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INTRODUCTION

The increasing competitiveness of the international market is compelling a large number of manufacturing firms to optimize their operations to succeed in business. Optimal production planning is one of the most useful instruments for achieving this objective. Researchers have suggested and implemented many optimal planning models for capacity planning, shopfloor scheduling, material purchasing, and other production functions (e.g., Beged-Dov, 1983; Gelders and Van Wassenhove, 1981; Hitomi, 1991; Kendall and Schniederjans, 1985; Martin et al., 1993; Miller and Liberatore, 1988). Integrated production planning problems in the manufacturing industry were discussed by Tang et al. (1970) and Ware (1992).

This article examines a medium-sized chair company faced with diminished demand for its products and excess operating capacity. The company has one central plant with nine manufacturing departments. Different factories, material suppliers in various territories, and customers in different geographic regions form an interactive material flow network. The chairs are manufactured in varying quantities throughout the work week, depending on the demand. The company employs 98 workers who are dispersed throughout the plant at various machines and stations. The chairs contain many pieces. The number of pieces varies depending on the style needed. These pieces are created in nine manufacturing departments. The cost and times of producing each style are dependent on the model. Each style is sold at a different price level. Production costs and throughput rates at different stages of production are considered in developing the model. The task confronting the company is to identify the mix of products that maximizes its expected profit and the level of resources required to support this level of production.

Linear programming (LP) is used to model the company's goals and its operating constraints. Generally, a business problem can be formulated in LP as two sets of equations. An objective function is used to represent an organizational goal such as profit maximization. A second set of equations is used to model factors that constrain the attainment of this goal, such as limited resources. The resulting system of equations is solved using an LP algorithm to find the mix of products or inputs that give the maximum value for the objective function. When the resulting LP is solved, unused resources, if any, will be determined as part of the optimal solution. LP may also be used to identify departments that are presently understaffed. In turn, this should help to prevent workforce reductions that would be insidious to the firm. Equally important, identifying understaffed departments creates opportunities for reallocating the underused resources of other departments to applications that strengthen the firm's financial performance.

MATHEMATICAL FORMULATION

A linear programming problem is formulated. Table 1 contains the decision variables expressed for each different product style.

TABLE 1

Product	Notation	Style	Symbol
Product 1	X_{i1}	Dinaire Ladderback	D
Product 2	X_{i2}	Hallmark Queenann	Q
Product 3	X_{i3}	Hardenside Armchair	R
Product 4	X_{i4}	Harden	H

For each X_{ij} subscript $j = 1...4$ represents the product in question and i, i = 1...3, represents different models in each category.

THE OBJECTIVE FUNCTION

The objective is to maximize profit. The mathematical expression for profit is as follows:

Maximize: $\Sigma_j(P_j \cdot (\Sigma_i\ x_{ij})$; (1)

where: P_j = profit of style j per unit

= revenue for product j - total cost of production for product j.

THE CONSTRAINTS

Raw Material Constraints

Raw material constraints are included, reflecting the goal of transforming all the purchased parts. The constraints are as follows:

$\Sigma_j\ (r_{ij} \cdot x_{ij}) = R_{ij}$, $i = 1,2,3$ (2)

R_i is the available raw material for products. Since some of the raw materials are lost during the operation, transformation factors are needed in order to have the entering raw material and the processed materials on the same weight scale. r_{ij} represents transformation factors that are obtained from the company.

The following constraints also were used:

$$\Sigma_j (p_{ij} \cdot x_{ij}) = P_i \ , \qquad i = 1,2,3 \qquad (3)$$

P_i is the available raw material for market product and p_{ij} are the transformation factors.

Net Distribution Constraints

In order to respect the product specifications using the weight distribution of different product parts, net distribution constraints are included. These constraints are presented in the following form:

$$X_{ij} = F_{ij} \Sigma x_{ij} \ , \qquad i = 1,2,3 \ , \quad j = 1,2,3,4 \qquad (4)$$

F_{ij} is the percentage of x_{ij} found in product j.

Labor Time Constraints

A detailed study of the company's products and operations shows that most processing times depend mainly on the productivity of labor. The nine manufacturing processes and number of employees for each product at each processing operation are recorded.

The accuracy of data is calculated using a t distribution. With a confidence interval of 90%, the maximum error that collected data have varies from 1.29% to 17.05%, with an average of 11.20%.

