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Foreign Currency Options: Ex Post and Ex Ante Market Efficiency Tests

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Introduction

In their studies, Biger and Hull (1983), Johnson (1986), and Tucker (1985) suggest that the Black and Scholes option pricing model (OPM), adjusted for foreign currencies, is a useful model in estimating the option's market price. Tucker (1985, p. 283), for example, concludes there is not a sufficient number of abnormal rates earned from hedging strategies using the Black and Scholes OPM with daily quotes to suggest the market lacks efficiency. What studies by Tucker, Galai, Johnson and Collins, and others may be showing, though, is that a sufficient number of market participants use a similar OPM (possibly a Black and Scholes model computed for very small periods and with a variance estimated using an implied variance technique).

However, in conducting efficiency tests, it is important to be aware of cases where the market is inefficient, even though empirically one observes a market value consistently equal to some OPM value. Such a condition could occur if option investors estimated the same, but incorrect, variances and/or used the same, but incorrect, OPM formulas and hedging strategies. If this existed, then investors would force the value of the option to equal the formula's value, and they would eliminate any abnormality in rates earned from the strategy. This result, however, would create only an artificial equilibrium and, as such, would not preclude other investors from earning abnormal rates by developing strategies based on the true variance (or better estimate of it) and/or a better specified model.

Given this possibility of inefficiency, the purpose of this paper is to test the efficiency of the currency option market.[1] To this end, two tests are employed: the first is an ex post one, similar to the one used by Galai (1977), where the closing prices on day t are used to determine both the trading rules for mispriced stock options and also to execute the appropriate hedging strategy; the second is an ex ante one, similar to Tucker's, where the trading rules for mispriced currency options are determined from closing prices on day t, but the transactions are not executed until day t + 1, using the closing prices for that day. The tests used in this analysis differ, however, from Galai's and Tucker's tests in five respects.

First, the length of each trading period in this analysis is weekly instead of the more traditional daily periods which Galai and Tucker used. This is done partly to determine if the OPM holds when the length of the period is extended and partly to determine if the market is efficient from the viewpoint of an institutional investor, who may find longer periods, which would reduce commission costs, more appropriate than daily or intra-daily periods used by market traders pursuing arbitrage opportunities.

Second, because of the fewer number of periods with longer lengths to expirations resulting from the use of weekly periods instead of daily, the Cox, Ross, and Rubinstein (1979) binomial (OPM) defined for a discrete number of periods is used to estimate the equilibrium call price, instead of the seminal Black and Scholes (1973) OPM used by Tucker and Galai.[2]

Third, in deriving the OPM for currencies and in identifying mispriced options, this study uses hedging strategies defined in terms of the spot exchange rates instead of exchange rate futures (or forward contracts on exchange rates) as was used in Tucker's study.[3] The latter hedging strategy is more applicable for market makers or arbitrageurs who are not able to sell foreign securities, and therefore must create an equivalent short position using the future or forward markets. For institutional investors, who can sell foreign securities or borrow foreign currency at money market rates, the use of the spot exchange market is appropriate.[4] Moreover, if the interest rate parity relation holds, the hedging strategies yield the same result.

Fourth, the use of the binomial model with a week as a length of the period requires that the variance of the rate of change of the exchange rates be stable. This is necessary in order to insure the condition that foreign exchange prices, on average, increase or decrease with fixed ratios and with fixed probabilities each period, a condition particularly important given that the number of periods to expiration is often less than 30 when weekly periods are used. Accordingly, this study estimates a number of variances based on different lengths of time and then selects the one to be used in the OPM based on the Goldfeld-Quandt (1965) variance stability test. This contrasts to the other studies which simply use the implied variance method.

Finally, this study, unlike Tucker's analysis, makes use of the closing currency and currency option quotes from the **Wall Street Journal** (WSJ) instead of the intra-daily quotes from the Philadelphia Stock Exchange (PHLX). As a result, non-simultaneity problems associated with WSJ data do exist with the ex post tests. The ex ante tests do serve, however, to compensate for biases created by the use of closing price data. In addition, problems of knowing if the prices are bid or asked also exist. The authors do not view this, however, as a problem since determining mispriced options and computing the returns from arbitrage strategies involves both short and long positions in currency call options. Thus, we believe the likelihood of an institutional investor not obtaining, on average, the returns based on the closing prices quoted in the WSJ is unlikely. In addition, the bid and asked spread may be smaller for an institutional investor than a market maker or other currency option investor since the institutional investor is often able to reduce the dealer's spread by trading in large volume.

