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SIMULACIÓN DE UN SISTEMA ELÉCTRICO RC CON UNA FUENTE DE VOLTAJE VARIABLE

SIMULATION OF A RC ELECTRICAL SYSTEM WITH A VARIABLE VOLTAGE SOURCE

Julio Cesar Romero Pabón
julioromero@mail.uniatlantico.edu.co
Universidad del Atlántico, Colombia.

Jorge Luis Rodríguez Contreras
jorge.jrodri@gmail.com
Universidad del Atlántico, Colombia.

Gabriel Mauricio Vergara Ríos
gabrielvergara@mail.uniatlantico.edu.co
Universidad del Atlántico, Colombia.

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Abstract

In this research work, the solution of a fundamental application problem in the theory of electrical systems is presented, such as the RC circuit, which contains an electrical resistance, a capacitor and a time-dependent voltage source, its solution contemplates the application of its physical and mathematical foundations, in order to carry out a qualitative and quantitative study that allows obtaining, solving and modeling in Simulink the differential execution and that governs the system.

Keywords: dynamic system, RC circuit, simulation, linear differential equation.

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1. RC CIRCUIT SIMULATION WITH VARIABLE VOLTAGE SOURCE.

The following series circuit will be analyzed with the following electrical devices: a Resistor (R), a Capacitor (c), a variable power source, and a switch [9], [11] y [13]. A qualitative and quantitative study of the differential equation that governs the dynamic system will be carried out. See the following figure.

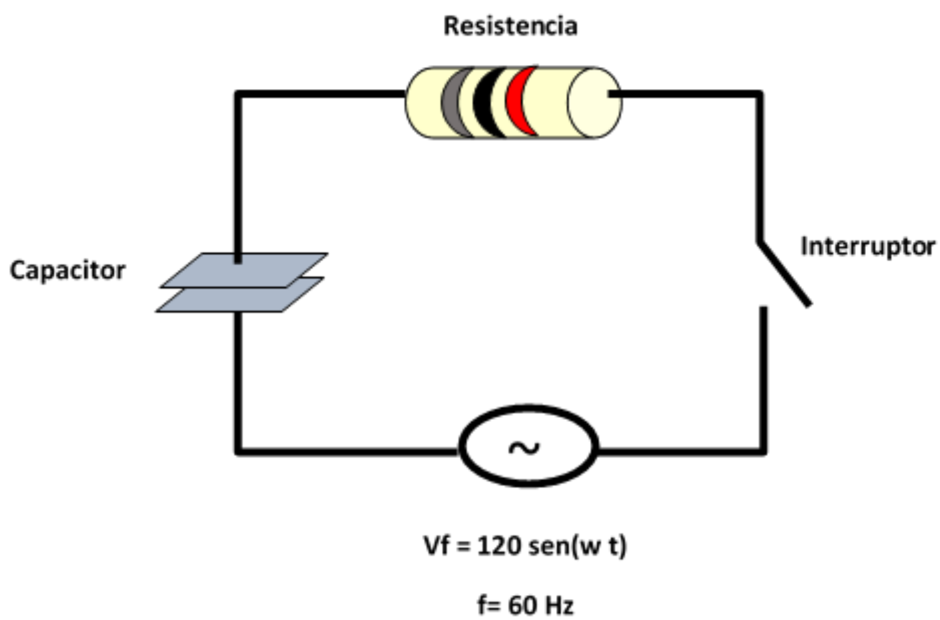

Figure 1. RC circuit with variable voltage source.

In addition, the simulation will be designed and implemented in Simulink for the following situations:

a) R = 100 Ω, C = 720 µf, Vf = 120 sin (wt) with f = 60 Hz, t = [0, 10] with h = 0.2
b) R = 600 Ω, C = 1200 µf, Vf = 100 sin (wt) with f = 50 Hz, t = [0, 20] with h = 0.2
c) R = 500 Ω, C = 900 µf, Vf = 110 sin (wt) with f = 50 Hz, t = [0, 30] with h = 0.2

Taking as suggestions the works of [1] and [2] to carry out a good simulation, the following steps will be carried out:

A. Qualitative analysis of the system
B. Obtaining the ODE that governs the system
C. Solution of the ODE that governs the system
D. System simulation design
E. Simulate the system with the following information:

a) R = 100 Ω, C = 720 µf, Vf = 120 sin (wt) with f = 60 Hz, t = [0, 10] with h = 0.2

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A. Qualitative analysis of the system

RC circuits are circuits that are made up of a resistor and a capacitor. It is characterized by the fact that the current can vary over time. When the time is equal to zero, the capacitor is discharged, at the moment that time begins to run, the capacitor begins to charge since there is a current in the circuit. Due to the space between the capacitor plates, no current circulates in the circuit that is why a resistor is used. When the capacitor is fully charged, the current in the circuit equals zero [8].

Charging a capacitor. Consider the series circuit in the figure. Initially the capacitor is discharged. If the switch I closes the charge begins to flow producing current in the circuit, the capacitor begins to charge. Once the maximum load is reached the current ceases in the circuit [13].

In the circuit of the figure we will have the sum

$$V_R + V_C = V_f$$

The voltage across resistor R according to Ohm's law is: $V_R = iR$

The voltage across the capacitor is given by: $V_C = \frac{q}{C}$

The voltage of the source is variable, as it is a source of alternating current:

$$V_f = 120\, sen(wt)$$

With: $w = 2\,\pi\, f = 2\,\pi\,(60) = 120\,\pi$

The equation of the circuit is:

$$iR + \frac{q}{C} = 120\, sen(wt)$$

B. Obtaining the ODE that governs the system

Taking into account that the intensity is defined as the load that crosses the section of the circuit in the unit of time, $i = \frac{dq}{dt}$ we will have the following differential equation.

$$R\,\frac{dq}{dt} + \frac{q}{C} = 120\, sen(wt)$$

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C. Solution of the ODE that governs the system

We will solve the general differential equation for any voltage and pulsation in the power source.

$$R\,\frac{dq}{dt} + \frac{q}{C} = V\,sen(wt)$$

The above is a linear ODE and its solution is given by [6]:

$$q(t) = C_1\; e^{-\frac{t}{R\,C}} - V\,C\,\frac{w\,C\,R\cos(wt) - \;sen(wt)}{1 + (w\,C\,R)^2}$$

Applying the initial conditions: that say that t = 0 t=0 then q(0)=0

$$q(t) = \frac{V\,C^2\,w\,R}{1 + (w\,C\,R)^2}\; e^{-\frac{t}{R\,C}} - V\,C\,\frac{w\,C\,R\cos(wt) - \;sen(wt)}{1 + (w\,C\,R)^2}$$

$$q(t) = \frac{V\,C}{1 + (w\,C\,R)^2}\left(C\,w\,R\;\; e^{-\frac{t}{R\,C}} - w\,C\,R\cos(wt) + sen(wt)\right)$$

D. System simulation design

To simulate the ODE: $R\,\frac{dq}{dt} + \frac{q}{C} = V\,sen(wt)$

We proceed as follows: Solve for the derivative of higher order

$$\frac{dq}{dt} = \frac{1}{R}\left(V\,sen(wt) - \frac{q}{C}\right)$$

As the derivative of higher order is equal to the sum of two terms, we will need an adder with two inputs (+ -), each term has coefficients which implies the use of two multipliers or amplifiers, one term is the alternate source or the another is depending on the load.

According to [4], [5] and [7], to implement a simulation of a dynamic system, a programming language with mathematical functions is recommended, such as Matlab, which brings with it a very versatile tool for this, such as Simulink. Now we open Matlab, and select Simulink model.

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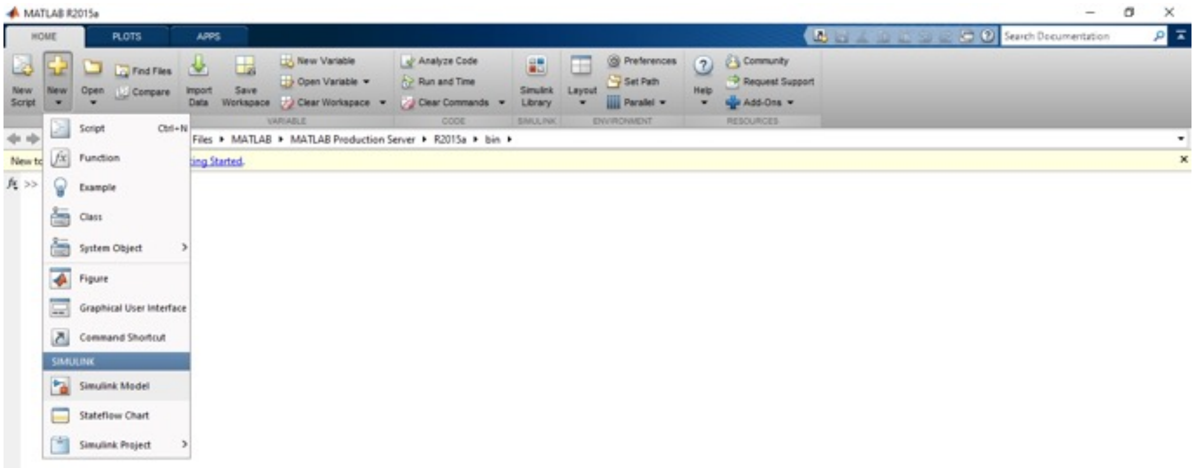


