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Maximising social welfare in selfish multi-modal routing using strategic information design for quantal response travelers

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MAXIMISING SOCIAL WELFARE IN SELFISH MULTI-MODAL ROUTING USING STRATEGIC INFORMATION DESIGN FOR QUANTAL RESPONSE TRAVELERS

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ABSTRACT

Traditional selfish routing literature quantifies inefficiency in transportation systems with single-attribute costs using price-of-anarchy (PoA), and provides various technical approaches (e.g. marginal cost pricing) to improve PoA of the overall network. Unfortunately, practical transportation systems have dynamic, multi-attribute costs and the state-of-the-art technical approaches proposed in the literature are infeasible for practical deployment. In this paper, we offer a paradigm shift to selfish routing via characterizing idiosyncratic, multi-attribute costs at boundedly-rational travelers, as well as improving network efficiency using strategic information design. Specifically, we model the interaction between the system and travelers as a Stackelberg game, where travelers adopt multi-attribute logit responses. We model the strategic information design as an optimization problem, and develop a novel approximate algorithm to steer **Lo**git **R**esponse travelers towards social welfare using strategic **I**nformation design (in short, LoRI). We tested the performance of LoRI and compare with that of a SSSP algorithm on a Wheatstone network with multi-modal routes. We improved LoRI and demonstrated the enhanced performance of LoRI V2 when compared to LoRI V1 in similar experiment settings. We considered a portion of Manhattan, New York, USA and presented the performance of LoRI on a real world multi modal transportation network. In all our simulation experiments, including real world networks, we find that LoRI outperforms traditional state of the art routing algorithms, in terms of system utility, and reduces the cost at travelers when large number of travelers on the network interact with LoRI.

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1. INTRODUCTION

Smart navigation systems (e.g. GPS devices, navigation applications on mobile/smart devices) have transformed the transportation domain in terms of reducing cognitive overload in travelers. However, such technological advancements have had little impact on several fundamental issues such as mitigating congestion [15] and reducing carbon emissions [19], which have only worsened over time. For instance, current state-of-the-art navigation systems employ traditional shortest paths algorithms, such as Dijkstra's algorithm [18], Bellman–Ford or Warshall-Floyd, and A^*-algorithms [10] to recommend routes and mitigate travelers' cognitive overload. On the other hand, selfish travelers exhibit multi-attribute preferences, which are typically misaligned from system's interests. As a result, travelers often reject route recommendations that involve non-personal transport modalities, such as public transportation, ridesharing services and other micro-mobility services [6, 26]. Although unintentional, people have steered away from personal car usage during the ongoing COVID-19 pandemic in 2020 [35], which have resulted in significant cost reductions in terms of congestion, carbon emissions as well as collisions. *Our goal in this thesis is to steer selfish travelers away from personal car usage (even under non-pandemic conditions), via offering them alternative routing choices in a persuasive manner.*

Selfish routing is a strategic framework where travelers employ their best-response routes selfishly according to their respective preferences to form an equilibrium. However, the central authority (e.g., a city transportation department) chooses a social-welfare objective that is not necessarily aligned with all travelers' interests. This leads to system inefficiency, which can be quantified by price-of-anarchy (PoA) [28]. Several techniques have been proposed to drive PoA towards unity, which happens when the equilibrium outcome is optimal in terms of the system's objective. A seminal example is *marginal cost pricing*, where selfish travelers are imposed taxes based on their marginal contribution to the system's objective [29]. Although the idea of marginal cost pricing has been floating around for several decades, the technique remains practically infeasible due to our inability to estimate marginal costs accurately.

In [31], the authors studied the effects of underestimating marginal costs on the optimality in terms of system objectives, and showed that taxing underestimating marginal costs produces an outcome that is at least as good as having no taxes. Although attempts have been made to implement such solutions by authoritarian regimes [38], the friction to adopt marginal cost pricing continues to persist due to various political reasons in democratic nations. Another powerful idea to influence traveler behavior is *Stackelberg routing*, where a fraction of agents are routed centrally, while the remaining agents are allowed to choose their routes selfishly [32]. A similar routing algorithm is proposed In [30] based on multi-objective A* with a goal to design routes that decrease the overall network congestion.

Meanwhile, information-revelation systems have also been proposed [1, 2, 22], where the traffic state is revealed to travelers as opposed to recommending routes. Although such systems do not mitigate cognitive-overload at the travelers, they have been found to generate a positive impact on traffic congestion and other global objectives even in non-strategic settings. However, these systems still suffer from poor persuasive ability, in terms of inducing behavior modification among travelers. A natural and effective solution is to design information strategically at the city transportation department, and present it to the travelers to steer their routing decisions towards socially optimal outcomes.

Recently, strategic information design has been studied in the transportation domain when the network congestion state is uncertainly available at the travelers. For example, in [11], the authors computed best-response signals under first-best, full information, public-signal and optimal information structure scenarios in the context of Wheatstone Network; they demonstrated that optimal information structures reveal only partial information revelation to mitigate network congestion. Similar results have been found in [36] in the case of Pigou networks (graphs with parallel routes between a single-source and a single-destination) in the presence of state uncertainty on one of the routes. Optimal information structures have been found using Bayesian persuasion framework to reduce average traffic spillover on a specific route in a Pigou network.

Despite the above development, existing works in strategic information design in transportation settings make several impractical assumptions. Since this is still a fledgling topic, almost all efforts assume that travelers are expected utility maximizers (EUM). However, there has been a strong evidence from real-world observations that travelers deviate from EUM behavior quite frequently. Such an effort was first made in [27], which studied strategic information design in a single-sender, single-receiver setting when both are prospect-theoretic agents. Nevertheless, this framework is not applicable to transportation domain where there are multiple receivers. Another impractical assumption is the consideration of single-attribute costs and unimodal transportation networks, all of which are far from reality. Therefore, in this thesis, we consider a more realistic transportation framework and develop a novel strategic information design framework as stated below.

First, we assume that the travelers' responses exhibit quantal response equilibrium (QRE), where deviations from EUM at each traveler are captured by the randomness within the stochastic utility maximization framework [21]. We model the strategic interaction with the system as a novel *Stackelberg-QRE game*, where the system (leader) exhibits EUM behavior, while the travelers (followers) exhibit logit responses. Second, we assume that both the system and travelers exhibit non-identically weighted multi-attribute preferences. Specifically, we assume that the system's motive is to reduce both network congestion (in terms of travel time) and carbon emissions on the entire transportation network, whereas the traveler wishes to minimize travel time and/or carbon emissions along his/her personal route.

Inspired from Bayesian persuasion [16] as well as the method in [4, 24], when there is a single sender and multiple receivers, we develop a novel, approximate strategic information design algorithm to steer **Lo**git **R**esponse travelers towards social welfare using strategic

Information design (in short, LoRI). Our proposed algorithm LoRI uses the predictor-corrector method to find quantal responses at the travelers, and finds a locally-optimal state-information signal using interior-point algorithms that minimizes a non-convex system cost.

2. LITERATURE REVIEW

2.1. STRATEGIC SIGNALING INTERVENTIONS

Strategic signaling interventions has been extensively studied by diverse researchers in various fields such as computer science, game theory and persuasion as signaling games [4, 7, 24, 32, 37, 41], algorithmic information structure design [1, 2, 11, 12, 16, 22, 27, 36] and recommendation systems [9, 14, 20, 22, 30, 40]. The reminder of this section reviews various techniques from papers specially relevant to this thesis.

While Stackelberg games are the most basic form of signaling games, Bayesian Stackelberg Games capture more real world scenarios. The notion of signaling in different models of Bayesian Stackelberg games and their computational complexity is investigated by Xu et al., in [37]. They show that the optimal combinations of mixed strategies and signaling schemes can be computed in polynomial time in the case of a single leader and multiple follower types. However, in security games, the problem is NP-hard in general, though a special case that can be solved efficiently is identified. For the case with multiple leader and a single follower types, it shows that the optimal combinations of mixed strategies and signaling schemes can also be computed in polynomial time. Moreover, the polynomial time solvability extends to security games in this setting. Also, these results can be easily generalized to the case with both multiple leader and follower types.

On the other hand, Dugmi et al., in [12] study algorithmic information design. Information structure design, also sometimes known as signaling or persuasion, refers to understanding the effects of information on the outcomes of strategic interactions, and in computing the information sharing strategies which optimize some design objective. This paper focuses on information structure design in single agent and multi-agent cases. The case of multi-agents is further divided into (i) multi-agents with public signals and (ii) multi-agents with private signals to each agent. For each of the aforementioned cases, this paper discusses (i) model and examples, (ii) characterization of different signaling schemes namely (a) no information, (b) full information and (c) optimal information, (iii) negative/positive results,(iv) algorithmic perspective and computational complexity and (v) future work and open questions. Motivated by specific applications, this paper talks about a number of works in the computer science community that explores variants and extensions of the basic models from a computational perspective.

