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## The Global Correspondence Principle: A Generalization

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# The Global Correspondence Principle: A Generalization

By JAGDISH N. BHAGWATI, RICHARD A. BRECHER, AND TATSUO HATTA\*

*This paper generalizes the Global Correspondence Principle by extending, in two major ways, Paul Samuelson's 1971 analysis of the exchange rate response to an international purchasing-power transfer. We analyze the price effect of a shift in any parameter, not necessarily a transfer. We then explore the resulting adjustments in any nonprice variable such as welfare. As our analysis shows, the direction of these adjustments depends neither on whether they are small or large nor on whether equilibrium is locally stable or unstable.*

The celebrated Correspondence Principle of Paul Samuelson (1947) highlighted the importance of local stability for deriving fruitful theorems in comparative statics based on "small" changes. Thus, theorists typically qualify their comparative-static conclusions with the proviso that equilibrium is locally stable.

As Samuelson (1971) subsequently pointed out, however, the comparative-static effects of a parametric shift upon relative prices do not depend qualitatively on whether the initial equilibrium is locally Walras stable, provided that Walrasian tâtonnement is invoked and "large" adjustments are taken properly into account. Thus, using Samuelson's recent (1983) terminology, we have the Global Correspondence Principle.

Samuelson's 1971 paper dealt explicitly with the case of an international purchasing-power transfer in a two-country, two-good model. He examined the effect of a transfer upon the exchange rate, that is, the price variable whose disequilibrium behavior

is directly specified by the adjustment mechanism postulated in the model.<sup>1</sup>

First, we shall analyze the global response of price to a shift in any parameter (not necessarily a transfer payment) within the two-good, general equilibrium model. Then, more remarkably, we shall also establish that the corresponding responses of nonprice variables, such as welfare, are qualitatively independent of both the magnitude of adjustment and the question of local stability. This result extends the Global Correspondence Principle to cover variables other than price. The extension enables us to prove that, in a two-agent, two-good world, a transfer of income always improves the recipient's welfare—regardless of whether the initial equilibrium is stable, and irrespective of the magnitude of the transfer. In this way we generalize Samuelson's (1947) well-known proposition that the recipient's welfare improves provided that the initial equilibrium is stable. To take another example, our analysis can also be utilized to show that local stability can be dropped from Bhagwati's (1958) well-known set of conditions for immiserizing growth.

In Section I, we set up the model and analyze the global effect of a parametric shift on the relative price. In Section II, the shift's

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<sup>1</sup>We may mention that the original draft of our present paper also happened to develop a global analysis of price changes in relation to the transfer problem. Samuelson was kind enough to draw our attention to the fact that his 1971 paper had already anticipated our findings. The present draft is thus focused on two other aspects of global analysis, as explained in the text.

effect on nonprice variables will be studied and our main results proved. Section III applies our results of Section II specifically to the example of a transfer payment. Concluding remarks are offered in Section IV.

### I. Price Effects

We adopt the standard competitive model of an economy in which two goods,  $X$  and  $Y$ , are produced and consumed. Let the economy's aggregate excess demand function for good  $X$  be  $x(p, \theta)$ , where  $p$  is the relative price of this good, and  $\theta$  is a shift parameter. This generic model can be interpreted in many different ways. For example,  $p$  and  $x$  may be domestic price and excess demand for  $X$  in a single-country economy, with  $\theta$  representing a sales tax, an internal transfer between agents of the country, or the endowment of a factor. Alternatively, in the case of more than one country,  $p$  and  $x$  may be the international terms of trade and world excess demand, while  $\theta$  could be an import tariff, an international transfer or domestic productivity.

The condition for market equilibrium is

$$(1) \quad x(p, \theta) = 0.$$

To guarantee existence of a positive equilibrium price, we make the following assumption:

**ASSUMPTION 1:** *The function  $x(p, \theta)$  is continuous with respect to each of its arguments, and for each value of  $\theta$  in the relevant domain there exist a  $\bar{p}$  and a  $\bar{p}$  such that  $x(p, \theta) < 0$  for all  $p \geq \bar{p}$  and  $x(p, \theta) > 0$  for all  $p \leq \bar{p}$ .*

To cover situations of disequilibrium, we postulate a dynamic adjustment process of Walrasian tâtonnement, characterized by the following assumption:

**ASSUMPTION 2:** *When market-clearing condition (1) is not satisfied,  $p$  continuously increases or decreases as  $x(p, \theta) \geq 0$ , respectively, ceasing to change when  $x(p, \theta) = 0$ .*

