In this case, supporting a startup does not pay off, and thus the venture capitalists reject all R&D proposals. When $\tilde{\alpha} \leq 1$, in contrast, the future profits of a new project cover its necessary expenses (i.e., $Q \geq C$), and thus the venture capitalists are prepared to support new startups. However, potential startups may refuse their offers if

$$\tilde{V}_I \equiv (1-\alpha)Q = \frac{\pi\beta(1-1/\lambda)N}{(1-\beta)(1-\beta+\beta\pi)} - C \leq \frac{1}{1-\beta}. \tag{32}$$

When (32) is valid, no entrepreneurs may find it attractive to engage in R&D activities. In fact, $\tilde{\alpha} > 1$ implies (32), and thus the zero-innovation equilibrium exists in this model if and only if (32) is true. The next proposition summarizes the argument thus far.

Proposition 2. *Define C_3 as*

$$C_3 \equiv \frac{\pi\beta(\lambda-1)N - \lambda(1-\beta+\pi\beta)}{\lambda(1-\beta)(1-\beta+\pi\beta)}. \tag{33}$$

(a) When the parameters satisfy

$$N \leq \frac{\lambda[(1-\beta)C_I + 1](1-\beta+\pi\beta)}{\pi\beta(\lambda-1)}, \tag{34}$$

there is a unique zero-innovation equilibrium in this model. (b) When (34) does not hold, there is a unique zero-innovation equilibrium in this model if and only if C_E satisfies $C_E \in [C_3 - C_I, +\infty)$.

[6] In a zero-innovation equilibrium, financial intermediaries provide no support for startups. Moreover, the indexed goods are produced competitively, and thus their producers earn zero profit. These jointly simplify (18) as $E = 1$.

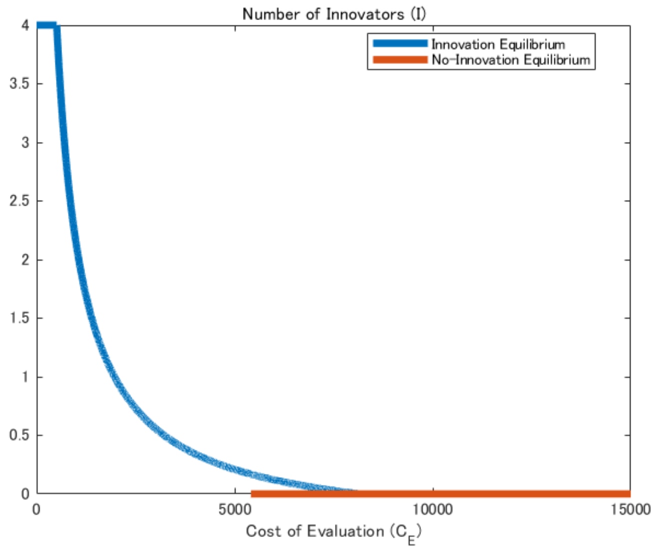

Figure 1: Equilibrium Values of I

Proof. It is easy to verify that $\tilde{V}_I = 1/(1-\beta)$ if $C = C_3$ and that $C \geq C_3$ is equivalent to (32), which is the existence condition for the zero-innovation equilibrium. When (34) is true, $C_I \geq C_3$, ensuring the existence of a zero-innovation equilibrium. When (34) does not hold, $C_I \leq C_3$, meaning that the model has a unique zero-innovation equilibrium if and only if $C_E \geq C_3 - C_I$. □

3.5 Essential Conditions for Innovation and Growth

Propositions 1 and 2 jointly imply the next proposition.

