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Investigating Cost Variances: A Markovian Approach

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INVESTIGATING COST VARIANCES: A MARKOVIAN APPROACH

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One constant factor in today's ever-changing business world is the pervasive interest in methods of controlling costs. This study analyzes various methods of evaluating efforts to determine the sources of cost variances and suggests that a Markovian decision process should be used when the underlying probability distributions are obtainable.

Many businesses use variance analysis for both process and model control. Three items are important here. Process control concentrates on controlling individual operations to reduce performance error costs. Process control variance analysis focuses on identifying problems signaled by deviations from a "standard." Model control implies that once a decision model is formulated, changes in the parameters of the model, as the business climate changes, may cause a new model to be preferred to the one currently employed. Model control variance analysis attempts to measure the cost of not revising or replacing the original model. The evaluation of alternative actions depends on the differences in incremental cost. Incremental cost represents the difference between actual performance and the performance suggested by revising the model in order to return to the optimal solution.

Methods of Analyzing the Significance of Cost Variables

Most literature deals with determining whether a process is in control and, hence, whether to investigate it. Kaplan (1975) has suggested that the investigation of cost variances should be classified along two dimensions. The first dimension relates to whether the investigative decision is based on a single or historical sequence of observations, including the most recent one, e.g. distinguishing between single and multi-period models. A standard Shewhart control chart approach in which a variance is investigated if it falls outside a pre-specified limit (e.g., 2 or 3 standard deviations from the expected value) exemplifies a single-period model. A multi-period model occurs if all of the most recent observations are used to estimate the current mean of the process to determine whether the process is within the pre-specified control limits.

The second dimension is based on whether the relevant model explicitly considers the expected costs of investigating itself relative to cost variance. Figure 1 identifies the two types of costs associated with model investigation.

Figure 1

	Investigate	Don't Investigate
In Control	Type I Error	O.K.
Out of Control	O.K.	Type II Error

Some models developed for cost-variance investigation have considered only Type I errors in their development. Using the two dimensions of Figure 1, the cost variance investigation models developed previously fall into the classes shown in Table 1. Table 1 represents an update of Kaplan's (1969) taxonomy.

Table 1
Variance Investigation Models

	Costs and Benefits of Investigation NOT Considered	Costs and Benefits of Investigation Considered
Single Period	Zannetos (1964) Juers (1967) Koehler (1968) Luh (1968) Probst (1971) Buzby (1974)	Duncan (1956) Bierman, Fouraker and Jaedicke (1961)
Multi Period	Page (1964) Barnard (1959) Chernoff and Zacks (1964)	Bather (1963) Duvall (1967) Kaplan (1969) Dyckman (1969) Magee (1976) Dittman and Prakash (1978) Dittman and Prakash (1979) Magee (1977) Buckman and Miller (1981) Waller and Mitchell (1984) Cheng, Jacobs and Marshall (1984) Gullege, Wormer and Tarimcilar (1985)

Note: Appendix A contains a review of the related literature.

Using a Markov process, with known transition matrices, our multi-period model considers the costs/benefits of investigation of cost variances. It should be noted that knowing the propensity to change does not reduce uncertainty of outcomes. Certainty concerning the transition matrix does not imply certainty concerning the decision. A good decision can be made in spite of bad data and a bad transition matrix, and vice versa. Thus, Types I and II errors may still occur.

The determination of whether to investigate can be controlled by statistical control limits. Statistical control limits are usually obtainable if the nature of the distribution of the variance is known. If the distribution is not known, Chebyshev's inequality or other non-parametric measures may be used to set control limits. Variances falling outside their predetermined control limits are said to be "significant" and require managerial attention.