From these data, the processing rates of each product at each operation are then calculated. From these processing rates, labor time constraints are included for each operation. The total time required in order to process all the products must be less than the available time. The total available time for each operation is the multiplication of the maximum number of employees possible for each operation by the annual number of minutes worked by an employee. The constraints are the following:

$$\Sigma_j \Sigma_i (e_{ijd} \cdot x_{ij}) \leq E_d \ , \qquad d = 1...3 \qquad (5)$$

The subscript d is the operation in question and e_{ijd} are the time parameters. These time parameters, expressed in minutes, are the required amounts of time needed by an employee to produce one x_{ij}. e_d is the total available time for the operation d. Values of e_{ijd} are the processing rates resulting from the study of the company's products and operations. Values of e_d are obtained from the company.

The total available time E_d, is expressed as a decision variable with the following mathematical expression:

$$\Sigma_j \Sigma_i (e_{ijd} \cdot x_{ij}) \leq T_d \ , \qquad d = 4...9 \qquad (6)$$

The subscript d represents the six operations having tasks that are shared among the employees. $T_4 \ldots T_9$ are the necessary times for each operation. However, these variables cannot exceed the total available time for each operation. Again, the total available time for each operation, T_{max}, is the multiplication of the maximum number of employees possible for each operation by the annual number of hours worked by an employee. These constraints are expressed in the following manner:

$$T_d \leq T_{d\,max}, \qquad d = 4 \ldots 9 \tag{7}$$

The variables $T_4 \ldots T_9$ are necessary since they are present in processing time constraint. The sum of these variables cannot exceed the total available time. The total available time, T_{max}, is the multiplication of the fixed number of employees who continually share the tasks of these nine operations by the annual amount of hours worked by an employee. This constraint is expressed in the following manner:

$$\Sigma_{d=4}^{9} \leq T_{max}. \tag{8}$$

RESULTS

The analysis of the data in the following tables may be extended by managers knowledgeable about demand for the firm's products and production process. Using LP, the optimal product mix and underutilized resources for different production scenarios can be determined. The financial implications of each scenario may be assessed using a product income statement. From analyses of these scenarios over the company's planning horizon, the tradeoffs in financial performance and the employment of the company's resources can be determined. Information developed from analyses of these scenarios helps in evaluating the economic feasibility of the production process as well as identifying how and what to produce.

The equations were solved using the POM Windows program. Analyses and interpretations of the information from an LP solution would generally take some time and effort due to the large number of coefficients generated and the indirect nature of many of the variables they present.

A discussion of the different sets of results obtained from various scenarios follows.

Scenario 1

The production and economic data in the above equations, converted to an equivalent set of LP equations, are shown in Table 2. The first equation (profit row) in Table 2, the objective function, specifies the goal that the company seeks to achieve. It integrates the firm's revenue and cost structure to give the expected contribution margin for any potential mix of products that the company may choose to produce. The objective function is formulated in terms of contribution margin to reflect those elements in the firm's operating structure (price and variable cost) over which the company's management has the most control. Fixed costs are relatively constant over a wide range of operating activities and can be evaluated easily outside the LP model. The coefficients of the objective function under X_1, X_2, X_3, and X_4 represent the amount of profit (contribution margin) per unit generated by each one of these products. The next equations represent the limited production time (shown by "RHS" which is the abbreviation for Right Hand Side) in each of the company's nine operating departments for producing a given mix of products. The coefficients of these constraints under X_1, X_2, X_3, and X_4 indicate the number of minutes required to produce each one of these products.

TABLE 2

Scenario 1	X_1	X_2	X_3	X_4	**RHS**
Profit	11.29	25.6	14.84	21.69	
100 Machining	1.88	5.17	1.02	1.8	112
200 Carving Front Legs	0	3.35	2.49	0	70
300 Machine Room	10.23	12.1	8.5	10.01	280
400 Sanding	3.98	10.48	10.13	12.11	350
500 Assembly	8.56	7.58	3.4	11.14	210
600 Sanding/Cleaning	2.43	5.73	6.24	8.24	168
700 Finish	4.83	6.25	0	0	140
800 Upholstery	0.33	0.43	0.36	0.5	42
900 Shipping/Packaging	2.33	0	2.5	2.7	98
Solution	0	17.26	0	7.1	596.03

Scenario 1 suggests that only products X_2 and X_4, in quantities of 17.26 and 7.10 (approximately 17 and 7), respectively, should be produced. The company would receive a profit of $596.03.

Scenario 2

Scenario 2 (Table 3) maintains that only the models X_1 and X_2 are being produced. The nine manufacturing constraints are still applicable as well as the limitations arising due to the number of minutes per day available. Using the X_1 and X_2 models, 1.80 and 21.00 chairs are produced. The profit resulting from the manufacture of these two models is $558.15.