The paper is organized as follows: a definition of the binomial OPM as it applies to currency options and the hedging strategies for mispriced currency options; a description of the data set, the estimated inputs, and the simulation program; the results of the ex post and ex ante tests; and, a conclusion.

Foreign Currency OPM

The binomial OPM used to determine mispriced currency options is:

$$(1)\ Co^* = \sum_{j=0}^{n} [n!/(n-j)!j!]\ p^j\ (1-p)^{n-j}\ \max[0, u^j d^{n-j} Eo - X]\}/r^n,$$

where:

Co^* = the value of the currency call option.

Eo = the spot exchange rate (dollar price of foreign currency).

n = the number of periods to expiration.

X = the exercise price.

u = one plus the proportional increase in the exchange rate possible in one period.

d = one minus the proportional decrease in the exchange rate possible in one period.

p = $[r-d]/rfEo(u-d)$.

r = 1 + domestic risk free-rate for the period (Rd).

rf = 1 + foreign risk free-rate for the period (Rf). Domestic country is the U.S.

Equation (1) is an adaptation of the OPM developed by Cox, Ross, and Rubinstein (1979). The model is based on the following assumptions: (1) the time to expiration for the option consists of n-periods of length h; (2) in each period the exchange rate either increases to equal u times its initial value or decreases to equal d times its initial value, with u and d constant for each period; (3) the risk-free interest rate in both the home country (U.S.) (R) and foreign country (Rf) are constant during the life of the option;[5] and (4) investors in the market seek out riskless hedging opportunities each period. (An explanation of the derivation of the currency OPM is in Appendix A.)

The only parameters in the OPM requiring estimation are u and d. Following the methodology of Cox, Ross, and Rubinstein (1979), the expressions for u and d are obtained by: (1) deriving the mathematical equations for the mean and variance of the growth rate in the exchange rate that results from assuming changes in exchange rates follow a binomial process; (2) setting the resulting equations for the mean and variance equal to their estimated values; and (3) solving the resulting equations simultaneously for u and d. This yields:

$$(2)\qquad u = e^{\sigma\sqrt{t/n}},\ d = e^{-\sigma\sqrt{t/n}},$$

where:

σ = annualized standard deviation in the growth rate of the exchanges rate.

t = time to expiration expressed as a proportion of the year.

n = number of periods of size h to expiration.

As noted in the above assumptions, opportunities for arbitrage returns or abnormal rates of return from market inefficiencies will exist if the market price of the call (Cmo) does not equal Co^*. For multiple periods, there are two riskless trading strategies that can be used.

First, if the call is initially overpriced, ($Cmo > Co^*$) then the overvalued strategy is followed where a currency call option is sold at Cmo, and Do units of foreign currency are bought at an exchange rate Eo and invested in a foreign risk-free security for the period; this strategy, in turn, can be financed by borrowing DoEo − Cmo dollars at the risk-free rate (or selling short or issuing a security). The position is then readjusted at the end of each period k if the option is overpriced ($Cm_k > C^*_k$) by buying or selling an amount of foreign currency necessary to obtain the D_k associated with that period and exchange rate, E_k. If additional foreign currency is needed ($D_k > D_{k-1}rf$), then funds equal to $(D_{k-1}rf - D_k)E_k$ are borrowed at a rate R; if foreign currency needs to be sold ($D_k < D_{k-1}\ rf$) to obtain the new hedge, then the proceeds from the sale $(D_k - D_{k-1}\ rf)E_k$ are invested at rate R. This readjustment occurs each period until a period z is reached where the option is undervalued. Closing the position at the end of period z yields the following return:

$$(3)\quad \text{Return} = D_{z-1}\ rf_{z-1}\ E_z - Cm_z - [DoEo - Cmo] \prod_{k=0}^{z-1} r_k - \sum_{k=1}^{z-1} [D_{z-k} - D_{z-k-1}\ rf_{z-k-1}]\ E_{z-k} \prod_{i=1}^{k} r_i$$

where:

z = the period the position is closed.

o = starting period (0).