Zhang et al., in their survey paper [40] presents an extensive review of recent research efforts on deep learning based recommendation systems. It presents a taxonomy of deep learning-based recommendation models, along with a comprehensive summary of the state of the art. First, traditional recommendation models such as content based, collaborative filtering and hybrid are reviewed. Then, basic deep learning techniques like multilayer perceptron, autoencoders, convolutional neural networks and restricted boltzmann machine are introduced. The paper then states some of the strengths and possible limitations of deep learning based recommendation models such as nonlinear transformation, representative learning, sequence modelling and flexibility. State of the art deep learning based recommendation systems are discussed and are categorized based on the deep learning techniques into (i) recommendation with neural building blocks and (ii) recommendation with deep hybrid models. The paper then concludes by discussing several promising prospective research directions.

2.2. STRATEGIC INTERVENTIONS IN TRANSPORTATION SYSTEMS

One of the first attempts to examine the question of designing information in games of congestion using the framework of Bayesian Persuasion is made by Das et al., in [11]. This paper considers a simple network of two routes P_1 and P_2 between origin and destination is considered (Pigou's example). The cost of travel depends on the state of the network ω

for the route P_1 and for P_2, it depends on the number of travelers taking P_2. This paper minimizes the expected aggregate travel costs via designing the following information structures,

- *First-Best:* where a central system mandates which route each traveler will take,
- *Full information:* where all the agents know the state ω,
- *Public Signal:* where the agents can observe a public signal about ω.
- *Optimal information structure:* where a central system only presents the information to socially optimal share of travelers.

this paper analyzes the aforementioned information structures and demonstrated that optimal information structures reveal only partial information revelation to mitigate network congestion. It also presents some practical issues of implementing information design to reduce congestion such as competition between two central planners/systems and fairness.

The problem of computing optimal ex ante persuasive signaling schemes in Bayesian Network Congestion Game (BNCG) was analyzed by Castiglioni et al., in [7]. First, they show that an optimal ex ante persuasive signaling scheme can be computed in polynomial time in symmetric BNCGs (i.e., where all the players share the same source and destination pair) with edge costs defined as affine functions of the edge congestion. Then, it is shown that symmetry is a required crucial property for efficient signaling by proving that it is NP-hard to compute an optimal ex ante persuasive signaling scheme in asymmetric BNCGs. These results also work for some simple class of asymmetric congestion games such as non-Bayesian singleton congestion games with affine costs. This paper also discusses a solution concept to this problem setting which is optimal coarse correlated equilibrium.

On the other hand, the problem of large-scale multi-modal transportation recommendation is explored by Liu et al., in [20]. A number of features are extracted from multiple perspectives based on the domain knowledge of traffic engineering, including user preference, mode accessibility, and location popularity. Additionally, a bipartite graph is designed to learn the embedded representation of user, origin, destination, and OD pair. Considering the inconsistency between the objective function and the evaluation metric, a post-processing algorithm is proposed to fine tune the predicted probability. Experimental results on the querying records in four cities all demonstrate significant improvements using the proposed model. platform.

To capture multiple modes of transportation in a transportation network, Samal et.al. in [30] models the multi-modal transportation network as multi-layered graph where different layers corresponds to the different modes supported by the network and travelers switch modes using switch edges. This paper proposes an algorithm based on multi-objective $A*$ which designs routes that decrease the overall network congestion. A User Optimal Multi-Modal Router (UO-MMR) is proposed which presents multi-modal paths to the user that maximizes their objectives. Extending UO-MMR, this paper develops a Social Optimal Multi-Modal Router (SO-UMMR) using a *proactive* approch to avoid congestion by recommending routes that are socially optimal and improve system-level performance. Using MATSim as a simulation tool, this paper shows that as the number of users using SO-MMR increases, the average travel time of all travelers decrease and SO-MMR increasingly offers transit route to the users.

While most of the literature assume utility maximizing agents in congestion games, Zhao et al., in [41] consider boundedly rational agents and investigate *quantal response equilibrium (QRE)* in congestion games. This paper establishes the travelers' route choice behavior with bounded rationality in the framework of QRE, where at equilibrium, the route choice probability for the travelers follows logit probability. These probabilities are obtained by assuming that each player chooses a "noisy" best response by maximizing the travel time $u_i + \epsilon_i$ instead of maximizing the travel time u_i. Additionaly, the paper enriches the QRE model by allowing the models' bounded rationality parameter λ to vary with time (increase exponentially with time). The model is further extended to the realm of different "types" of travelers, where the type corresponds to different values for the parameter $\lambda \in [0, \infty)$. The relationship between travel time and bounded rationality parameter λ is studied. Following the same direction, the authors in [8] compute the optimal strategies to commit to against boundedly rational agents in sequential games. It first proves that the aforementioned problem is NP-hard in general. To enable further analysis, this paper introduces a non-fractional reformulation of the direct non-concave representation of the equilibrium. Furthermore, using Dinkelbach-Type formulation of quantal stackelberg equilibrium (QRE), this work identifies the conditions under which the problem can be approximated in polynomial time in the size of representation. They show that a MILP can approximate the reformulation with a guaranteed bounded error. The experimental results demonstrate that this algorithm computes higher quality results, several orders of magnitude faster than a baseline method for general non-linear optimization.

Considering a completely different signaling strategy, Chen et al., in [9] propose and investigate a novel Dynamic Pricing Strategy (DPS) to price travelers' trips in intelligent transportation platforms. To solve the problem, first a route pricing model is designed to compute the congestion contribution to global urban traffic systems made by a route. The dynamic pricing strategy retrieves a matching between n travelers' trips and the potential travel routes (each trip has k potential routes) to minimize the global traffic congestion. This is challenging due to its high computation complexity (there exist k^n matching possibilities). To solve this, an efficient and effective approximate matching algorithm based on local search, as well as pruning techniques is developed to further enhance the matching efficiency. Experiments on two real-life data sets show that the proposed swap based algorithm is capable of achieving both high efficiency and high accuracy compared against the exact route matching algorithm.

In an attempt to reduce traffic pollution, Parchuri et al., in [17] model the problem of managing urban traffic pollution as a Maximum Flow Problem (MFA). This work contributes to the literature of transportation research by developing a Pareto-optimal Max

Flow Algorithm to suggest multiple max flow solutions. In this paper, Pareto Optimal max flow refers to the fact that given a (max) flow solution, we cannot increase flow on one route without decreasing flow on at least one other route. To compute maximum flow in a flow network, this paper uses the popular Ford-Fulkerson Algorithm (FFA). Pareto Max Flow Algorithm (PMFA) is developed that works on a directed graph to find all the possible solutions that allow the maximum number of vehicles to flow from origin to destination. Next, PMFA is extended to k-PMFA where the notion of k-optimality is introduced. In particcular, it aims to filter the PMFA solution set so that the remaining solutions in the Pareto solution set are k-distant from each other. Experiments are performed on New York city map, simulated using SUMO. The experimental results showcases that the k-PMFA indeed spreads pollution better, but with a cost of 21.75% increase in travel time on an average for vehicles going from origin to destination.

In the remaining sections of this thesis, we model the problem, design and develop an approximate response signaling algorithm - LoRI and validate the performance by simulating multiple travelers on a multi modal Wheatstone transportation network and real world data.

3. STRATEGIC INFORMATION DESIGN FOR QUANTAL RESPONSE TRAVELERS

3.1. MODELING MULTI-MODAL TRANSPORTATION NETWORK

The literature of modeling multi-modal networks is vast and various techniques are used to represent multi-modal transportation networks using graphs. Some of the prominent models are discussed below:

- **Multi Layered Network**: A multimodal transport network is modeled as a graph $G(V, E)$ with $|V| = n$ such that each layer corresponds to a mode $m \in M$ [3] as shown in the Figure 3.1. Hence, a mode $m_i \in M$ is defined for each node $i \in V$ while a travel time $d_{i,j}$ is associated to each arc $(i, j) \in E$. An arc (i, j) such that $m_i \neq m_j$ is called a transfer arc. In terms of multi-modal characteristics, each path in G yields a sequence (or string)of modes.