Figure 1. Matlab environment

The Simulink window opens

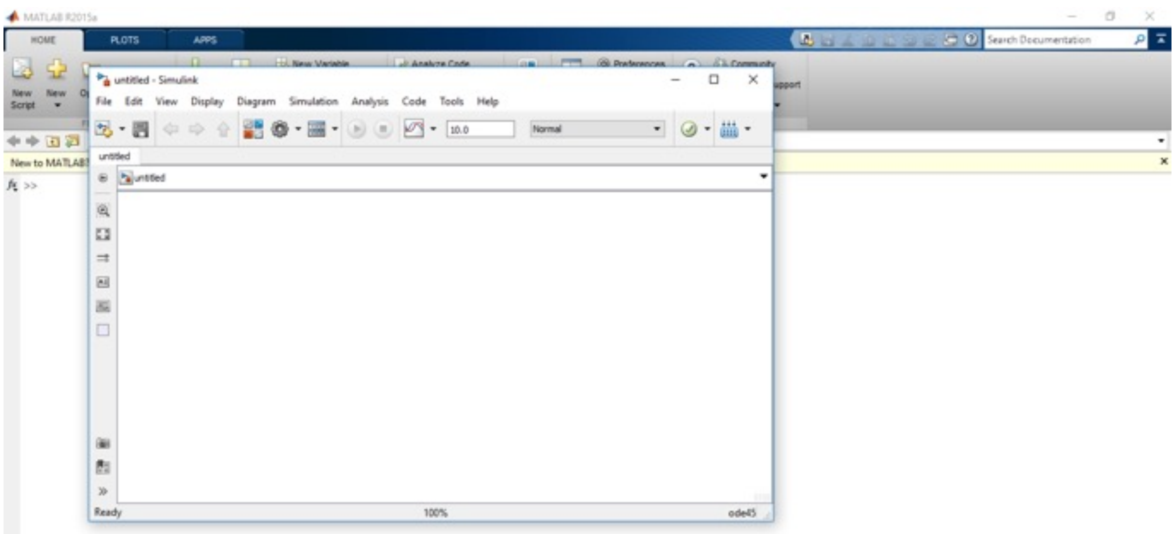
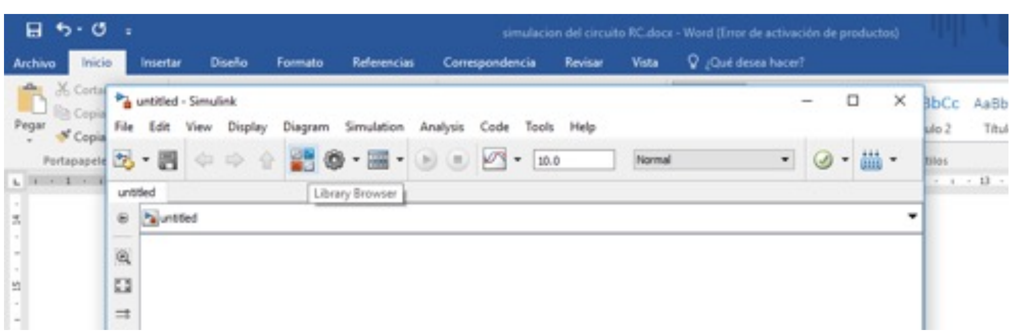


Figure 2. Simulink simulation window

We click on the Simulink library



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Simulink library

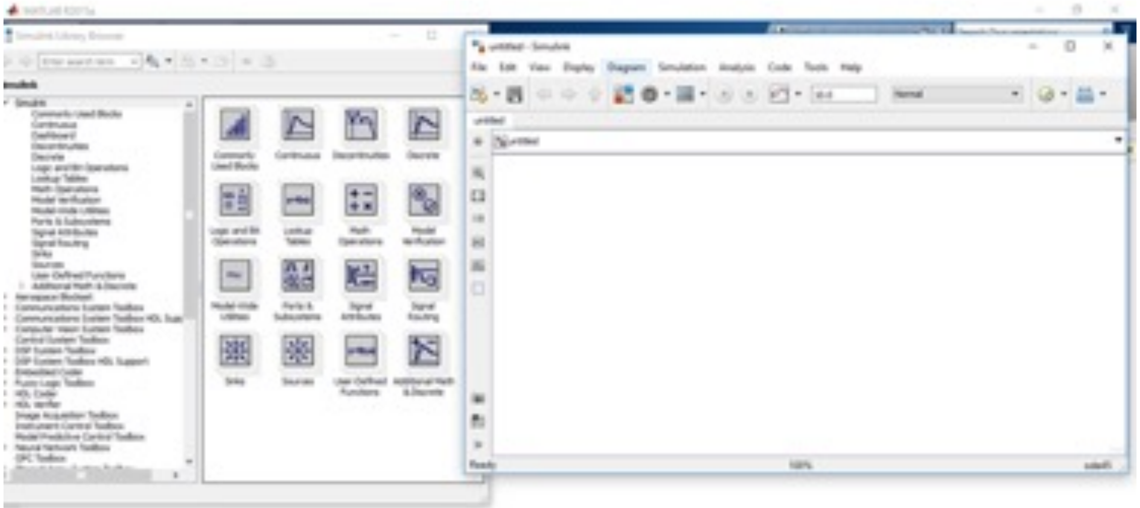

Figure 3. Simulink library

We choose our elements for the simulation of the library:
First we select the input source. We drag the source to the Simulink window

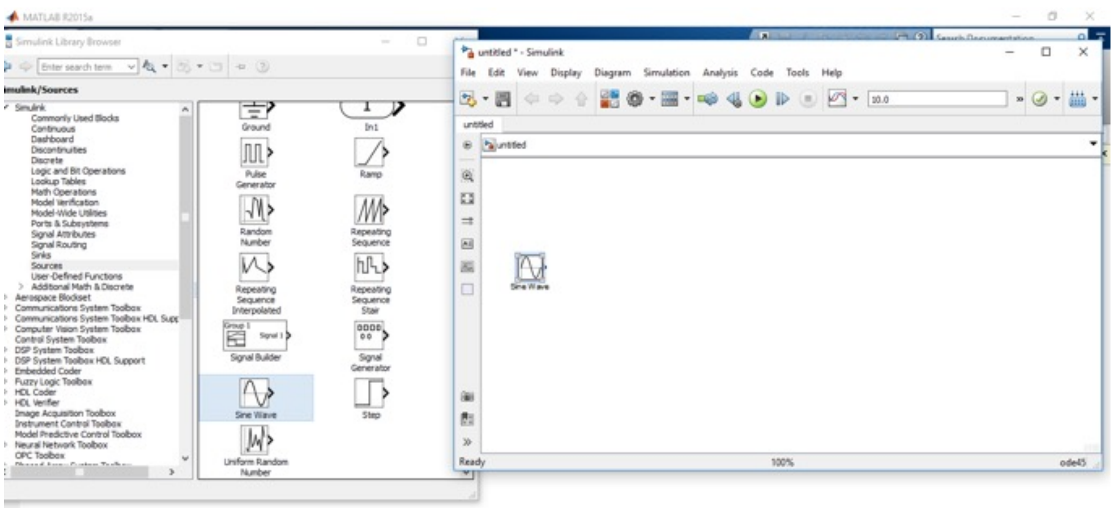

Figure 4. The source sine wave in the Simulink library.

We choose the output element, which in this case will be an oscilloscope, to see the graphical behavior of the system. For this we go to the Simulink library and select sinks, then drag the oscilloscope to the simulation window.

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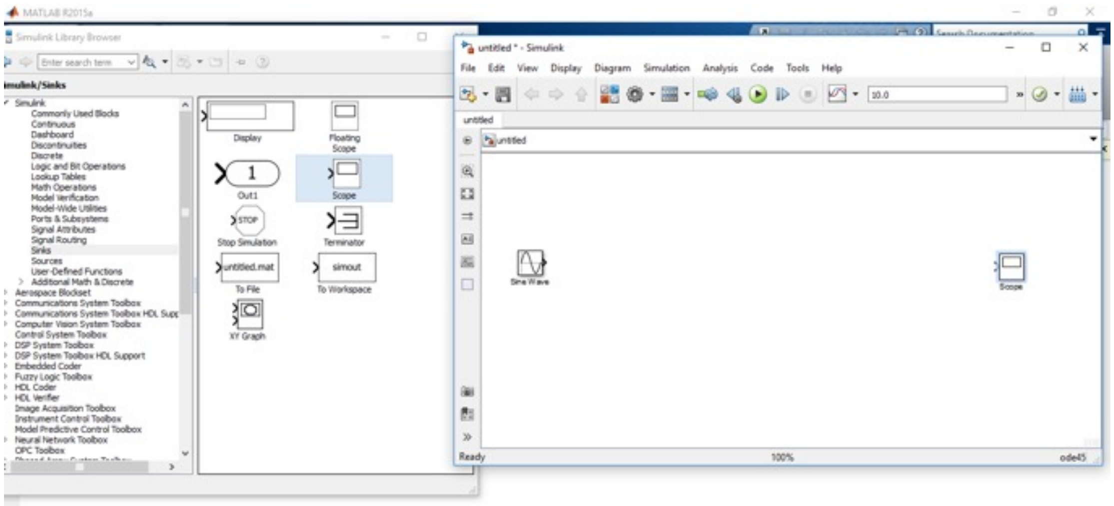

Figure 5. The oscilloscope in the Simulink library.

From the library we now choose the mathematical operators menu. We select the sum or add and drag the adder to the Simulink window

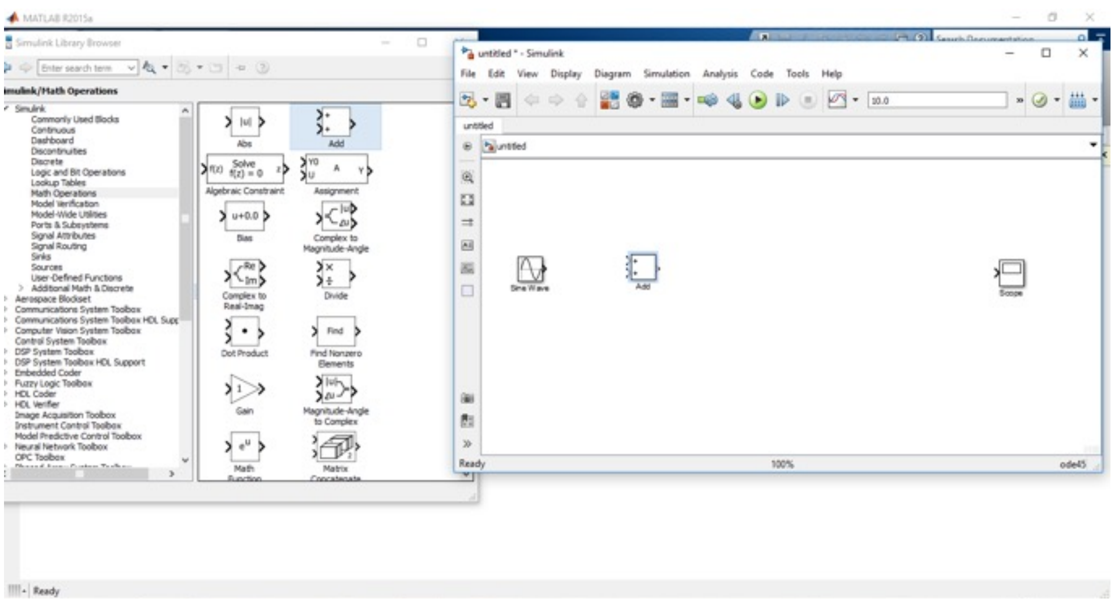

Figure 6. The adder in the Simulink library.