TABLE 3

Scenario 2	X_1	X_2	RHS
Profit	11.29	25.6	
100 Machining	1.88	5.17	112
200 Carving Front Legs	0	3.35	70
300 Machine Room	10.23	12.1	280
400 Sanding	3.98	10.48	350
500 Assembly	8.56	7.58	210
600 Sanding/Cleaning	2.43	5.73	168
700 Finish	4.83	6.25	140
800 Upholstery	0.33	0.43	42
900 Shipping/Packaging	2.33	0	98
Solution	1.8	21	558.15

Scenario 3

The company data suggest that the production of models X_3 and X_4 would create an output of 3.4 and 17.81, respectively. In this scenario (see Table 4, below), the company would receive a daily profit of $436.83.

TABLE 4

Scenario 3	X_3	X_4	RHS
Profit	14.84	21.69	
100 Machining	1.02	1.8	112
200 Carving Front Legs	2.49	0	70
300 Machine Room	8.5	10.01	280
400 Sanding	10.13	12.11	350
500 Assembly	3.4	11.14	210
600 Sanding/Cleaning	6.24	8.24	168
700 Finish	0	0	140
800 Upholstery	0.36	0.5	42
900 Shipping/Packaging	2.5	2.7	98
Solution	3.4	17.81	436.83

Scenario 4

Table 5 examines the output if only X_1 and X_4 are manufactured. This analysis suggests that the company would be better off producing only X_4 rather than a X_1, X_4 mix. Daily profit received would be $408.87.

TABLE 5

Scenario 4	X_1	X_4	RHS
Profit	11.29	21.69	
100 Machining	1.88	1.8	112
200 Carving Front Legs	0	0	70
300 Machine Room	10.23	10.01	280
400 Sanding	3.98	12.11	350
500 Assembly	8.56	11.14	210
600 Sanding/Cleaning	2.43	8.24	168
700 Finish	4.83	0	140
800 Upholstery	0.33	0.5	42
900 Shipping/Packaging	2.33	2.7	98
Solution	0	18.851	408.87

Scenario 5

In this scenario (Table 6), sensitivity analysis is carried out to investigate the impact of the input parameters on the optimality of the solution. This analysis provides the information regarding individual impacts on the result when some parameters of the model vary. In this model, an additional sixty minutes are added to each of the nine time constraints in the first scenario (with the highest profit). By adding these additional minutes, the units of products X_2 and X_4 manufactured are increased by almost 1 and 5 units, respectively. This increase generates an additional profit of $129.30 ($725.33 - 596.03).

TABLE 6

Scenario 5	X_1	X_2	X_3	X_4	RHS
Profit	11.29	25.6	14.84	21.69	
100 Machining	1.88	5.17	1.02	1.8	172
200 Carving Front Legs	0	3.35	2.49	0	130
300 Machine Room	10.23	12.1	8.5	10.01	340
400 Sanding	3.98	10.48	10.13	12.11	410
500 Assembly	8.56	7.58	3.4	11.14	270
600 Sanding/Cleaning	2.43	5.73	6.24	8.24	228
700 Finish	4.83	6.25	0	0	200
800 Upholstery	0.33	0.43	0.36	0.5	102
900 Shipping/Packaging	2.33	0	2.5	2.7	158
Solution	0	18.41	0	11.7	725.33

CONCLUSION

In conducting this research project, real data were acquired and used with the help of the company's planning managers and staff. Different scenarios are presented. The company's management can decide which scenario to adopt.

Scenario 1 shows the impact of the nine manufacturing constraints and the number of minutes available per day on the production of the four products. The company can decide to increase its efforts in order to increase both the market demand for these products and/or the amount of labor time available for production.

Scenario 1 yields a high profit from the production of models X_2 and X_4. The manufacturing of these two products will generate a profit of $596.03. Analysis of the individual products in Table 2 suggests that products X_1 and X_3 should not be produced. Therefore, their direct fixed cost represents another potential layer of resources that may be eliminated to improve firm profitability. However, analysis of products X_1's and X_3's roles in the firm's strategic plan, as well as the potential for reallocating its resources to other products and functional areas, needs to be considered first.

The amount of X_4 produced is higher in Scenario 4 than it is in Scenario 2. However, the product mix for Scenario 4 means only 18.85 units will be manufactured per day, bringing the profit to an all time low for this scenario. Profit would only be $408.87 per day.

In Scenario 2, approximately one X_1 was considered profitable along with 21.00 X_2. From this data, the profit for the day would be \$558.15.

In the final scenario, the results of sensitivity analysis indicate that adding an hour to these constraints will result in an additional 1 unit of product X_2 and 5 units of product X_4. This analysis translates into an additional profit of \$129.30.

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