Suggested Model

Markovian decision processes are stochastic processes that describe the evolution of dynamic systems controlled by sequences of decisions or actions. This paper focuses on the cost-control system which is observed periodically, and influenced at the time of observation by taking one of several possible actions. The evolution of the system results from the interaction between the "laws of motion" of the system and the sequence of actions taken over time. The different paths of the system will have associated economic consequences. The ultimate aim is to determine a policy which establishes criteria that will direct the firm to take those actions that control the system in an optimal manner based on current conditions (states). Optimality will be defined relative to a stipulated criterion. Our model considers tbe three states listed in Figure 2 (Dopuch, et al. 1967).

Figure 2
A Taxonomy of Variances

Type of Deviation	Cause	Action
1	Stochastic Nature of the Controlled Process	No Action
2a	Error in the Process	Restore to Expected Performance Level
2b	Permanent Change in the Process	Management Must Incorporate This Permanent Change Into the Decision-Making Process

A Type 1 deviation in Figure 2 results from the stochastic nature of the controlled process. Statistically insignificant responses require no management actions. Type 2 deviations result from a temporary or permanent change in the process. Type 2a is a controllable deviation where the error can be corrected and the expected performance level restored. Type 2b results from a permanent change in the process. In this situation, the deviation is uncontrollable, but management response is required to incorporate this permanent change into the decision-making process. Traditional accounting systems focus on Types 1 and 2a deviations. This focus is essential for process control. In contrast, Type 2b deviations are central to the control of decision models.

Control limits are established through statistical analysis to distinguish between Type 1 and Type 2 deviations. Two prerequisites effectively control the decision models. First, the firm's control system, designed around the formal decision model, should include the identification of the variances for the decision variables. Second, the control system must be able to distinguish between Type 1 and Type 2 deviations. Firms failing to correct significant Type 2 deviations will incur incremental costs, which are the differences between the actual costs and the costs indicated by the optimal solution for the revised decision model (or ex-post optimal cost).

Therefore, the amount of incremental cost depends on the decision model's sensitivity to the change signaled by Type 2b deviations. Traditional operation research sensitivity analysis techniques can then help determine the significance of the Type 2b deviations. In this sense, effective sensitivity analysis can normally be performed only on well-defined decision models. Zannetos (1964) illustrated these incremental costs, first with an economic order quantity (EOQ) inventory model and then with a resource allocation linear programming model. Sensitivity analysis was performed on the coefficients of the objective function to study the post-optimal behavior of the model.

For the general model, we assume that a system is observed at discrete time t, $t \in (0, 1, 2, \ldots, T)$, and classified into one of a finite number of states (S_i), $i \in (0, 1, \ldots, M)$. Let s_T, $t \in (0, 1, \ldots T)$, denote an observed state at time t and call the sequence of observed states as $S_t = s_0, s_1, \ldots, s_t$ with $s_t \in (s_{0t}, s_{1t}, \ldots, s_{Mt})$. After each observation, a set of finite possible decisions, d_k, is taken where $k \in (1, 2, \ldots, K)$. In general, the number of possible decisions depends upon the state of the system, but overall there are still K decisions. Let D_T, ($D_T = d_0, d_1, \ldots, d_t$ and $d_t \in (d_{1t}, d_{2t}, \ldots, d_{Kt})$) denote the sequence of actual decisions made.

A policy, denoted by R, is a rule for making decisions at each point in time. In principle, a policy could use all previously observed information up to time t, that is, the entire "history" of the system consisting of $s_t, s_{t-1}, d_{t-2}, \ldots$ and $d_t, d_{t-1}, d_{t-2}, \ldots$. However, for most problems encountered, it is sufficient to confine consideration to those policies that only depend upon s_t (the observed state of the system at time t) and D_k (the possible decision available at any time) since the adoption of a set of policies incorporates historical information. Hence a policy R can be viewed as a rule that prescribes decision d_{ik} when the system is in state i, $i \varepsilon (0, 1, \ldots, M)$, and k represents a possible action with $k \in (0, 1, \ldots, K)$. Thus, R is completely characterized by the values $R(d_{0k}), R(d_{1k}), \ldots, R(d_{mk})$. A system evolves over time according to the joint effect of the probabilistic laws of motion and the sequence of decisions made with its path dependent on its initial state, s_0. Assuming that when decision $R(d_{ik})$ is made, the system moves to a new state j, with a known transition probability $P_{ij}(k)$; $i, j \in (0, 1, \ldots, M)$ and $k \in (1, 2, \ldots, K)$. Thus, if policy R is followed, the resultant stochastic process is a Markov chain with a known transition matrix (dependent on the policy chosen). The known transition matrix can be derived from the observation of the history of this system.

