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Optimal Inspection under Moral Hazard and Limited Liability of Polluter

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Abstract.

We have considered an environmental pollution that seldom occurs but catastrophically destroys the environment once it occurs. While this kind of pollution might be avoided to some extent through precaution activity, the effort to prevent pollution could not be observed by the government without inspection. In addition, the polluter might not afford to compensate for the damage. The first best has not been achieved in the literature when moral hazard and limited liability are considered at the same time. By generalizing other policies including strict liability and negligence rule, we derive an optimal inspection policy under moral hazard and limited liability. The optimal policy is composed of advance payment and ex-post payment after inspection. In other words, we can consider the optimal policy as a deposit/refund system. The first best will always be achieved under the optimal policy as long as the liability covers the first-best effort if inspection cost is negligible. We also derive the second-best policy by taking account of inspection cost.

Key words: Moral hazard, Limited liability, Inspection, Environmental accident

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1. Introduction

We consider an environmental pollution that seldom occurs but catastrophically destroys the environment once it occurs, like accidents in nuclear power plants and oil tanker accidents. In those environmental pollution, the polluters might not afford to compensate for such a serious damage. In addition, even though such an environmental accident might be avoided by precaution activity, it is quite difficult for the government to observe the actual effort to prevent an accident.

The problem is that we need to cope with two obstacles or moral hazard and limited liability at the same time. Even if the effort to prevent the pollution is not observable, it is possible to attain the social optimum, when the polluter can afford to compensate for the damage. The government can require the polluter to compensate for the entire damage. In other words, the first best can be achieved by the strict liability rule. On the other hand, suppose that the polluter cannot afford to compensate for the damage. It is still possible to attain the first best, if the effort to prevent pollution is observable. As shown in the next section, the social optimum can be attained by subsidizing the effort and imposing a fixed amount of penalty in case of pollution.

Since the seminal papers were published by Summers (1983) and Shavell (1986), it is well known that it is difficult to attain the first best, if we have to cope with moral hazard and limited liability at the same time. There are many policies to alleviate this problem. Extending liability to the third party is one of them (Pitchford (1995), Boyer and Laffont (1997), Lewis and Sappington (1999), Balkenborg (2001), Lewis and Sappington (2001), Dionne and Spaeter (2003), Hutchinson and van't Veld (2005), Hiriart and Martimort (2006), Che and Spier (2008)). The extended liability is a rule where the parties such as lenders, who have the contractual relationship with insolvent polluters have to be liable for the remaining liability. Partial liability may achieve the second-best care when care is not observable, while extending all the liability to the lender induces the potential injurers to implement the first-best care under perfect information (Boyer and Laffont (1997), Dionne and Spaeter (2003)).

Financial responsibility is another remedy to mitigate the inefficiency caused by limited liability and moral hazard. The most common instrument of financial responsibility is compulsory liability insurance. But as it is well known, the compulsory liability insurance induces the efficient level of prevention, only when the insurer is able to observe the prevention level performed by the firm (Shavell (1986), Jost (1996), Polborn (1998)). On the other hand, minimum asset requirement as a remedy for the limited-liability problem is investigated by Shavell (2005). Such a requirement can mitigate the problem, while it might decrease social welfare through banning operation of some firms in their businesses. It is shown that compulsory liability insurance may be inferior to asset requirement when insurers cannot observe levels of care.

Some researchers relax the assumption of moral hazard. They assume that the effort to prevent accidents is observable only if inspection is conducted. The Negligence rule is advocated by Shavell (1986) and Miceli and Segerson (2003). This rule suggests inspection only in case of accident. If insufficient effort compared with the first best is detected, then the firm will be required to pay the maximum amount of money that she can afford to pay. Otherwise, any penalty will never be imposed on the firm. The social optimum will not always be attained under this rule, although it can mitigate the inefficiency.

In this paper, we will hold the same assumption that the effort to prevent accidents is observable only if inspection is conducted. We will derive the optimal inspection policy under moral hazard and limited liability. Inspection rule considered in this paper is more general than that under the strict liability rule and the negligence rule. Inspection is a probabilistic event in our model, which means that frequency of inspection might be equal to or less than 1. In addition, inspection can be conducted even in no-pollution case.

The optimal policy is composed of advance payment and ex-post payment after inspection. The optimal policy can be considered as a deposit/refund system. Firm is required to pay a fixed amount of money or deposit in advance but refund will be returned to the firm if it is found to be compliant with the standard through inspection. The optimal policy does work well. Any target effort can be achieved under the policy. We show that there is a trade-off between the magnitude of liability and frequency of inspection. As liability imposed on firm increases, the inspection frequency can be reduced. In addition, we show that if liability is larger than twice as much as the target effort, inspection is *almost unnecessary* to attain the target effort. It is also shown that the optimal policy is superior to the strict liability rule and the negligence rule. We also derive the second-best policy by taking account of inspection cost. We show that the second-best effort is smaller than the first best.

The rest of the paper is organized as follows. The next section specifies theoretical background of the problem that we try to cope with in the paper. Our scheme is proposed in Section 3. Section 4 gives conclusion.

2. The Background

We consider an environmental pollution that seldom occurs but catastrophically destroys the environment once it occurs. We assume that such a pollution might be avoided to some extent by precaution activity. We consider a simple model. Let x stand for precaution expenditure to prevent accidents. Probability of pollution is represented by a decreasing and convex function $p(x)$. We assume that pollution incurs a fixed amount of damage represented by D. The socially optimal level of care will minimize the expected social cost $x + p(x)D$. The socially optimal care represented by x_{fb} satisfies the following first-order condition:

$1 + p'(x)D = 0,$ (1)

where the subscript *fb* represents first best.

If the business which a potential polluter engages in is sufficiently profitable so that the polluter can compensate for the damage, the strict liability will encourage such the firm to take the socially optimal level of precaution. To see this, note that polluter will choose x to minimize the total cost represented by $x + p(x)D$ under the strict liability.