We make two clicks on the adder or add to configure it

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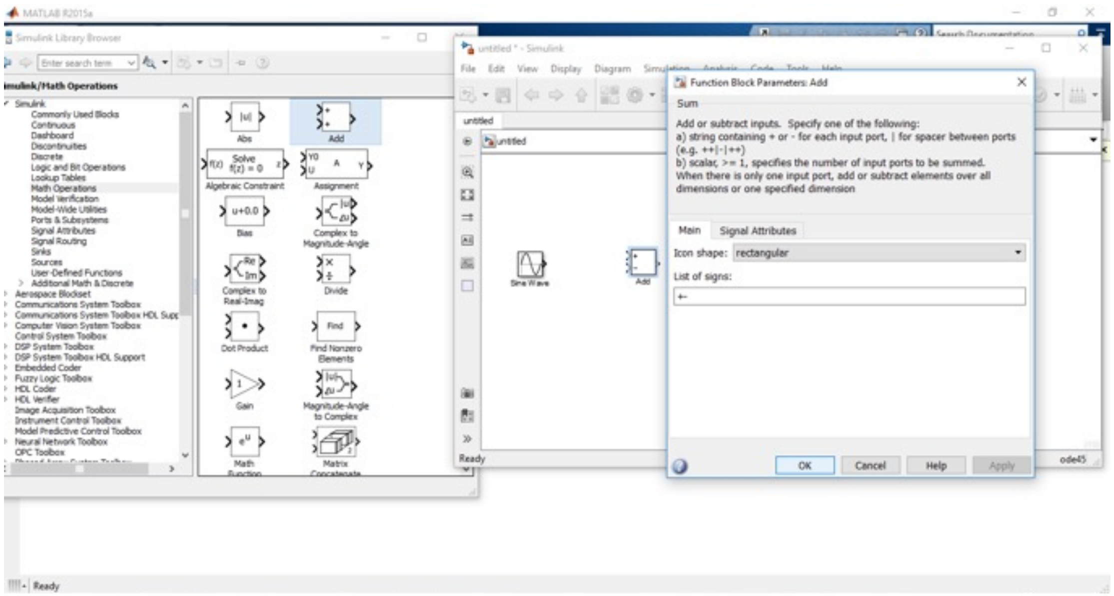

Figure 7. The adjusting the adder in the Simulink library.

We apply and we give ok so that the changes in the signs (+ -) are supplied

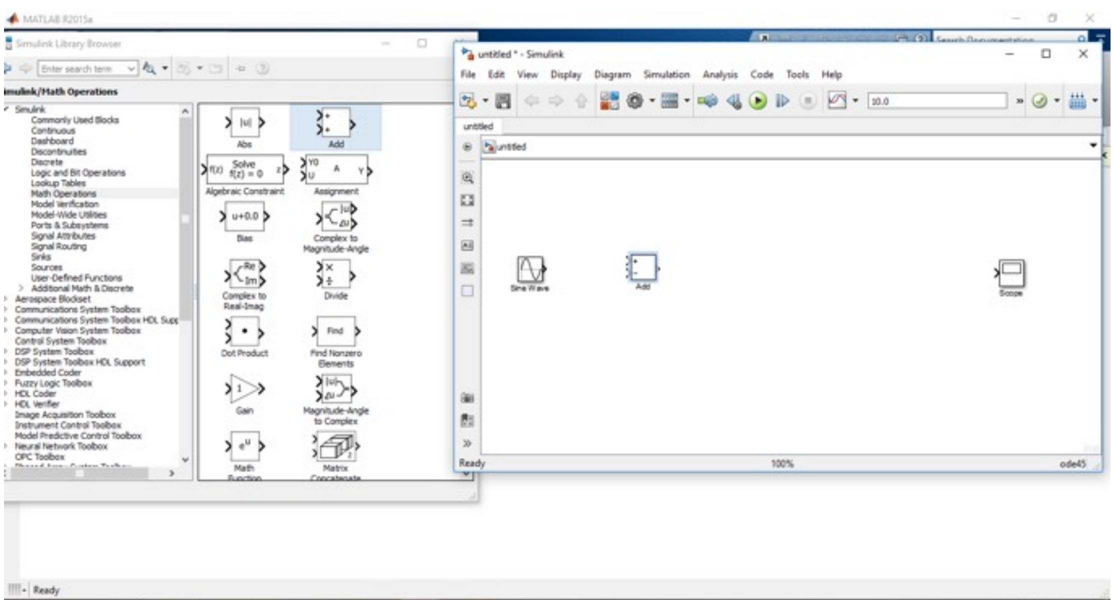

Figure 8. The adjusting the adder in the Simulink library.

We go to the bookstore and on the continuous menu. We select the integrator element and drag the integrator to the Simulink window

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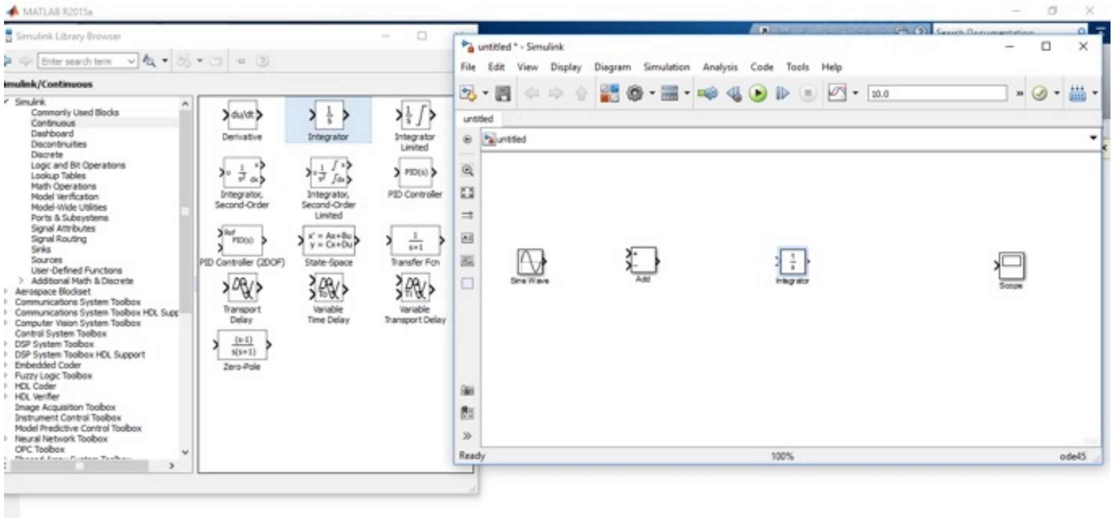

Figure 9. The adjusting the adder in the Simulink library.

Now we draw the lines of the variables or terms. First we draw the line from the source to the adder, for this we click and hold at the beginning of the source and join it at the end of the adder.

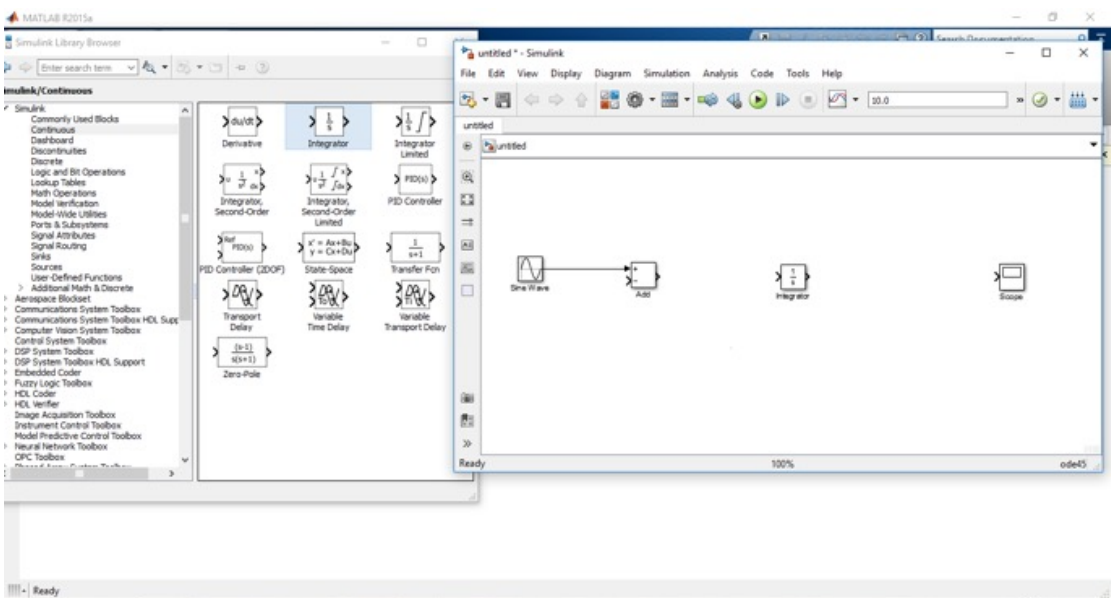

Figure 10. . Connecting the components in Simulink.

We draw the line that goes from the output of the adder to the input of the integrator.