A known cost C_{ik} is assumed to have been incurred when decision $R(d_{ik})$ is made following policy R. That is, taking action k when the system is in state i, net cost C_{ik} is incurred, which equals the net benefits of investigation. This cost may represent an expected rather than an actual cost; i.e., C_{ik} = Known expected cost incurred during the next transition if the system is in state i and decision k is made. Thus, the action to follow is dependent on the R with the lowest C_{ik}.

It is necessary to settle on an appropriate cost measure to compare policies. One such measure associated with a policy is the (long-run) expected average cost per unit of time. The expected average cost per unit of time, E(C), for any policy can be calculated from the expression:

$$E(C) = \sum_{i=0}^{M} C_{ik} R_j(\pi_i)$$

where k are the possible decisions made with respect to policy R for each state $i \varepsilon (0, 1, \ldots, M)$ and $(\pi_0, \pi_1, \ldots, \pi_M)$ represents the steady state distribution of the system under policy R_j being evaluated (long-term probabilities for each state that occurs over time).

Thus, the objective of this model is to obtain the policy that minimizes E(C).

To summarize, given a distribution $P(S = i)$ over the initial states of the system and a policy R, a system evolves over time according to the joint effect of the probabilistic laws of motion (the transition matrix) and the sequence of decisions made (actions taken). In particular, when decision $R(d_{ik})$ is made, the probability that the system is in state j at the next observed time period is given by $P_{ij}(k)$. This results in a sequence of decisions made, $D_T = d_0, d_1, \ldots, d_t$. This sequence of observed states and the sequence of decisions is called a Markovian decision process, because of underlying assumptions made about the probabilistic laws of motion, and the effect caused by the transition matrix.

Example

The following example helps illustrate the suggested model. A production process contains a sequence of operations. Cost reports are obtained periodically. The cost reporting period may range from 1 to n days based on the convention established by management. The convention may be based on something as simple as the convenience of compiling and producing this report or a quantitative analysis of the optimal cost reporting period, which could be produced exogenously using expected costs and steady state probabilities based on information in a current cost report. The cost reports are used until the next report period to establish the initial state of the process.

The process is classified into one of four possible states with an assumed cost. Figure 3 contains the classification of possible states.

Figure 3

State	Condition
0	Process in Control
1	Out of Control - Minor Problems
2	Out of Control - Major Problems
3	Out of Control - Major Process Revision

NOTE: In reality, many possible states which lie upon a continuum of states could have an expected cost [E(C)]. This continuum of possible states would also likely be subdivided into a set of discrete classificatory states whose lines of demarcation are cognizably drawn in order to be practically usable.

Let s_t denote the observed state of the process after inspection at the end of the t^{th} day. Assuming that the state of the system evolves according to some probabilistic "laws of motion," the sequence of states (S_T, $S_T = s_0, s_1, \ldots, s_k$) can be viewed as a stochastic process. In addition, assume a finite-state Markov chain with a known transition probability matrix given by:

State	To:	0	1	2	3
From:					
0		$\frac{4}{8}$	$\frac{3}{8}$	$\frac{1}{16}$	$\frac{1}{16}$
1		0	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
2		0	0	$\frac{1}{2}$	$\frac{1}{2}$
3		0	0	0	1

A decision must be made at the end of the day based on the current observed state. Thus, the process will not correct itself without outside action. The possible decisions are:

Decision	Action
1	Do Nothing
2	Minor Adjustment (return to state 1)
3	Investigation (return to state 0)

The costs incurred while this system evolves contain several components. When the process is in state 0, 1, 2 or 3, assume the following expected costs per day:

State	Expected Cost Per Day
0	$0
1	$2,000 <---> 3,999
2	$4,000 <---> 5,999
3	$6,000 <---> 10,000

NOTE: These expected costs represent average expected costs which may also influence the analysis of cost variance and ultimate policy decision as they vary. Changes in average expected costs may require that this analysis be reapplied as expected costs change to obtain a more current policy statement for actions to be taken when certain states occur.

The four policies considered are:

Policy	Verbal Description	Decisions d_0^*	d_1^*	d_2^*	d_3^*
R_a	Investigate in state 3	1	1	1	3
R_b	Investigate in state 3 Minor adjustment in state 2	1	1	2	3
R_c	Investigate in states 2, 3	1	1	3	3
R_d	Investigate in states 1, 2, 3	1	3	3	3

$d^* = f[S_i, R_j]$

If a given policy R is followed, the resultant stochastic process is a Markov chain with a known transition matrix dependent upon the policy chosen. Assume that the following transition matrices are obtained for the above example:

	Policy R_a				Policy R_b			
State	0	1	2	3	0	1	2	3
0	4/8	3/8	1/16	1/16	4/8	3/8	1/16	1/16
1	0	3/4	1/8	1/8	0	3/4	1/8	1/8
2	0	0	1/2	1/2	0	1	0	0
3	1	0	0	0	1	0	0	0

	Policy R_c				Policy R_d			
State	0	1	2	3	0	1	2	3
0	4/8	3/8	1/16	1/16	4/8	3/8	1/16	1/16
1	0	3/4	1/8	1/8	1	0	0	0
2	1	0	0	0	1	0	0	0
3	1	0	0	0	1	0	0	0

In addition, assume that costs for the four maintenance policies can be obtained from the following information:

Decision	State	Expected Cost Due to Process Out of Control	Maintenance Cost	Cost Due To Investigation	Total Cost
1	0	0	0	0	0
	1	2,000<---> 3,999	0	0	3,000
	2	4,000<---> 5,999	0	0	5,000
	3	6,000<--->10,000	0	0	8,000
2	0,1,2,3	0*	3,000	8,000	11,000
3	0,1,2,3	0*	8,000	8,000	16,000

*Decision 2 and 3 incur no expected cost due to the process being out of control, since the "action" places the system back into control.

As a result, the total expected cost/day is summarized as follows:

	C_{ik} (in $1,000) Decision		
State	**1**	**2**	**3**
0	0	11	16
1	3	11	16
2	5	11	16
3	8	11	16

The long-run expected average cost/time will be used to compare policies (see Appendix B for policy notations).

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$$E(C) = \sum_{i=0}^{M} C_{ik}\pi_i$$

where $\pi_i = P_{ij}$

Policy	**E(C)**
R_a (Investigate in State 3)	$4,286
R_b (Investigate in State 3; adjust in State 2)	$4,364
R_c (Investigate in States 2 and 3)	$4,167
R_d (Investigate in States 1, 2 and 3)	$5,333

Thus, R_c is the optimal policy (i.e., the minimum expected cost) based solely on the transition matrix for each policy. The optimal policy calls for doing nothing if the process is in states 0 or 1 and investigating when the process is in states 2 or 3.

Influences of the Decision Makers Learning Curve

The proposed procedures can also be dynamically adjusted based on the influence of the decision maker's learning process. Individual learning is improvement that results from a person repeating a process and gaining skill or efficiency from his or her experience (Moriarity and Allen, 1987). As a decision maker performs particular actions, data would likely be more validly ascertained and interpreted. Thus, the experience of the decision maker would influence the decision making process.