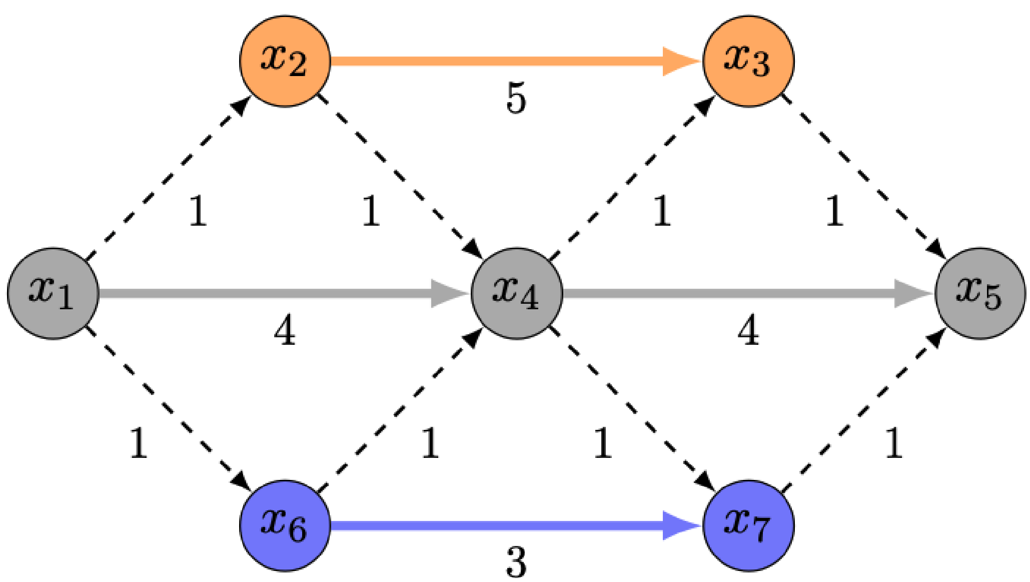

Figure 3.1. Multi-Layered Graph

The acceptable mode sequences are represented via a non-deterministic finite state automaton (NFA) as shown in Figure 3.2, possibly issued from a user-defined regular expression. This NFA is given by a 5-tuple $A = (S, M, \delta, s_0, F)$ where $S = \{1, \cdots, |S|\}$ is the set of states, s_0 is the initial state, F is the set of final states and $\delta : M \times M \times S \to 2S$ is the transition function such that $\delta(m, m', s)$ gives the set of states obtained when traversing, from state s, an arc (i, j) with $m_i = m$ and $m_j = m'$. We assume that $\delta(m, m', s) = \emptyset$ denotes the case where the transition is infeasible. Note that the case were $\delta(m, m', s)$ is either the empty set or a singleton yields a deterministic finite state automaton (DFA).

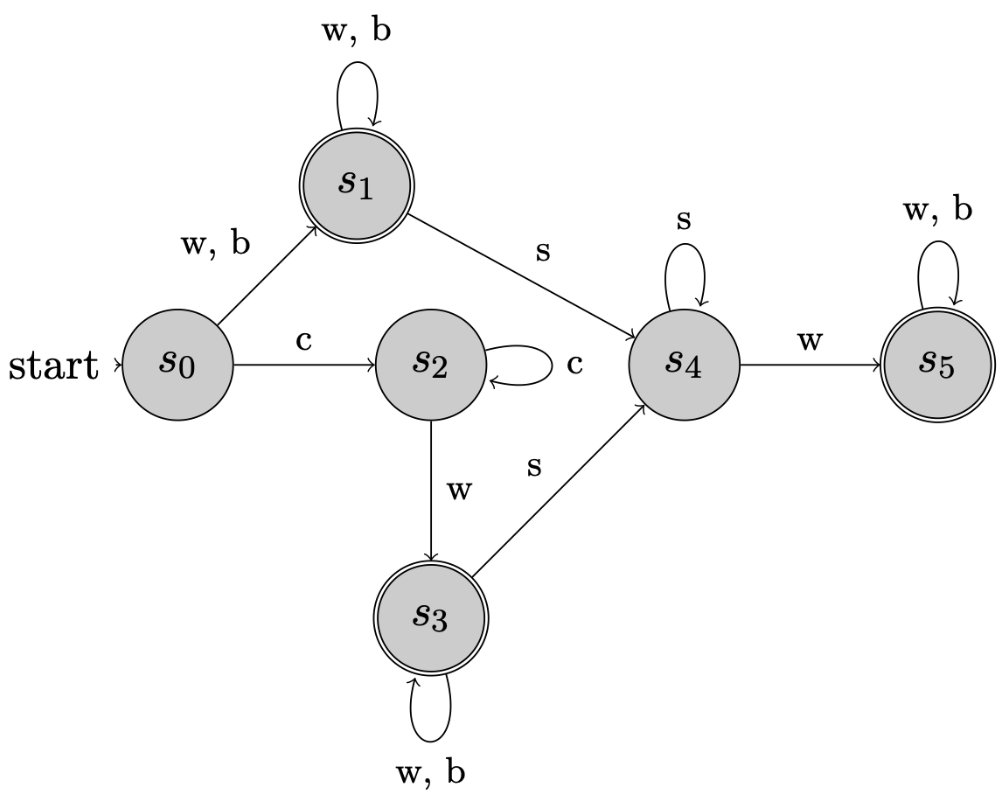

Figure 3.2. Initial NFA for mode scenario $M_3 = \{w, n, c, s\}$

- **Time-Expanded model** [14]: In this model, public transport stops (e.g., bus stops) are simply modeled as "stop"- labeled nodes. Other nodes represent time events from the timetable. Directed edges between these nodes are added whenever it is possible to transfer from one to the other (i.e., the departure of one transport vehicle is after the arrival of the other). These time nodes are also linked to nodes of their owning stops. While these models often lead to a large number of nodes and edges, standard route planning algorithm and their speedup techniques can be directly applied on the resulting graph. Multimodal route planning can be achieved by merging the involved network graphs into a single graph, and applying routing algorithms on the merged graph. A common approach for merging different networks is to solve the nearest neighbor problem, which simply connects spatially close stops from different networks. Figure 3.3, Figure 3.4, Figure 3.5 and Figure 3.6 show the time-expanded models under different scenarios.

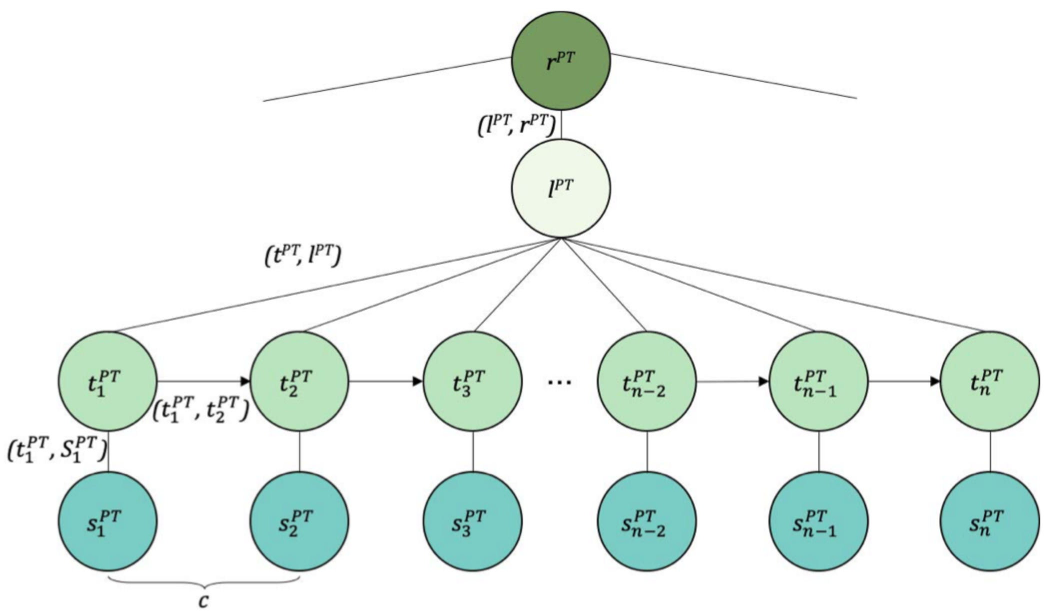

Figure 3.3. A time-expanded model of a public transport network: A trip is composed of a sequence of time nodes (t_i^{PT}) at different stop nodes (s_i^{PT}), and it belongs to a route.

After modeling public transport and carpooling as time-expanded graphs, the next step is to merge and link these graphs.

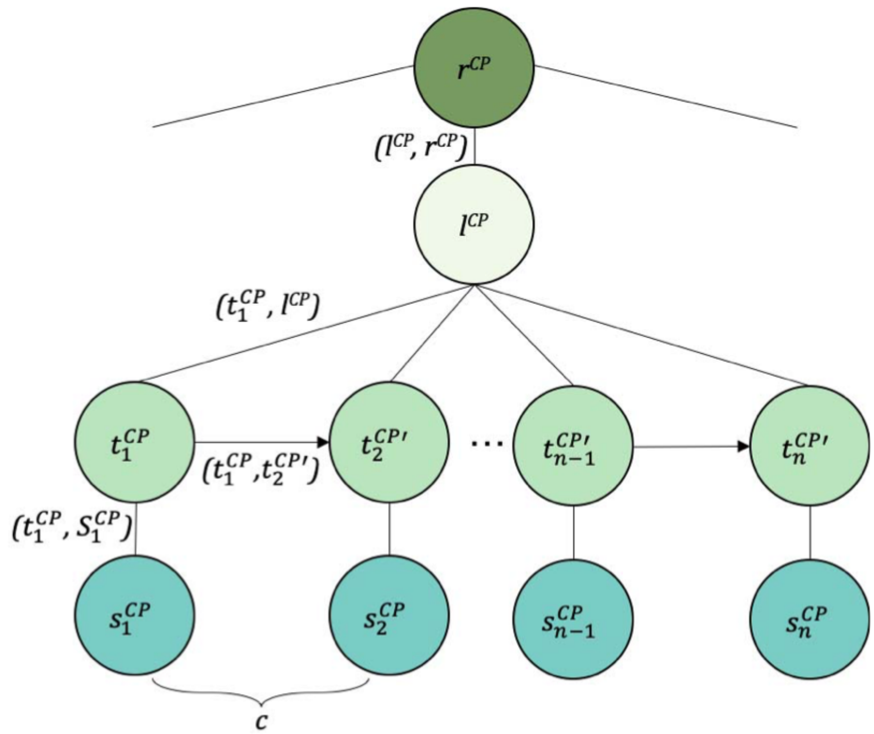

Figure 3.4. A time-expanded model of carpooling: A trip is composed of a sequence of time nodes (t_i^{CP}) at different stop nodes (s_i^{CP}), and it belongs to a route.