We can now show that the direction of change in the price ratio depends only on the sign of the aggregate excess demand created by the parametric shift at the initial equilibrium price. This result is stated as the following proposition:

**PROPOSITION 1:** *In the model characterized by equation (1) and Assumptions 1 and 2, suppose that  $x(p^0, \theta^0) = 0$  for some pair  $(p^0, \theta^0)$  of  $p$  and  $\theta$ . Let  $\theta$  be shifted from  $\theta^0$  to  $\theta'$  where  $x(p^0, \theta') \neq 0$ . Then the price will reach a new equilibrium value, denoted  $p'$ , such that*

$$(2) \quad (p' - p^0)x(p^0, \theta') > 0.$$

**PROOF:**

First suppose that

$$(3) \quad x(p^0, \theta') < 0.$$

Then, from Assumption 2,  $p'$  must be the maximum  $p$  satisfying both

$$(4) \quad x(p, \theta') = 0;$$

$$(5) \quad p < p^0.$$

Since Assumption 1 ensures that  $x(\bar{p}, \theta') > 0$  for a  $\bar{p} < p^0$ , the Intermediate Value Theorem and inequality (3) together imply that there is at least one value of  $p$  satisfying both conditions (4) and (5). Denoting the highest such value  $p'$ , and invoking Assumption 2, we find that  $p'$  is the new equilibrium price. Thus, inequalities (3) and (5) yield (2). Similar reasoning establishes this proposition in the alternative case where  $x(p^0, \theta') > 0$  instead of inequality (3).

Proposition 1 states that the price increases if and only if the parametric shift creates a positive excess demand at the initial equilibrium price. Moreover, this result holds for large as well as small parametric changes of any kind, and regardless of whether local stability obtains, in our two-good model with Walrasian tâtonnement. Correspondingly, the conditions that have been derived in the comparative-static theo-

retical literature for determining price change from the sign of  $x(p^0, \theta')$  following specific parametric shifts, and that have been considered valid only for small changes from a locally Walras-stable equilibrium, are equally valid globally. That is, these conditions hold for changes of any size, with or without local stability. For example, the well-known transfer problem criterion for price adjustments—according to which the donor's terms of trade will improve or worsen as the sum of the marginal propensities to consume importables of the two countries is respectively greater or less than unity<sup>2</sup>—is immediately seen to hold globally, and not just for small changes from a locally stable equilibrium as conventionally established.

Figure 1 illustrates why Proposition 1 holds even when the initial equilibrium is unstable. The solid and dashed curves respectively represent the aggregate excess demand function for good  $X$  after and before the parametric shift. Since the dashed curve is drawn upward sloping at  $p^0$ , the initial equilibrium is unstable. (To avoid cluttering the diagram, most of this curve is not shown.) At  $p^0$ , the shift in  $\theta$  then creates an excess supply of good  $X$  in the case depicted. Thus, the price of this good falls by Walrasian tâtonnement to  $p'$ , the new equilibrium.<sup>3</sup>

By contrast, the traditional technique of differential calculus evidently cannot be used to infer the shift of an unstable equilibrium, since a small change in  $\theta$  at such an equilibrium will lead to a large adjustment in  $p$ . The calculus technique leads to the following well-known formula derived from equation (1):

$$(6) \quad dp/d\theta = -x_\theta/x_p,$$

where subscripts denote partial derivatives. At an unstable equilibrium, we have  $x_p > 0$ ,

<sup>2</sup>Of course, this condition on marginal propensities determines the sign of excess demand created at the initial equilibrium price (i.e., the sign of  $x(p^0, \theta')$ ) when the specific parametric shift is a transfer.

<sup>3</sup>If the solid curve were slightly redrawn to touch (rather than cross) the vertical axis at  $p'$ —in which case the new equilibrium would no longer be fully stable—the essence of our above argument would be unaffected.