$D_{z-k} - D_{z-k-1}\ rf_{z-k-1} > 0$, funds are borrowed, and if

$D_{z-k} - D_{z-k-1}\ rf_{z-k-1} < 0$, funds are invested.

By contrast, if the option is initially underpriced, then the undervalued hedging strategy is used. This strategy requires (1) borrowing Do units of foreign currency at a rate Rf (or selling a foreign security short or issuing a foreign security), (2) converting to dollars at Eo, and then (3) using the proceeds to invest in one call and a domestic risk-free security. This strategy is then readjusted at the end of each period k if the option is undervalued, $Cm_k < C^*_k$. Specifically, if $D_k > {}_{Dk-1}\ rf_{k-1}$, then $(D_k - D_{k-1}\ rf_{k-1})$ additional foreign currency is borrowed, converted to dollars at E_k, and invested in the domestic risk-free security. If $D_k < D_{k-1}\ rf_{k-1}$, then $(D_k - D_{k-1}\ rf_{k-1})E_k$ dollars are borrowed at rate R, converted to foreign currency and used to repay $(D_k - D_{k-1}\ rf_{k-1})$ of the foreign currency loan. The position is closed at the end of period z when the option is overvalued. Closing the position at the end of period z yields the following return:

$$(4)\quad \text{Return} = -D_{z-1}\ rf_{z-1}\ E_z + Cm_z + [DoEo - Cmo] \prod_{k=0}^{z-1} r_k + \sum_{k=1}^{z-1} [D_{z-k} - D_{z-k-1}\ rf_{z-k-1}]\ E_{z-k} \prod_{i=1}^{k} r_i$$

where:

$D_{z-k} - D_{z-k-1}\ rf_{z-k-1} > 0$, foreign currency is borrowed, and if

$D_{z-k} - D_{z-k-1}\ rf_{z-k-1} < 0$, foreign currency is repaid.

Efficiency Tests

PHLX currency option prices and exchange rates from the WSJ are used in the efficiency tests. Twelve currency options make up the sample for both the ex post and ex ante hedge tests. Options are defined in terms of their exercise prices, exercise month, and exercise year. Using weekly periods, there are approximately 21 quotes per option. Thus, the total number of option investments evaluated is approximately 228 for each test. The sample covers the year 1985 and includes the currency options for the West German mark and British pound. The options are shown in Table 1.[6]

The average U.S. and foreign treasury bill rates for the simulation period are used as R and Rf in the OPM (adjusted to be a weekly rate) and also in computing investment returns for the hedging strategies and the interest costs and/or interest earned from the readjustments (Data Source: **International Financial Statistics**). Using the average rates for the simulation period makes the analysis ex post. The differences between the rates for the beginning of each period and the averages for the simulation periods, however, were found to be very small.

The last required input parameter is the annualized standard deviation.[7] The method used to determine σ is the historical method given by

$$\sigma = [(\mathrm{Var}(\ln(E_w/E_{w-1}))52]^{1/2},$$

where E_w and E_{w-1} are the spot exchange rates at the end of week w and w−1.

Using the Friday exchange rate quotes from the WSJ and varying the sample periods to include the last 50, 100, 150, and 200 weeks prior to the beginning of each simulation period for the option, a number of standard deviations were obtained using both methods. From this group, the standard deviations selected for the mark and pound for each of the simulation runs were the ones with the most stability as determined by the lowest F-statistic obtained from a Goldfeld-Quandt test (1965).[8] The standard deviations and interest rates used in the tests are shown in columns 13, 14, and 15 in Table 1.

The ex post and ex ante computer simulation programs written to compute the rates for hedging strategies of mispriced options are based on Equation (1) for the OPM, Equation (2) for u and d, and Equations (3) and (4) for determining the returns for the hedging strategies. Specifically, each program first computes the OPM values and D's for each option at each date using the option and exchange rate data base and the inputs shown in Table 1. The OPM values are determined by Equation (1) and the D's for each date are based on the estimated call values and exchange rates estimated by the OPM for the end of the next period. The ex post and ex ante programs next are used to compare the OPM values with the market prices at each date to determine the appropriate strategy to implement. Third, the programs compare the OPM values and market prices at each subsequent date to determine if the option is to be closed or readjusted, and if readjusted, what the cost or additional return is. Finally, the programs calculate the rates earned from the strategies. For Strategy 1, the rate is computed as the return (Equation (3)) expressed as a proportion of the investment of DoEo − Cmo; for

