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A Linear Programming Model for the Selection of Optimal Government Bond Portfolios

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A LINEAR PROGRAMMING MODEL FOR THE SELECTION OF OPTIMAL GOVERNMENT BOND PORTFOLIOS

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INTRODUCTION:

Trading of marketable U.S. Treasury securities involves billions of dollars every business day, a magnitude of transaction matched by few industries [3]. Most of these transactions involve treasury security dealers, a relatively small field composed of less than 30 firms in the entire United States.

Experience and judgement seem to be the tools used by dealers to make a profit in the highly sensitive, fast moving environment of the treasury market. No evidence was found to indicate that any quantitative model was in use by the dealers. There is a need to provide a practical model which can be used by the security dealers on a day-to-day basis. The purpose of this research was to develop the basis for an optimization model (using linear programming techniques) which could be used by security dealers. Though we had to make several assumptions in developing the model, we attempted to retain as much of the reality of a bond portfolio decision as possible.

A study of the data provided by the First Boston Corporation [4] indicates that over the period 1965 to 1975, there have been significant shifts in volume and maturity structures of treasury securities. The Treasury seems to have shortened the maturity of its outstanding debt. A reduced volume of outstanding long-term securities over this period might create an increase in price volatility of long-term securities. An increase in volume of the short-term instruments (T-Bills) over this period might decrease the price volatility of these securities. Security dealers keep an inventory of securities with various maturities to satisfy the demand of their clients. The various holdings in Treasury securities must be closely monitored on a risk-return basis in order to minimize losses and maximize returns.

Of the various models developed by researchers, the one developed by Yawitz, Hempel and Marshall (YHM) [11] was the most promising model for practical applications in this field. We plan to update the YHM model.

The objectives of this paper are as follows. First, update the risk-maturity relationships of Treasury security price fluctuations; second, incorporate portfolio liquidity and inventory constraints; and third, develop the concept of break-even yields.

Literature Review

Although no models were found to be in use by Treasury dealers to evaluate the risk-return characteristics of these securities, considerable work has been done in the area of general portfolio optimization models. The most notable studies are the ones by Crane [2], Wolf [9], Porter [6], Cheng [1], Watson [8], and Yawitz, Hempel and Marshall [11]. These studies incorporate risk as primarily a function of liquidity. Some studies have used security price variability as a measure of risk.

Porter's study [6] was primarily concerned with the development of a model designed to optimize a bank's investment in general classes of assets (cash, securities and loans). His model did not consider any explicit measure of risk, and there was no attempt made to look at the practicality of the model. Cheng's study [1] was in the area of optimum bond portfolio selection. He defined risk as the variance in portfolio returns, and assumed that securities will be held to maturity. This assumption is quite unrealistic in the real world, especially in the field of treasury security transactions.

Wolf attempted to optimize a bank's government security portfolio by incorporating uncertainty in a multi-period decision making process [9]. He found that "barbell" portfolios consisting of one short-term and one long-term security consistently provided a higher return than the corresponding intermediate maturity bonds with the same average maturity. A similar conclusion was reached by Crane [2]. In a later study, Yawitz, Hempel and Marshall [10] demonstrated the serious shortcomings from the use of a bond portfolio's average maturity as a risk proxy. Crane's model is similar to Wolf's model and treats future cash flows and interest rates as random variables. However, Crane's model maximized expected return subject to a loss limit constraint. This, in effect, limited the downside losses on the sale of securities. Crane assumed that investors face a **linearly** increasing yield curve and that a disturbance to the bond market leads to an **equal change in yields** along the entire term structure.

Watson used simulation techniques to investigate the optimal maturity combination for a bank's government bond portfolios [8]. He used standard deviation in a portfolio's return as relevant risk measure, and developed a general model for optimizing government bond portfolios. He failed to consider whether his sample of yield curves is consistent with long-run equilibrium in the bond market. He also ignored the fact that the shape of the current yield curve is partly determined by an investor's expectations for future interest rate movements.