Proposition 3. *Let C_2 and C_3 be as defined in (26) and (33), respectively. (a) When the parameters satisfy (27), this model exhibits a unique zero-innovation equilibrium. (b) When the parameters satisfy (28) and (34), this model exhibits both innovation and zero-innovation equilibria if $C_E \in [0, C_2 - C_I)$ and a unique zero-innovation equilibrium if $C_E \in [C_2 - C_I, +\infty)$. (c) When (34) does not hold, the model exhibits a unique innovation equilibrium if $C_E \leq C_3 - C_I$, both innovation and zero-innovation equilibria if $C_E \in [C_3 - C_I, C_2 - C_I)$, and a unique zero-innovation equilibrium if $C_E \in [C_2 - C_I, +\infty)$.*

Proof. Recall that this model has an innovation equilibrium only when $C < C_2$ and a zero-innovation equilibrium only when $C_3 \leq C$. $C = C_I + C_E$ by definition then yields the result in Proposition 3. □

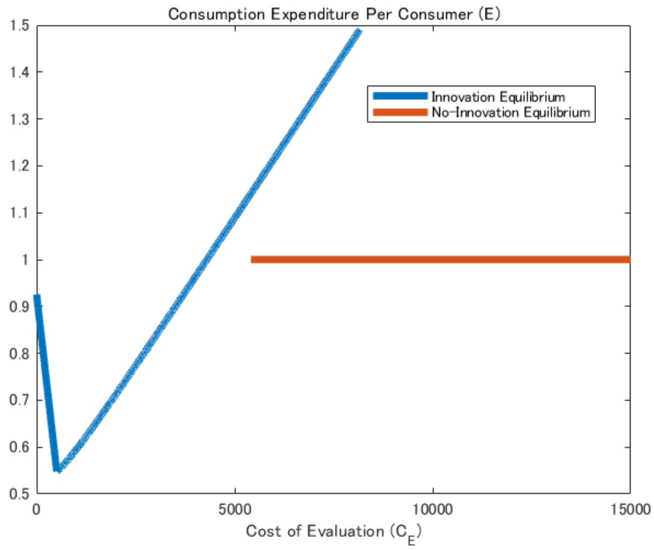

Figure 2: Equilibrium Values of E

Figures 1–3 depict the equilibrium values of I, E, and α, respectively, for every integral value of C_E in $[0, 15000]$.[7] Other parameter values are set as

$$\beta = 0.99,\ \pi = 0.05,\ \lambda = 1.5,\ N = 200,\ M = 10,\ C_I = 100$$

which satisfy (30) and yield the values of C_1, C_2 and C_3 as

$$C_1 = 258.99, \qquad C_2 = 8269.33, \qquad C_3 = 5512.89.$$

These figures show that the innovation equilibrium exists when C_E takes a relatively small value and that the zero-innovation equilibrium exists when C_E takes a rather significant value, which coincide with the statements of Proposition 3. Proposition 3 implies that a sufficiently large N relative to C_I and a small enough C_E are essential to this economy's innovation and growth. Specifically, the first and foremost condition for innovation is that N must satisfy the following inequality:

$$N \geq \frac{[(1-\beta)C_I + \lambda](1 - \beta + \pi\beta)}{\pi\beta(\lambda - 1)}.$$

When this condition is not met, the population of non-entrepreneurs, who buy and consume the indexed goods, is so small that innovators cannot

[7] Since in the zero-innovation equilibrium, $\tilde{\alpha} \leq 1$ (as discussed in (31)) only for a slight range of C($\in [5411, 5462]$), I did not present $\tilde{\alpha}$ in figure 3.

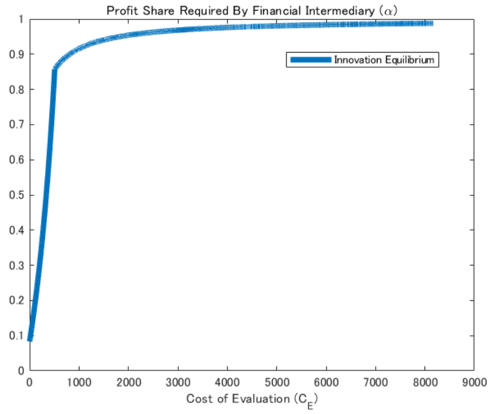

Figure 3: Equilibrium Values of α

make a considerable net profit even if they succeed at innovation. Thus, the entrepreneurs lose their interest in innovation. Even when N satisfies the above condition, no innovation occurs unless C_E satisfies $C_E < C_2 - C_I$. When $C_E \geq C_2 - C_I$, venture capitalists would require most of the profits from successful innovations to cover their expenses, which discourages entrepreneurs from engaging in R&D activities.