Table 1
Summary of Ex Post and Ex Ante Simulation Tests

				Strategy										
	Exercise*			1			2			3		Inputs		
1 RUN	2 CUR	3 PRICE	4 MO	5 N	6 Ra	7 σa	8 N	9 Ra	10 σa	11 N	12 NEG	13 R	14 Rf	15 σ
Ex Post Tests														
1	DM	33	12	26	.516	.686	6	2.910	1.823	1	0	.078	.057	.1667
2	DM	32	9	12	.540	.620	7	1.350	1.640	2	0	.078	.057	.1667
3	DM	32	6	12	.690	.880	12	1.610	2.770	4	4	.078	.057	.1667
4	DM	33	6	8	.510	.390	7	.970	1.187	4	0	.078	.057	.1667
5	DM	33	9	19	.470	.380	12	2.520	5.570	2	0	.076	.057	.1667
6	DM	29	6	10	.200	.160	1	.620	—	2	0	.078	.057	.1667
7	DM	35	3	18	.500	.410	6	.960	1.020	1	1	.075	.057	.1667
8	Pound	110	6	4	.985	1.30	14	1.260	.770	1	1	.076	.119	.1816
9	Pound	110	9	16	.278	.268	12	2.880	2.550	1	0	.076	.119	.1816
10	Pound	110	12	16	.268	.372	1	.660	—	1	0	.076	.119	.1816
11	Pound	115	12	15	.430	.750	2	2.530	1.350	2	0	.076	.119	.1816
12	Pound	115	9	17	.620	.621	16	4.512	5.490	2	0	.076	.119	.1816

Ex Ante Tests

1	DM	33	12	19	.225	.232	12	.383	.553	2	3	.078	.057	.1667
2	DM	32	9	14	.166	.098	17	.650	1.86	1	8	.078	.057	.1667
3	DM	32	6	4	.240	.529	14	.489	.625	6	4	.078	.057	.1667
4	DM	33	6	11	.344	.285	13	1.321	.189	4	3	.078	.057	.1667
5	DM	33	9	14	.891	2.49	18	.518	1.064	0	8	.076	.057	.1667
6	DM	29	6	3	.081	.113	3	.349	.184	3	1	.078	.057	.1667
7	DM	35	3	7	.368	.235	6	1.218	1.253	3	0	.075	.057	.1667
8	Pound	110	6	18	.183	.050	9	5.905	6.643	0	0	.076	.119	.1816
9	Pound	110	9	10	.101	.029	9	1.220	.989	2	0	.076	.119	.1816
10	Pound	110	12	14	.103	.031	0	—	—	1	0	.076	.119	.1816
11	Pound	115	12	16	.100	.060	0	—	—	0	1	.076	.119	.1816
12	Pound	115	9	17	.136	.079	10	4.841	6.050	0	1	.076	.119	.1816

N = number of investments in the strategy for the run.
Ra = average annual rate for the run expressed as a proportion.
σa = average standard deviation of the annual rates for the strategies in the run.
Neg = number of simulations in the run with negative rates.
R = average U.S. treasury bill rate for the simulation period.
Source: **International Financial Statistics.**
Rf = average foreign rate for the simulation period for treasury bills (G.B.) and call money rate (Germany).
Source: **International Financial Statistics.**
*All options had expiration dates in 1985.

Strategy 2, the rate is the return (Equation (4)) expressed as a proportion of an assumed collateral requirement of 0.5(DoEo) used to define the investment.[9]

In the ex post simulation program, the rates obtained from the hedging strategies are generated by using the same option premiums and exchange quotes to define the initial and readjustment strategies and to execute them. In the ex ante program, though, the initial and readjustment strategies are defined with the prices for day t but are executed with the prices for the next trading day. (The ex ante simulation program is explained in terms of one of the option runs in Appendix B.)

The other option strategy appearing in Table 1 is labeled number 3. Strategy 3 is defined as an initial Strategy 1 or 2 which never becomes undervalued or overvalued and is therefore closed on the last date in the data base (which is not necessarily the expiration date) at the prices prevailing on that date. Because of the way these strategies are closed, their results are not summarized in Table 1.