Perhaps, one of the pioneering studies done in the area of bond portfolio selection is the study by Yawitz, Hempel and Marshall [11]. They developed a model using data over the period 1957 through 1972 to generate an "efficient frontier," (i.e. obtain maximum yield for each level of risk exposure). Risk was defined as mean absolute change in monthly price, or MAC (P). MAC (P) was defined as follows:

$$MAC\,(P) = 100\,(1/n) \sum_{i=1}^{n} \frac{\lfloor Pi - Pi - 1 \rfloor}{Pi - 1}$$

where n = number of months

Pi - 1, Pi = bond's price at the beginning and end of the period

Each portfolio's risk and return was obtained as a weighted average of the risks and returns of the bonds represented. The performance of the portfolios selected by the YHM technique was far superior to the performance of the "barbell" portfolios (consisting of one 30-day and one 30-year security). Their "optimal" portfolio provided annual returns ranging from 0.5328 to 1.2239 percent above that obtainable on the barbell portfolio. Their main contribution to the field is the placing of the bond acquisition decision within the context of a risk-return tradeoff.

We used the YHM study as the foundation for further analysis. We modified the YHM study in two areas. Because of the presence of liquidity and inventory constraints on the portfolio manager, each security was bound between a minimum and a maximum. The YHM study stopped in the year 1972. Security markets and interest rate conditions have changed considerably since then. We replicated and extended the YHM study for the period 1972 to 1978.

Methodology

The basic measure of risk used in the analysis was mean absolute change in price (MAC (P)). The procedure used to calculate MAC (P) was determined by contacting one of the authors of the YHM study [5]. YHM assumed that securities were selling at par (so that coupon = yield to maturity) and derived monthly yields by dividing coupon by 12. The sum of the present values of the monthly cash flow streams and the par value was equal to the price at the end of the month. MAC (P) was calculated for periods of one year and for 55 months by averaging monthly changes in price for the previous 12 months and previous 55 months, respectively. This was done for the period 1958 to 1972 by YHM. They also calculated a relative risk index (RRI) by dividing a given maturity's MAC (P) by the MAC (P) of the 30-year bond (the riskiest security). We updated the YHM work through the year 1978 using basically the same procedure. We used 60-month intervals instead of the 55 months used by YHM and also began the calculations in January of each period, instead of November as was done by YHM. The results are reported in the next section. Data was obtained from Solomon Brothers [7].

"LP Model"

Next, we developed a Linear Programming (LP) model to optimize Treasury Security portfolios. The LP portfolio model maximizes the following objective function.

$$Y_p = \sum_{i=1}^{n} X_i Y_i$$

$$\text{Subject to } R_p = \sum_{i=1}^{n} X_i R_i$$

$$\sum_{i=1}^{n} X_i = 1.00$$

$$A \leq X_i \leq B$$

where:

- Y_p is the portfolio yield
- X_i is the proportion of each security
- Y_i is the yield of each security
- n is the total number of securities
- i denotes individual securities
- R_p is the portfolio risk
- R_i is the risk of each security
- A is the minimum proportion of each security
- B is the maximum proportion of each security

The optimization model was set up as follows:

Row	Bond i	Bond i + 1...	Bond j	Total		RHS
Yield	*	*	*			
Risk	*	*	*		=	*
Money	1.0	1.0	1.0	-1.0	=	0.0
	1.0				≥	*
	1.0				≤	*
		1.0			≥	*
		1.0			≤	*
			1.0		≥	*
			1.0		≤	*
				1.0	=	*

*Represents coefficients of the equations.

"BREAK-EVEN YIELDS"

The LP model described above will provide the highest yielding or optimum portfolio. It also provides insight into any potential alternative portfolio adjustments. The value of the alternate portfolio adjustments is reflected in the break-even yields of securities in the portfolio. The break-even yield (BEY) for a security is defined as follows:

$$\text{BEY} = \text{Current yield of security} + \left[\frac{\text{Change in portfolio yield}}{\text{Change in proportion of security in portfolio}}\right]$$

The LP optimized portfolio will provide three classes of results: 1) Securities optimized at their minimum allowable proportion in the portfolio; 2) securities optimized at their maximum allowable proportion in the portfolio; and 3) securities optimized between the minimum and maximum allowable proportions in the portfolio.