Conversely, suppose that the polluter cannot afford to compensate for the damage. Let us suppose in turn that the effort to prevent pollution is observable. In this case, it is still possible to attain the social optimum. To see this, suppose that a subsidy represented by t will be provided for any unit of effort that exceeds the first-best level or x_{fb}. In addition, the firm will be required to pay a fixed amount of penalty represented by T in case of pollution. Under this policy, the firm will choose effort x to minimize the cost represented by $x - t(x - x_{fb}) + p(x)T$. The first-order condition for this problem will be written as

$1 - t + p'(x)T = 0$ (2)

By comparing (1) and (2), the socially optimum can be attained if we set the subsidy rate at $t = 1 - \frac{T}{D}$. Note that we can choose any positive value for T.

However, it is quite difficult to attain the social optimum, if we have to cope with moral hazard and limited liability at the same time.

3. Optimal inspection policy

We relax the assumption of moral hazard following Shavell (1986) and Miceli and Segerson (2003). It is assumed that the effort to prevent accidents is observable only if inspection is conducted. We derive the optimal inspection policy that will achieve the target level of effort with minimizing the inspection cost under several conditions including limited liability and budget constraint for the government.

We consider a more general inspection rule compared with the strict liability rule and the negligence rule, which require inspection with probability one only in the case of pollution. In our model, inspection is a probabilistic event. Probability of inspection in case of pollution is represented by s, while that in no-pollution case is represented by q. Under the policy considered in this paper, payment is composed of advance payment and ex-post payment, which are represented by K_0 and K_1, respectively. Firm is required to pay K_0 in advance prior to decision of effort. Note that it cannot be dependent on the actual effort represented by x and it must be set at a fixed level. In addition to advance payment K_0, ex-post payment K_1 will be imposed on firm after a level of effort is chosen and when inspection is conducted. Note that it can depend on the actual effort x, since the effort is revealed through inspection. We assume that the effort can be represented by expenditure on precaution activity or it is pecuniary[1]. Total payment of firm including effort, advance payment, and ex-post payment should not exceed a fixed amount of money represented by L. This setting of the policy reflects limited liability of firm. We assume that liability L is exogenously given. Timing of events under the policy is described in Figure 1.

However, inspection might be costly in some circumstances. In this section we are supposed to seek for the least frequency of inspection that will make a target level of effort represented by x_t be realized in the equilibrium. The target level of effort x_t can be different from the first best effort x_{fb}, which is defined by (1). The problem to be solved is written as follows:

$$\min_{s,q} \pi = sp(x_t) + q(1 - p(x_t))$$

subject to

$$\operatorname*{argmin}_x \ C(x, s, q) = x_t \qquad (3)$$

$$x_t + K_0 \leq L \qquad (4)$$

$$x + K_0 + K_1 \leq L, \qquad (5)$$

$$K_0 + \big(1 - p(x_t)\big)qK_1 + p(x_t)sK_1 \geq 0 \qquad (6)$$

where $C(x, s, q)$ represents the cost of the firm and it is composed of effort, advance payment, and the expected ex-post payment, in other words,

$$C(x, s, q) = x + K_0 + \big(1 - p(x)\big)qK_1 + p(x)sK_1. \qquad (7)$$

The inspection policy should encourage the firm to make the target level of effort in the equilibrium. Constraint (3) means that the cost is minimized when the firm complies with the standard or the target effort. Constraints (4) and (5) represent limited liability. Constraint (4) means that advance payment K_0 plus the target level of effort should not exceed the liability L. In other words, the government is allowed to request the firm to pay $L - x_t$ in advance before the level of effort is chosen. Note that the standard of effort (x_t), advance payment (K_0), ex-post payment (K_1), and inspection frequency (s and q) are announced in the beginning. On the other hand, if inspection is conducted, the government is also allowed to impose ex-post payment K_1 on the firm depending on the actual effort which is revealed by inspection. The constraint (5) requires that total expenditure including effort, advance and ex-post payments should not exceed

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[1] Some authors assume that precaution effort is non-monetary (Summers (1983) and Shavell (1986)), while others assume a pecuniary effort (Beard (1990) and Dari-Mattiacci and Geest (2005)). The main results in this paper do not depend on the assumption.

the liability represented by L. The constraint (6) is the budget constraint of the government, which means that the expected payment of the firm or the expected revenue of the government must be non-negative.

There is a possibility that the firm has an incentive to violate the standard by choosing an insufficient effort. Therefore, there must be a mechanism that can encourage the firm to comply with the standard in the equilibrium. Our strategy is to increase the cost of the firm in case of non-compliance on one hand and on the other hand to give a reward to the firm if compliance is proved by inspection. As a result, the government can reduce inspection frequency until the firm becomes indifferent between compliance and non-compliance.

We will characterize the optimal inspection rule as Proposition 1, which consists of the following eight lemmas. Prior to proving them, we will pose the following assumptions:

Assumption 1

Liability must be large enough to cover the target level of effort but it cannot cover the damage. In other words, we assume that $x_t \leq L < D$.

Assumption 2

We assume that $p'(x) < 0$ and $p''(x) > 0$.

The first lemma shows that ex-post payment should be set at the maximum if an insufficient effort is detected. The higher ex-post payment is imposed on the firm, the more the firm should pay if she does not comply with the standard. Consequently, the firm is incentivized to comply with the standard in the equilibrium.

Lemma 1

In order to increase the cost in case of non-compliance, K_1 should be set at the maximum if an insufficient effort is detected through inspection. In other words, it must be that

$K_1 = L - K_0 - x > 0$ for $x < x_t$. (8)

Proof.

We suppose that the firm will choose an insufficient level of effort or $x < x_t$ to minimize the cost represented by (7). Then we can see that the minimized cost will increase as K_1 increases. To see this, we substitute $K_1 = l - x$ into (7), where l is a parameter and it is equal to $L - K_0$ or less. We apply the envelope theorem to the cost minimization to obtain the following inequality:

$$\frac{\partial C(x,s,q,l)}{\partial l} = \big(1 - p(x)\big)q + p(x)s > 0.$$

It means that K_1 should be set at the maximum in order to increase the cost in case of non-compliance. In other words, it must be that

$$K_1 = L - K_0 - x \qquad (8)$$

under the optimal policy as claimed.