Results

An examination of the ex post simulation tests in Table 1 shows a large number of abnormal rates. For Strategy 1, the annualized rates range from 20% to 98.5%, with the average rate equal to 50%. For Strategy 2, the rates range from 62% to 450%, with the average equal to 190%. However, an examination of the standard deviations of the rates from the simulation runs indicates a relatively high level of risk. Many of the deviations, though, are distorted by several large positive rates associated with their runs. Accordingly, more significant than the standard deviations is the fact that 97% of the ex post runs had positive rates of return, as can be seen by the small number of negative rates indicated in column 12 of Table 1; this indicates positive skewness. In summary, the results of the ex post simulation tests show abnormal rates earned from weekly hedging strategies involving currency options. Moreover, these results suggest inefficiency existed in the currency option market in 1985.

The ex ante simulation tests in Table 1 show similar results to the ex post tests. Specifically, for Strategy 1, the annualized rates range from 8.1% to 89.1%, with the average rate equal to 24.5%. For Strategy 2, they range from 34.9% to 590%, with the average equal to 169%. The standard deviations are high, but 87% of the runs yielded positive rates; this again indicates positive skewness. In summary, the ex ante tests moderately support the ex post test's inference of market inefficiency.

Conclusion

This analysis is not without certain limitations. For one, the study could be broadened to include more options and more years. In addition, commission costs need to be incorporated to determine whether the abnormal earnings suggested by the simulations are available to exchange members, non-members, or institutional investors who can buy in large quantity.

Nevertheless, this study is noteworthy in that it does show that when one extends the length of the period and select variances based on a stability test, abnormal rates are possible. As a result, this study, unlike previous ones, questions whether the currency option market is efficient.

Footnotes*

[1]The Philadelphia Stock Exchange (PHLX) was the first organized market trading currency options in the United States. Options traded include the pound, Canadian dollar, Japanese yen, Swiss franc, West German mark, and French franc. Some other organized markets trading currency options are located in Amsterdam, Montreal, and Vancouver. In addition, there is an interbank market in foreign currency options.

[2]The limiting case of the binomial model is the Black and Scholes. However, in this analysis the number of periods to expiration is often less than 30. As a result, the binomial model should be thought of as an estimate of the equilibrium call price.

[3]For detailed discussions of foreign currency option pricing models see Feiger and Jacquillat (1979), Biger and Hull (1983), Garman and Kohlhagen (1983), Johnson (1986), and Johnson, Zuber and Loy (1986). Feiger and Jacquillat were the first to show that a combination of a forward contract and an option on currencies provided a hedge; Biger and Hull showed that such a hedge needed to include the foreign rate; and Johnson, Zuber, and Loy showed the currency option strategy in terms of spot exchange rates.

[4]Unlike the U.S. security market, selling foreign securities short in many foreign countries is nonexistent. As a result, setting hedging strategies using short positions in foreign securities must be done either by using the forward market and the domestic money markets or by issuing foreign securities. The latter alternative could be accomplished by an institutional investor (e.g. bank) by selling foreign securities or borrowing through its foreign subsidiary or through a correspondent foreign bank.

[5]Using the Black-Scholes OPM, Biger and Hull (1983) show the assumption of a constant foreign rate is not necessary if the forward rate follows a Geometric Brownian Motion Process.

[6]It should be noted that suspension of trade by the Philadelphia Exchange or no trading resulted in several gaps in the weekly data. These gaps, in turn, often made it difficult to find options with two consecutive trading days, a condition necessary to conduct the ex ante test.

[7]For a discussion of this method, see Cox and Rubinstein (1985).

[8]The Goldfeld-Quandt test first takes the observations in their chronological order and deletes the middle M observations; where M = .2n and n = sample size. The remaining observations are then equally divided into two blocks and a linear regression of each block's observations against its chronological number is run. Finally, to test stability, the F-statistic SSE1/SSE2, where SSE is the sum of squared errors from the regression for the first (SSE1) and second (SSE2) blocks are used to test the hypothesis of a constant variance (homoscedasticity) versus the alternative hypothesis of non-constant variance. Accordingly, if the F-statistic (with $[n-M-4]/2$ degrees of freedom in the numerator and denominator) is greater than the critical value of F, the null hypothesis is rejected.