For the first two categories of securities described above, the BEY's for any given securities are the value to which the yield of each security must change before the optimum portfolio would be altered. If the current yield is below the BEY, the security is overpriced and the proportion of the security should be reduced below the minimum allowable. Conversely, a security with a current yield above the BEY is underpriced, and the proportion in the portfolio should be raised above the maximum allowable proportion. The third category of securities—those below the maximum and above the minimum allowable proportions—carry current yields that are equal to the BEY. Thus, an increase in the proportion of a security in the first category would decrease the total portfolio yield by an amount proportional to the difference between the BEY and the current yield. Or, an increase in the proportion of securities in the second category would increase the portfolio yield by an amount proportional to the difference between the BEY and the current yield. The third category of securities represents the optimized proportions, and any change in the proportion would result in a decrease in the portfolio yield.

By utilizing the BEY analysis, the relative underpricing or overpricing of different securities can be used to make the decisions regarding particular securities to trade in restructuring the portfolio. Thus, the BEY analysis can be a very practical tool for evaluating the relative value of various securities.

Assumptions Used in the Development of the Model

Several assumptions were made in order to test the model for applicability. These assumptions were as follows:

1. Security yield to maturity data are based on monthly average yields for maturities of 1, 2, 3, 4, 5, 10, 20, and 30 years. Using average yields rather than individual yields eliminated distortion effects due to peculiarities of individual securities.

2. To reflect the sensitivity of overall portfolio risk on BEY's, the optimizations made were for three levels of portfolio risk: conservative, average, and high risk. These levels were arbitrarily set for purposes of testing the model. A conservative risk approach was represented by a RRI of 0.4, corresponding to about 3 years average portfolio maturity. An average risk approach was represented by a RRI of 0.5, which is about 6 years average portfolio maturity. The high risk approach was represented by a RRI of 0.6, which is about 8 years average portfolio maturity. Each level of risk represents expectations of future events. The BEY's from each optimization reflect yield sensitivity, or potential price variability, associated with each level of overall portfolio risk.

3. The RRI for each security was based on the updated RRI data described earlier. The RRI factors for the entire year of 1978 were calculated from MAC(P) data for the 60 months prior to January, 1978.

4. For portfolio liquidity and inventory purposes, there was a maximum and minimum amount of any one maturity in the portfolio. These limits were arbitrarily set at 25% for the maximum and 2% for the minimum of each maturity structure.

RESULTS

First, we will present the results of the YHM study which was extended through the year 1978 by us. Tables 1, 2, and 3 present the values of mean absolute change in price (%), Relative Risk Index and a comparison of the YHM results and our results, respectively. There are some differences between the two sets of results. The differences are partly due to the slightly different procedures used by YHM and us, and partly due to different levels of interest rates that existed in their study and our study.

In an attempt to compare the two results further, MAC(P) and RRI data were calculated for each individual year from 1954 through 1978. Figures 1 and 2 present the differences in YHM results and the updated average MAC(P) and RRI data for several time periods. The differences between the two studies seem to drop considerably when compared on the RRI basis.

It is obvious from the results shown that any model which attempts to develop an optimum portfolio of treasury securities, should use as much current data as possible rather than some historical averages.

Next, the portfolio optimization model discussed earlier was put to test. We arbitrarily chose the month of June 1978 to test the model (i.e. the average yields to maturity for June 1978 were used for the test). The risk index (RRI) data used in the test were obtained for the period 1973 to 1977 from Table 2.

We set limits of 2 (minimum) and 25 (maximum) percent for the security vector used in the LP model. The objective of the model was to maximize portfolio yield. The expected yield and the associated risk index (RRI) are shown in Tables 4, 5, and 6 for risk levels of 0.4, 0.5, and 0.6. The tables also show the maturity of security, the risk level associated with that security, the minimum, maximum and optimum percent of each security in the portfolio, the actual average yield to maturity (as of June 1978), the break-even yield (BEY) and the difference between actual and break-even yields respectively.

The two most significant results worth noting are the optimum percent in portfolio and the difference in BEY and actual yields. When the difference is positive, it indicates that the security is underpriced at that point, and when it is negative, the indication is that the security is overpriced. Figure 3 shows a plot of maturity of security against actual and break-even yields, for the three levels of risk used in the study.

