Southern Business Review

Volume 14 | Issue 2 Article 7

September 1988

Commodity-Linked Bonds: A Pedagogical Note

Jacques A. Schnabel
Wilfrid Laurier University

Ebrahim Roumi
Universiy of New Brunswick

Brian Warrack
Wilfrid Laurier University

COMMODITY-LINKED BONDS: A PEDAGOGICAL NOTE

Jacques A. Schnabel
Ebrahim Roumi and
Brian Warrack

Introduction

In their classic paper on the valuation of options, Black and Scholes [1] demonstrated the isomorphic relationship which exists between call options and common stock, **viz.**, the stockholders own a call option on the firm's assets with exercise price equal to the promised payment to the bondholders. The debt instrument assumed in the Black-Scholes development was the simplest possible, the pure discount bond. Very few firms issue debt of this simple type. Bonds usually contain numerous "bells and whistles," i.e., options granted to either the debt-issuing firm or the bondholders. For example, the call feature grants the firm the right to redeem the bonds prematurely, whereas the retraction or put feature grants the bondholders the right to demand that the bonds be redeemed prematurely. In this note we focus on one example of these "bells and whistles," the commodity-linked bond. Specifically, we attempt to answer the following question. How is the classic Black-Scholes isomorphism altered by the presence of the commodity-linked feature? A simple firm whose sole source of debt financing is a pure discount commodity-linked bond is assumed. It is shown that the value of the common stock of such a firm is equal to a Margrabe [4] call option on the firm's assets with stochastic exercise price equal to the value of the underlying commodity bundle truncated below at the promised payment to the bondholders. As a corollary, we demonstrate the existence of a new type of agency problem unique to commodity-linked bonds. Although the characteristics and valuation of commodity-linked bonds have been discussed by numerous authors such as Brauer and Ravichandran [2], Carr [3], O'Hara [5], and Schwartz [6], they have failed to detect the agency problem implied by the simple isomorphism we derive here.

Commodity-linked bonds are bonds for which the coupon interest and/or principal payments are indexed to the price of a commodity. Corporations are showing increasing interest in the use of this type of debt financing. Several examples of outstanding commodity-linked bond issues are given below. To date, the most popular commodities used for this purpose have been crude oil, gold and silver.

Examples of Commodity-Linked Bonds

Since 1977, the government of Mexico has successfully issued a series of "petrobonds." The 1977 issue had the principal payment indexed to the (U.S. dollar) market price of crude oil, while both the interest payments and the principal payment were so indexed in a 1983 issue. The pioneer issue of corporate oil-indexed notes appeared in June of 1986, when Standard Oil of Ohio (Sohio) came to the market with a $375 million package of debentures and oil-indexed notes. Holders of the notes receive no interest payments but,

at maturity, will receive the face value of $1000 plus an amount determined by the average price of West Texas Intermediate crude oil during the year prior to maturity. Shortly after the Sohio issue, Interprovincial Pipe Line Ltd. used an oil-indexed debenture to partially finance its purchase of Home Oil Co. Ltd. from Hiram Walker Resources Ltd.

Perhaps the best known gold-linked bonds are those issued by the French government in 1973, when Valery Giscard d'Estaing was France's finance minister. Commonly known as the "Giscards," these bonds have both the interest and principal payments indexed to the price of gold bullion. Although gold-linked bonds haven't yet become popular in Canada, there have been numerous public offerings of gold-linked equity issues since 1981. It was in that year that the Canadian market was introduced to commodity-linked securities, when Echo-Bay Mines Ltd. issued a preferred share package that included detachable gold purchase warrants.

Our final example involves the operator of the largest U.S. silver mine. In 1980, Sunshine Mining Co. issued $1000 face value silver-linked bonds carrying a 8½% coupon, for which the principal payment is the greater of the $1000 face value and the market value of 50 troy ounces of silver.