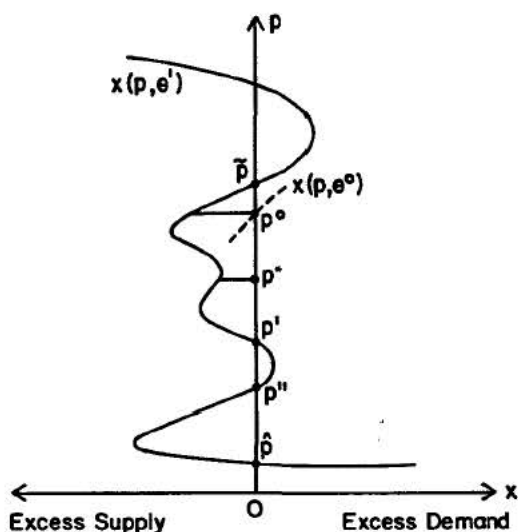


FIGURE 1

in which case  $dp/d\theta > 0$  when  $x_\theta < 0$ . Thus, in Figure 1, we can interpret formula (6) as simply telling us that a price rise from  $p^0$  to  $\bar{p}$  would be necessary to eliminate the excess supply created (at the initial price) by a parametric shift from  $\theta^0$  to  $\theta'$ . It would, however, be wrong to infer, as some statements in the literature by many can suggest, that  $\bar{p}$  will in fact be the new equilibrium in this case. Given the Walrasian price-adjustment mechanism,  $p$  will actually fall in response to the excess supply. Hence  $\bar{p}$  will not be the new equilibrium price; rather, the price will adjust globally to  $p'$  in this case.

To put the matter somewhat differently, contrary to customary interpretations, it is sufficient to look at the sign on the numerator alone in formula (6) to determine the direction of change in the terms of trade, in the event of a small parametric shift. For  $x_\theta$ , the numerator, is nothing but the excess demand at initial terms of trade when  $\theta$  shifts; that is, it corresponds to  $x(p^0, \theta')$  in formula (2). As Proposition 1 shows, moreover, the sign of this excess demand exclusively determines the direction of change in the terms of trade, that is, the sign of  $(p' - p^0)$  in formula (2). Thus, the sign of the denominator in formula (6), that is, of  $x_p$ , or equivalently the stability term, is irrelevant

to the direction of price change. The conventional criteria for terms-of-trade adjustment under different parametric shifts, which relate to the sign of the numerator in formula (6) and have been stated as valid subject to Walras-stability restriction to sign the denominator in formula (6), are therefore valid quite generally.

## II. Welfare and Other Nonprice Effects

Proposition 1, like Samuelson's Global Correspondence Principle, characterizes the global effect of a parametric shift upon the price variable. An intriguing question is: can the Global Correspondence Principle also be extended to the effects on nonprice variables? For example, the criteria for determining the effects of transfers or growth on a country's welfare have been established, invoking again Walras stability. Can these results also be shown to be independent of such a stability restriction? If so, the demonstration would be truly remarkable, since the dynamic adjustment behavior is specified on prices, not on the nonprice variables.

In this section, we indeed demonstrate that the criteria established for changes in nonprice variables in response to a small parametric shift, using differential calculus methods and considered valid in the presence of Walras stability, are valid in the global context under certain reasonable conditions.

To carry out this demonstration, we resort to an analogue of the technique utilized in the price-change problem above, where the excess demand induced by the parametric shift at initial prices (i.e.,  $x(p^0, \theta')$ ) played the key role. Since the objective now is to analyze nonprice change instead, we reassign this role to excess demand at the initial value of the nonprice variable. To maintain the value of the nonprice variable at the initial level, of course, we shall assume that price adjusts as required (to a level to be denoted as  $p^*$  below). Thus, for example, if welfare is the nonprice variable and domestic growth is the shift parameter, the focus now would be on the excess demand where the price has been adjusted (to  $p^*$ ) to leave welfare unchanged after growth.

The criteria derived by examining the factors governing the sign of this excess demand at constant value of the nonprice variable correspond then to the ones stated as the conventional comparative-static results, using differential calculus methods.<sup>4</sup> Although these traditional results are stated as valid provided that Walras stability obtains, our analysis below will demonstrate that the results so derived are valid globally and independently of local stability, under specific restrictions.

Our demonstration proceeds by first deriving the conventional differential calculus results for nonprice-variable change in a general form, for any shift parameter, equivalently to the price-change formula (6). We then examine the precise manner in which these results generalize in the global context, that is, for large changes and regardless of Walras stability.