This line indicates the variable $\frac{dq}{dt}$

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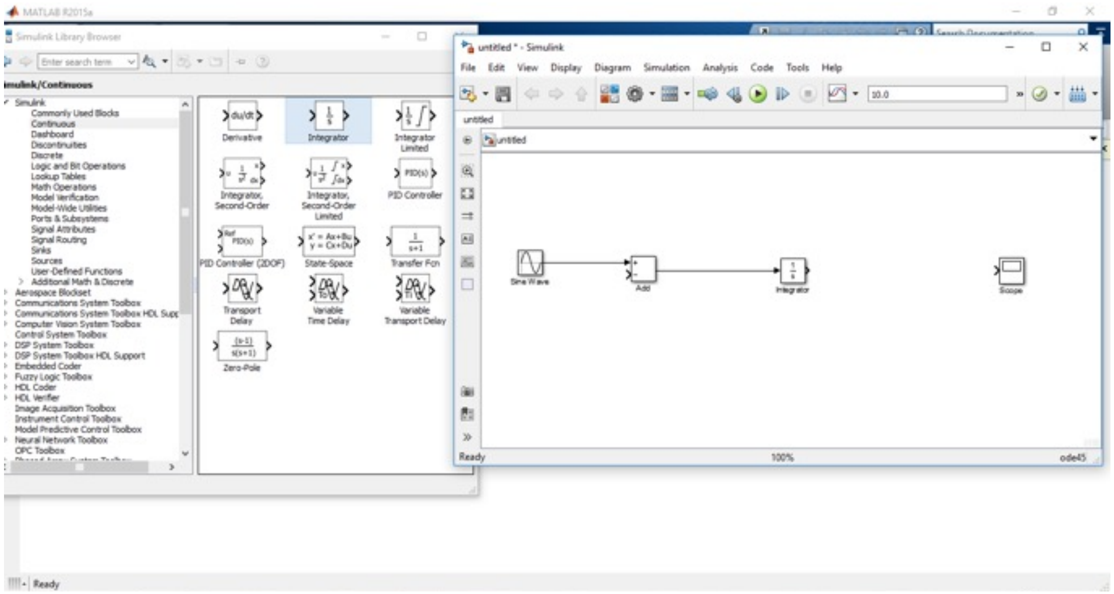

Figure 11. Connecting the components in Simulink.

We draw the line that goes from the output of the integrator to the input of the adder. This line indicates the variable q.

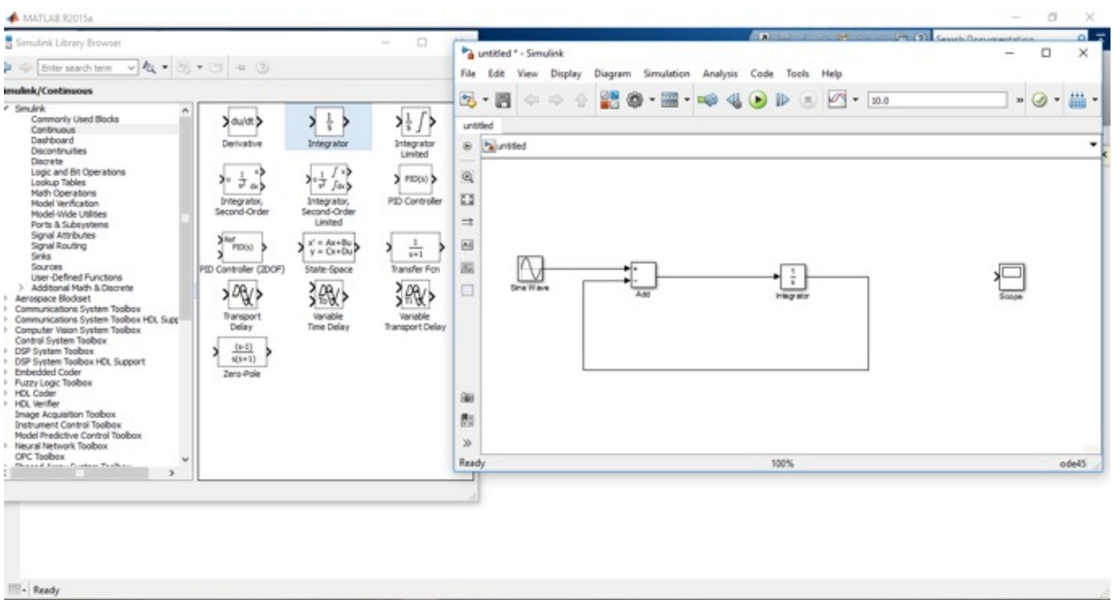

Figure 12. Connecting the components in Simulink.

We identify the variables, making two quick clicks next to each line to be able to write.

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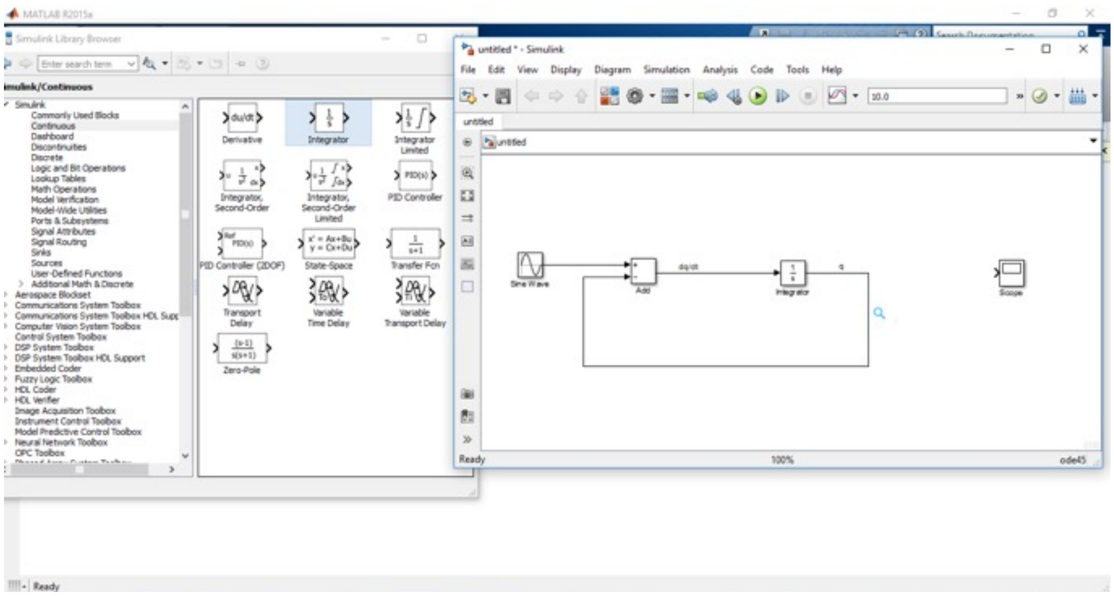

Figure 13. Labeling the variables.

Now we will place the multipliers (gain) or coefficients of the system, we go to the library, we look for the mathematical operators menu. We select the gain element and drag the multipliers or winners to the lines or variables to the Simulink window.

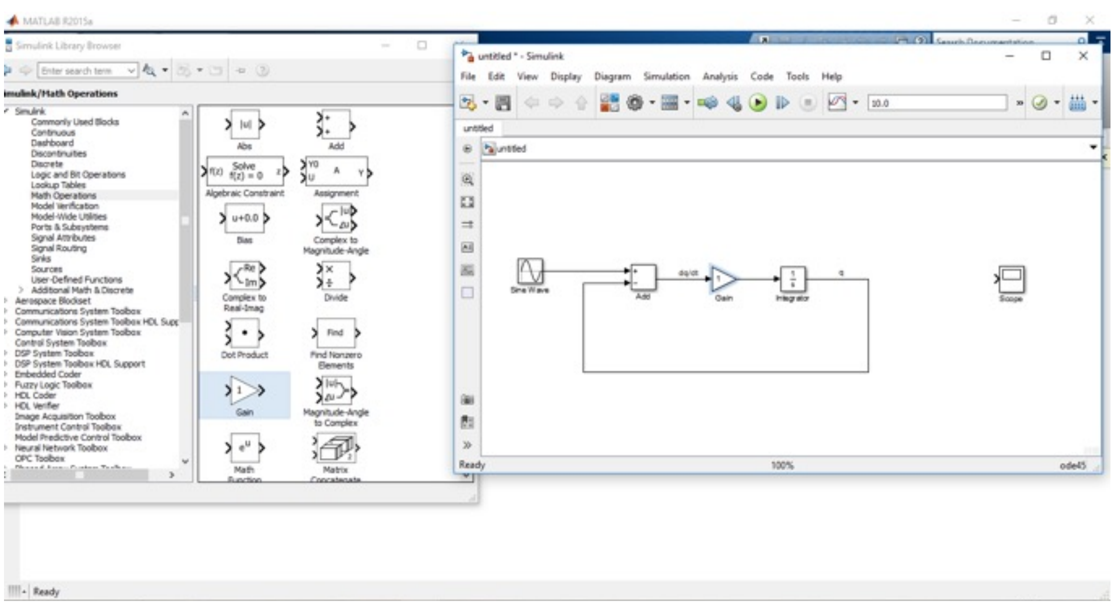

Figure 14. Placing the gains in the simulation.

We identify the multiplier or gain, for this we click on the name and change it to the system name

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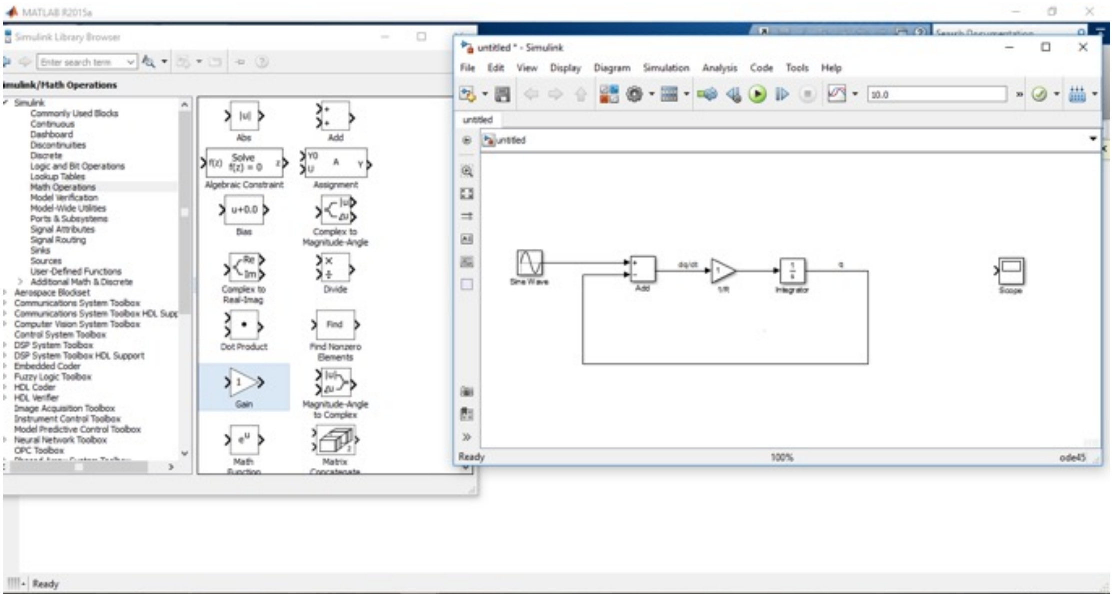

Figure 15. . Labeling the gain of the simulation.