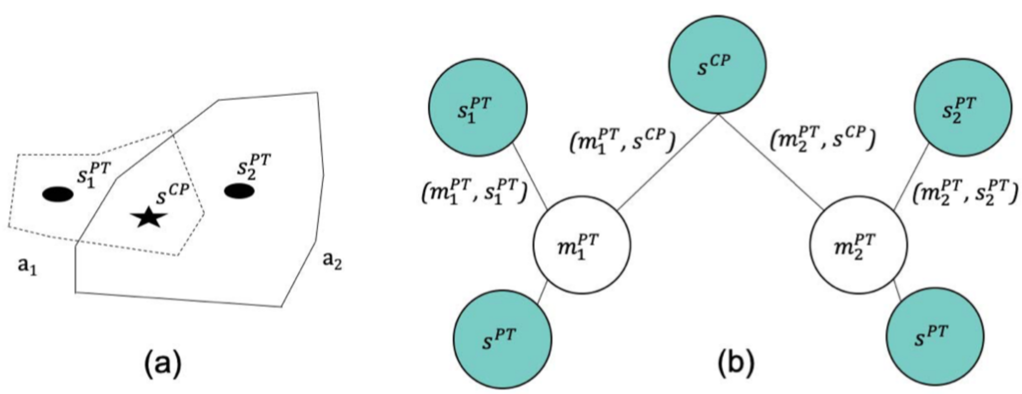

Figure 3.5. A carpooling stop is contained by drive-time areas of two public transport stops. (a) the example; (b) schematic representation of the carpooling stop linked to the meta-stops of the two public transport stops.

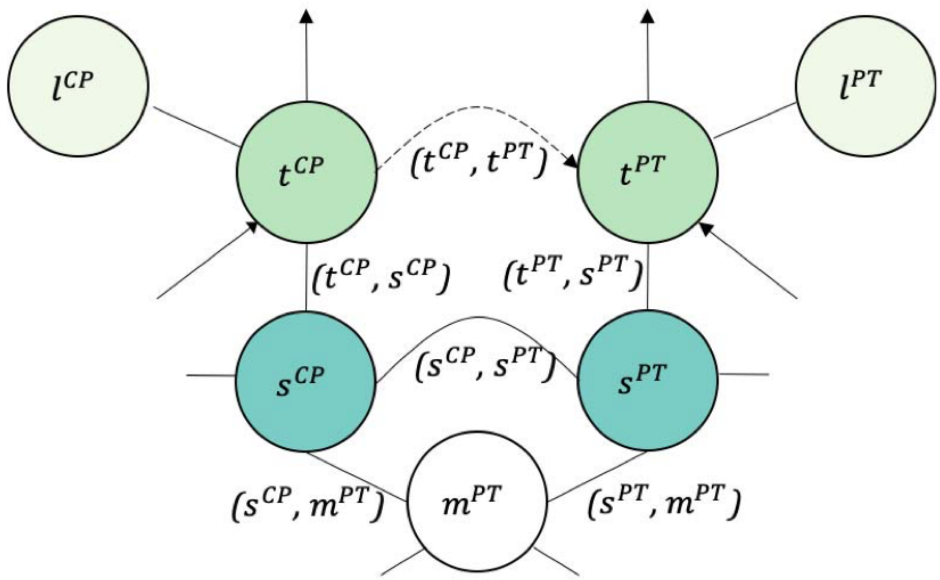

Figure 3.6. Schematic representation of a transfer between carpooling and public transport. If a given conditional is true, a transfer edge (t^{CP}, t^{PT}) is created.

In this work, we model the multi-modal transportation network as multi-layer graph. Let a multi-modal transportation network consisting of Λ_t travelers at time t, be represented as a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{0, 1, \cdots, N\}$ represents the set of physical locations (vertices), and $\mathcal{E}$ represents the transport interconnections (edges) between various locations in $\mathcal{V}$. Let $\mathcal{G}$ support a gamut of transport modalities $\mathcal{M} = \{1, \cdots, M\}$. For the sake of convenience, we expand the network $\mathcal{G}$ into a multi-layered graph $\mathcal{G}_{exp.}$ using unimodal subgraphs $\{\mathcal{G}_m\}_{m \in \mathcal{M}}$, and switch edge sets $\mathcal{E}_{i,j}$ which interconnect i^{th} modality to j^{th} modality within each vertex. For example, consider a Wheatstone road network with four vertices and ten edges, as illustrated in Figure 3.7. Consider $M = 3$ transport modalities on this network, and $\mathcal{M}$ ={*Private Car* (colored black), *Metro Train* (colored blue) and *Walking* (colored green)}.Using unimodal subgraphs and switch edges (depicted using dashed lines), we expand the example network into a multi-layered graph $\mathcal{G}_{exp.}$, as shown in Figure 3.8. We model the network state as $s_t = \left\{c_{e,t}\right\}_{e \in \mathcal{E}}$, where $c_{e,t}$ is the number of travelers on edge $e \in \mathcal{E}$ at time t.

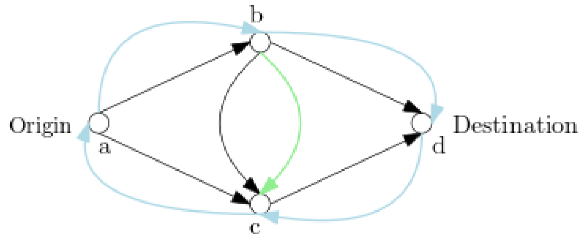

Figure 3.7. An Example Multi-Modal Transportation Network

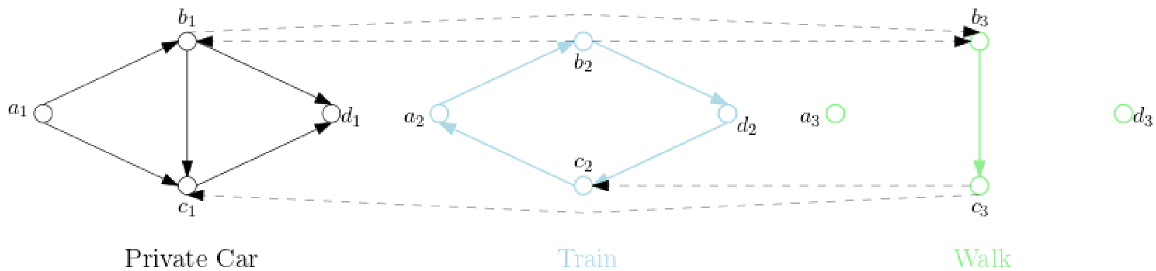

Figure 3.8. Multi-layered expansion of the Multimodal Transportation Network shown in Figure 3.7

3.2. PROBLEM FORMULATION

Assume a central entity (*a.k.a.* the system), which evaluates the network state in terms of the overall traffic congestion and carbon emissions using a weighted multi-attribute cost. Assuming that there are K attributes, each edge $e \in \mathcal{E}$ has a multi-attribute cost vector $\boldsymbol{x}(c_{e,t}) = [x_1(c_{e,t}), \cdots, x_K(c_{e,t})]$. The system evaluates the cost of each edge e at time t as

$$y(c_{e,t}) = \sum_{k=1}^{K} a_k \cdot x_k(c_{e,t}). \tag{3.1}$$

Since centralized systems typically have access to sensing infrastructure across the network to measure the network state in real-time, we assume that the system has greater information regarding the current state s_t than the travelers.

In this thesis, we assume that the system constructs a multi-dimensional signal $\boldsymbol{\mu}_{\ell,t} = \left[\mu_{\ell,e,t}\right]_{e\in\mathcal{E}}$ to steer ℓ^{th} traveler's decision, where

$$\mu_{\ell,e,t}(\eta,\lambda) = [\mathbb{P}_\ell(c_{e,t+1} = \lambda | c_{e,t} = \eta)]_{\lambda=0}^{c_e}, \tag{3.2}$$

is the state transition probability shared by the system to the ℓ^{th} traveler. The system constructs this signal with the goal of steering travelers' decisions towards system's optimal (a.k.a. social welfare).