4 Optimal Growth

Define the growth rate g to be the rate of increase in X_t. When interpreting the ωs as indexed goods, g corresponds to the growth rate in a quality-adjusted consumption index.

To calculate g, note that in a stationary equilibrium, $\log X_t$ can be expressed as

$$\log X_t = \log E - \log \lambda + \int_0^1 \log q_t(\omega) d\omega$$

Since in equilibrium the same intensity of $R\&D$ applies to all products and by assumption $q_t(\omega) \sim Bi(t, \Pi)$, the above equation can be rewritten as

$$\log X_t = \log E - \log \lambda + \Pi t \log \lambda \tag{35}$$

Hence, the growth rate of X_t is given by $\Pi \log \lambda = \pi I \log \lambda$. One is now prepared to study issues of welfare. Using (2), (35) and that E and I are constant in a stationary equilibrium, U_n can be derived as:

$$(1-\beta)U_n = \log E - \log \lambda + \frac{\beta}{1-\beta}\Pi \log \lambda \tag{36}$$

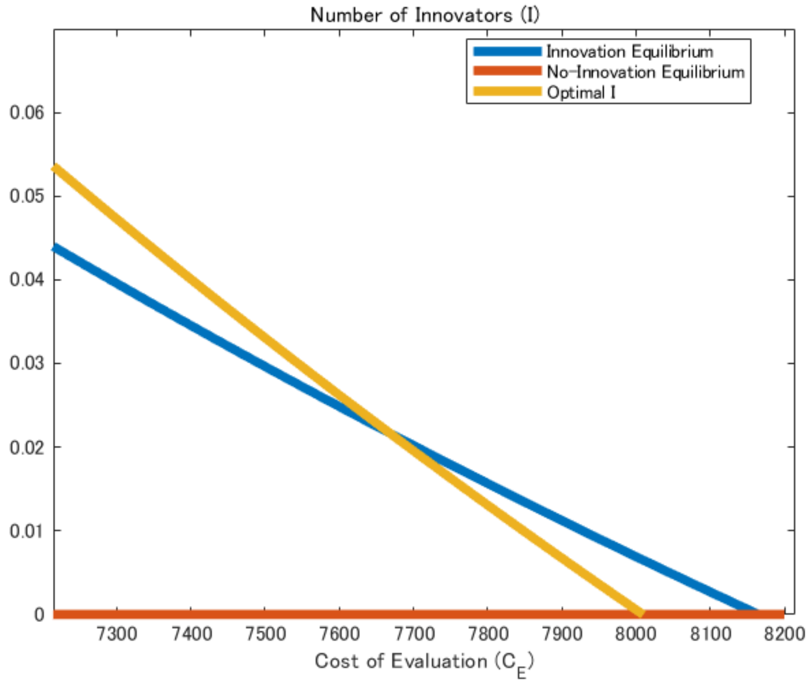

Figure 4: Optimum and Decentralized I

The optimization problem of a social planner is to maximize (36) subject to (18) and $0 \leq I \leq M-1$. The optimal I is given by

$$I^* = \max\left\{0, \min\left\{M-1, \frac{1}{\pi C}\left(N - \frac{1-\beta}{\beta}\frac{C}{\log\lambda}\right)\right\}\right\} \tag{37}$$

$I^* = 0$ means that it is optimal for the economy to maintain the zero innovation equilibrium. The next proposition is straightforward:

Proposition 4. *Define* C_4,C_5 *as*

$$C_4 \equiv \frac{\beta N \log\lambda}{\pi\beta(M-1)\log\lambda + 1 - \beta}, \quad C_5 \equiv \frac{\beta N \log\lambda}{1-\beta}. \tag{38}$$

both of which are positive, and $C_5 > C_4$*. (a) When* $C_I \in (0, C_4]$*, for* $C_E \in [0, C_4 - C_I]$*,*$I^* = M-1$*; for* $C_E \in (C_4 - C_I, C_5 - C_I)$*,* $I^* = \frac{1}{\pi C}(N - \frac{1-\beta}{\beta}\frac{C}{\log\lambda})$*; and for* $C_E \geq C_5 - C_I$*,* $I^* = 0$*. (b) When* $C_I \in (C_4, C_5)$*, for* $C_E \in [0, C_5 - C_I)$*,* $I^* = \frac{1}{\pi C}(N - \frac{1-\beta}{\beta}\frac{C}{\log\lambda})$*; for* $C_E \geq C_5 - C_I$*,* $I^* = 0$*. (c) When* $C_I \geq C_5$*,* $I^* = 0$*.*