We drag the other multiplier or gain

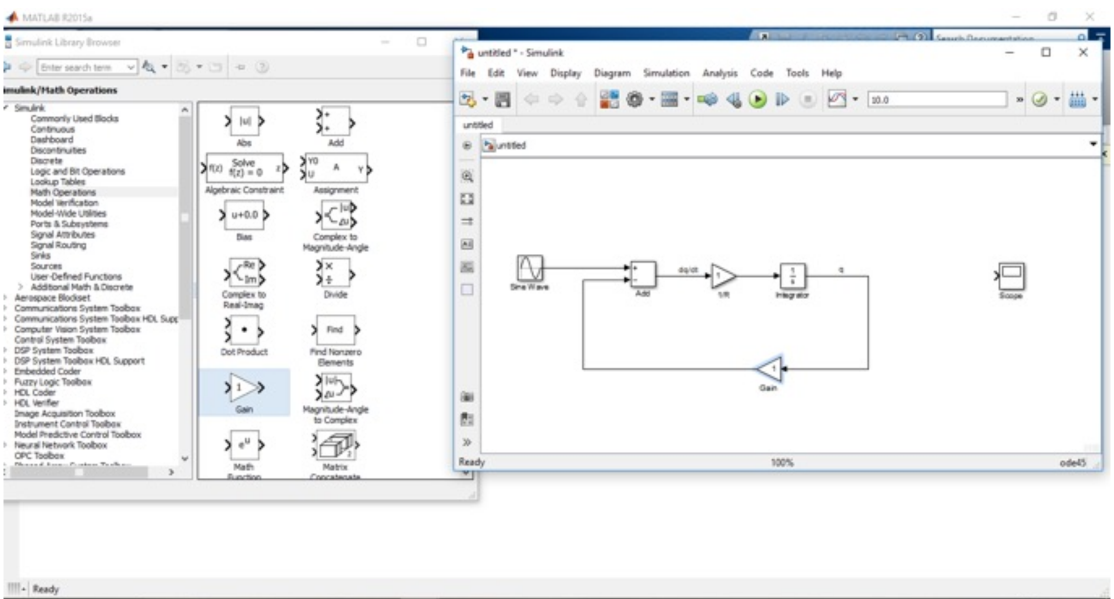

Figure 16. Placing the second gain in the simulation.

We identify the multiplier or gain of the line of q.

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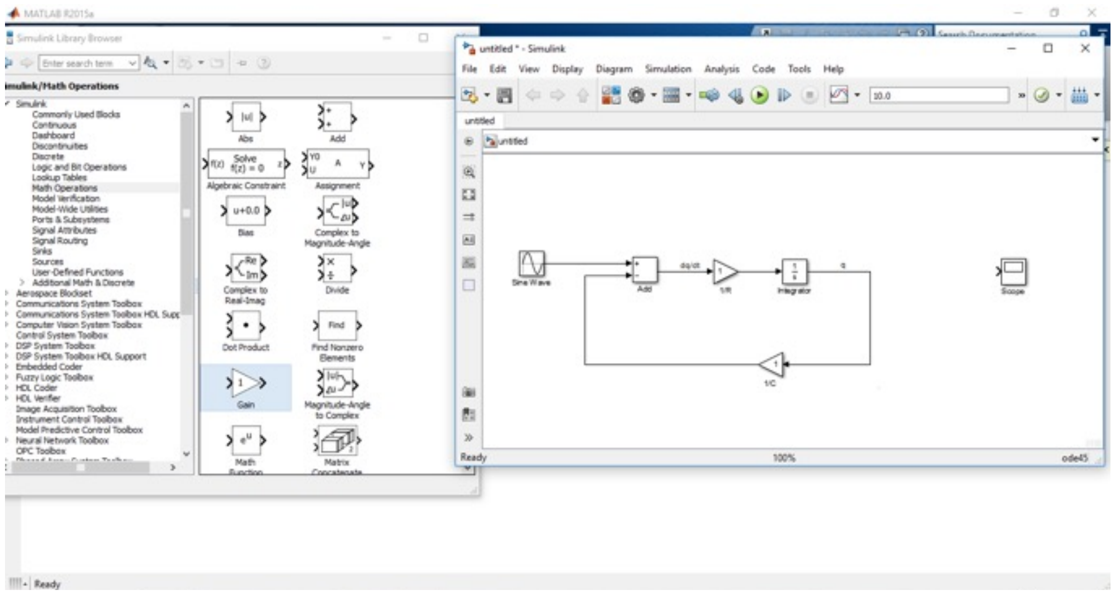

Figure 17. Labeling the second gain of the simulation

Finally we connect the oscilloscope to the system.

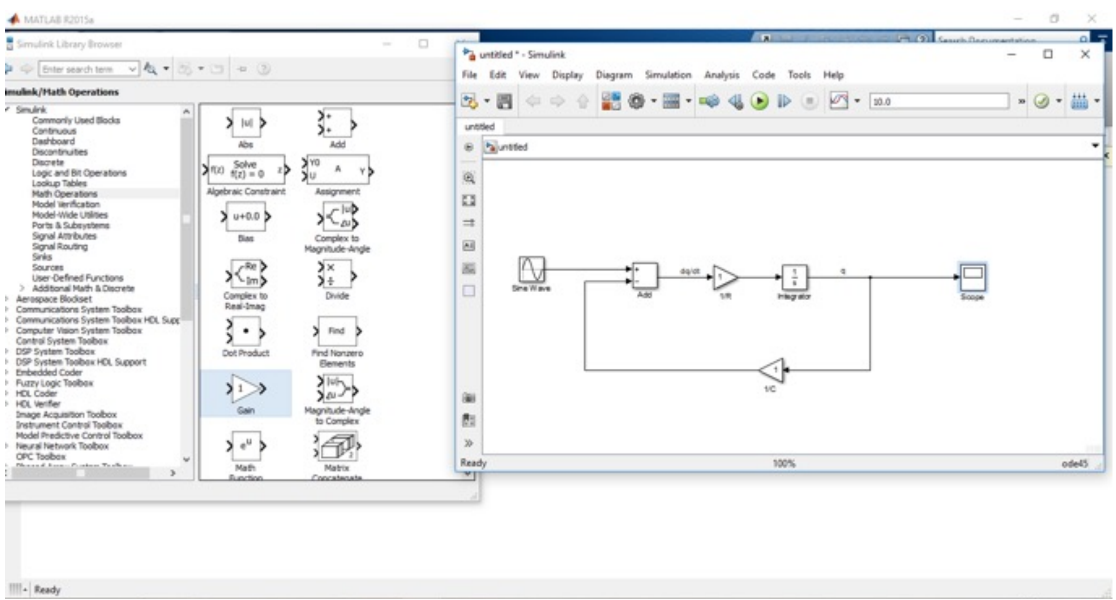

Figure 18. Connecting the oscilloscope to the simulation.

We identify the multiplier or gain of the line of q.

E. Simulate the system with the following information:

To carry out the simulation for each of the following situations, it is necessary to introduce these parameters in the simulator elements. For case 1) we have to:

R = 100 Ω, C = 720 μf, Vf = 120 sin (wt) with f = 60 Hz, t = [0, 10] with h = 0.2

We will configure the value of R, for this we make two clicks on the Resistance element.

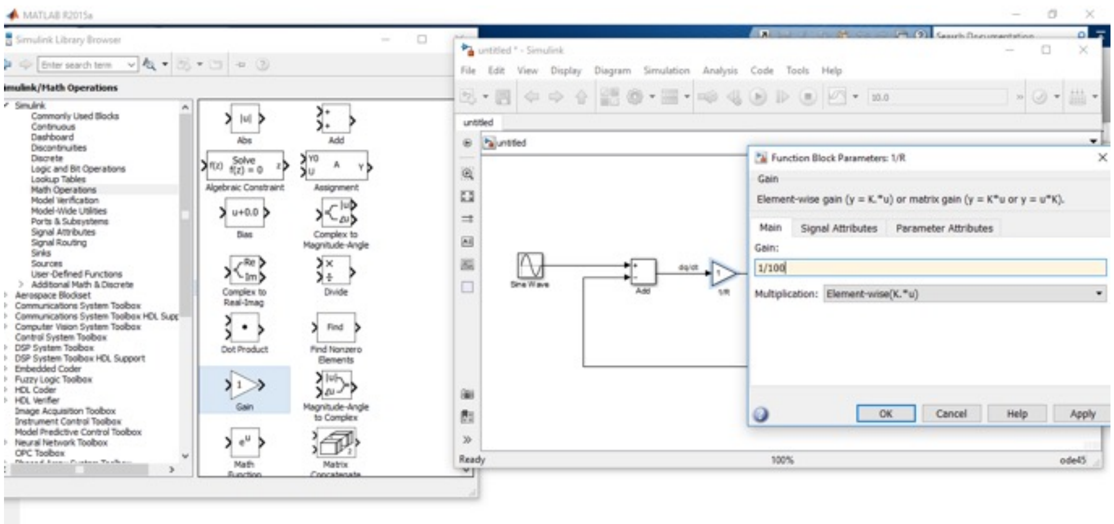

Figure 19. . Assigning value to the first gain.

We apply and accept to supply the changes in the resistance multiplier. To set the value of C = 720 μf = 720 * 10 ^ -6, for this we make two clicks on the capacitor element.