Note that the overall system cost after a finite time horizon T depends on decisions taken by all the active travelers and all the signals presented to the active travelers. It comprises of both past and future costs, and is given by

$$U_{0,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right) = \sum_{t=1}^{T}\sum_{e\in\mathcal{E}}\sum_{\lambda=1}^{\infty}\delta_{e,t}(\lambda)y(\lambda), \tag{3.3}$$

where $\boldsymbol{\mu}_T = [\boldsymbol{\mu}_{1,T}, \cdots, \boldsymbol{\mu}_{\Lambda_T,T}]$ is the signal profile sent to all the travelers in the network; $\boldsymbol{p}_T = [p_{1,T}, \cdots, p_{\Lambda_T,T}]$ is the path profile chosen by the travelers; and $\delta_{e,t}(\lambda)$ denotes the *a priori* system's belief probability regarding the state of edge e being $c_{e,t} = \lambda$ at time t. Then, we define the system's rationality as follows:

Definition 1. The system's motive is to minimize its cost function that depends on all the travelers' decisions and the signals presented by the system. The motive is given by:

$$\min_{\boldsymbol{\mu}_T} U_{0,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right) \tag{3.4}$$

Although these signals can be revealed by the system at any time, the travelers can take advantage of this information and change their path only when they are present at some node. We label such agents as *active* travelers. In other words, we can define the state of

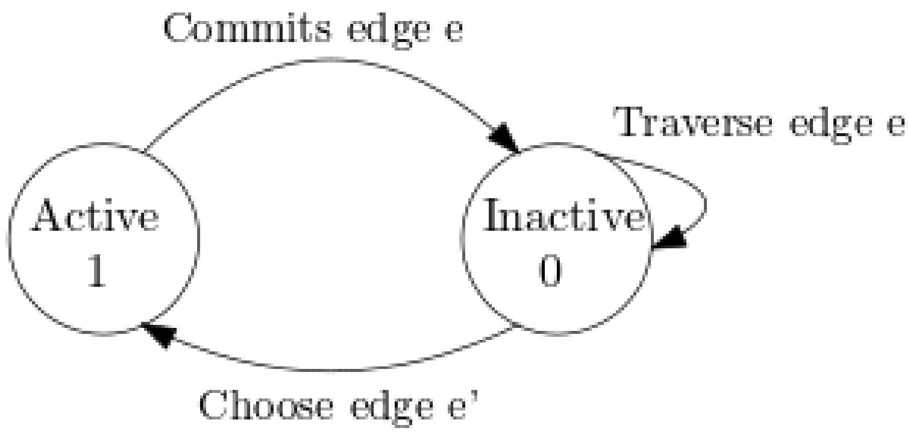

Figure 3.9. State Transitions at the ℓ^{th} Traveler

the ℓ^{th} traveler at time t as

$$\alpha_{\ell,t} = \begin{cases} 1 & \text{if the } \ell^{th} \text{ traveler is active,} \\ \\ 0 & \text{if the } \ell^{th} \text{ traveler is inactive.} \end{cases} \tag{3.5}$$

In other words, an active traveler's state gets updated to an inactive state as soon as an active traveler chooses the next edge, and remains so until he/she traverses that edge completely and reaches the other vertex as shown in Figure 3.9. That is, $c_{e,t}$ is equal to the total number of inactive travelers $\{\alpha_{\ell,t} = 0\}$ on edge e at time t.

Furthermore, we assume that the travelers cannot fully observe the true network state s_t at any given time, but can construct a multi-dimensional belief $\phi_{\ell,t} = \left[\phi_{\ell,e,t}\right]_{e\in\mathcal{E}}$ about s_t at time t based on prior experiences, where

$$\phi_{\ell,e,t} = \left\{\phi_{\ell,e,t}(c)\right\}_{c=0}^{\infty} \tag{3.6}$$

is the traveler's belief vector regarding the state of the edge $e \in \mathcal{E}$ at time t, and $\phi_{\ell,e,t}(c) = \mathbb{P}_\ell(c_{e,t} = c)$. Assuming that the ℓ^{th} traveler's multi-attribute cost[1] on edge e at time t is a weighted linear combination of all attribute-wise edge costs $\boldsymbol{x}(c_e, t)$, as given by

$$z_\ell(c_{e,t}) = \sum_{k=1}^{K} b_{\ell,k} \cdot x_k(c_e, t), \tag{3.7}$$

we model the ℓ^{th} traveler's stochastic expected cost for choosing a path $p_{\ell,T}$ as

$$V_{\ell,T}(\boldsymbol{\mu}_T, \boldsymbol{\pi}_T) = \mathbb{E}_{\boldsymbol{p}_T \sim \boldsymbol{\pi}_T} \left[U_{\ell,T} \left(\boldsymbol{\mu}_T, \boldsymbol{p}_T \right) \right] + \epsilon_{p_{\ell,T}}, \tag{3.8}$$

where $\boldsymbol{\pi_T}$ is a set of probability distributions over the set of all paths $\mathcal{P}_\ell$ at every traveler $\ell \in \Lambda_T$, $U_{\ell,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right)$ denotes the nominal (known) expected cost of the traveler, and $\epsilon_{p_{\ell,T}}$ is the noise (random parameter) term that captures any uncertainty regarding ℓ^{th} traveler's rationality. The decision policy adopted by the ℓ^{th} traveler at time t is denoted as the path $\boldsymbol{p}_{\ell,t} \in \mathcal{P}_\ell$, where $\mathcal{P}_\ell$ represents the set of all paths available for the l^{th} traveler.

Let $\boldsymbol{p}_{\ell,1:T}$ denote the sequence of edges that the ℓ^{th} traveler has already taken (committed) until time T. Then, the ℓ^{th} traveler's expected cost $U_{\ell,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right)$ comprises of two terms: the incurred (deterministic) cost from traversed, and the future (unknown) cost from the remaining path to be traversed. In other words, we have

$$\begin{aligned} U_{\ell,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right) = & \sum_{e \in \boldsymbol{p}_{\ell,1:T}} z_\ell(c_{e,t_{\ell,e}}) \\ & + \sum_{e \in \boldsymbol{p}_{\ell,T} - \boldsymbol{p}_{\ell,1:T}} \left(\sum_{\lambda=1}^{\infty} \phi_{\ell,e,t}(\lambda) \cdot z_\ell(\lambda) \right), \end{aligned} \tag{3.9}$$

where $t_{\ell,e}$ is the time at which the traveler is at the head of edge e, and $\boldsymbol{p}_{\ell,T} - \boldsymbol{p}_{\ell,1:T}$ represents the sequence of edges that the traveler will travel in the future, if he/she continues to stay on the same decision policy $\boldsymbol{p}_{\ell,T}$. Then, the traveler's rationality is defined as follows:

[1]If some attribute k is not applicable to a given edge $e \in E$, then we let $x_k(c_e) = 0$. For example, the attribute 'CO emissions' is not applicable to all the edges of mode "walking", for these edges, we let $x_{CO}(c_e) = 0$.

Definition 2. The traveler's motive is to minimize the random cost function that depends on the signals presented by the system and the path chosen by the traveler, which is given as:

$$\min_{\pi_{\ell,T} \in \Delta(\mathcal{P}_\ell)} V_{\ell,T}\left(\boldsymbol{\mu}_T, \boldsymbol{\pi}_T\right). \tag{3.10}$$

Given that both the system and travelers have non-identical utilities (i.e., mismatched motives), it is natural to model their interaction as a one-shot Stackelberg-Quantal-Response (SQR) game, where the system commits to its signaling strategy as defined in Definition 1, before travelers choose their stochastic policies as per Definition 2 [13].

Definition 3. The equilibrium of an SQR game between the system and travelers is defined as the pair $(\boldsymbol{\mu}_t^*, \pi_{\ell,t}^*)$, where

$$\begin{aligned} \boldsymbol{\mu}_t^* &\triangleq \arg\min_{\boldsymbol{\mu}_t} \quad U_{0,T}\left(\boldsymbol{\mu}_t, \boldsymbol{p}_t^*\right), \text{where } \boldsymbol{p}_t^* \sim \boldsymbol{\pi}_t^*, \text{ and} \\ \pi_{\ell,t}^* &\triangleq \arg\min_{\pi_{\ell,t}} \quad V_{\ell,t}\left(\boldsymbol{\mu}_t^*, \pi_{\ell,t}, \pi_{-\ell,t}^*\right), \text{for all } \ell \in \Lambda_T. \end{aligned} \tag{3.11}$$

Similar to solving traditional Stackelberg-Nash games, we propose a novel solution approach named *LoRI* based on backward induction, which evaluates travelers' quantal response equilibrium as a function of system's signal $\boldsymbol{\mu}_T$, and then evaluate the best response signal at the system. We present the technical details of our approach in the following section, and later analyze its performance in simulation experiments.