There are a variety of ways of specifying the nature of the contingency of a commodity-linked bond's cash flows on the price of some commodity. But whatever are the specifics of the issue, the appeal to investors is clear. These bonds offer the investor an opportunity to participate in the price appreciation of a commodity, thereby providing the bondholder with a much-needed hedge against inflation, while retaining income and safety of principal. From the borrower's standpoint, Schwartz [6] has noted that the appeal of these bonds stems from such factors as "a lower coupon rate, more favorable bond indentures or the acceptance of a weaker currency by foreign investors." The aforementioned Home Oil Co. transaction suggests yet another reason for using commodity-linked bonds. They provide an interesting vehicle for bridging the gap between the respective valuations of a firm's assets by the buyer and seller, a gap that arises due to differing forecasts of the future price of a commodity.

Development of the Model

Consider a firm that is financed by debt and common equity, the current market values of which are D_0 and E_0, respectively. The firm's sole source of debt financing consists of pure discount commodity-linked bonds with a total face value of B. The bonds mature in T years, at which time the firm will be liquidated and all claims settled. At maturity, the bonds will be redeemed at the greater of B and C_T (if possible), where C_t is the market value at time t of a specified commodity bundle. The shareholders will receive any residual. The values of the commodity (C_t) and of the firm's assets (A_t) at time t are assumed to be jointly lognormally distributed, for $0 < t \leq T$, and ϱ denotes the correlation between them. Let σ_A and σ_C be the standard deviations of the logarithmic growth rates in the market values of the firm's assets and the commodity, respectively.

The structure of the cash flows to the firm's claimants at time T, shown in Tables 1 and 2, depends on whether the end-of-period commodity price C_T is "low" ($C_T \leq B$) or "high" ($C_T \geq B$) in relation to the face value of B.

Table 1

Cash Flows if Commodity Price is "Low" ($C_T \leq B$)

	(Bankrupt) $A_T \leq B$	(Not Bankrupt) $A_T \geq B$
Bondholders receive	A_T	B
Shareholders receive	0	$A_T - B$

Table 2

Cash Flows if Commodity Price is "High" ($C_T \geq B$)

	(Bankrupt) $A_T \leq C_T$	(Not Bankrupt) $A_T \geq C_T$
Bondholders receive	A_T	C_T
Shareholders receive	0	$A_T - C_T$

Combining these two situations, we obtain the cash flows shown in Table 3, where $S_T = \max(B, C_T)$.

Table 3

Cash Flows Available to Claimants at Time T

	(Bankrupt) $A_T \leq S_T$	(Not Bankrupt) $A_T \geq S_T$
Bondholders receive	A_T	S_T
Shareholders receive	0	$A_T - S_T$

Consider the situation depicted in Table 3. Following the suggestion of Black and Scholes [1], this situation can be viewed as one in which the bondholders own the firm and the shareholders hold a European call option to buy back the firm at time T upon payment of the exercise price S_T. As S_T is the greater of a constant value B and a stochastic value C_T, the probability distribution of the uncertain exercise price of the shareholders' call option is the probability distribution of the value of the commodity bundle at the bond's maturity date truncated below at the face value of the debt. Mar-

grabe's model for the valuation of a call option with a stochastic exercise price may be directly applied to value the shareholders' equity.

$$E_0 = A_0N(f_1) - S_0N(f_2) \qquad (1)$$

where $f_1 = \dfrac{\ln(A_0/S_0) + \sigma^2A/S^T}{\sigma_{A/S}\sqrt{T}}$

$$f_2 = f_1 - \sigma_{A/S}\sqrt{T}$$

$$\sigma^2A/S = \sigma^2A - 2\varrho S\sigma A\sigma S + \sigma^2S.$$

The parameter σ^2A/S is the variance of the logarithmic growth rate of A_t/S_t, the ratio of the market value of the firm's assets to the truncated market value of the commodity. S_0 equals max(B,C_0), i.e., the value of the commodity bundle at time 0 truncated below at the face value of the debt, and ϱ_S is the correlation between A_t and S_t. The current market value of the firm's debt is therefore