The rationality of raising ex-post payment up to the maximum is that it can encourage the firm to choose compliance in the equilibrium by increasing the cost of the firm in case of non-compliance. As a result, the government can decrease frequency of inspection until the firm will becomes indifferent between compliance and non-compliance. To see this, note that the envelope theorem applied to the cost minimization also yields

$$\frac{\partial C(x,s,q)}{\partial s} = p(x)K_1 > 0 \qquad (9)$$

$$\frac{\partial C(x,s,q)}{\partial q} = \big(1 - p(x)\big)K_1 > 0. \qquad (10)$$

Note also that K_1 is positive for $x < x_t$, since the following result follows from (8) and (4):

$$K_1 = L - K_0 - x \geq x_t - x > 0.$$

So far we have considered non-compliance case or $x < x_t$. However, the firm should be encouraged to comply with the standard in the equilibrium. To achieve that goal, the government should subsidize the firm if she is found to be compliant with the standard through inspection or $x \geq x_t$. We represent the subsidy *r* as a fixed and negative payment of the firm. In other words,

$$K_1 = r < 0 \quad \text{for } x \geq x_t.$$

The budget constraint of the government represented by (6) is rewritten as

$$K_0 + \{\big(1 - p(x_t)\big)q + p(x_t)s\}r \geq 0. \qquad (6)'$$

Immediately, we can see that advance payment K_0 should be set at the maximum, since the government can decrease inspection frequency until the budget constraint becomes bind. Therefore, we can obtain the next lemma.

Lemma 2

The advance payment should be set at the maximum, in other words,

$$K_0 = L - x_t. \qquad (11)$$

Now we move on to seeking for the optimal frequency of inspection. Suppose that the firm does not comply with the standard. Then, the less frequently inspection is conducted, the less compliant the firm becomes. It implies that the government can reduce inspection frequency by encouraging the firm to make zero-effort if she decides to become non-compliant with the standard. It should be stressed that the firm must be indifferent between zero-effort and the standard for effort in the equilibrium, so that the firm can be expected to be compliant with the standard. It is possible by giving the firm a reward or subsidy if she is found to be compliant with the standard through inspection.

By substituting (8) and (11) into (7), the cost function can be rewritten as

$$C(x,s,q) = x + L - x_t + \left(\left(1-p(x)\right)q + p(x)s\right)(x_t - x) \qquad \text{for } x < x_t. \tag{12}$$

It immediately follows from (12) that

$$L - x_t < C(0,s,q) = L - \left(1 - \left(1-p(0)\right)q - p(0)s\right)x_t < \lim_{x \to x_t} C(x_t,s,q) = L. \tag{13}$$

It can also be shown that the cost function is increasing at $x = x_t$, since the derivative of the function with respect to x yields

$$\left.\frac{\partial C(x,s,q)}{\partial x}\right|_{x=x_t} = 1 - \left(\left(1-p(x_t)\right)q + p(x_t)s\right) > 0. \tag{14}$$

Therefore, assuming convexity of the cost function[2], the firm will choose zero-effort if and only if

$$\left.\frac{\partial C(x,s,q)}{\partial x}\right|_{x=0} = 1 - \left(\left(1-p(0)\right)q + p(0)s\right) + p'(0)x_t(s-q) \geq 0. \tag{15}$$

Figure 2 illustrates the cost function without reward for compliance.

Next lemma gives the positive minimum frequency of inspection that can motivate the firm to choose zero-effort ($x = 0$) if she becomes non-compliant.

Lemma 3

Suppose that the firm becomes non-compliant or $x < x_t$. The firm will always choose zero-effort under any level of inspection frequency, if $p(0) - p'(0)x_t < 1$. Otherwise, the frequency of inspection represented by (s,q) that makes the firm choose $x = 0$ should satisfy the following inequality.

$$q \geq -\frac{1}{p(0)-p'(0)x_t-1} + \frac{p(0)-p'(0)x_t}{p(0)-p'(0)x_t-1}s. \tag{16}$$

Proof.

The firm will choose $x = 0$ when she decides to deviate from the standard, if and only if (15) is satisfied. This inequality can be rewritten as (16) if $p(0) - p'(0)x_t > 1$. This case is illustrated by Figure 3. On the other hand, if $p(0) - p'(0)x_t < 1$, it can be shown that the whole rectangle $[0,1] \times [0,1]$ is included in the area which is represented by (16) as illustrated by Figure 4.

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[2] We will make sure the convexity of the cost function under the optimal policy by (24).

Let us call (15) the zero-effort constraint from here on. A frequent inspection is not necessary to lead the firm to zero-effort in case of non-compliance, which means that the government can reduce inspection frequency. Another advantage of zero-effort in non-compliance case is that the effectiveness of the ex-post payment or the penalty imposed on non-compliant firm will be enhanced through zero-effort, since the probability of pollution goes up to the maximum or $p(0)$.

Note that inspection is *almost unnecessary* if we are allowed to ignore the budget constraint for the government which is to be considered later, since (s,q)=(0,0) satisfies the inequality represented by (15). This result is summarized in the following lemma.

Lemma 4

If the budget constraint does not bind, inspection is *almost unnecessary* to encourage the firm to choose zero-effort in case of non-compliance.

Lemma 4 seems obvious, since the firm is expected to make zero-effort with less frequent inspection if she is not compliant with the standard. However, when we say that inspection is *almost unnecessary*, it does not mean that $(s, q) = (0,0)$. A positive frequency of inspection is still necessary even though it is infinitesimal. Our strategy is to motivate the firm to comply with the standard in the equilibrium by giving her a reward if she is found to be compliant through inspection.

The next lemma will give us another piece for the optimal policy or the subsidy to the firm which is found to be compliant with the standard through inspection.

Lemma 5

Under the optimal policy, K_1 should be set at a certain negative fixed level represented by r if the firm is found to be compliant with the standard. In other words, it must be that

$$K_1 = r^* < 0 \text{ for } x \geq x_t, \tag{17}$$

where r^* is given by

$$r^* = -\left(\frac{1-(1-p(0))q-p(0)s}{p(x_t)s+(1-p(x_t))q}\right)x_t < 0. \tag{18}$$

Proof.