The results show that the longer maturity (20, 30 year) portfolios are all overpriced at all levels of risk and the portfolios get optimized at their minimum constraint level of 2 percent. For the 30-year security, the lower the risk level, the higher the break-even yield. This indicates that the price of the 30-year securities must drop considerably at lower RRI values before they become attractive investments.

For the intermediate levels of maturity (5, 10 years), the portfolios' yields indicate underpricing at risk levels of 0.5 to 0.6. The optimization occurred at the maximum constraint level of 25 percent. If security dealers perceive that risk levels are likely to be in the area of 0.5 to 0.6, then the intermediate level maturity securities offer some good potential for increased profitability.

Table 1
Update of MAC(P) by Periods

Period	1 Year	2 Year	3 Year	4 Year	5 Year	10 Year	20 Year	30 Year
1954-58	.1756	.3302	.4535	.5658	.6657	.8116	1.1600	1.3114
1955-59	.1922	.3641	.4942	.6074	.7107	.8773	1.2481	1.3659
1956-60	.2111	.4335	.5635	.7030	.8380	1.0303	1.4766	1.5621
1957-61	.1987	.4114	.5220	.6527	.7718	.9796	1.3381	1.4651
1958-62	.1867	.3865	.4744	.5833	.6829	.8283	1.1352	1.3236
1959-63	.1344	.3025	.3658	.4502	.5431	.7233	.9530	1.0289
1960-64	.1127	.2613	.3174	.4056	.4829	.6132	.8174	.8890
1961-65	.0755	.1702	.2146	.2734	.3121	.4139	.5139	.6041
1962-66	.0970	.2127	.2861	.3547	.4055	.5657	.6834	.7749
1963-67	.1256	.2613	.3677	.4643	.5281	.7628	.9609	1.1077
1964-68	.1444	.2914	.4226	.5259	.5887	.8422	1.1545	1.3834
1965-69	.1828	.3877	.5641	.6923	.7914	1.1413	1.5430	1.7442
1966-70	.2449	.5134	.7404	.9023	1.0454	1.5329	2.0449	2.3126
1967-71	.2845	.5834	.8422	1.0106	1.1508	1.6773	2.1106	2.3951
1968-72	.2891	.5834	.8056	.9643	1.0943	1.5295	1.9071	2.1080
1969-73	.3411	.6429	.8678	1.0406	1.1860	1.5461	2.1727	2.3891
1970-74	.3789	.6527	.8488	1.0172	1.1566	1.4840	2.0979	2.3941
1971-75	.4014	.6687	.8637	1.0478	1.1357	1.2679	1.9204	2.1096
1972-76	.3835	.6202	.7619	.9226	.9886	1.0762	1.7467	1.8905
1973-77	.3753	.6121	1.7740	.9467	1.0111	1.1606	1.8268	1.9501
1974-78	.3379	.5675	.7398	.9051	.9536	1.1998	1.5694	1.5866

Table 2
Update of RRI by Periods

Period	1 Year	2 Year	3 Year	4 Year	5 Year	10 Year	20 Year	30 Year
1954-58	.1339	.2518	.3458	.4313	.5076	.6819	.8846	1.0000
1955-59	.1407	.2666	.3618	.4447	.5203	.6423	.9138	1.0000
1956-60	.1351	.2775	.3607	.4500	.5365	.6596	.9453	1.0000
1957-61	.1356	.2808	.3563	.4455	.5268	.6686	.9133	1.0000
1958-62	.1411	.2920	.3584	.4407	.5159	.6258	.8477	1.0000
1959-63	.1306	.2940	.3555	.4376	.5278	.7030	.9262	1.0000
1960-64	.1268	.2939	.3570	.4562	.5432	.6898	.9195	1.0000
1961-65	.1250	.2817	.3552	.4526	.5166	.6852	.8507	1.0000
1962-66	.1252	.2745	.3692	.4577	.5233	.7300	.8819	1.0000
1963-67	.1134	.2359	.3319	.4192	.4768	.6886	.8675	1.0000
1964-68	.1044	.2106	.3055	.3802	.4255	.6088	.8345	1.0000
1965-69	.1048	.2223	.3234	.3969	.4537	.6543	.8846	1.0000
1966-70	.1059	.2220	.3202	.3902	.4520	.6628	.8842	1.0000
1967-71	.1188	.2436	.3516	.4219	.4805	.7003	.8812	1.0000
1968-72	.1371	.2768	.3822	.4574	.5191	.7256	.9047	1.0000
1969-73	.1428	.2691	.3632	.4356	.4964	.6471	.9094	1.0000
1970-74	.1583	.2726	.3545	.4249	.4831	.6199	.8763	1.0000
1971-75	.1903	.3170	.4094	.4967	.5383	.6010	.9103	1.0000
1972-76	.2029	.3281	.4030	.4880	.5229	.5693	.9239	1.0000
1973-77	.1925	.3139	.3969	.4855	.5185	.5951	.9368	1.0000
1974-78	.2130	.3577	.4663	.5705	.6010	.7562	.9892	1.0000