$$D_0 = A_0 - [A_0N(f_1) - S_0N(f_2)] \qquad (2)$$
$$= A_0N(-f_1) + S_0N(f_2).$$

These equations should be viewed as merely approximations to the market values of the equity and debt, respectively. This is because the probability distributional assumptions of Margrabe's model do not fit exactly our development. Specifically, Margrabe assumes that the exercise price is lognormally distributed whereas the exercise price assumed here is truncated (from below) lognormally distributed.

An Agency Problem

Recall that ϱ is the correlation between the values of the firm's assets and the commodity to which the redemption value of the bond is linked. Consider the effect of changes in ϱ on the values of the firm's equity and debt. Let N′(•) denote the standard normal density function. Since

$$\frac{\partial E_0}{\partial \sigma_{A/S}} = A_0N'(f_1)\sqrt{T} > 0$$

$\dfrac{\partial \varrho_S}{\partial \varrho}$ is positive and $\partial\sigma_{A/S}/\partial\varrho_S$ is negative, it follows that

$$\frac{\partial E_0}{\partial \varrho} < 0, \text{ and that therefore,}$$

$$\frac{\partial D_0}{\partial \varrho} > 0.$$

In other words, if the value of the correlation coefficient ϱ increases, the commodity-linked bonds become more valuable while the firm's equity becomes less valuable. This result suggests a strategy available to shareholders that increases their wealth at the expense of bondholders. If a firm has commodity-linked bonds outstanding, the shareholders could undertake an

investment strategy that reduces the correlation between the value of the firm's assets and that of the commodity. As shareholders would thus expropriate wealth from the bondholders, this creates an agency problem between these two groups. The foregoing is analogous to the well-known strategy, which has a similar detrimental effect on bondholders, of shareholders in a levered firm undertaking investments that increase the risk of the firm.

Conclusion

In this pedagogical note, the Black-Scholes isomorphism was refined to render it applicable to a firm with commodity-linked bonds outstanding. A simple firm whose sole source of debt financing is a pure discount commodity-linked bond was assumed. It was shown that the value of the common stock of such a firm equals that of a Margrabe call option on the firm's assets with stochastic exercise price equal to the value of the underlying commodity bundle truncated below at the promised payment to the bondholders. We derived a novel insight regarding the effect of changes in the correlation between the value of the firm's assets and that of the commodity bundle on the values of the firm's debt and equity. We showed that stockholders can expropriate wealth from bondholders by reducing this correlation. Thus, a new type of agency problem unique to commodity-linked bonds was demonstrated.

References

1. Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," **Journal of Political Economy**, 1973, 81, 637-659.

2. Brauer, G.A. and R. Ravichandran, "How Sweet is Silver?" **The Journal of Portfolio Management**, 1986, 13, 33-42.

3. Carr, P., "A Note on the Pricing of Commodity-Linked Bonds," **Journal of Finance**, 1987, 42, 1071-1076.

4. Margrabe, W., "The Value of an Option to Exchange One Asset for Another," **Journal of Finance**, 1978, 33, 177-186.

5. O'Hara, M., "Commodity Bonds and Consumption Risks," **Journal of Finance**, 1984, 34, 193-205.

6. Schwartz, E.S., "The Pricing of Commodity-Linked Bonds," **Journal of Finance**, 1982, 37, 525-539.

Jacques A. Schnabel is Professor of Business and Head of Finance Area in the School of Business and Economics, Wilfrid Laurier University, Waterloo, Ontario, Canada. Ebrahim Roumi is an Associate Professor of Opera-

tions Management in the Division of Administration, University of New Brunswick, Saint John, New Brunswick, Canada. Brian Warrack is an Associate Professor of Finance in the School of Business and Economics, Wilfrid Laurier University, Waterloo, Ontario, Canada.