Suppose that the firm is not compliant with the standard. Under the policy proposed by previous lemmas the firm will choose $x = 0$ and the expected cost is given by (13). On the other hand, if the firm decides to be compliant with the standard x_t, the expected cost will be calculated as

$C(x_t, s, q) = L + (p(x_t)s + (1 - p(x_t))q)r.$

If the subsidy represented by *r* is determined so that the two expected costs are equivalent to each other, the firm is expected to comply with the standard. Such a critical value for *r* can be calculated as claimed.

The idea of the optimal policy is illustrated by Figure 5. The cost of the firm drops discontinuously at $x = x_t$ due to the subsidy.

The budget constraint represented by (6) has not been considered yet. The constraint requires that the expected payment of the government must be non-negative in the equilibrium. To realize the reason why the budget constraint is necessary, let us consider how the situation where inspection is *almost unnecessary* looks like. The reward for compliance goes to infinity in the absolute value as frequency of inspection becomes infinitesimal (see Lemma 5). The government can finance such an extremely big reward if it does not confront the budget constraint.

By substituting (11), (17), and (18) into (6), the budget constraint can be rewritten as

$$q \geq \frac{2x_t - L}{(1-p(0))x_t} - \frac{p(0)}{1-p(0)}s. \qquad (19)$$

The next lemma shows that inspection is still *almost unnecessary* if a sufficient liability can be imposed on the firm.

Lemma 6

If $2x_t < L$, inspection is *almost unnecessary* to make the firm choose zero-effort in case of non-compliance.

Proof.

According to Lemma 4, inspection is *almost unnecessary* as long as the budget constraint is satisfied. We can easily see that the area which is represented by (19) includes the origin or (0,0) in (*s*,*q*) plain if $2x_t < L$.

However, inspection with positive frequency is necessary, when we suppose that $x_t \leq L \leq 2x_t$. The next two lemmas show that frequency of inspection will increase as liability decreases.

Lemma 7

Assume that $p(0) - p'(0)x_t < 1$. Then, the optimal inspection policy is characterized by

$$s^* = \frac{2x_t - L}{p(0)x_t} \text{ and } q^*=0, \text{ if } (2-p(0))x_t < L < 2x_t, \qquad (20)$$

$$s^*=1 \text{ and } q^* = \frac{(2-p(0))x_t - L}{(1-p(0))x_t}, \text{ if } x_t \leq L < (2-p(0))x_t. \qquad (21)$$

Proof.

According to Lemma 3, the zero-effort constraint represented by (15) will not bind if $p(0) - p'(0)x_t < 1$. Only the budget constraint represented by (19) will bind. The area which satisfies (19) in the rectangle $[0,1] \times [0,1]$ is illustrated by Figure 6. It can be shown that the optimal frequency of inspection is given either by (s,0) or by (1,q). To see this, note that the slope of iso-inspection line is represented by $-\frac{p(x_t)}{1-p(x_t)}$ in the equilibrium and it is larger than that of the line represented by the budget constraint or $-\frac{p(0)}{1-p(0)}$. The condition that $(2-p(0))x_t < L$ means that $s < 1$. As illustrated by Figure 6, if L decreases further less than $(2-p(0))x_t$, inspection in no-pollution case will be implemented or $q > 0$ while keeping $s = 1$ as claimed.

Figure 6 illustrates the optimal path of inspection frequency when the liability decreases. The optimal path suggests that inspection in case of pollution should be prioritized over one in no-pollution case. The next lemma copes with the remaining case which is represented by $p(0) - p'(0)x_t > 1$.

Lemma 8

Assume that $p(0) - p'(0)x_t > 1$.

The optimal inspection policy is characterized by

$$s^* = \frac{2x_t - L}{p(0)x_t} \text{ and } q^*=0, \text{ if } 2x_t - \frac{p(0)x_t}{p(0)-p'(0)x_t} < L < 2x_t, \quad (22)$$

$$s^* = \frac{(1-p(0))(L-x_t) - p'(0)x_t(2x_t - L)}{-p'(0)x_t^2} \text{ and } q^* = \frac{2x_t - L}{(1-p(0))x_t} - \frac{p(0)}{1-p(0)}s^*,$$

$$\text{if } x_t \le L < 2x_t - \frac{p(0)x_t}{p(0)-p'(0)x_t}. \quad (23)$$

Proof.

Not only the budget constraint represented by (19) but also the zero-effort constraint represented by (15) or (16) will bind in this case as shown by Lemma 3. The feasible area is illustrated by Figure 7. We can see that the optimal point is the intersection point of the line represented by (19) and the s axis if the liability is sufficiently high. The optimal point or the intersection point will move in the right direction from the origin as the liability decreases from $2x_t$ to $2x_t - \frac{p(0)x_t}{p(0)-p'(0)x_t}$. However, when the liability becomes even smaller than $2x_t - \frac{p(0)x_t}{p(0)-p'(0)x_t}$, the zero-effort constraint will bind. The optimal point will move along the line represented by (15) toward the corner represented by (1,1). Note that full inspection represented by (1,1) is the optimal policy if the liability is equal to the standard or $L = x_t$.

By putting all lemmas together, we can summarize the optimal policy which can minimize the expected frequency of inspection or the inspection cost as the next proposition.

Proposition 1

The payment rule under the optimal policy will be characterized by

$K_0 = L - x_t$

$$K_1 \begin{cases} = r^* \leq 0 & \text{if } x \geq x_t \\ = x_t - x & \text{otherwise,} \end{cases}$$

where r^* is given by (18). The optimal inspection policy depends on the target level of effort (x_t) and the liability (L) as follows: The optimal frequency of inspection is given by (20) or (21) if $p(0) - p'(0)x_t < 1$, while it is characterized by (22) or (23) otherwise.

The optimal inspection problem remains to be examined in some respects. One of them is convexity of the cost minimization problem. In other words, we need to check whether the cost function remains convex with respect to x under the optimal policy. The second derivative of the cost function with respect to x can be calculated as

$$\frac{\partial^2 C}{\partial x^2} = (s^* - q^*)\big(-2p'(x) + p''(x)(x_t - x)\big) \quad \text{for } x < x_t. \qquad (24)$$

It follows from (24) that convexity is guaranteed if $s^* - q^* > 0$. We can see that this inequality is assured under the optimal frequency of inspection which is given by Lemma 7 or Lemma 8.