Figure 4 depicts I and I^* for the integral value of C_E in $[7300, 8200]$ in the numerical example discussed above. In this case, $C_4 = 421.13$, $C_5 = 8028.21$ and $C_1 < C_4 < C_3 < C_5 < C_2$. According to proposition 1, when $C_E \in$

$(0, 158.99]$, $I = M - 1 = I^*$ and the optimal growth rate coincides with the decentralized growth rate. When $C_E \in (158.99, 7668.21]$, there is excessive innovation, and when $C_E \in (7668.21, 8169.33)$, innovation is insufficient. Specifically, it is noteworthy that $I^* = 0$ for $C_E \geq C_5 - C_I = 7928.21$, while the decentralized I is still positive for $C_E \in (7928.21, 8169.33)$.

The inefficiency in the economy stems from three main reasons. First, although R&D projects with $Q > C_I$ make society better off and thus should be financed, only projects supported by venture capital (i.e., projects such that $\alpha Q \geq C$) are executed, resulting in deficient innovation in decentralized equilibria. The second and the third reasons are externalities generated by innovation, referred to as the *consumer surplus effect* and *business-stealing effect*, as mentioned in Grossman and Helpman (1991). Their aggregate impact is typically ambiguous. The optimum can be decentralized here through a tax on entrepreneurs who choose to work for others or a subsidy on those involved in R&D activities when innovation is insufficient. An adverse policy is desirable when innovation is excessive.[8] By imposing a tax on or compensation for entrepreneurs, the government can achieve the socially optimal growth rate.

5 Conclusion

This paper presents a model in which venture capital has significant ramifications for innovation and long-run growth. In this model, an innovation equilibrium exists only when N takes a sufficiently large value relative to the project cost C_I and C_E takes a relatively small value. Since $1/C_E$ can be interpreted as a measure of the competency of venture capitalists, this result means that innovation and economic growth can occur only when venture capitalists are sufficiently competent and the market (i.e., the population of non-entrepreneurs) is sufficiently large.

Given a sufficiently large market size N, when C_E takes a reasonably large value, venture capitalists reject all R&D proposals, as their future profits cannot cover the expenditure. Even when future profits cover costs, entrepreneurs prefer working for others to engaging in R&D activities. For these reasons, the innovation equilibrium does not exist for such a large value of C_E. In contrast, when C_E takes a sufficiently small value, venture capitalists are willing to grant certificates to R&D proposals, as their future profits cover the expenditure with ease. Given this attitude of venture capitalists, entrepreneurs also find it more profitable to engage in R&D activities than to work for others. These jointly lead to a continuous process of innovation and economic growth. That demonstrates why the zero-innovation equilibrium does not exist for such a small value of C_E. When C_E takes a medium

[8] See Philippon (2010) for a discussion of taxes and subsidies in financial and nonfinancial sectors.

value, both innovation and zero-innovation equilibria coexist. Which occurs depends on entrepreneurs' expectations of the net worth of an innovator. This feature of the model is related to entrepreneurs self-fulfilling expectations about the implementation of innovations discussed in Shleifer (1986). Nonetheless, if the population of non-entrepreneurs who buy and consume indexed goods is small, no entrepreneurs can make a considerable profit even if they succeed at innovation. Consequently, no innovation occurs in the economy.

An attractive extension to this model is to make C_E endogenous. It takes an *ex ante* cost C_E to evaluate and support startup R&D projects in the model. However, technological innovation itself may substantively affect the operation of financial systems by, for example, transforming the acquisition, processing, and dissemination of information.[9] Additional research needs to explore the co-evolution of venture funding and growth.