### A. Local Effects Reconsidered

First, we introduce the necessary terminology. Let  $u$  denote the nonprice variable that we examine, and suppose that it is functionally dependent on both  $p$  and  $\theta$ , as follows:

$$(7) \quad u = v(p, \theta).$$

We call the direct impact of  $\theta$  upon  $u$  the "primary impact" (given by  $v(p^0, \theta') - v(p^0, \theta^0)$ ) and the indirect impact of  $\theta$  upon  $u$  through  $p$  the "secondary impact" ( $v(p', \theta') - v(p^0, \theta')$ ). When primary and secondary impacts are opposite in direction and the latter outweighs the former, we shall say that the overall effect on  $u$  is "paradoxical." Otherwise, the overall effect will be called "normal."

<sup>4</sup>It should be stated that the comparative-static results on welfare have typically been derived directly, rather than by using the technique of investigating excess demand at constant value of the nonprice variable—a technique that we use to advantage here in generalizing the Global Correspondence Principle to the nonprice domain. Bhagwati's (1958) analysis of immiserizing growth (and, more recently, several analyses of international transfers) did, however, use the technique that we further develop here.

For example, we may interpret  $u$  and  $\theta$  as the welfare level and the transfer receipt, respectively, of one country in the standard model of international trade. Then the primary impact is the direct effect of the transfer upon the recipient's welfare at constant terms of trade, and the secondary impact is the transfer-induced terms-of-trade effect. Since the primary effect increases  $u$  in this case, the paradox implies a terms-of-trade deterioration so great as to make the overall effect on  $u$  negative. Other examples of paradoxes defined in this way are Bhagwati's immiserizing growth, or a welfare loss from abolishing a tariff.

Now, define the function  $\bar{x}$  by

$$(8) \quad \bar{x}(p, u) \equiv x[p, c(p, u)],$$

where  $\theta = c(p, u)$  is the inverse function of equation (7) with respect to  $\theta$ .<sup>5</sup>

Then, in view of equation (1) and identity (8), the equilibrium values of  $p$  and  $u$  must satisfy  $\bar{x}(p, u) = 0$ . Thus, we obtain  $du/dp = -\bar{x}_p/\bar{x}_u$ . This result and equation (6) then yield  $du/d\theta = (du/dp)dp/d\theta = x_\theta \bar{x}_p/\bar{x}_u x_p$ . Hence, noting that identity (8) implies that  $\bar{x}_u = x_\theta c_u$ , we have  $(du/d\theta)c_u = \bar{x}_p/x_p$ . Since the sign of  $c_u$  indicates the direction of the primary impact, a paradox implies that  $(du/d\theta)c_u < 0$ . Now,  $x_p < 0$  is the Walras-stability assumption. Therefore, we get the central result that, for small changes in the presence of local stability,

$$(9) \quad \bar{x}_p > 0$$

is a necessary and sufficient condition for a paradoxical outcome. In other words,  $\bar{x}_p \leq 0$  is a necessary and sufficient condition for a normal outcome, when Walras stability is assumed.

Analyses of specific parametric shifts then relate to  $\bar{x}_p$ , which is the shift-induced change

<sup>5</sup>When equation (7) can be interpreted as an indirect utility function,  $c(p, u)$  is the corresponding expenditure function. When  $\theta$  is interpreted as the transfer receipt and  $u$  the welfare of the recipient agent, then  $c(p, u)$  is the expenditure function minus the revenue function of this agent; we called this difference the *overspending function* in our 1983 article.

in excess demand at constant  $u$ . Thus, Bhagwati, who analyzed the paradox of immiserizing growth, found that  $\bar{x}_p > 0$  could arise from ultra-biased expansion or inelastic demand, despite Walras stability. Moreover, to obtain Samuelson's (1947) result that a transfer between two agents engaged in free trade could not yield welfare paradoxes in the presence of Walras stability, we could show that necessarily  $\bar{x}_p < 0$  in this case.

### B. Global Effects

Can formula (9) be generalized to cover both large (as well as small) parametric changes and situations where Walras stability does not obtain? In the general case, we shall show (via Proposition 2) that when (9) is suitably rewritten in equivalent noncalculus terms, it becomes only a necessary (but not sufficient) condition for a paradox to arise (and hence  $\bar{x}_p \leq 0$  becomes correspondingly a sufficient, but not necessary, condition for a normal outcome). Also, if we add certain restrictions (to rule out multiple values of  $p^*$  and to ensure monotonicity in the relationship of  $v$  to  $p$ ), the noncalculus version of (9) can indeed be shown (via Lemma 2 and Proposition 3) to become a necessary and sufficient condition for a paradox.