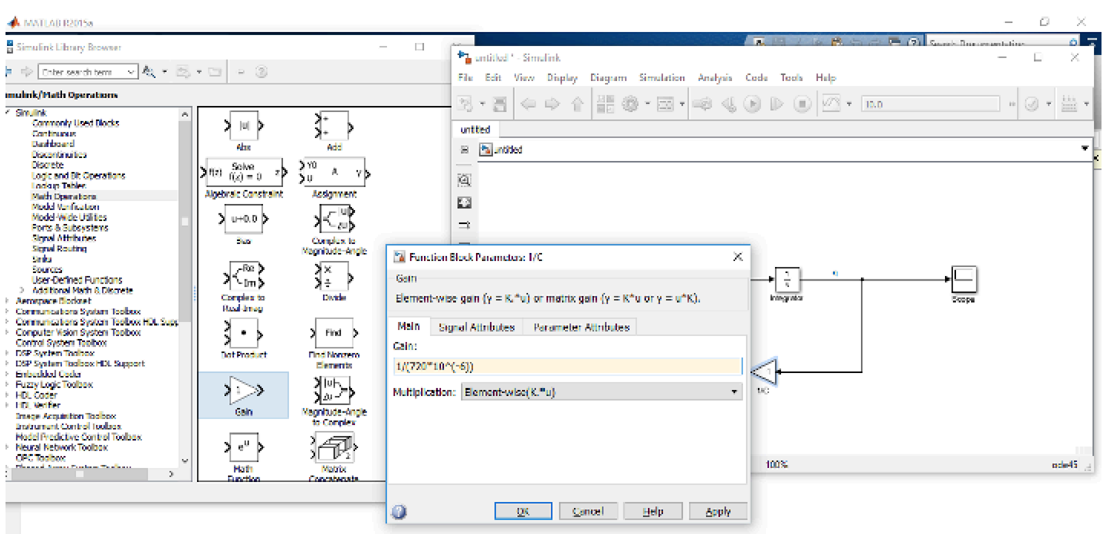

Figure 20. Assigning value to the second gain.

We apply and accept so that the changes in the multiplier apply.

Finally we configure the source, Vf = 120 sin (wt), we make two clicks on the source, in amplitude we digitize 120, in the frequency we write 2 * pi () * 60.

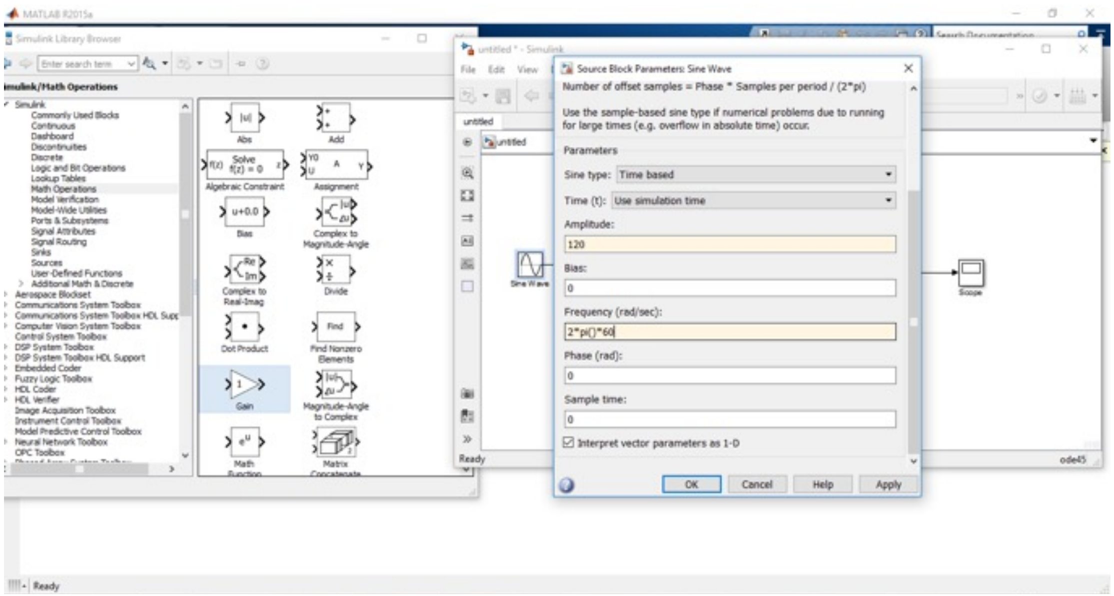

Figure 21. Assigning value to the source sine wave.

To change the jump in t and select the type of numerical solution for the ODE, do the following steps:

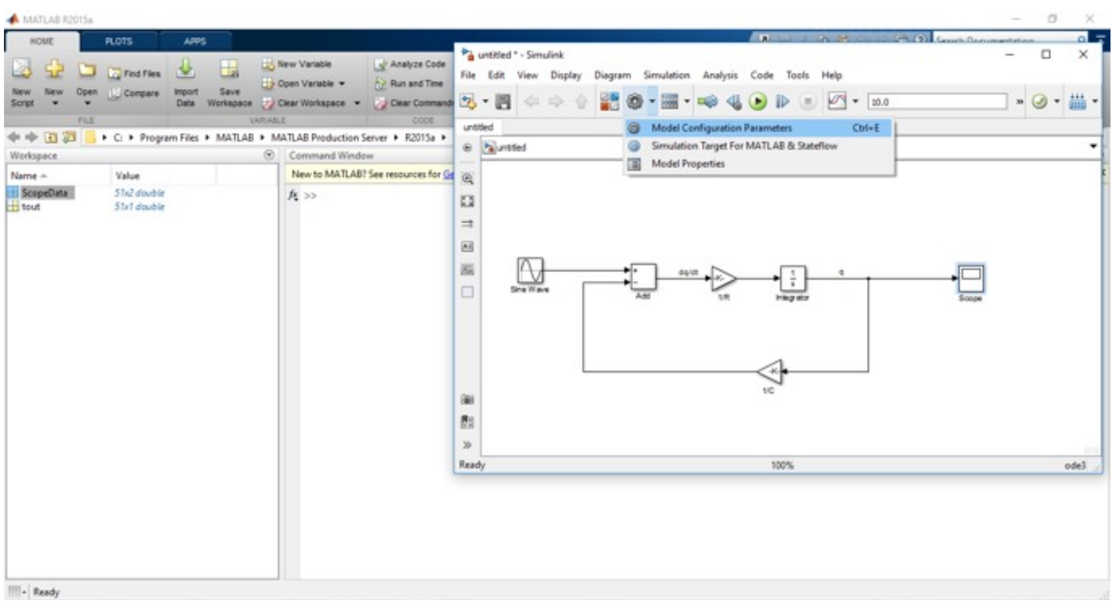

Figure 22. . Configuring the parameters of the simulation.

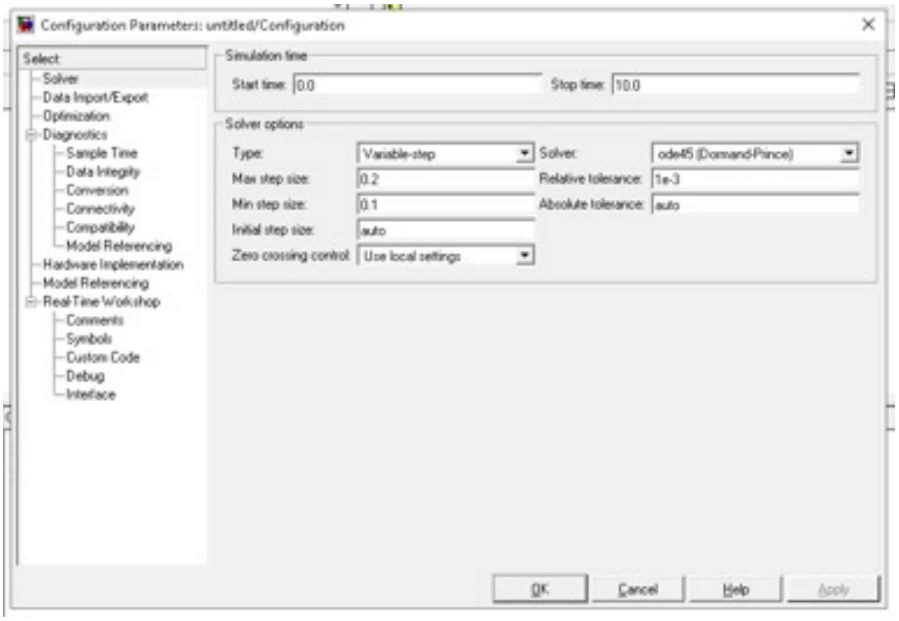

Figure 23. Configuring the jumps and solutions of the simulation.

The desired jump is given, in this case a 0.1 jump and a numerical solution of ode45 (Domand Price) was chosen. To run our simulation we click on the play button.

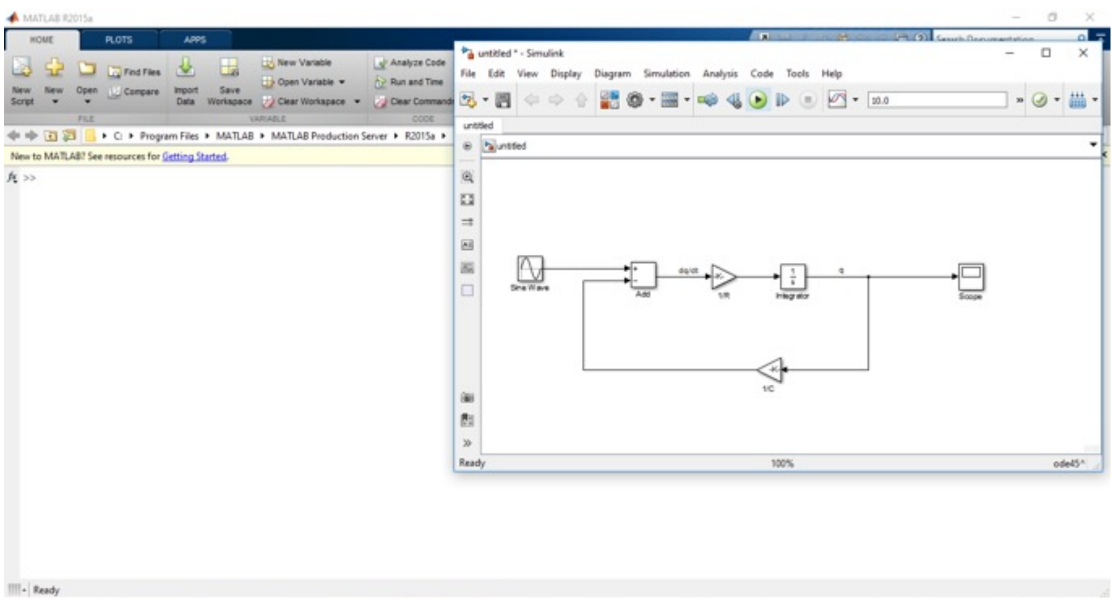

Figure 24. Running the simulation.