We should also examine whether the optimal policy could prevent the firm from making an excessive effort. It can be shown that the cost function is an increasing function if $x \geq x_t$. To see this, the first derivative of the cost function can be calculated as

$$1 + (s^* - q^*)p'(x)r^* > 0 \quad \text{for } x \geq x_t. \qquad (25)$$

It follows that it is suboptimal for the firm to make an excessive effort. Beard (1990) shows that it is possible for polluter (injurer in his model) to take too much care compared with the first best under strict liability. In our model, the firm will never do it, since excessive compliance is not compensated by subsidy.

The optimal policy derived in Proposition 1 can be considered as a deposit-refund system. The firm is requested to pay a deposit represented by K_0 in advance. Afterwards, a refund represented by r will be returned to the firm, if she is found to be compliant with the standard through inspection. Figure 5 illustrates how the optimal policy works to encourage the firm to comply with the standard.

One of the most remarkable features of the optimal policy is that it successfully breaks through the limited-liability restriction that has precluded the social optimum from being attained in the literature. Assumption 1 just requires that liability L can cover the standard for effort. Any level of liability L which is larger than x_t can be applied. However, the less liability becomes, the more frequently inspection should be conducted. A trade-off between magnitude of liability and frequency of inspection is implied by the optimal path of inspection frequency which is illustrated by Figure 6 and 7. Conversely, when the liability becomes large, the deposit-refund mechanism embedded in the policy will be enhanced. The government can take advantage of the enhanced mechanism to decrease inspection frequency as shown by the next proposition.

Proposition 2

Assume that $L \leq 2x_t$. As the liability increases, the advance payment (deposit) and the subsidy for compliance (refund) will increase under the optimal policy while inspection frequency will decrease.

Proof.

According to Proposition 1, the advance payment K_0 is an increasing function of L. It follows from Lemma 7 and Lemma 8 that both s^* and q^* are decreasing with respect to L in any case.

On the other hand, suppose that liability is more than twice as much as the target effort even though it cannot cover the damage (see Assumption 1). Asset constraint of the firm or limited-liability restriction usually emerges in this case. For example, the strict liability rule cannot attain the first best. However, inspection is *almost unnecessary* in this case (see Lemma 6), in other words, the optimal policy derived in Proposition 1 can restore the first best without inspection as long as $L > 2x_t$. In addition, inspection is still *almost unnecessary* to attain the first best if the budget constraint does not bind (see Lemma 4).

Let us consider the optimal policy when inspection is *almost unnecessary.* According to Lemma 5, the reward for compliance represented by r^* goes to infinity in the absolute value as frequency of inspection becomes infinitesimal. Therefore, this policy could be interpreted as a kind of lottery. In other words, if the firm complies with the standard and if compliance is proved by very less frequent inspection, the firm will be given a big bonus as the reward for compliance.

The optimal paths of inspection frequency suggest that inspection should be prioritized in case of pollution over in no-pollution case. Note that $s^* > q^*$ along the optimal path (see Figure 6 and 7). In order to understand the rationality of prioritizing inspection in case of pollution over one in case of no-pollution, remember that the firm is encouraged to make zero-effort when she chooses non-compliance. As a result, inspection in case of pollution will be enhanced by the maximum probability of pollution or $p(0)$. In other words, inspection in case of pollution will effectively increase the cost in case of non-compliance.

Another remarkable feature of the policy is that the cost of the firm is fixed at x_t in the equilibrium, if liability is *less than* twice as much as the target effort, as stated by the next proposition. Note that the cost will usually increase in addition to effort under more prevalent policies. For example, some firms might be encouraged to exit the market when Pigouvian tax increases the cost in addition to effort. In this respect, the optimal policy derived in this paper could avoid the distortion that would be generated by other policies.

Proposition 3

The expected cost of the firm in the equilibrium, which is represented by $C(x_t, s, q)$, is fixed at x_t, if $L \leq 2x_t$. Otherwise, it is fixed at $L - x_t$.

Proof.

Suppose that $L \leq 2x_t$. In this case the budget constraint of the government always binds (see Figure 6 and 7). It follows from (6) and (7) that the expected cost of the firm in the equilibrium can be calculated as

$$C(x_t, s, q) = x_t. \qquad (26)$$

On the other hand, if $L > 2x_t$, substituting (18) into (7) yields the following result:

$$\lim_{(s,q)\to(0,0)} C(x_t, s, q) = L - x_t. \qquad (27)$$

Next two propositions cope with another special cases. One is the strict liability rule and another is the negligence rule. Under the strict liability rule, inspection will always be conducted in the case of pollution. Even though a firm which caused pollution is required to compensate for the damage, as a matter of fact, the firm will pay less than the damage because of limited liability. Next proposition shows that the strict liability is never optimal.

Proposition 4

The strict liability is not optimal.

Proof.

The strict liability rule is a special case of inspection policy. To see this, note that the same result under the strict liability can be replicated, if we set $(s, q) = (1,0)$, $K_0 = 0$, and $K_1 = L - x$. However, it follows from Proposition 1 that this set of policy variables is not optimal.

Another special case is the negligence rule. Under this rule, inspection will also be conducted only when pollution occurs. When the polluter is found to be non-compliant with the standard, the maximum amount of payment which is equal to the liability will be confiscated.

Proposition 5

The negligence rule is not optimal either.

Proof.

The negligence rule is also a special case of inspection policy. The negligence rule can be replicated, if we set $(s,q)=(1,0)$, $K_0=0$, and

$$K_1 \begin{cases} = r = 0 & \text{if } x \geq x_t \\ = L - x & \text{otherwise.} \end{cases}$$

However, this set of policy variables is not contained in the optimal policy.

The final question to be raised is how the target level of effort represented by x_t should be determined. It seems reasonable to think that the second-best effort will be determined so that the social cost including inspection cost can be minimized. We assume a fixed unit cost for inspection which is represented by *I*. Then the social cost represented by *SC* can be written as

$$SC = x_t + p(x_t)D + I\{p(x_t)s^* + (1-p(x_t))q^*\}. \qquad (28)$$

The terms in the bracket in (28) represent the expected inspection cost. The final proposition characterizes the second-best effort.