The learning curve would cause a modification of the transition matrix and may also affect E(C). As a result, a different R may become optimal. To illustrate, assume that the decision maker's learning curve is represented by the following matrix:

.25	.75	0	0
.25	.75	0	0
0	.75	.25	0
0	0	0	1

New transition matrices (and subsequently new steady-state equations) would be obtained by multiplying the original transition matrix of each R by the learning curve matrix as indicated in Figure 4.

Figure 4
New Transition Matrix

Learning Curve				R_j Current Transition Matrix					R_j New Transition Matrix			
.25	.75	0	0	.5	.375	.0625	.0625		.375	.46875	.078125	.078125
.25	.75	0	0	0	.75	.125	.125	=	0	.1875	.40625	.40625
0	.75	.25	0	0	0	.5	.5		0	.5625	.21875	.21875
0	0	0	1	1	0	0	0		1	0	0	0
.25	.75	0	0	.5	.375	.0625	.0625		.375	.46875	.078125	.078125
.25	.75	0	0	0	.75	.125	.125	=	0	.9375	.03125	.03125
0	.75	.25	0	0	1	0	0		0	.8125	.09375	.09375
0	0	0	1	1	0	0	0		1	0	0	0
.25	.75	0	0	.5	.375	.0625	.0625		.25	.5625	.09375	.09375
.25	.75	0	0	0	.75	.125	.125	=	.125	.65625	.109375	.109375
0	.75	.25	0	1	0	0	0		.25	.5625	.09375	.09375
0	0	0	1	1	0	0	0		1	0	0	0
.25	.75	0	0	.5	.375	.0625	.0625		.625	.28125	.046875	.046875
.25	.75	0	0	1	0	0	0	=	1	0	0	0
0	.75	.25	0	1	0	0	0		1	0	0	0
0	0	0	1	1	0	0	0		1	0	0	0

The long-run expected average cost/time for each policy with the learning curve adjusted transition matrix are as follows (see Appendix C for policy notations).

$$E(C) = \sum_{i=0}^{M} C_{ik}\pi_i$$

Policy	E(C)
R_a (Investigate in State 3)	\$4,020
R_b (Investigate in State 3; adjust in State 2)	\$4,448
R_c (Investigate in States 2 and 3)	\$4,861
R_d (Investigate in States 1, 2, and 3)	\$1,939

Thus, R_d is now the optimal policy (i.e., the minimum expected cost) based on adjusting the transition matrix for the decision maker's learning curve.

Limitations and Implications

Similar to other Markov models, the developed model contains three potentially unrealistic assumptions: (1) a Markovian process, (2) known transitional probabilities, and (3) steady-state conditions. All Markov chains have a "lack of memory" assumption which assumes that the state of the process is independent of its past history. Measuring transitional probabilities is always difficult and the steady-state assumption may not always be satisfied given such changing conditions as personnel turnover, equipment efficiency changes, raw material quality variations and weather changes. However, the model's consideration of the cost of investigation and incorporation of the decision maker's learning respond somewhat to these assumptions.

The use of the decision maker's learning curve in our model has implications concerning a possible method for obtaining the transition matrix. Propensity to change, as proposed by the transition matrix, is influenced by an individual's experience regarding conditions faced by the individual. The authors suspect that as the model is employed, the continuous application of the learning curve of the decision maker will force a given transition matrix to converge on a true transition matrix just as the application of a transition matrix forces the problem into a steady state transition matrix. Discerning the learning curve of a decision maker may prove to be more trackable. Thus, the derivation of the transition matrix may be more simplified. An implication which may respond to the main criticism of using a Markov process in the investigation of problems facing corporate decision makers. It is suggested that further research be conducted to substantiate these implications.

A second major contribution of the model is that the model further develops the advantage of the Markovian process. This advantage includes a less laborious determination of computational costs than other more traditional methods (e.g., Bayesian updating for each period).