Algorithm 1: LoRI

Data: Travelers Λ_t, Network State s_t

for *time* $t = 1$ *to infinity* **do**

  forall $\ell \in \Lambda_t$ **do**

    if $\alpha_{\ell,t} = 1$ **then**

      ; /* If traveler is active */

      $activeTravelers.add(\ell)$

    end

  end

  forall $\ell \in activeTravelers$ **do**

    $cost \leftarrow costMatrix(\ell)$;

    $\boldsymbol{\pi}_t^* \leftarrow QRE(cost)$;

    $\boldsymbol{\mu}^* \leftarrow \arg\min_{\boldsymbol{\mu}_t} U_{0,t}\left(\boldsymbol{\mu}_t, \boldsymbol{p}_t\right)$ $chosenPath \leftarrow path(\ell, \mu^*, s_t)$;

    $e \leftarrow chosenPath[currentEdge]$;

    if $\ell.location = \ell.destination$ **then**

      $activeTravelers.remove(\ell)$

    end

  end

end

3.3. STRATEGIC INFORMATION DESIGN USING LORI

Given $c_{e,t}$ at time t on every edge $e \in \mathcal{E}$, the state transition probability is defined as

$$\begin{aligned} \psi_{e,t+1}(\eta, \lambda | \Lambda_t) &= \mathbb{P}\left(c_{e,t+1} = \lambda \;\middle|\; c_{e,t} = \eta\right) \\ &= \mathbb{P}\left(\sum_{\ell=1}^{\Lambda_{t+1}} \mathbb{1}(e_{\ell,t+1} = e) = \lambda \;\middle|\; \sum_{\ell=1}^{\Lambda_t} \mathbb{1}(e_{\ell,t} = e) = \eta\right). \end{aligned} \tag{3.12}$$

Let $\rho_{\ell,t}(e, e')$ denote the probability that the ℓ^{th} traveler is present on edge e at time t given that he is on edge e' at time $t-1$. Then, the state transition probability $\psi_{e,t+1}$ can be evaluated using the following recursive relation:

$$\begin{aligned} \psi_{e,t+1}(\eta, \lambda | \Lambda_t) &= \rho_{\ell,t+1}(e, e') \cdot \psi_{e,t+1}(\eta, \lambda - 1 | \Lambda_t - 1) \\ &+ (1 - \rho_{\ell,t+1}(e, e')) \cdot \psi_{e,t+1}(\eta, \lambda | \Lambda_t - 1) \end{aligned} \tag{3.13}$$

where

$$\rho_{\ell,t+1}(e, e', \alpha_{\ell,t}) = \sum_{p_{\ell,t} \in \mathcal{P}_{l,t}} \pi_{\ell,t}(p_{\ell,t} | \mu_{\ell,t}, e \in p_{\ell,t}, e' \in p_{\ell,t-1}, \alpha_{\ell,t} = 1) \tag{3.14}$$

is the probability that the ℓ^{th} traveler switches from edge e' to e at time t, and the ℓ^{th} traveler's logit choice probability [21] for the path $p_{\ell,t}$ at time t is given by

$$\pi_{\ell,T}(p_{\ell,T}) = \frac{\exp\left(\alpha \cdot U_{\ell,T}(\boldsymbol{\mu}_T, \boldsymbol{p}_T)\right)}{\sum\limits_{p'_{\ell,T} \in \mathcal{P}_{\ell,T}} \exp\left(\alpha \cdot U_{\ell,T}(\boldsymbol{\mu}_T, \boldsymbol{p}_T)\right)}, \tag{3.15}$$

where $\alpha \geq 0$ is the parameter of the quantal response model. Note that these logit probabilities depend on the traveler's utilities $U_{\ell,T}$, which in turn depends on the posterior belief $\phi_{\ell,e,t}$ as defined in Equation (3.9).

Let every traveler's belief regarding the future state of the network remains stationary until the system presents a signal. Then, we assume that the traveler updates his prior belief defined in Equation (3.6) using Bayes rule to obtain the following posterior belief regarding the network state:

$$\phi_{\ell,e,t+1}(\lambda) = \frac{\phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}{\sum\limits_{\lambda=0}^{\infty} \phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}. \tag{3.16}$$

Without any significant loss in practical applicability, we assume that the denominator in Equation (3.16) always converges to some value in the region $[0, 1]$.

To compute the Quantal Response Equilibrium for the travellers, we use Gambit [25]. Gambit is a library of game theory software and tools for the construction and analysis of finite extensive and strategic games. We build a strategic game (Normal-Form game) between all the travellers and use Gambit's tool $gambit - logit$ to solve for QRE. Gambit computes the principle branch of the (logit) quantal response correspondence using the predictor-corrector method based on the procedure described in [33]. The predictor-

$$
\begin{aligned}
U_{0,T} &= \sum_{t=1}^{T}\sum_{e\in\mathcal{E}} y(c_{e,t}) + \sum_{t=T+1}^{\infty}\sum_{e\in\mathcal{E}}\sum_{\lambda=1}^{\infty}\Big[\rho_{\ell,t}(e,e')\psi_{e,t}(\eta,\lambda-1|\Lambda_{t-1}-1) \\
&\qquad\qquad +\Big(1-\rho_{\ell,t}(e,e')\Big)\psi_{e,t}(\eta,\lambda|\Lambda_{t-1}-1)y(\lambda)\Big] \\
&= \sum_{t=1}^{T}\sum_{e\in\mathcal{E}} y(c_{e,t}) + \sum_{t=T+1}^{\infty}\sum_{e\in\mathcal{E}}\sum_{\lambda=1}^{\infty}\left[\left(\sum_{p_{\ell,t}\in\mathcal{P}_{l,t}}\pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t})\right)\cdot\psi_{e,t}(\eta,\lambda-1|\Lambda_{t-1}-1)\right. \\
&\qquad\qquad \left.+\left(1-\sum_{p_{\ell,t}\in\mathcal{P}_{l,t}}\pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t})\right)\cdot\psi_{e,t}(\eta,\lambda|\Lambda_{t-1}-1)y(\lambda)\right].
\end{aligned}
\tag{3.17}
$$

corrector method first generates a prediction using differential equations describing the branch of the correspondence, followed by a corrector step which refines the prediction using Newton's method for finding a zero of a function.

The leader's optimal strategy is to minimize its cost $U_{0,T}$ which can be computed as:

$$
\min_{\mu_{\ell,T}} \quad U_{0,T}\left(\mu_\ell, \mu_{-\ell}, \pi_{\ell,T}(p_{\ell,T}|\mu_{\ell,T}), p_{-\ell,T}\right) \tag{P1}
$$

Using Equation (3.13), we write the term $\psi_{e,t}(\lambda)$ and expand $U_{0,T}$ as shown in Equation (3.17).

Upon computing the travelers' QRE, the system can evaluate its optimal strategy via minimizing $U_{0,T}$. Using Equation (3.13) and (3.14), we expand the term $U_{0,T}$ as shown in Equation (3.17), where $\Lambda_{t-1,-\ell} = \Lambda_{t-1} - \{\ell\}$ is the set of travelers excluding the ℓ^{th} traveler. Since $\boldsymbol{\mu}_T$ is a right stochastic matrix, the feasibility (search) space is convex. However, it is analytically hard to verify whether or not, the objective function $U_{0,T}$ stated in Equation (3.17) is convex in μ. Note that the term $\pi_{\ell,t}$ represents logit probabilities which are known to be non-convex. Equation (3.17) comprises of convex combination of sum

Algorithm 2: Computing the Cost Matrix - Version 1

Data: Traveler ℓ, Network State s_t, Λ_t
Result: cost matrix
for $p_{\ell,t} \in \mathcal{P}_{\ell,t}$ **do**
  for $profile \in pathProfiles(\Lambda_t)$ **do**
    $pathCost \leftarrow V_{\ell,t}(\boldsymbol{\mu}_t, \boldsymbol{p}_t)$;
    $costMatrix[p_{\ell,t}][profile].update(pathCost)$;
  end
end

of logit probabilities whose convexity properties are hard to verify. Therefore, we employ interior point algorithms using CVX* package [5] to compute the approximate signal that minimizes expected cost at the system.

In order to design strategic information at the system, we evaluate the cost of traversing every feasible path $p_{\ell,t} \in \mathcal{P}_{\ell,t}$ at the ℓ^{th} traveler using Algorithm 2.

3.4. SCALABLE SID USING LORI-V2

The first version of LoRI has a significant bottle neck while computing the cost matrices in Algorithm 2. This bottleneck is caused because size of cost matrix increases exponentially in number of active travelers i.e., if we consider Λ active travelers, each with p number of possible route choices, then the size of the cost matrix will be $p \times p^{\Lambda}$. To address this concern, we consider a reduced cost matrix by considering only the number of active travelers as shown in Algorithm 3. Specifically, we assume that traveler j can influence the cost matrix of traveler i only if $IncidenceSet(i) \cap IncidenceSet(j) \neq \emptyset$, where $IncidenceSet(i)$ represents the set of all edges that are incident to traveler i's current node. If Algorithm 3 is used to computing costs in Algorithm 1, we call this new algorithm as *LoRI-v2*.

Algorithm 3: Computing the Cost Matrix - Version 2

```
Data: Traveler ℓ, Network State s_t, Λ_t
Result: cost matrix
for i ∈ Λ_t do
    if IncidenceSet(i) ∩ IncidenceSet(j) ≠ ∅ then
        Λ_reduced.add(i)
    end
end
for p_{ℓ,t} ∈ P_{ℓ,t} do
    for profile ∈ pathProfiles(Λ_reduced) do
        pathCost ← V_{ℓ,t}(μ_t, p_t);
        costMatrix[p_{ℓ,t}][profile].update(pathCost);
    end
end
```

3.5. RESULTS AND DISCUSSIONS

In this section, we present the simulation results of LoRI V1 and V2 and compare their performances across various scenarios with two single attribute (travel time) algorithms: (i) A route recommendation algorithm that uses Dijkstra's routing algorithms (***SSSP***), (ii) An information revelation algorithm that reveals the true full network state information to the travelers (***Full_Info***).