Proposition 6

The second-best target represented by x_{sb} that will minimize the social cost defined by (28) is smaller than the first-best effort, in other words, $x_{fb} > x_{sb}$, where the subscript *sb* represents second best.

Proof.

The second-best target represented by x_{sb} will be derived by the following first-order condition:

$$\frac{\partial SC}{\partial x_t} = \frac{\partial(x_t + p(x_t)D)}{\partial x_t} + I\left\{\left(\frac{\partial s^*}{\partial x_t} - \frac{\partial q^*}{\partial x_t}\right)p(x_t) + \frac{\partial q^*}{\partial x_t} + (s^* - q^*)p'(x_t)\right\} = 0. \qquad (29)$$

Note that the first term is equal to zero at the first-best effort or $x_t = x_{fb}$. Even though it looks complicated to calculate the terms in the bracket of (29), we can easily examine whether the inspection cost in the equilibrium will increase as the target effort increases. Suppose that $p(0) - p'(0)x_t < 1$. According to Lemma 3, the zero-effort constraint represented by (15) will not bind in this case. The feasible area for s and q is subject solely to the budget constraint or (19) as illustrated by Figure 6. It is easy to see that the line represented by (19) will shift upward as x_t increases, which means that the feasible area will shrink. It follows that the minimized inspection cost will also increase as x_t increases. It means that the sum of the terms in the bracket in (29) is positive. Therefore, we can conclude that the second-best effort is smaller than the first-best effort, in other words, $x_{fb} > x_{sb}$ as illustrated by Figure 8.

Conversely, suppose that $p(0) - p'(0)x_t > 1$. In this case, the feasible area is constrained by both (15) and (19). We can see that the line represented by (15) will rotate clockwise on the point (1,1) as the target effort increases as illustrated by Figure 9. On the other hand, the line represented by (19) will shift upward as the target effort increases. Those observations suggest that the feasible area will shrink as the target effort increases (see Figure 9). It also means that the minimized inspection cost will increase. Therefore, we can derive the same conclusion as in the first case that the second-best target of effort is smaller than the first-best effort.

4. Conclusion

We found how the optimal policy based on inspection looks like in the situation where the effort to prevent pollution is not observable and the damage caused by pollution cannot be compensated by the polluter due to the asset constraint. We have coped with moral hazard and limited liability at the same time.

We have derived an optimal inspection policy that will achieve the target effort with the minimum frequency of inspection under moral hazard and limited liability. The policy is composed of the standard for effort, the advance payment, the ex-post payment imposed on non-compliance, and the reward for compliance aside from inspection frequency. The optimal policy is considered as a deposit-refund system. Our strategy is to increase the cost of the firm in case of non-compliance on one hand and on the other hand to give a reward to the firm if compliance is proved by inspection. As a result, the government can reduce inspection frequency until the firm becomes indifferent between compliance and non-compliance. At the same time, the firm is encouraged to make zero-effort in case of non-compliance. Zero-effort increases the probability of pollution goes up to the maximum and it makes more effective to inspect in case of pollution. This feature of the policy is consistent with prioritizing inspection in case of pollution.

The optimal policy does work to make any standard for effort be realized in the equilibrium. It will also break through the liability restriction. Liability can be set at any level as long as it covers the standard effort. However, if the government is allowed to raise the liability, the deposit-refund system embedded in the policy can be enhanced and the inspection frequency can be reduced. In particular, if the liability is liability is more than twice as much as the target effort, the first best can be attained without inspection, even though the firm cannot compensate for the damage. It was also shown that the optimal policy is superior to the well-known policies, in other words, the strict liability rule and the negligence rule. We also derived the second-best standard that can minimize the social cost including the inspection cost.

We have considered an environmental pollution that seldom occurs but catastrophically destroys the environment once it occurs. However, it should be stressed that our contribution can be applied to other areas in economics. Analysis of the effect of limited liability was initiated in law and economics (Summers (1983) and Shavell (1986)) and pollution in the model could be replaced by accidents in general.

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References

Balkenborg D (2001) How liable should a lender be? The case of judgment-proof firms and environmental risk: Comment. *American Economic Review* 91: 731-738.

Beard TR (1990) Bankruptcy and care choice, *RAND Journal of Economics* 21:626-634.

Boyer M, Laffont JJ (1997) Environmental Risks and Bank Liability, *European Economic Review* 41: 1427-1459.

Che YK, Spier KE (2008) Strategic judgment proofing. *RAND Journal of Economics* 39:926-948.

Dionne G, Spaeter S (2003) Environmental risk and extended liability: The case of green technologies, *Journal of Public Economics* 87:1025-1060.

Feess E, Hege U (2003) Safty monitoring, capital structure and financial responsibility. *International Review of Law and Economics* 23: 323-339.

Hiriart Y, Martimort D (2006) The Benefits of Extended Liability. *RAND Journal of Economics* 37: 562-582.

Hutchison E, van't Veld K (2005) Extended liability for environmental accidents: what you see is what you get, *Journal of Environmental Economics and Management* 49: 157-173.

Jost PJ (1996) Limited liability and the requirement to purchase insurance. *International*

Review of Law and Economics 16: 259-276.

Lewis TR, Sappington DEM (1999) Using decoupling and deep pockets to mitigate judgment-proof problems. *International Review of Law and Economics* 19: 275-293.

Miceli and Segerson (2003), A note on optimal care by wealth-constrained injurers, *International Review of Law and Economics* 23: 273-284.

Pitchford R (1995) How Liable Should a Lender be? The Case of Judgment-Proof Firms and Environmental Risk. *American Economic Review* 85: 1171-1186.

Polborn MK (1998) Mandatory insurance and the judgment-proof problem. *International Review of Law and Economics* 18: 141-146.

Shavell S (1986) The Judgment Proof Problem. *International Review of Law and Economics* 6: 45-58.