In this analysis, the standard deviations used were the ones from the various sample sizes with the lowest F-statistic. These standard deviations also had values less than the critical F at a 5% level of significance. Thus, the null hypothesis of a constant variance was not rejected for each currency's standard deviation.

[9]Although it does not make a difference, the rates of return were calculated instead of returns since the strategies represent investments rather than arbitrage strategies from an institutional investors viewpoint. Also, for Strategy 2, the investment is viewed as the cost of setting up a short sale in a foreign security; as such, the investment is defined in terms of collateral needed to execute a short sale.

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Appendix A
Binomial Foreign Currency OPM

As noted earlier, when the option is overpriced (Cmo > Co*) investors will find arbitrage returns by following a strategy of borrowing dollars (or selling a domestic security short or issuing a security) and selling a call in order to buy a proportion D of foreign currency units to a written call at a price of Eo, with the funds invested for the period in a foreign risk-free security yielding a rate Rf. By contrast, when the option is underpriced (Cmo < Co*), investors will find arbitrage returns by reversing the strategy by borrowing D units of foreign currency at a rate Rf (or selling a foreign security short or issuing a foreign security) and converting the currency to dollars units at Eo, with the proceeds invested in a call and a domestic risk-free security (or a multiple of this strategy). In executing either strategy, arbitrageurs will attempt to make each strategy risk-free by choosing a D such that the return at the end of the period is known. Algebraically, this requires finding D where the following condition is satisfied:

(A-1) $$Drf\ uEo - Cu = Drf\ dEo - Cd,$$

where:

Cu = OPM call value associated with the echange rate uEo, with n−1 period to expiration.

Cd = OPM call value associated with the exchange rate dEo, with n−1 period to expiration.

Solving (A-1) for D thus yields:

(A-2) $$D^* = [Cu - Cd]/[Eo\ rf\ (u-d)].$$

Given the opportunity for a riskless return, arbitrageurs will pursue the appropriate over or undervalued strategy, in the process selling currency call options (or buying them), until a call premium of Co* is attained where return from the strategy is zero. Algebraically, the equilibrium price is found by solving for Co* where the following condition is satisfied:

(A-3) $$D^*\ uEo\ rf - Cu - [D^*Eo - Co^*]\ r = 0$$

Solving (A-3) for Co* thus yields:

(A-4) $$Co^* = (1/r)\ [D^*\ rf\ uEo + Cu + D^*r\ Eo],$$

or, alternatively, substituting (A-2) for D* in (A-4) and rearranging, one obtains:

(A-5) $$Co^* = [p\ Cu + (1-p)Cd]/r,$$

where:

(A-6) $$p = [r - rf\ d]/[rf(u-d)].$$

Finally, given the equilibrium call price in (A-5), the price of a call with n-periods to expiration is obtained by substituting the equations (similar to (A-5)) for all of the equilibrium call prices to expiration, where at expiration the possible call values will equal their intrinsic values. This successive substitution yields Equation (1).

Appendix B
Ex Ante Simulation Program

The nature of the ex ante simulation program can be seen in the case of the call option on the mark with an exercise price of 33 (cents) expiring in December of 1985 that is summarized in Table B-1. The first, third, and fourth columns in the table show the dates, the reported call premiums for the option, and selected exchange rates. The second and fifth columns show the OPM values (C*) and D's computed for each date. The sixth column indicates the strategy used where: 1 indicates the strategy used when the option is overvalued, 2 signifies the undervalued strategy, and 3 indicates that the option was closed on expiration or at the last date in the option's data base. In columns 7 and 8, the rates earned for the period (PERrate) and the annual rates (ANNrate) are shown for each option run. Finally, in columns 9 and 10 select call premiums and exchange rates for dates t+1 are shown.