Appendix A Local Instability of the Stationary Equilibria

To explore local stability, rewrite the previous equation with time label

$$V_I = \beta[\pi V_{P,t+1} + (1-\pi)V_{I,t+1}] \quad (39)$$
$$V_P = (1-\alpha_t)\delta_t + \beta[\Pi_t V_{I,t+1} + (1-\Pi_t)V_{P,t+1}]. \quad (40)$$

Using the following notation, (39) and (40) can be written in matrix form (41).

$$V_t = \begin{pmatrix} V_{I,t} \\ V_{P,t} \end{pmatrix}, A_{I_t} = \begin{pmatrix} \beta(1-\pi) & \beta\pi \\ \beta\pi I_t & \beta(1-\pi I_t) \end{pmatrix},$$
$$\Delta_t = \begin{pmatrix} 0 \\ (1-\alpha_t)(1-1/\lambda)NE_t \end{pmatrix}.$$
$$V_t = A_{I_t}V_{t+1} + \Delta_t \quad (41)$$

Note that I_t is a jump variable such that:

$$I_t = \begin{cases} M-1 & \text{if } V_{I,t} > 1/(1-\beta) \\ \in [0, M-1] & \text{if } V_{I,t} = 1/(1-\beta) \\ 0 & \text{otherwise} \end{cases} \quad (42)$$

and that A_{I_t} depends on I_t, while Δ_t depends on both α_t and E_t (and thus I_t). The relationship determined in (41) varies through time. First, consider the case in which the economy starts from $I = 0$ and some V_0. $C \in (C_3, C_2)$

[9] For instance, Philippon and Reshef (2012) documents a set of new, interrelated stylized facts about the evolution of skill intensity, wages, organization, and occupational complexity in the financial industry.

for which the innovation equilibrium and zero innovation equilibrium coexist. Specifically, let $I = 0$, $E = 1$, and α be given by that in the zero innovation equilibrium; $\tilde{\alpha}$ in (31). (41) becomes:

$$V_{t+1} = A_0^{-1}(V_t - \Delta_0) \tag{43}$$

Calculation shows that

$$V_{t+1} - \tilde{V} = A_0^{-1}(V_t - \tilde{V})$$

where

$$\tilde{V} = \begin{pmatrix} \frac{\pi\beta(1-1/\lambda)N}{(1-\beta)(1-\beta+\beta\pi)} - C \\ -\frac{1}{\pi(1-\beta)}(\pi\beta(1-1/\lambda)N - (1-\beta)(1-\beta+\pi\beta)C) \end{pmatrix}$$

correspond to the V_I and V_P in the zero innovation equilibrium,

$$A_0^{-1} = \frac{1}{\beta(1-\pi)} \begin{pmatrix} 1 & -\pi \\ 0 & 1-\pi \end{pmatrix}.$$

Two eigenvalues of A_0^{-1} are $\Lambda_1 = \frac{1}{\beta(1-\pi)}$ and $\Lambda_2 = \frac{1}{\beta}$, both of which have a modulus greater than 1. Thus, starting from V_I in some right neighborhood of $\tilde{V_I}$, $V_{I.t}$ increases. (Note that $C > C_3$ implies that $\tilde{V_I} < 1/(1-\beta)$.) Starting from V_I in some left neighborhood of $\tilde{V_I}$, $V_{I.t}$ decreases. Therefore, the zero innovation equilibrium is locally unstable. Typically, for given $I \in [0, M-1]$,

$$A_I^{-1} = \frac{1}{\beta(1-\pi(I+1))} \begin{pmatrix} 1-\pi & -\pi \\ -\pi I & 1-\pi I \end{pmatrix}$$

Eigenvalues of A_I^{-1} are $\Lambda_1 = \frac{1}{\beta(1-\pi(I+1))}$ and $\Lambda_2 = \frac{1}{\beta}$. By assumption, $0 < \pi(M-1) < 1$ and $I \in [0, M-1]$; thus $1 > |1 - \pi(I+1)| > \pi$, meaning that $|\Lambda_1|, |\Lambda_2| > 1$. The same argument then applies, which proves the statement in section 3.

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