Summary

The model uses a Markovian decision process approach to investigate cost variances where the underlying probability distributions are obtainable. This approach assists accountants in the investigation of cost variances. The decision maker can develop and use a predetermined model, which can then be updated and reapplied to reflect environmental changes in the problem situation (e.g., changes in expected cost for possible states, changes in the number of identifiable states or decisions or changes in the transition matrix). Since a transition matrix may be difficult to develop in practice as the number of states and policies increase, the authors suggest that a more efficient method (e.g., linear programming technique) be used to arrive at an optimal policy as the number of states and policies increase. Miller (1956) recommends that decision makers have the capacity to process information up to a reasonable upper limit of states (S_i) and policies (R_j), $i, j = 7 \pm 2$. However, further research is necessary to determine a more specific upper limit for both states and policies.

A further area of future study concerns the construction of the transition matrix. This matrix represents the decision maker's propensity for change. We hypothesize that as decision makers continue to learn, a true transition matrix may be obtained by the continuous application of the influence of the learning curve of the decision maker on a selected transition matrix. Thus, the structure of the elusive transition matrix could be derived more scientifically by determining the decision maker's learning curve.

Recommendations

An example of the application of the model indicates its value in determining optimal policies. We recommend that further research be performed to determine a reasonable limit of states and policies and of the value of using learning curves to obtain transition matrices. We also suggest that further research determine if implications concerning the derivation of a true transition matrix prove to be useful.

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A Review of the Related Literature
Appendix A

Zannetos (1964) was among the first to apply the statistical control concept to cost variance analysis, the deviation of actual cost from a standard cost. He asserted that systematic control of a formal decision model requires changes in both the type and method of assessing the significance of these variances. A process is said to be in a state of statistical control if it falls within pre-specified statistical control limits. These limits are set to minimize the total costs of two types of error—adjusting an "in-control" process (a Type I error) and failing to adjust an "out-of-control" process (a Type II error).

Bierman, et al. (1961) introduced the use of costs and benefits of an investigation into the decision concerning whether to investigate. They point out that knowing when to investigate is an important part of the control process. Specifically, when deciding whether to investigate a variance, the investigator should consider the following three factors:

1. the probability that this variance results from the random aspect of the process (e.g., a sampling error).
2. the expected reward of investigation, and
3. the expected cost of investigation.

These three factors coupled with the size of the variance determine whether to investigate the process.

Dopuch, et al. (1967) extended standard cost analysis to monitor both performance and the decision process by starting from a taxonomy of variances, as illustrated in Figure 2.

Kaplan (1969) adopted the Girshick and Rubin (1952) procedure for the variance investigation decision. Rather than deriving a cost from operating out of control, Kaplan used the actual costs when operating in or out of control to derive optimal policies. Therefore, a decision to delay investigation for one period incurred the risk of operating one more period out of control; that is, the decision led to obtaining a cost realization from the higher cost, out-of-control distribution, rather than from lower cost, in-control distribution. Balanced against this risk was the certain cost of an investigation which might find that the system was still in control, or that the gain from controlling is less than the cost of investigation. The loss function in the accounting variance setting arises directly from the nature of the problem.

A key feature of the two-state Markov model used by Girshick and Rubin and by Kaplan is that all relevant historical information may be summarized by a single state variable—the probability that the system is currently operating in control. This probability is revised after each observation via Bayes' theorem, to incorporate information from the most recent observation.

Dyckman (1969) dealt with a model similar to Kaplan's except that the multi-period cost structure was suppressed. Using a Markov process with Bayesian updating to describe transitions between in-control and out-of-control states, Dyckman assumes a constant saving, "L", from investigating an out-of-control situation, which also is the "L" constant saving originated by Bierman, et al. (1961). Dyckman offers little guidance for interpreting or estimating "L". He calls it the "present value of the savings obtainable from an investigation when the activity is out of control." He then notes that "where a corrective action is not forever binding, the calculation of "L" should be adjusted to reflect the possibility of future out-of-control periods," and then concludes that "the precise determination of the savings for each future period is not an easy matter."