Table 3
Update of MAC(P) and RRI Averages

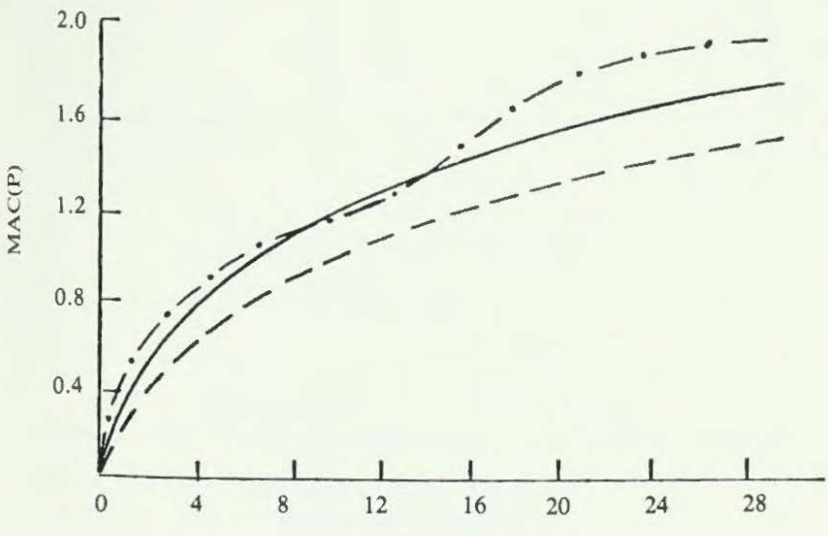
Years to Maturity	1954 — 1972* MAC(P)	1954 — 1972* RRI	1973 — 1978 MAC(P)	1973 — 1978 RRI	1954 — 1978 MAC(P)	1954 — 1978 RRI
1	.1874	.1303	.3511	.1887	.2267	.1472
2	.3813	.2650	.5714	.3071	.4269	.2772
3	.5176	.3598	.7354	.3952	.5699	.3701
4	.6348	.4413	.8962	.4871	.6975	.4530
5	.7342	.5104	.9558	.5237	.7874	.5213
10	.9868	.6859	1.1443	.6990	1.0246	.6954
20	1.2905	.8971	1.7546	.9430	1.4019	.9104
30	1.4386	1.0000	1.8606	1.0000	1.5399	1.0000

*See Yawitz, Hempel, Marshall (11).

One year (short-term) securities were overpriced at risk levels of 0.5 and 0.6. The optimization occurred at the 2% level. If the dealers expect such risk levels, they might consider selling these securities to a minimum level. The results were mixed at maturity levels of 2, 3, and 4 years.

These results indicate that based on anticipated risk levels, the dealers in the U.S. Government security markets can use this LP model to identify possible avenues for increasing their returns, as well as avoid suffering excessive losses.