We proceed by first obtaining the following lemma:

**LEMMA 1:** *A necessary condition for the overall effect of  $\theta$  upon  $u$  to be paradoxical is that*

$$(10) \quad (p' - p^*)(p^* - p^0) > 0,$$

for a  $p^*$  satisfying

$$(11) \quad v(p^*, \theta') = v(p^0, \theta^0).$$

**PROOF:**

Without loss of generality, let us assume that the primary impact of the parametric shift is positive in the sense that  $v(p^0, \theta') > v(p^0, \theta^0)$ . When a paradox occurs, we must then have

$$(12) \quad v(p^0, \theta') > v(p^0, \theta^0) > v(p', \theta').$$

This result and the Intermediate Value Theorem imply that there must be a  $p^*$  between  $p^0$  and  $p'$  such that equation (11) is satisfied. Thus, we have either  $p^0 < p^* < p'$  or  $p' < p^* < p^0$ , which implies inequality (10).

Equation (11) indicates that  $p^*$  is a price that would leave welfare at the initial level after the shift in  $\theta$ . Inequality (10) implies that such a  $p^*$  must exist between  $p^0$  and  $p'$ . In other words, when a paradox occurs as a result of a parametric shift, the price must pass through  $p^*$  before it reaches the final equilibrium level.

Intuitive appreciation of this proposition can be strengthened with the help of Figure 1. Inequalities (12) imply that, after the shift in  $\theta$ , the level of  $u$  at  $p^0$  is higher than the initial level, while that at  $p'$  is lower. Hence, between  $p^0$  and  $p'$ , there must be some level of  $p$  (denoted  $p^*$ ) causing equation (11) to hold and making the level of  $u$  equal to the initial one.

Lemma 1 yields the following:

**PROPOSITION 2:** *A necessary condition for the overall effect of  $\theta$  upon  $u$  to be paradoxical is that*

$$(13) \quad (p' - p^0)x(p^*, \theta') > 0,$$

for a  $p^*$  satisfying equation (11).

**PROOF:**

Take the  $p^*$  defined in Lemma 1. Then, from the lemma,  $p^*$  is between  $p^0$  and  $p'$ . Thus, in view of Assumption 2, the sign of the excess demand  $x(p^*, \theta')$  at  $p^*$  must be equal to that of  $x(p^0, \theta')$  created by the direct impact of the parametric shift. This result and inequality (2) in Proposition 1 immediately yield (13).

Inequality (13) in this proposition—a result that holds evidently for large and small changes regardless of local stability—can now be formally related to  $\bar{x}_p$  in (9).

Since  $x(p^0, \theta^0) = 0$ , and since  $(p^* - p^0)$  and  $(p' - p^0)$  have the same sign when a paradox occurs (in view of Lemma 1), in-

equality (13) can be alternatively written as

$$(14) \quad [x(p^*, \theta') - x(p^0, \theta^0)] / (p^* - p^0) > 0.$$

Recalling identity (8), we can then rewrite condition (14) and hence (13) as

$$(15) \quad [\bar{x}(p^*, u^0) - \bar{x}(p^0, u^0)] / (p^* - p^0) > 0,$$

where the initial value of  $u$  is denoted  $u^0 [ \equiv v(p^0, \theta^0) ]$ .<sup>6</sup> Condition (9), however, is nothing but the calculus version of (15) and hence of (13) in Proposition 2.<sup>7</sup>

Therefore, it follows from Proposition 2 that the conventional necessary condition for paradoxical nonprice adjustments (derived for small changes and local stability) holds equally for large changes and regardless of stability. This proposition immediately implies that Bhagwati's necessary condition for immiserizing growth holds for changes of any magnitude, with or without stability. A similar generalization can be developed for Samuelson's (1947) theorem regarding the impossibility of a welfare-paradoxical transfer (as in Section III below).

Our generalization of the conventional comparative-static results is, however, not total. While (9) is both a necessary and sufficient condition for a paradox with small changes and local stability, its global counterpart in terms of condition (13) is only necessary but not sufficient.

A full generalization is possible, however, if added restrictions are imposed. These are suggested by noting that there are two reasons why necessary condition (13) is not also

<sup>6</sup>Note that from identity (8) we have  $\bar{x}(p^*, u^0) = x[p^*, c(p^*, u^0)] = x(p^*, \theta')$ .