Ozan and Dyckman (1971) expand on Dyckman's model by defining different types of controllable and noncontrollable variances. They suggest how to estimate some of the different probabilities required and eventually derive a reward function similar to that used by Duvall (1967).

Duvall (1967) assumes that in-control costs are normally distributed with a mean μ, equal to standard costs, and variance equal to $\sigma^2 w$. An observed deviation from standard cost consists of a noncontrollable component, w (with $w \approx N(0, \sigma^2 w)$) and a controllable component, y. The controllable component, y, is also assumed to be normally distributed and statistically independent of the noncontrollable component, w. Duvall (1967) then developed procedures which allow the parameters of the distribution of y to be estimated from the observed deviation.

After describing the estimation procedure, an inference is performed only on the most recent observation. This assists in determining whether to investigate. Duvall's assumption of stationarity is possibly unrealistic, because he uses only the most recent observation. If the process is stationary, then the investigation decision should be based on all observations, not just on the latest one. If, however, the most recent observation is deemed to be more informative than prior observations, there is a strong presumption of nonstationarity which implies that the procedure to estimate the parameters of the process is incorrect.

Bather (1963) presents a model which overcomes the previously described difficulties in Duvall's procedure. Bather, like Duvall, has a state described by a single continuous variable which represents the performance level of this continuous variable. Y_t is the unknown performance level of the system at time t and X_t is the observation of time t. As before, $X_t \approx N(Y_t, \sigma^2)$. Bather, however, postulates a process by which the performance level changes from period to period. $Y_t = Y_{t-1} + Z_t$, where $Z_t \sim N(0, \sigma^2)$.

Dittman and Prakash (1978) developed a two-stage Markovian process transition matrix to test the cost-benefits of the system being either in or out of control. Dittman and Prakash (1979) later compared the effectiveness of their model with the "best" Bayesian policy. However, their model considered only two states and contained no provisions for the process correcting itself or suffering an irreversible shift, or for the transition probabilities changing with time (i.e., accounting for the decision maker's learning curve).

Buckman and Miller (1981) present a model whereby each statistically independent cost process is assumed to satisfy Kaplan's assumptions (1969). The cost processes are related by the assumption that corrective action takes place for all n processes at once, and the decision problem is to determine when investigation and correction should take place given the vector of probabilities that each cost process is "in control." They propose a myopic procedure which is optimal for certain problems (parameters) and heuristic otherwise. They solve optimally a model with twenty cost processes. In order to see how well their method performs when it is not optimal, they solved the same examples that Dittman and Prakash (1979) used to investigate the nonoptimality of their procedure. Buckman and Miller conclude that the Dittman and Prakash algorithm is faster, while theirs has the advantage that it can be generalized to n cost processes.

Cheng, Jacobs and Marshall (1984) present a variance investigation model based on a two-action multiperiod model developed by Kaplan (1969). Kaplan used Bayesian updating of the manager's probability assessment that the process is in control, and an investigation is signalled whenever this probability assessment is less than or equal to a certain critical value, a value determined by means of dynamic programming. The Cheng, Jacobs and Marshall (1984) model determines this critical value using linear programming in conjunction with rounded Bayesian updating of the manager's in-control probability assessments.

Gulledge, Wormer and Tarimcilar (1985) present a dynamic programming solution for the model to help solve the order production problem. The model is intended for use as a planning tool to assess the cost impact of extensions and compressions of the contracted production time horizon. The model was subjected to extensive sensitivity analysis. The behavior of the solution points after parametric changes indicate that the model is capable of providing data on variables that are important in planning for made-to-order programs.