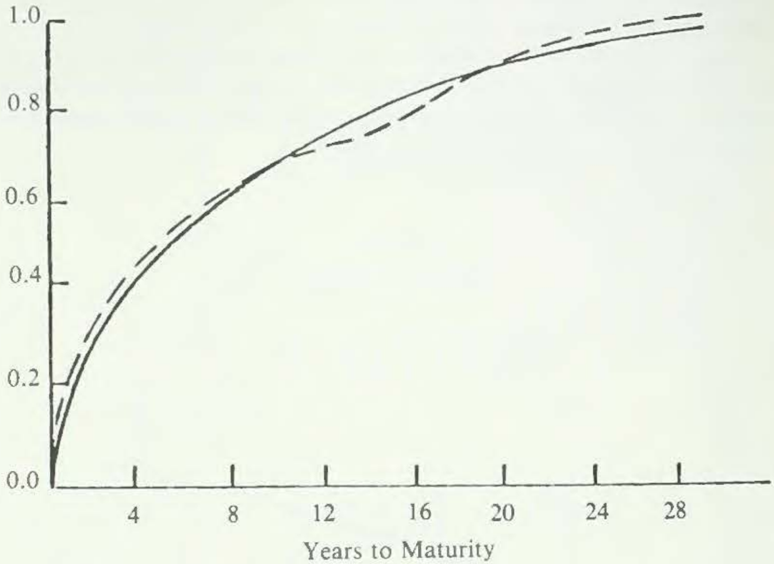
Figure 1
MAC (P) Averages



——YHM Average MAC(P) from 1953-72
-------Updated Average MAC(P) from 1954-72
-.-.-.-Updated Average MAC(P) from 1973-78

Figure 2

RRI Averages



____YHM and Average RRI from 1954-72
(both curves are essentially the same)
------Update RRI from 1973-78

Table 4

Portfolio Optimization Test at 0.4 RRI

Portfolio Yield is 8.140 PCT
Portfolio Risk is 0.400 RRI

		PCT.IN PORTFOLIO			BOND YIELDS		
MATURITY	RRI	MIN	MAX	OPT	ACT	BEY	DIFFER
01 Year	.192	2.00	25.00	25.00	7.90	7.89	0.01
02 Year	.341	2.00	25.00	25.00	8.08	8.04	0.04
03 Year	.397	2.00	25.00	17.84	8.14	8.14	0.00
04 Year	.485	2.00	25.00	2.00	8.20	8.25	-0.05
05 Year	.518	2.00	25.00	2.00	8.23	8.29	-0.06
10 Year	.595	2.00	25.00	24.16	8.38	8.38	0.00
20 Year	.937	2.00	25.00	2.00	8.47	8.79	-0.32
30 Year	1.000	2.00	25.00	2.00	8.49	8.87	-0.38

Table 5
Portfolio Optimization Test at 0.5 RRI

Portfolio Yield is 8.234 PCT
Portfolio Risk is 0.500 RRI

		PCT. IN PORTFOLIO			BOND YIELDS		
Maturity	RRI	MIN	MAX	OPT	ACT	BEY	DIFFER
01 Year	.192	2.00	25.00	2.00	7.90	8.00	-0.10
02 Year	.314	2.00	25.00	7.26	8.08	8.08	0.00
03 Year	.397	2.00	25.00	25.00	8.14	8.14	0.00
04 Year	.485	2.00	25.00	11.74	8.20	8.20	0.00
05 Year	.518	2.00	25.00	25.00	8.23	8.22	0.01
10 Year	.595	2.00	25.00	25.00	8.38	8.28	0.10
20 Year	.937	2.00	25.00	2.00	8.47	8.52	-0.05
30 Year	1.000	2.00	25.00	2.00	8.49	8.56	-0.07

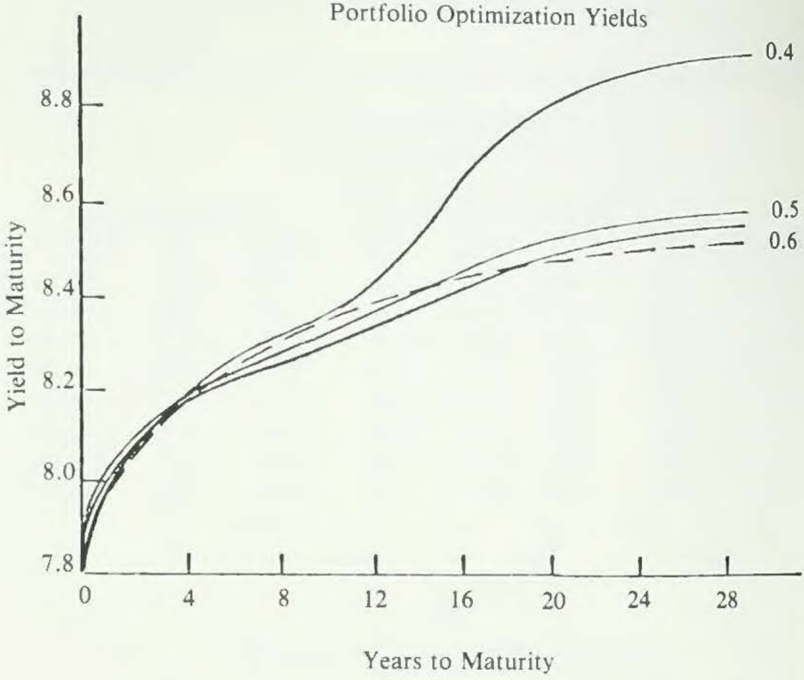
Table 6
Portfolio Optimization Test at 0.6 RRI