Shavell S (2005) Minimum asset requirements and compulsory liability insurance as solutions to the judgment-proof problem. *RAND Journal of Economics* 36: 63-77.

Summers JS (1983), The case of disappearing defendant, *University of Pennsylvania Law Review* 132: 145-185.

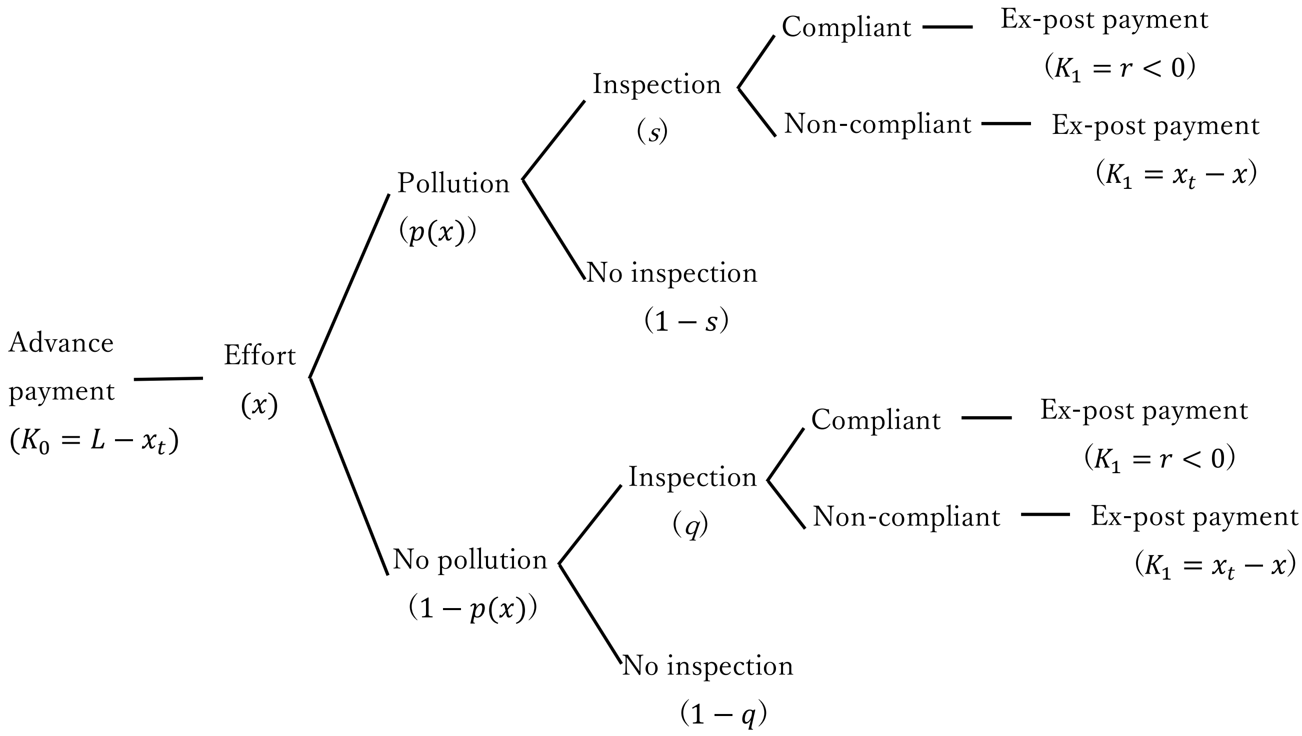

Fig.1　Timing of events

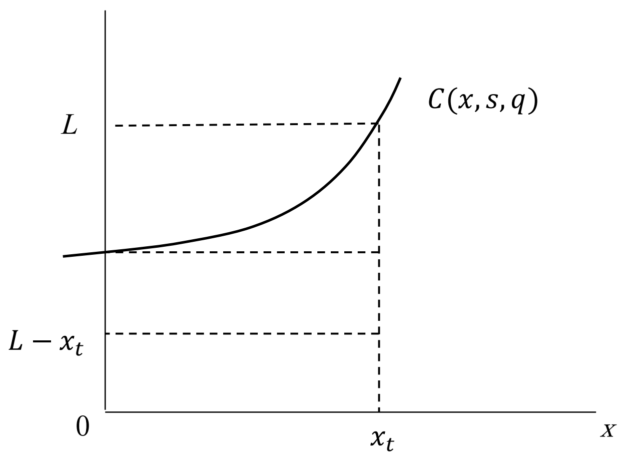

Fig.2　The cost function without reward for compliance

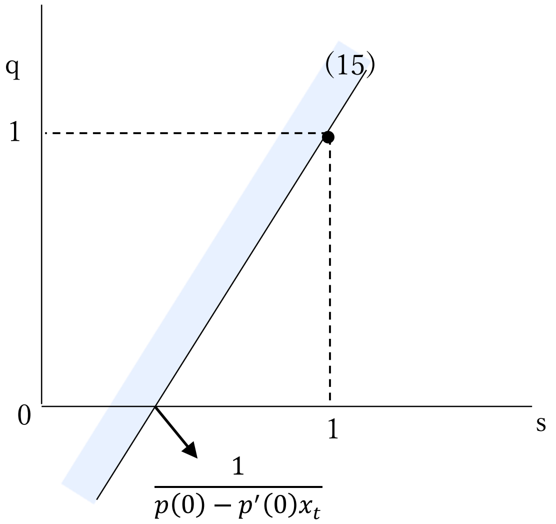

Fig.3 The zero-effort constraint with $p(0) - p'(0)x_t > 1$

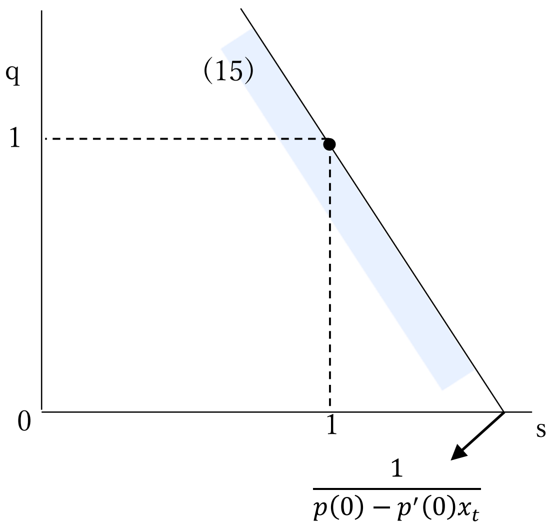