Once the simulation is running, we make two clicks on the oscilloscope.

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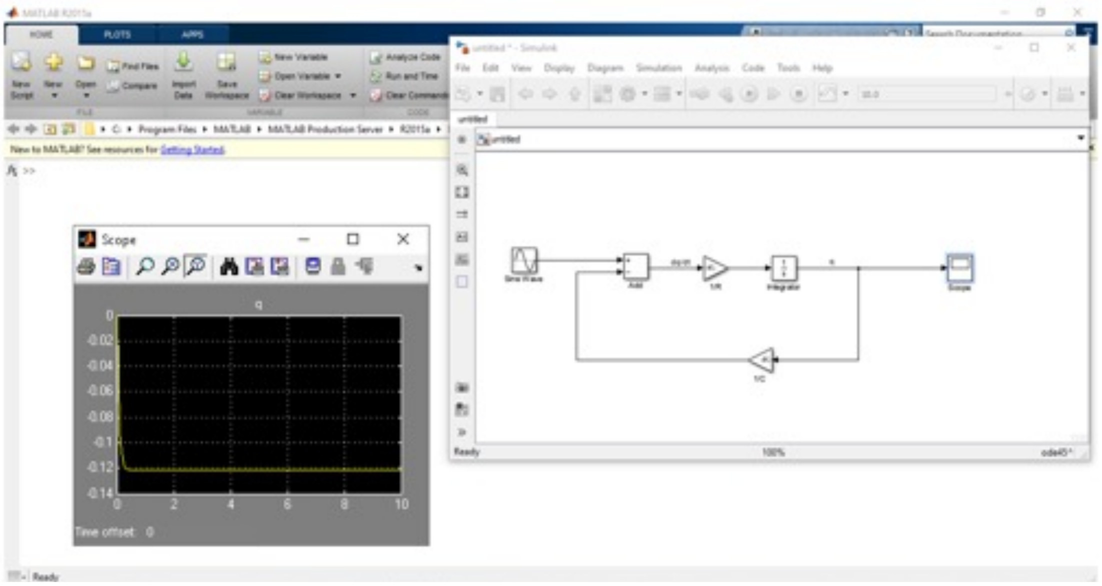

Figure 25. . Results of running the simulation.

We expand the oscilloscope.

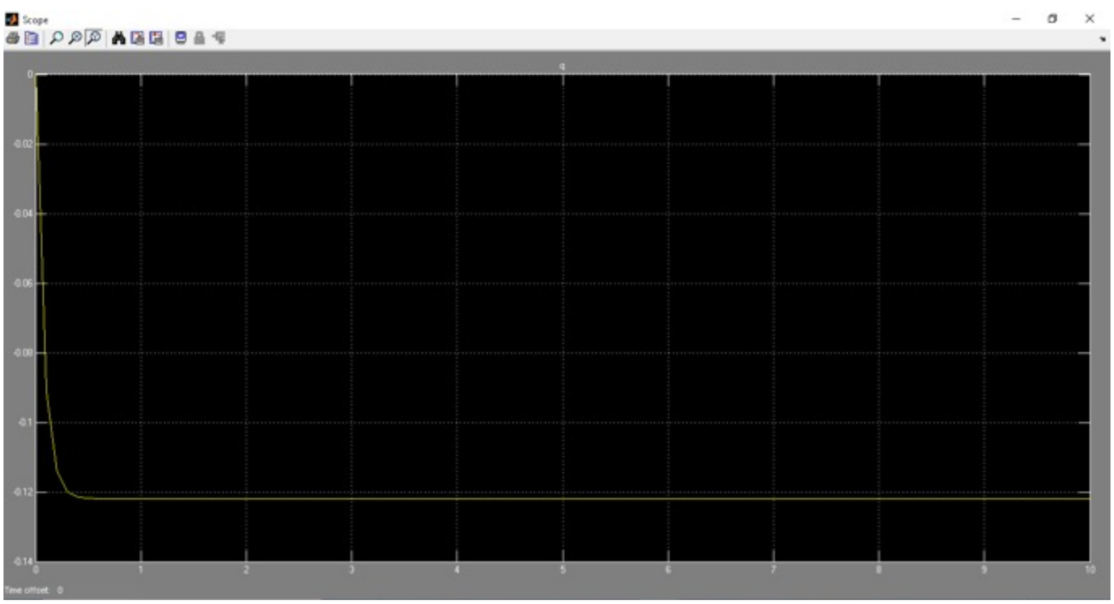

Figure 26. Output of the results in the oscilloscope.

And we can appreciate the behavior of the output in the system, which in this case is the accumulated charge in the capacitor.
To save the time and load data of the simulation, go to the oscilloscope window menu, and click on parameters.

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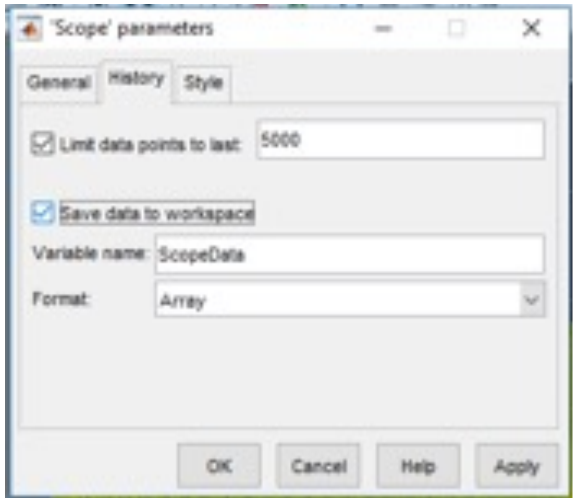
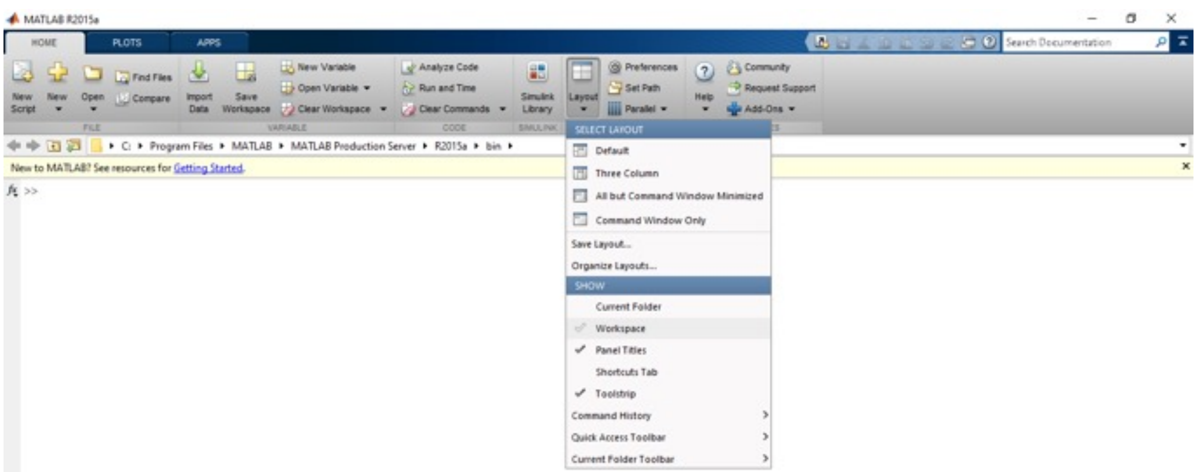
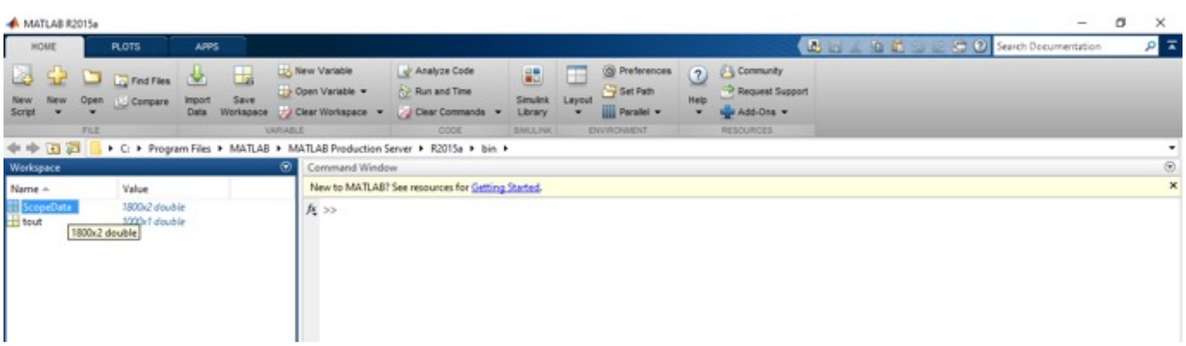


Figure 27. Save the oscilloscope output results.