<sup>7</sup>Inequality (15) implies that, according to Proposition 2,  $\bar{x}$  is (on average) an increasing function with respect to  $p$  in the interval between  $p^0$  and  $p^*$  when the overall effect of  $\theta$  upon  $u$  is paradoxical. That is, as the price is increased (decreased) from  $p^0$  to  $p^*$ , the excess demand has to increase (decrease) if  $\theta$  is adjusted to keep  $u$  constant at  $u^0$ .

a sufficient condition for a paradox. First, even when (13) holds,  $p^*$  may fail to satisfy inequality (10) in Lemma 1; that is,  $p^*$  may not lie between  $p^0$  and  $p'$ . Hence, a normal outcome may arise. This situation would occur in Figure 1 if  $p^*$  were relocated to lie between  $p''$  and  $\hat{p}$ , still leaving  $x(p^*, \theta') < 0$ .

Second, even if inequality (13) holds for a  $p^*$  satisfying (10), a normal result may arise if  $v(p, \theta')$  is not monotonic with respect to  $p$ . To see this possibility in Figure 1, note that (by definition) the price movement from  $p^0$  to  $p^*$  keeping  $\theta$  at  $\theta'$  would just offset the primary impact on  $u$ . Thus, if  $v(p, \theta')$  is monotonic with respect to  $p$ , the further price movement from  $p^*$  to  $p'$  will continue to change  $u$  in the same direction; that is, the secondary will now outweigh the primary impact, necessarily creating a paradox. This outcome need not arise, however, if  $v(p, \theta')$  is not monotonic, as may happen in the case when  $u$  represents welfare and distortions are present.

Thus, we immediately have the following:

**LEMMA 2:** *Assume that  $v(p, \theta')$  is monotonic with respect to  $p$  in the interval between  $p^0$  and  $p'$ . Then a necessary and sufficient condition for the overall effect of  $\theta$  upon  $u$  to be paradoxical is that inequality (10) hold for a  $p^*$  satisfying equation (11).*

This lemma yields the following:

**PROPOSITION 3:** *Assume that: (i) the function  $v(p, \theta')$  is monotonic with respect to  $p$ , so that  $p^*$  is unique; (ii) the primary and secondary impacts of the shift from  $\theta^0$  to  $\theta'$  are opposite in direction (since otherwise a paradox would clearly be impossible); and (iii) the shift from  $\theta^0$  to  $\theta'$  is small enough to insure that  $x(p, \theta')$  does not change sign more than once between  $p^0$  and  $p^*$ . Then, inequality (13) is a necessary and sufficient condition for the overall effect of  $\theta$  upon  $u$  to be paradoxical.*

**PROOF:**

In view of Proposition 2 and Lemma 2, we only have to prove that inequality (13) implies (10) under the assumptions of

the present proposition. Now suppose that the function  $x(p, \theta')$  changes sign between  $p^0$  and  $p^*$  exactly once. Then we have  $x(p^0, \theta')x(p^*, \theta') \leq 0$ . This result and inequality (13) yield  $(p' - p^0)x(p^0, \theta') \leq 0$ , a contradiction of (2). In view of Proposition 3, assumption (iii), therefore, the function  $x(p, \theta')$  cannot in fact change sign between  $p^0$  and  $p^*$ . Thus,  $p'$  must be outside the interval between  $p^0$  and  $p^*$ , which means that

$$(16) \quad (p' - p^0)(p' - p^*) > 0.$$

On the other hand, we know that  $(p' - p^0)(p^* - p^0) > 0$  from assumption (ii). This result and inequality (16) immediately yield (10).

Proposition 3, we may remark, does not revert trivially to local analysis. It is important to realize that restricting attention to small shifts in  $\theta$  does not automatically imply small adjustments in  $p$  and  $u$ . More specifically, if the initial equilibrium is unstable, these adjustments will be large even when the parametric shift is infinitesimal. Thus, our analysis is still global in essence.

### III. An Application: Transfers and Welfare

Samuelson (1947) proved that a purchasing-power transfer from donor to recipient never has a paradoxical effect on welfare in the standard two-agent, two-good model if the initial equilibrium is Walras stable. Our Proposition 2 now enables us to establish that his theorem is valid regardless of the magnitude of the transfer and the stability of initial equilibrium.<sup>8</sup>

Let an increase in  $\theta$  represent a purchasing-power transfer from the donor to the recipient, and have  $u$  denote the utility level of the latter agent. Without loss of generality we may assume that, in trading with the donor, the recipient is a net seller of good  $X$ .