3.5.1. Simulating Using Wheatstone Network. We first test the performance of LoRI on a simple Wheatstone network as shown in Figure 3.8 and then demonstrate the performance of LoRI on a real world data set. Depending on the transport mode, we employed well-known cost models found in the literature, to carry out our simulation experiments. For example, travel time TT_e on edge e can be calculated for transport modes serviced on a road network (e.g. car, taxi, bus) using Bureau of Public Roads (BPR) formula [23]:

$$TT_e(c_{e,t}) = f_e\left[1 + a\left(\frac{n_{e,t+1}}{c_e}\right)^b\right], \tag{3.18}$$

where $n_{e,t}$ is the number of vehicles at time t, c_e is the capacity, of the edge on edge e, f_e denotes the free-flow travel time of edge e. a and b ares constants in the BPR function (usually a is 0.15 and b is 4). Similarly, the rate of carbon emissions per vehicle can be calculated using a a non-linear, static emission model for network links proposed by Wallace et al. [34], as shown below:

$$CO_e(TT_e(n_{e,t})) = 0.2038TT_e(n_{e,t}) \exp \frac{0.7962l_e}{TT_e(n_{e,t})} \tag{3.19}$$

where l_e is the link length (in kilometers), $T_e(n_{e,t})$ is the travel time (in minutes) for link e, and CO_e is measured in grams per vehicle per hour.

We combine the two costs by evaluating travel time and CO emissions in terms of their monetary value, as discussed in [39].

3.5.1.1. Scenario 1: Comparing agents costs across different motives. We simulate travelers with unique origin-destination pairs, each of whom interacts with each of the algorithms network state information. In our first experiment, we assume the LoRI's weight for travel time to be 0.7. We compute the empirical average costs across different traveler motives at both traveler and system. Due to the computational bottle neck of LoRI V1, we run the simulation for three travelers, and plot the results as a bar plot as shown in Figure 3.10. The system's cost reduces by 25% when the travelers interact with the LoRI V1 in lieu of SSSP/Full_Info. We also compute the run-times of LoRI V1, as shown in Table 3.1 to study the scalability of LoRI V1. The table compares the run times of LoRI V1 and SSSP when they interact with different number of travelers. In the case of LoRI V2, we consider 35 different travelers under scenario 1 and plot the graph as shown in the Figure 3.11.

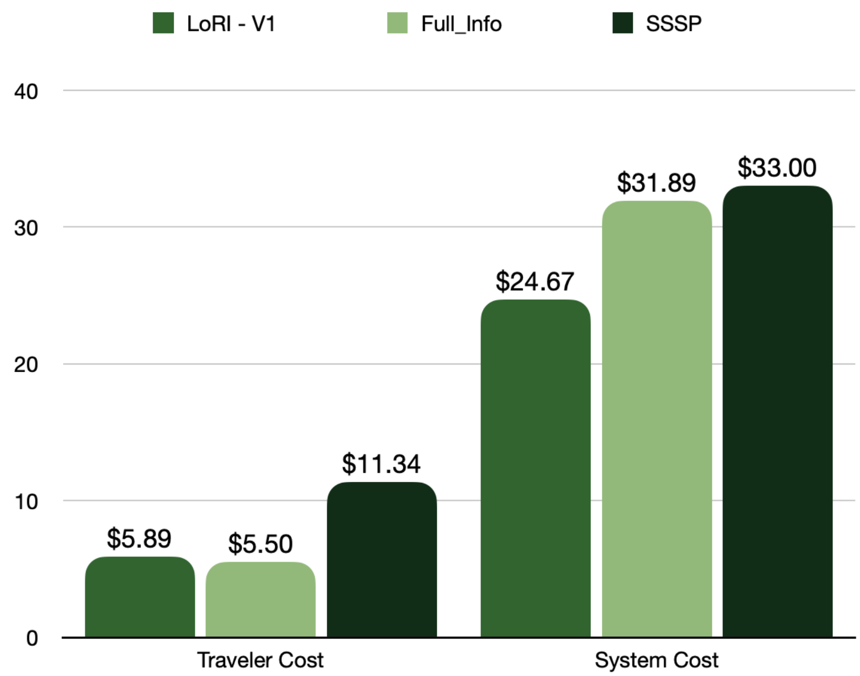

Figure 3.10. Comparison of agents costs due to LoRI - V1, Full-Info and SSSP under Scenario 1

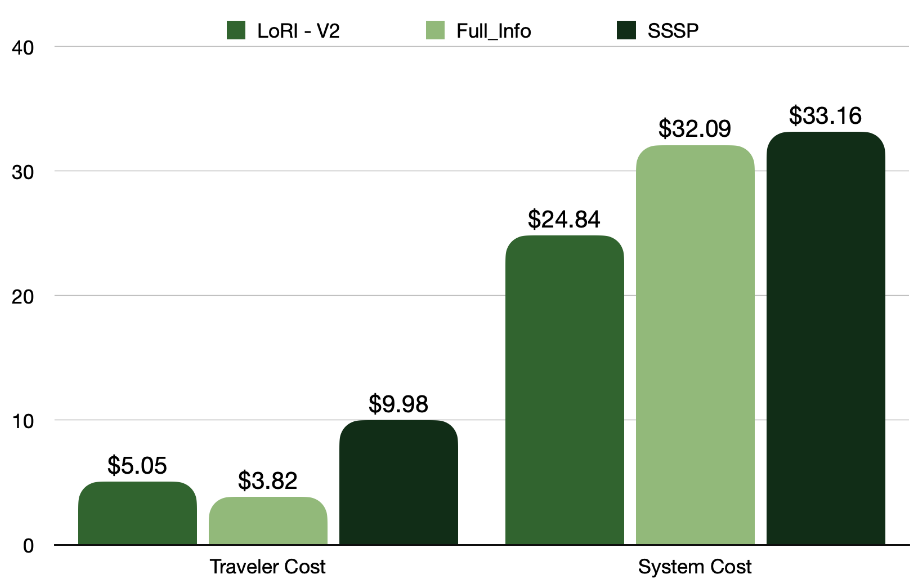

Figure 3.11. Comparison of agents costs due to LoRI - V2, Full-Info and SSSP under Scenario 1

Table 3.1. Run Time under Scenario 2

Number of Travelers	LoRI V1	SSSP
1	0.2467	0.00112
2	0.41092	0.00200
3	2.24961	0.00280
4	909.98529	0.00276

We can observe that the performance of LoRI V2 is similar to V1, even when the number of travelers jumped from 3 to 35. We also compare the run times LoRI V1 and LoRI V2 as shown in Figure 3.12.

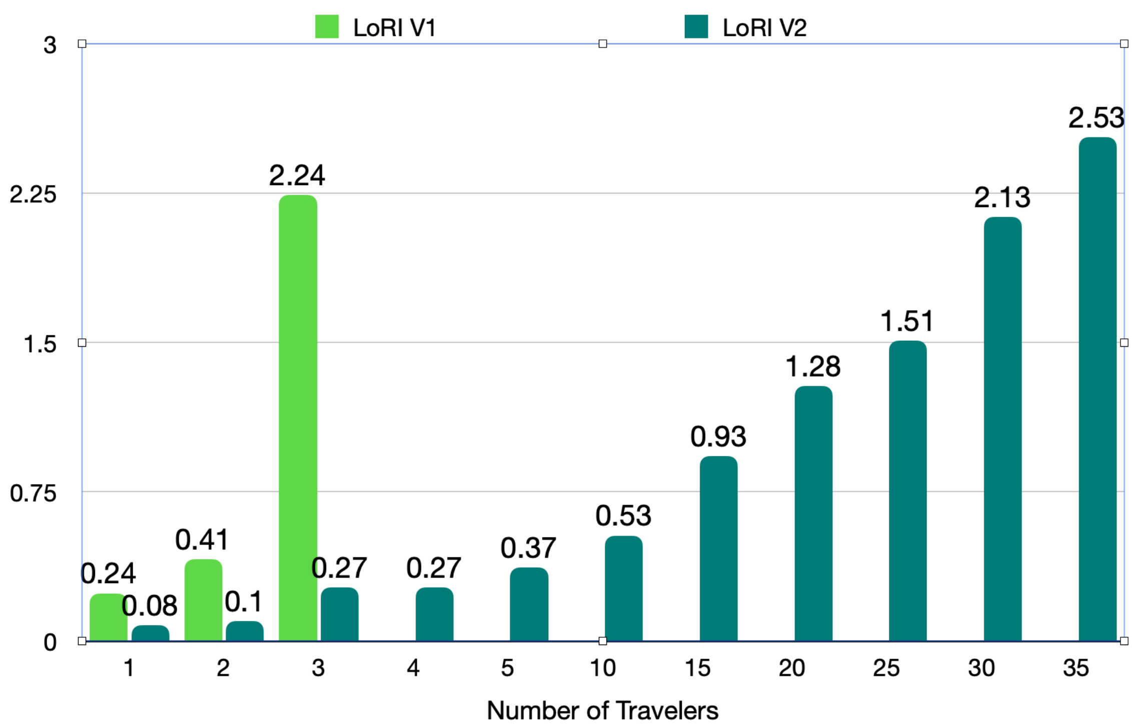

Figure 3.12. Comparison of run times of LoRI - V1 and LoRI V2, when they interact with different number of travelers.