Fig.4 The zero-effort constraint with $p(0) - p'(0)x_t < 1$

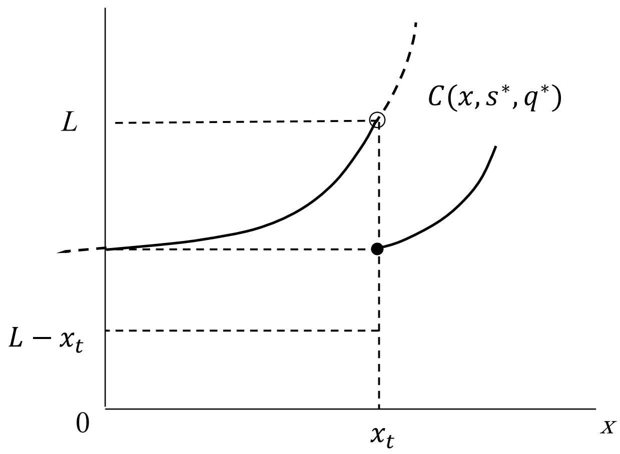

Fig.5　The cost function under the optimal policy

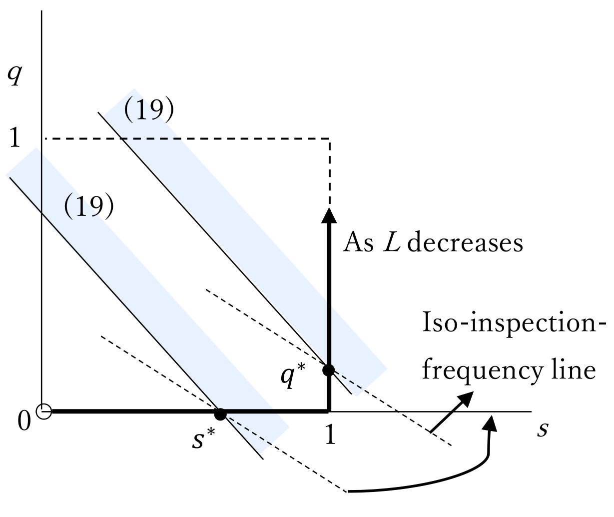

Fig.6　Optimal path of inspection frequency with $p(0) - p'(0)x_t < 1$

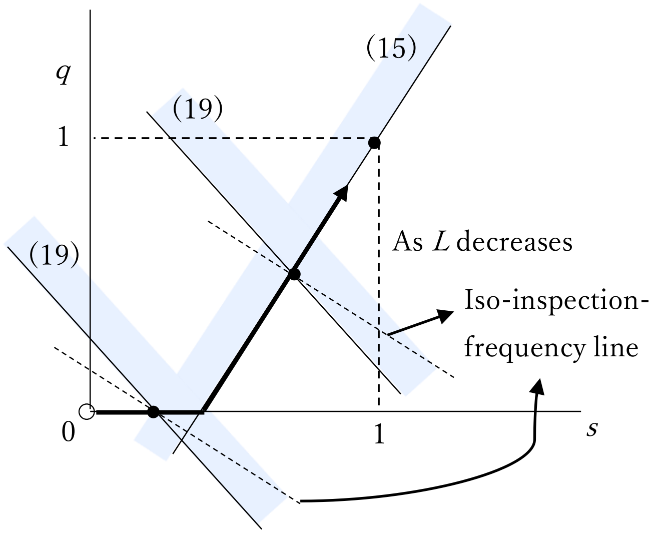

Fig.7 Optimal path of inspection frequency with $p(0) - p'(0)x_t > 1$

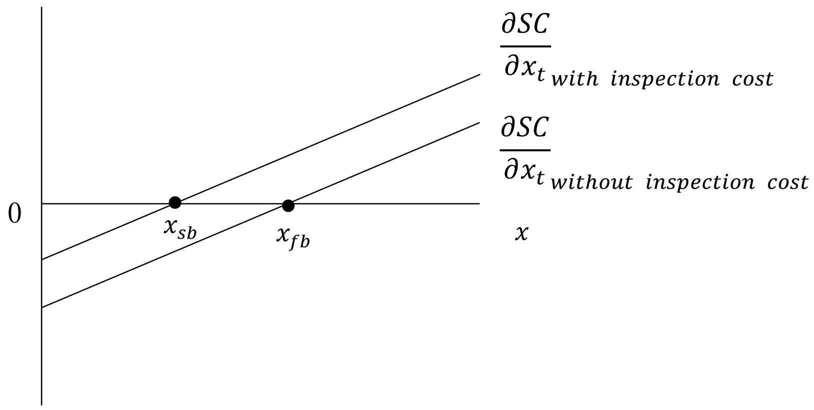

Fig.8 The second-best effort

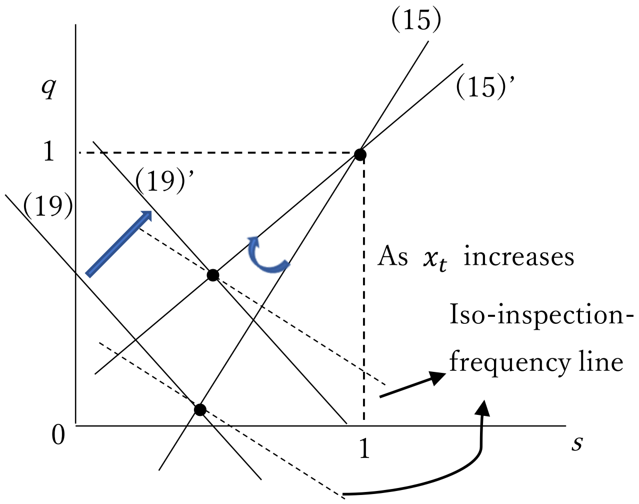

Fig.9　The effect of increasing x_t on the feasible area