<sup>8</sup>See also our (1984) Theorem 3 and our related discussion of Yves Balasko (1978). Recall that the analysis of the transfer problem was the fertile ground for Samuelson (1971) and us (in our original version of the present paper) to raise the global issues discussed here.



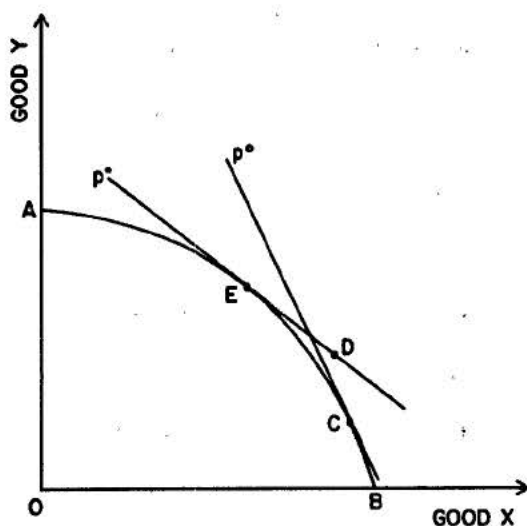


FIGURE 2

Noting that the primary impact of the transfer upon  $u$  is clearly positive, we suppose that the secondary impact is negative, since otherwise the possibility of a paradox would be trivially precluded.

In Figure 2, curve  $AB$  is the production-possibility frontier for the two-agent economy as a whole; point  $C$  is the initial equilibrium for aggregate production and consumption at price  $p^0$ ; while points  $E$  and  $D$  represent aggregate production and consumption, respectively, if the recipient's terms of trade were to deteriorate to  $p^*$ . (At this stage in the argument,  $D$  may be located anywhere on the  $p^*$  line.) Since convexity of the production-possibility set ensures that  $C$  lies below the  $p^*$  line, a well-known theorem due to John Hicks (1940)<sup>9</sup> implies that no distribution of bundle  $C$  can make both agents as well off as with the actual distribution of bundle  $D$ . Thus, recalling that  $v(p^*, \theta')$  at  $D$  equals  $v(p^0, \theta^0)$  at  $C$  (by definition of  $p^*$  as the welfare-preserving price in the face of a transfer), we immediately see that the donor's welfare must

be greater at  $D$  than at  $C$ . In light of this information, reapplication of the Hicks theorem implies that  $D$  cannot lie below the  $p^0$  line. Therefore, noting that  $E$  does lie below this line (by convexity once again), we conclude that  $D$  must be located southeast of  $E$ . In other words, the necessary condition (13) for a welfare paradox is not satisfied, since  $(p' - p^0) < 0$  and  $x(p^*, \theta') > 0$ .

Hence, Proposition 2 implies that the transfer's overall effect on welfare is normal.<sup>10</sup> This result, moreover, holds for a transfer of any magnitude, regardless of whether the initial equilibrium is stable.

#### IV. Concluding Remarks

The present paper has generalized the Global Correspondence Principle by extending, in two major ways, Samuelson's 1971 analysis of the exchange-rate response to an international purchasing-power transfer. First, we analyzed the price effect of a parametric shift by allowing the shift to occur in any parameter, not necessarily a transfer payment. Second, we explored the resulting adjustments in any nonprice variable such as welfare. As our analysis showed, the direction of these adjustments is independent of both their magnitude (small or large) and the local nature of equilibrium (stable or unstable). Thus, we have generalized the conventional algebra of comparative statics, which typically assumes small shifts from a stable equilibrium.

What about "higher dimensionality"? Our generalization is by no means limited to two agents. In fact, Proposition 1 is really about the aggregate economy, without reference to the number of agents included; while Propositions 2 and 3 focus on one agent as distinct from the rest of the aggregate economy, whose underlying composition is not essential to the analysis.<sup>11</sup>

<sup>10</sup>This result need not hold if there were tax distortions, as shown by Brecher and Bhagwati (1982) and our paper (1985).

<sup>9</sup>As Hicks states: "Thus if we start from any actual

Generalization to more than two goods, however, is another matter,<sup>12</sup> as noted also by Samuelson (1971). Under what restrictions such a generalization may be possible in the many-good case is an interesting question for further research.

<sup>12</sup>In the two-good case, the tâtonnement process is always stable if the equilibria are isolated. This special property, however, does not hold in general when we introduce additional goods. See Kenneth Arrow and F.H. Hahn (1971, pp. 282-85).

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