We activate the save data to workspace option in the history tab, we run the simulation again
You can see the data now in the workspace window, to see it do the following steps:





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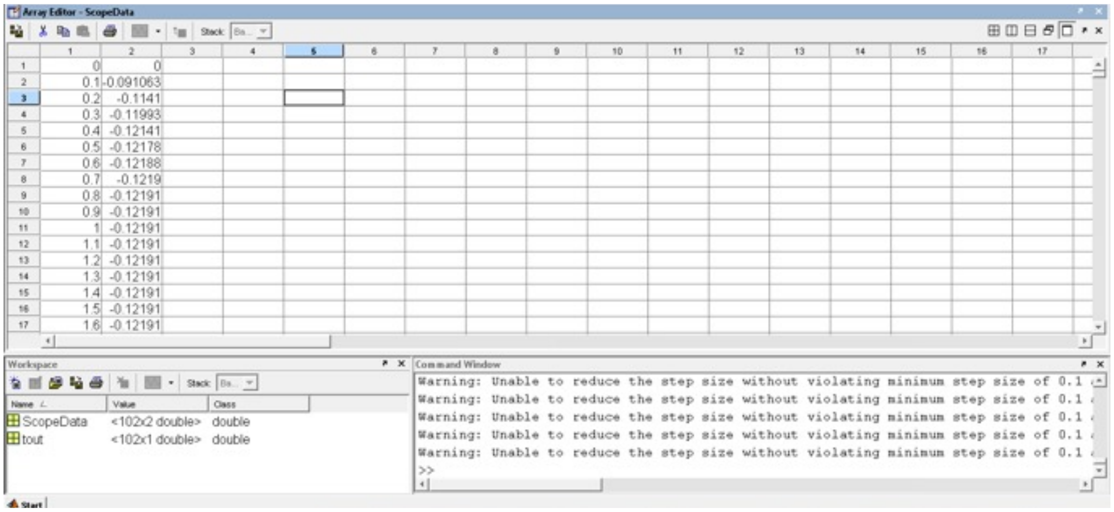

Figure 28. view of data in workspace

The data in column 1 and 2 of the ScopeData matrix represent the time and charge of the capacitor respectively. You can select the data and take it to Word, Excel, or Matlab.

2. ANALYSIS AND RESULTS OF THE SIMULATION

In the following table, you can see the results obtained with the simulation and the analytical solution of the differential equation that governs the system. In this case the solution of the ODE represents the charge of the capacitor at time t. The time interval in which the simulation was analyzed was from 0 to 10 seconds with a 0.1 jump.

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Table 1. Simulation results

Time t	Numerical Solution q(t)	Exact Solution qe(t)	Error=|q - qe|
0	0	0	0
0,1	-0,091063	-0,00238615	0,088676853
0,2	-0,1141	-0,00298114	0,111118861
0,3	-0,11993	-0,0031295	0,116800499
0,4	-0,12141	-0,0031665	0,118243505
0,5	-0,12178	-0,00317572	0,11860428
0,6	-0,12188	-0,00317802	0,11870198
0,7	-0,1219	-0,00317859	0,118721406
0,8	-0,12191	-0,00317874	0,118731263
0,9	-0,12191	-0,00317877	0,118731228
1	-0,12191	-0,00317878	0,118731219
1,1	-0,12191	-0,00317878	0,118731216
1,2	-0,12191	-0,00317878	0,118731216
1,3	-0,12191	-0,00317878	0,118731216
1,4	-0,12191	-0,00317878	0,118731216
1,5	-0,12191	-0,00317878	0,118731216
1,6	-0,12191	-0,00317878	0,118731216
1,7	-0,12191	-0,00317878	0,118731216
1,8	-0,12191	-0,00317878	0,118731216
1,9	-0,12191	-0,00317878	0,118731216
2	-0,12191	-0,00317878	0,118731216
2,1	-0,12191	-0,00317878	0,118731216
2,2	-0,12191	-0,00317878	0,118731216
2,3	-0,12191	-0,00317878	0,118731216
2,4	-0,12191	-0,00317878	0,118731216
2,5	-0,12191	-0,00317878	0,118731216
2,6	-0,12191	-0,00317878	0,118731216
2,7	-0,12191	-0,00317878	0,118731216
2,8	-0,12191	-0,00317878	0,118731216
2,9	-0,12191	-0,00317878	0,118731216

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Time t	Numerical Solution q(t)	Exact Solution qe(t)	Error=|q - qe|
3	-0,12191	-0,00317878	0,118731216
3,1	-0,12191	-0,00317878	0,118731216
3,2	-0,12191	-0,00317878	0,118731216
3,3	-0,12191	-0,00317878	0,118731216
3,4	-0,12191	-0,00317878	0,118731216
3,5	-0,12191	-0,00317878	0,118731216
3,6	-0,12191	-0,00317878	0,118731216
3,7	-0,12191	-0,00317878	0,118731216
3,8	-0,12191	-0,00317878	0,118731216
3,9	-0,12191	-0,00317878	0,118731216
4	-0,12191	-0,00317878	0,118731216
4,1	-0,12191	-0,00317878	0,118731216
4,2	-0,12191	-0,00317878	0,118731216
4,3	-0,12191	-0,00317878	0,118731216
4,4	-0,12191	-0,00317878	0,118731216
4,5	-0,12191	-0,00317878	0,118731216
4,6	-0,12191	-0,00317878	0,118731216
4,7	-0,12191	-0,00317878	0,118731216
4,8	-0,12191	-0,00317878	0,118731216
4,9	-0,12191	-0,00317878	0,118731216
5	-0,12191	-0,00317878	0,118731216
5,1	-0,12191	-0,00317878	0,118731216
5,2	-0,12191	-0,00317878	0,118731216
5,3	-0,12191	-0,00317878	0,118731216
5,4	-0,12191	-0,00317878	0,118731216
5,5	-0,12191	-0,00317878	0,118731216
5,6	-0,12191	-0,00317878	0,118731216
5,7	-0,12191	-0,00317878	0,118731216
5,8	-0,12191	-0,00317878	0,118731216
5,9	-0,12191	-0,00317878	0,118731216
6	-0,12191	-0,00317878	0,118731216
6,1	-0,12191	-0,00317878	0,118731216
6,2	-0,12191	-0,00317878	0,118731216
6,3	-0,12191	-0,00317878	0,118731216
6,4	-0,12191	-0,00317878	0,118731216
6,5	-0,12191	-0,00317878	0,118731216
6,6	-0,12191	-0,00317878	0,118731216
6,7	-0,12191	-0,00317878	0,118731216
6,8	-0,12191	-0,00317878	0,118731216
6,9	-0,12191	-0,00317878	0,118731216
7	-0,12191	-0,00317878	0,118731216

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Time t	Numerical Solution q(t)	Exact Solution qe(t)	Error=|q - qe|
7,1	-0,12191	-0,00317878	0,118731216
7,2	-0,12191	-0,00317878	0,118731216
7,3	-0,12191	-0,00317878	0,118731216
7,4	-0,12191	-0,00317878	0,118731216
7,5	-0,12191	-0,00317878	0,118731216
7,6	-0,12191	-0,00317878	0,118731216
7,7	-0,12191	-0,00317878	0,118731216
7,8	-0,12191	-0,00317878	0,118731216
7,9	-0,12191	-0,00317878	0,118731216
8	-0,12191	-0,00317878	0,118731216
8,1	-0,12191	-0,00317878	0,118731216
8,2	-0,12191	-0,00317878	0,118731216
8,3	-0,12191	-0,00317878	0,118731216
8,4	-0,12191	-0,00317878	0,118731216
8,5	-0,12191	-0,00317878	0,118731216
8,6	-0,12191	-0,00317878	0,118731216
8,7	-0,12191	-0,00317878	0,118731216
8,8	-0,12191	-0,00317878	0,118731216
8,9	-0,12191	-0,00317878	0,118731216
9	-0,12191	-0,00317878	0,118731216
9,1	-0,12191	-0,00317878	0,118731216
9,2	-0,12191	-0,00317878	0,118731216
9,3	-0,12191	-0,00317878	0,118731216
9,4	-0,12191	-0,00317878	0,118731216
9,5	-0,12191	-0,00317878	0,118731216
9,6	-0,12191	-0,00317878	0,118731216
9,7	-0,12191	-0,00317878	0,118731216
9,8	-0,12191	-0,00317878	0,118731216
9,9	-0,12191	-0,00317878	0,118731216
10	-0,12191	-0,00317878	0,118731216
10	-0,12191	-0,00317878	0,118731216

The trend of the points obtained in the simulation tend to follow the behavior of the analytical solution, which can be observed in the graphs of the exact solution and the numerical solution.

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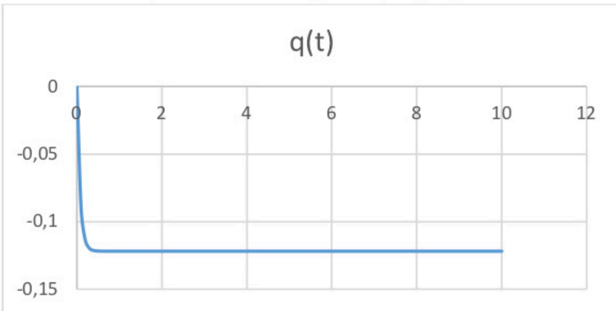
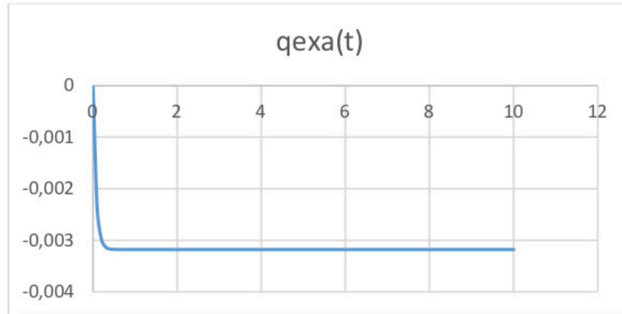


Graph 1. Numerical or simulated solution



Graph 2. The analytical solution

3. CONCLUSIONS

To perform real-time simulations of systems that are governed by a differential equation, it can be modeled and implemented with the Simulink simulator, which is a tool that brings the Matlab programming language. In addition, Simulink has a number of mathematical functions, logic blocks, sources and connectors that allow visual programming in real time. It is for these reasons that the simulations of dynamic systems and the design of models are very worked in Simulink, since this provides a diversity of facilities and options for the programmer or the researcher such as:

- Have a graphic editor for the construction and manipulation of block diagrams.
- Have a library of predefined blocks to model continuous and discrete time systems.
- Ordinary and partial differential equation solvers with fixed and variable step size.
- Output screens for the oscilloscope and data capture (oscilloscope and data display) to display the results.
- Project and data management tools for managing models and data files.
- Model analysis tools to refine your architecture and increase simulation speed.
- Matlab function block for importing Matlab algorithms into models.
- Legacy code tool for importing C and C ++ code into models.

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