Portfolio Yield is 8.296 PCT
Portfolio Risk is 0.600 RRI

		PCT. IN PORTFOLIO			BOND YIELDS		
Maturity	RRI	MIN	MAX	OPT	ACT	BEY	DIFFER
01 Year	.192	2.00	25.00	2.00	7.90	8.02	-0.12
02 Year	.314	2.00	25.00	2.00	8.08	8.10	-0.02
03 Year	.397	2.00	25.00	2.00	8.14	8.15	-0.01
04 Year	.485	2.00	25.00	24.36	8.20	8.20	0.00
05 Year	.518	2.00	25.00	25.00	8.23	8.22	0.01
10 Year	.595	2.00	25.00	25.00	8.38	8.27	0.11
20 Year	.937	2.00	25.00	17.64	8.47	8.48	-0.01
30 Year	1.000	2.00	25.00	2.00	8.49	8.51	-0.02

Figure 3

Portfolio Optimization Yields



____Break-even Yields

------Actual Security Yields

SUMMARY CONCLUSION, AND LIMITATIONS

The primary objective of this research was to develop the framework for a linear programming model which could be used by treasury security dealers. No evidence was found to indicate that any sophisticated model was being used by the dealers. The basic foundation of this study came from an article by Yawitz, Hempel and Marshall (YHM) on "A Risk Return Approach to the Selection of Optimal Government Bond Portfolios," [11]. We replicated as well as extended the YHM study through the year 1978, developed the LP model and the break-even yield (BEY) analysis, and tested the model for an arbitrarily chosen time period. The results showed that the model could indeed be useful in identifying overpriced/underpriced securities. This should be useful to dealers, brokers and other investors. By using the break-even analysis presented here, dealers could restructure their portfolios and maximize yields under constraints such as liquidity, price fluctuations and inventory requirements.

The LP model provided the following results after testing (for June 1978):

1) The long-term maturity (20, 30 years) portfolios were overpriced at all levels of risk tested, and they optimized at their lowest constraint level of 2 percent. The results indicate that unless their prices drop considerably, they should not be considered for purchase.
2) For intermediate maturity levels (5, 10 years), portfolios were underpriced at risk levels of 0.5 and 0.6 (Relative Risk Index). This indicates that the opportunity to increase profit exists at this maturity level, if the expected risk levels materialize.
3) Short-term (1 year) securities were overpriced at risk levels of 0.5 and 0.6. Dealers might consider lowering their inventory of 1 year securities, if they expect risk levels of 0.5 or 0.6 to materialize.

There are several limitations to this study and they are listed below.

First, the portfolio model was tested using agregate maturities to represent Treasury securities in the portfolio. In actuality, many different securities would be traded, each having unique features that could alter the optimization result. However, the purpose of this study was to construct and test a basic model, not replicate an actual portfolio problem.

Second, a sixty month period was used as the time span for calculating the risk coefficients for price variability. Using longer or shorter time spans may change the risk factors and thus alter the optimization results. In this study, the sixty month period was intuitively selected to represent the approximate time span considered by many securities traders.

Third, this study was concerned only with U.S. Treasury securities. Treasury security traders may buy and sell other forms of debt when restructuring their portfolios. This model can be expanded to include other forms of deht without major modifications to the matrix structure.

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