3.5.1.2. Scenario 2: Comparing systems costs across different motives. In our second experiment, we vary LoRI V2 weights across $\{0, 0.1, 0.2, \cdots, 1.0\}$. We evaluated average system costs across different travelers with varied origin-destination pairs and plot them as shown in Figure 3.13. It is quite evident that LoRI performs better than

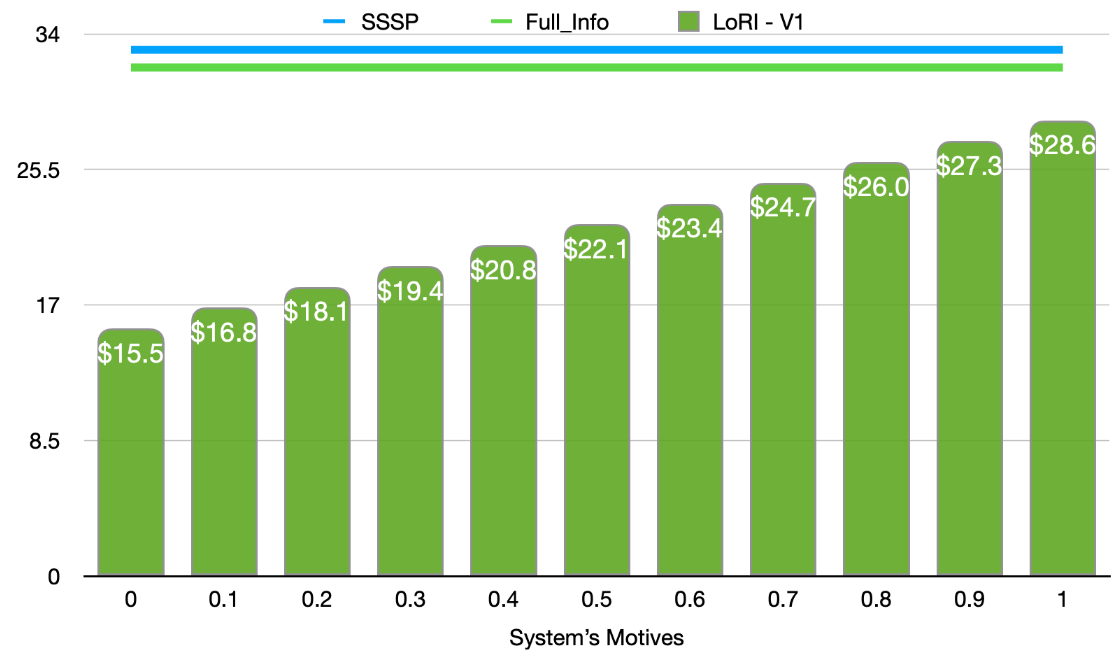

Figure 3.13. Comparison of system costs across different motives due to LoRI and SSSP in second experiment under Scenario 1

SSSP/Full_Info, across all possible motives of the system. Specifically, system obtains a tremendous gain by adopting LoRI when there is a motive mismatch between SSSP and the system. For example, when the system's weight for travel time is 0, the adoption of LoRI reduces the overall network congestion by more than **50**%.

3.5.2. Simulating Using Real World Data Set. To demonstrate the performance of LoRI on a real world transportation network, we designed multi-modal transportation network for a small selected portion of Manhattan, New York, USA as shown in Figure 3.14. In this simulation, we considered two modes of transportation $\{Car, Walk\}$ and designed the multi layered multi modal transportation network. This transportation network consists

Figure 3.14. Portion of Manhattan, New York, USA selected for the simulation experiment

of 4410 vertices and 15089 edges of car, walk and switch modes. We consider 10 different travelers with varying motives and origin-destination pairs such that the potential paths between these 10 travelers overlap at some point and there is a game among the active travelers.

For the real world data set, we compare LoRI with SSSP and Full_Info. Under scenario 1 as discussed above, we assume that the 10 travelers interact with LoRI, SSSP and Full_Info and plot the average travelers and system costs for different algorithms, as shown in Table 3.2.

We can observe that the system level costs reduces by almost 14% when the travelers interact with LoRI in lieu of SSSP/Full_Info. Under scenario 2, We evaluated average system costs across different travelers with varied origin-destination pairs and plot them as shown in Figure 3.15.

Table 3.2. Comparison of agents costs due to SSSP, Full-Info and LoRI

Algorithm	Agent Cost	System Cost
SSSP	\$3.276	\$32250.03
Full-Info	\$1.26	\$31122.4
LoRI	\$1.350	\$27572.46

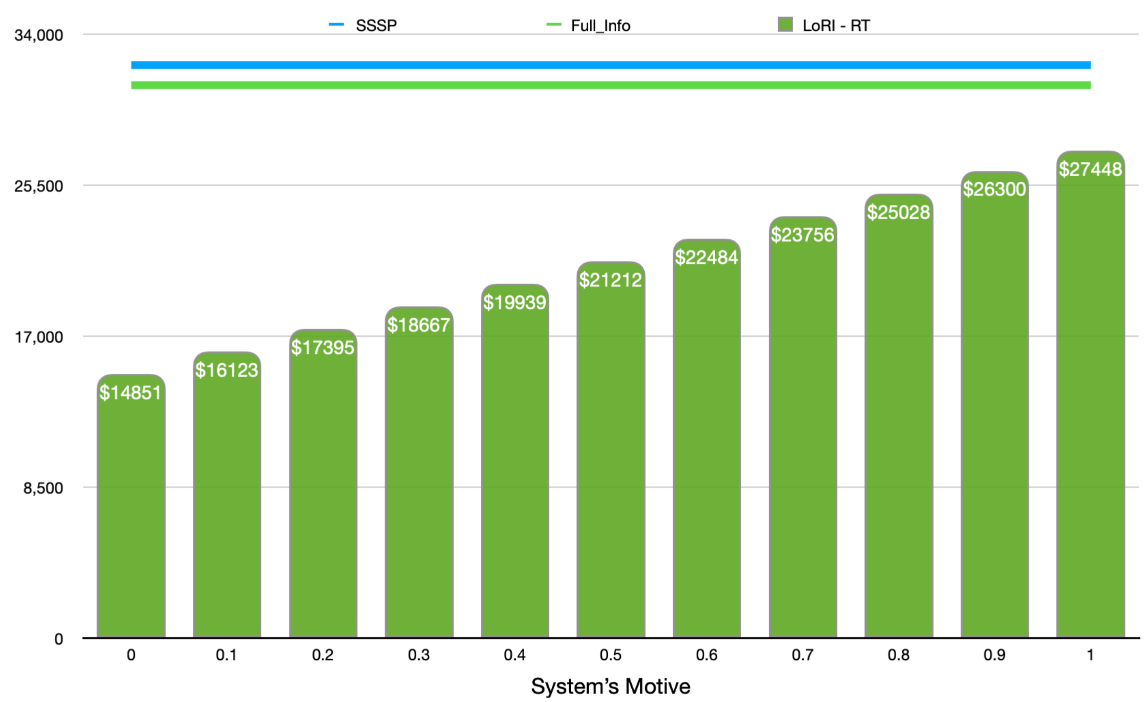

Figure 3.15. Comparison of system costs across different motives due to LoRI and SSSP/Full-Info in second experiment under Scenario 1 for the Manhattan Network

4. CONCLUSION AND FUTURE WORK

In summary, we proposed a novel Stackelberg signaling framework to improve the inefficiency of selfish routing in the presence of behavioral agents. We modeled the interaction between the system and quantal response travelers as a Stackelberg game, and developed a novel approximate algorithm *LoRI* that constructs strategic, personalized information regarding the state of the network. The system presents this information as a private signal to each traveler to steer their route decisions towards socially optimal outcomes. We tested the performance of LoRI and compare with that of a SSSP algorithm on a Wheatstone network with multi-modal routes. We improved LoRI and demonstrated the enhanced performance of LoRI V2 when compared to LoRI V1 in similar experiment settings. We considered a portion of Manhattan, New York, USA and presented the performance of LoRI on a real world transportation network. In all our simulation experiments, including real world multi modal transportation networks, we find that LoRI outperforms traditional state of the art routing algorithms in terms of system utility, and reduces the cost at travelers when large number of travelers on the network interact with LoRI.

In all the current versions of LoRI, information about the number of travelers on every edge of the network is sent as a signal to each traveler interacting with LoRI. Nevertheless, it is intuitive that information about the entire network is not required and information about only potential paths of a traveler is relevant and sufficient. For example, in our current experiment set up, LoRI presents information about all 15089 edges as a signal to every traveler, whereas only information about the edges that would potentially effect the cost of traveler would suffice. This particular bottleneck has accounted for significant increase in the run time of LoRI. In the future, we will solve the aforementioned bottleneck and design computationally efficient, approximate algorithms at the system that can support large number of travelers and higher number of transportation modalities in a real world network . We will also consider the problem of strategic information structure design for travelers with diverse rationalities.

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