As shown in row one of Table B-1, on date 3/19/85 the December 33 call option on the mark is undervalued. As detailed in Table B-2, Strategy 2 is implemented where .372027 marks are borrowed at an annual rate of 5.716% (adjusted for the period), and converted to dollars on date 3/20 at the exchange rate of $0.3058. The dollars are used to buy the December 33 call

Table B-1
Simulation Runs for 33 Call on the
Mark Expiring the 12th Month, 1985

1 DATE	2 C*	3 Cmkt	4 Select Et	5 D	6 St	7 ANNrate	8 PERrate	9 Ct+1	10 Select Et+1
+ 3 19 85	.0108611	.0108	.3088	.372027	2	.252682	.434179E−02	.0124	.3058
3 26 85	.0116771	.0138	.3113	.391483	1	.311458	.535256E−01	.0175	.3189
4 2 85	.0155217	.016		.470885	1	.202217	.323886E−01		
4 9 85	.0137612	.0164		.440047	1	.181623	.260077E−01		
4 16 85	.020095	.0215		.550823	1	.279002	.336809E−01		
4 23 85	.0156741	.016		.481853	1	.108448	.011951		
4 30 85	.0141309	.0165		.458731	1	.164541	.147547E−01		
5 7 85	.0119095	.0137		.416772	1	.447938	.288815E−01		
5 14 85	.0175205	.018		.52644	1	.513503	.241973E−01		
+ 5 21 85	.0163745	.0188	.3226	.512396	1	.712713	.020911	.0173	.3250
5 28 85	.013331	.0145	.3212	.459457	1	.829500	.116839E−01	.0163	.3257
6 4 85	.0168896	.0162	.3290	.533154	2	.278295	.473289E−02	.0155	.3270
6 11 85	.0141725	.0145		.484896	1	.053559	.100386E−02		
6 18 85	.0160098	.0127		.526973	2	−.038534	−.286757E−02		
6 25 85	.0146125	.0138		.502242	2	.237320	.115325E−01		
7 1 85	.0153158	.0150		.523485	2	.624223	.188301E−01		
7 8 85	.0194242	.019		.603656	2	.449259	.716114E−02		
7 15 85	.0264934	.0265		.710165	1	.433202	.694537E−02		
7 22 85	.0264163	.0252		.712329	2	.071194	.132343E−02		
7 29 85	.0290528	.0315		.750222	1	.077650	.605845E−02		
8 6 85	.0273604	.0276		.742514	1	.049505	.279152E−02		

Table B-1 (Continued)

8 13 85	.0349604	.0352	.827156	1	.065323	.243676E−02
8 19 85	.0365846	.0372	.842138	1	.018977	.433922E−03
8 27 85	.0034924	.0345	.84524	2	.274295	.842585E−02
9 4 85	.0267442	.0265	.776328	2	.885080	.734183E−02
9 9 85	.0182744	.019	.653963	1	.001783	.548363E−04
9 19 85	.0213759	.0209	.72395	2	1.91912	.208153E−01
9 26 85	.0449506	.049	.936933	1	.148156	.117588E−01
9 30 85	.0438955	.0458	.943775	1	.135515	.98238 E−02
10 28 85	.0490592	.0488	.982768	2	−.05092	−.341183E−02
11 4 85	.0552578	.055	.992757	2	−.30138	−.164134E−01
11 20 85	.0528084	.056	.995307	3	−.06702	−.346279E−02
12 2 85	.0671535	.070	.997011	3	.069207	.128770E−02
12 9 85	.0646037	0	.998077	0	NO	NUMBERS

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Statistics

STRATEGY	AvgAnnRet	AvgPerRet	AnnStdDev	PerStdDev	SkewnessAnn
1	.225562	.0141483	.232248	.0141815	.310586
2	.383390	.00515099	.552782	.00962158	3.24619
3	.0010914	−.00108755	.0681156	.00237525	

****

St = strategy
Et = exchange rate for date indicated.
Et+1 = exchange rate for the next trading day.
Ct+1 = call premium for the next trading day.
+ see Table B-2.

Table B-2
Rate Calculations for Strategies
3/19/85 and 5/21/85 on Table B-1

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3/19/85: Cm = .0108, C* = .01086; undervalued; strategy # 2.
Strategy:

(1) Sell Delta (D) = .372027 units of foreign currency (FC) short at rate Rf.
(2) Convert FC at 3/20 exchange rate: (.372027)(.3058) = \$.113766.
(3) Buy one call at 3/20 premium: .0124.
(4) Invest remainder in domestic risk-free security: \$.113766 − \$.0124 = \$.101366.

3/26/85: Cm = .0238, C* = .011677; overvalued; close.