Appendix B

$P_{ij}(R)$ for all i and j for existing transition matrix.
Steady-state equations based solely on the transition matrix can be written as:

R_a

$$\pi_0 = 4/8\pi_0 + \pi_3$$
$$\pi_1 = 3/8\pi_0 + 3/4\pi_1$$
$$\pi_2 = 1/16\pi_0 + 1/8\pi_1 + 1/2\pi_2$$
$$\pi_3 = 1/16\pi_0 + 1/8\pi_1 + 1/2\pi_2$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .2857142$, $\pi_1 = .4285713$, $\pi_2 = .1428571$, $\pi_3 = .42854$

R_b

$$\pi_0 = 4/8\pi_0 + \pi_3$$
$$\pi_1 = 3/8\pi_0 + 3/4\pi_1 + \pi_2$$
$$\pi_2 = 1/16\pi_0 + 1/8\pi_1$$
$$\pi_3 = 1/16\pi_0 + 1/8\pi_1$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .181818$, $\pi_1 = .636363$, $\pi_2 = .09090$, $\pi_3 = .090909$

$$\pi_0 = 4/8\pi_0 + \pi_2 + \pi_3$$
$$\pi_1 = 3/8\pi_0 + 3/4\pi_1$$
R_c
$$\pi_2 = 1/16\pi_0 + 1/8\pi_1$$
$$\pi_3 = 1/16\pi_0 + 1/8\pi_1$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .3333333,\ \pi_1 = .5,\ \pi_2 = .0833333,\ \pi_3 = .0833333$

$$\pi_0 = 4/8\pi_0 + \pi_1 + \pi_2 +_3$$
$$\pi_1 = 3/8\pi_0$$
R_d
$$\pi_2 = 1/16\pi_0$$
$$\pi_3 = 1/16\pi_0$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .6666667,\ \pi_1 = .25,\ \pi_2 = .0416667,\ \pi_3 = .0416667$

Appendix C

$P_{ij}(R)$ for all i and j using the new transition matrix.
Steady-state equations based upon transition matrix adjusted by the learning curve can be written as:

$$\pi_0 = .125\pi_0 + .125\pi_1 + \pi_3$$
$$\pi_1 = .65625\pi_0 + .65625\pi_1 + .5625\pi_2$$
R_a
$$\pi_2 = .10937\pi_1 + .10937\pi_1 + .21875\pi_2$$
$$\pi_3 = .109375\pi_0 + .109375\pi_1 + .2187\pi_2$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .2070313,\ \pi_1 = .5742186,\ \pi_2 = .1093749,\ \pi_3 = .1093749$

$$\pi_0 = .125\pi_0 + .125\pi_1 + \pi_3$$
$$\pi_1 = .65625\pi_0 + .65625\pi_1 + .8125\pi_2$$
R_b
$$\pi_2 = .109375\pi_0 + .109375\pi_1 + .09375\pi_2$$
$$\pi_3 = .109375\pi_0 + .109375\pi_1 + .09375\pi_2$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .1979167,\ \pi_1 = .6076389,\ \pi_2 = .0972222,\ \pi_3 = .0972222$

$$\pi_0 = .125\pi_0 + .125\pi_1 + .25\pi_2 + \pi_3$$
$$\pi_1 = .65625\pi_0 + .65625\pi_1 + .5625\pi_2$$
R_c
$$\pi_2 = .109375\pi_0 + .109375\pi_1 + .09375\pi_2$$
$$\pi_3 = .109375\pi_0 + .109375\pi_1 + .09375\pi_2$$
$$1 = \pi_0 + \pi_1 + \pi_2 = \pi_3$$

Solution $\pi_0 = .2222222,\ \pi_1 = .5833333,\ \pi_2 = .0972222,\ \pi_3 = .0972222$

$$\pi_0 = .875\pi_0 + .875\pi_1 + \pi_2 + \pi_3$$
$$\pi_1 = .09375\pi_0 + .09375\pi_1$$
R_d
$$\pi_2 = .015625\pi_0 + .015625\pi_1$$
$$\pi_3 = .015625\pi_0 + .015625\pi_1$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3$$

Solution: $\pi_0 = .878788,\ \pi_1 = .090909,\ \pi_2 = .0151514,\ \pi_3 = .0151514$

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