(1) Repay FC loan by buying FC at 3/27 exchange rate: $(.3189)(.372027)(1.05716)^{1/52}$ = \$.118766.
(2) Sell call at 3/27 premium: \$.0175.
(3) Investment return = \$.101366 (1.0785) = \$.101513.
    Return: −.118766 + .0175 + .101513 = \$.000247
    Period rate as a proportion of collateral (.5(\$.113766)) = .000247/.056883 = .00434
    Annualized Rate $(1.00434)^{52} - 1 = .253$.

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5/21/85: Cm = .0188, C* = .0163745; Overvalued; Strategy # 1
Strategy:

(1) Buy .512396 units of FC at 5/22 exchange rate: (.512396)(.325) = \$.16653.
(2) Sell one call at 5/22 premium: \$.0173.
    Investment = \$.16653 − \$.0173 = .14922.

5/28/85: Cm = .0145, C* = .0133; Overvalued; Readjust.

(1) Sell $((.512396)(1.05716)^{1/52} - .459457) = .053487$ FC at 5/29 exchange rate: (.053487)(.3257) = \$.01742
(2) Invest \$.01742 in domestic risk-free security.

6/4/85: Cm = .0162, C* = .01689; Undervalued; Close.

(1) Sell FC: at 6/5 exchange rate: $(.327)(.459457)(1.05716)^{1/52}$ = \$.150403.
(2) Buy call at 6/5 premium: \$.0155.
(3) 5/29 Investment principal and return: $(1.0785^{1/52}(.01742)$ = \$.01744.
    Rate = [(1/\$.14922)[\$.150403 − \$.0155 + \$.01744]] − 1 = .0209.
    Annualized Rate = $(1.0209)^{26} - 1 = .713$

****

for \$.0124 (date 3/20) and to invest (\$.101366 = .113766 − .0124) in a U.S. treasury bill yielding an annual rate of 7.85%. On date 3/26 (the end of the first weekly period), the December 33 call option is undervalued. As a result, the position is closed on date 3/27 by buying Drf = $(.372027)(1.05716)^{1/52}$ units of foreign currency at an exchange rate of .3189 in order to repay the loan (cost = \$.118766), selling the call at a premium of \$.0175, and liquidating the investment to obtain \$.101513 ($\$.101366(1.0785)^{1/52}$). Closing the position yields a dollar return of \$.000247. The rate of return, expressed as a proportion of the collateral (.5(\$.113766) = .056883), is .434%, which equates to an annual rate of 25.3%.

By contrast, on date 5/21/85 the December 33 option is overvalued. As is also detailed in Table B-2, Strategy 1 is implemented where, on date 5/22, .512396 marks are purchased at an exchange rate of \$.325 and invested in a foreign treasury security yielding a rate of 5.716%, and the December 33 call is sold for \$.0173. This equals an investment of \$.14922. At the next date (5/28) the option is overvalued. The strategy is therefore readjusted by selling (Dt rf − Dt+1) = ($.512396(1.05716)^{1/52}$ − .459457) = .053487 marks at the exchange rate of \$.3257 prevailing on date 5/29 and investing the proceeds (\$.01742) in a U.S. treasury bill. Finally, on date 6/4 the option is undervalued. The strategy is closed by selling $(.459457)(1.05716)^{1/52}$ = .459948 marks at the 6/5 exchange rate of \$.327 (\$.150403 proceeds), purchasing the December 33 call for \$.0155, and liquidating the investment made on 5/29 to obtain (.01742)(1.0785)**1/52 = \$.01744. The return from the investment is \$.15234 (.150403 − .0155 + .01744), which equals a rate of 2.09% for the two-week period and an annual rate of 71.3%.

All the simulation runs conducted for the December 33 call option on the mark are summarized in the first row of ex ante strategies in Table 1. In summary, for the simulation on the December 33 call, there are 33 strategies. Of the 33, 19 strategies are number 1, where the average annual rate is 22.5% and the average standard deviation is .2322, and 12 are Strategy 2, where the average annual rate is 38.3% and the standard deviation is .5528. Of the 31 Strategies 1 and 2, only three have negative rates. This, in turn, is reflected in the positive skewness of .31058 for Strategy 1 and 3.24619 for 2.

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