American Academic Scientific Research Journal for Engineering, Technology, and Sciences

ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Internal Newtonian Flow Due to a Cylinder Undergoing Longitudinal and Torsional Oscillations of Different Frequencies

Robin Ramlal[a*], Karim Rahaman[b]

[a]*Senior Lecturer in Mathematics, College of Science, Technology and Applied Arts of Trinidad and Tobago - COSTAATT*

[b]*Senior Lecturer in Mathematics, Department of Mathematics and Statistics, The University of the West Indies, Trinidad and Tobago*

[a]*Email: robinramlal@outlook.com*

[b]*Email: karim.rahaman@sta.uwi.edu*

Abstract

The unsteady flow of an incompressible viscous fluid contained in a cylinder of infinite length, subject to longitudinal and torsional oscillations of different frequencies is examined. Analytical expressions for the velocity field, shear stresses, drag on the cylinder, work done and the drag coefficients are obtained.

Keywords: Viscous; unsteady flow; Longitudinal; Torsional; Oscillation; Different Frequencies.

1. Introduction

As early as 1886, an exact solution for the velocity field due to an infinite rod rotating in a Newtonian fluid was determined by Stokes [1]. Later, Casarella and Laura [2] determined analytical expressions for the velocity components of a Newtonian fluid and the viscous drag forces acting on a cylindrical rod-like cable which is undergoing both longitudinal and torsional oscillations. Ramkissoon and Majumdar [3] looked at the corresponding internal problem to that done in [2], and determined the corresponding results. A modification to [3], which considered independent amplitudes of the oscillations, was looked at by Phillips and Rahaman [4], here, expressions for the velocity field, shear stresses, drag forces, drag coefficient and work done by the drag forces were obtained. Due to much interests in the field of non-Newtonian fluids and the important applications of such, Calmelet-Eluhu and Majumdar [5] considered the problem for a micropolar fluid. Analytical expressions of the fluid velocity and micro-rotation were obtained, along with explicit expressions for the shear stresses and drag force acting at the wall of the cylinder. Rahaman [6] considered a similar situation using an upper-convected Maxwell fluid; the velocity field, shear stresses and drag were obtained and comparisons made with its Newtonian counterpart.

---

* Corresponding author.

Reference [7] considered the case for different frequencies of oscillations with an Oldroyd-B fluid, analytic solutions were obtained for the velocity components, shear stresses and drag on the cylinder. Different frequencies was also considered by [8] for a micropolar fluid, analytical expressions for the velocity and microrotation components were obtained in terms of modified Bessel's functions, along with the drag force acting on the wall of the cylinder. The main objective of this research is to investigate an extension to that done by [3], in particular, the longitudinal and torsional oscillations of the cylinder is considered to have different frequencies. Analytical expressions for the velocity field, shear stresses, drag on the cylinder, work done and the drag coefficient are obtained. The behaviour of the velocity components, the drag and the work done are illustrated graphically and conclusions made.

2. Statement of the Problem.

The unsteady flow of an incompressible viscous fluid contained in a cylinder which is infinite in length and radius ‘a’, is undergoing longitudinal and torsional oscillations with different frequencies. Due to the nature of the flow, cylindrical polar coordinates, (R, θ, z), will be used, with the axis of the cylinder coinciding with the z axis. Similar to that done in [2] and [3], the velocity of the cylinder, $\underline{q_b}$, at $R = a$ takes the form,

$$\underline{q_b} = q_0 \cos\beta \cos(\Omega_1 t)\,\hat{\theta} + \ q_0 \sin\beta \cos(\Omega_2 t)\hat{z} \tag{2.1}$$

where q_0, β, Ω_1 and Ω_2 are real constants.

It is noted that when $\beta = 0$ or π the oscillations are purely torsional and when $\beta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ they are purely longitudinal. Due to the motion of the cylinder, it is fair to assume that the radial component of the fluid’s velocity is zero. Further, it will be assumed that the flow is axisymmetric about the z-axis. Hence, the velocity of the fluid takes the form,

$$\underline{q} = \mathrm{v}(R,t)\hat{\theta} + w(R,t)\,\hat{R} \tag{2.2}$$

Since the fluid is incompressible, the continuity equation [9] is,

$$\nabla \cdot \underline{q} = 0 \tag{2.3}$$

which is satisfied by (2.2).

In the absence of external forces, the Navier-Stokes equation [9] to be solved is,

$$-\frac{1}{\rho}\nabla p + \nu\nabla^2 \underline{q} = \frac{\partial \underline{q}}{\partial t} + \left(\underline{q} \cdot \nabla\right)\underline{q} \tag{2.4}$$

where ρ is the density, p is the pressure and ν is the kinematic viscosity.

3. Velocity Components, Stresses and Drag

On substituting (2.2) into (2.4) gives the following linear system of equations,

$$-\frac{1}{\rho}\frac{\partial p}{\partial R} = -\frac{\mathrm{v}^2}{R} \tag{3.1}$$

$$\nu\left(\frac{1}{R}\frac{\partial \mathrm{v}}{\partial R} + \frac{\partial^2 \mathrm{v}}{\partial R^2} - \frac{\mathrm{v}}{R^2}\right) = \frac{\partial \mathrm{v}}{\partial t} \tag{3.2}$$

$$\nu\left(\frac{1}{R}\frac{\partial w}{\partial R} + \frac{\partial^2 w}{\partial R^2}\right) = \frac{\partial w}{\partial t} \tag{3.3}$$

Assuming that the $\hat{\theta}$ component of the velocity field takes the form,

$$\mathrm{v}(R,t) = \mathrm{Re}[f(R)e^{i\Omega_1 t}] \tag{3.4}$$

where Re refers to the real part, along with the fact that the velocity must remain finite as $R \to 0$, one gets, on solving (3.2) and using the no-slip condition with (2.1),

$$\mathrm{v}(R,t) = \mathrm{Re}\left[q_0 \cos\beta \frac{I_1(\sqrt{i}\sigma_1 R)}{I_1(\sqrt{i}\sigma_1 a)} e^{i\Omega_1 t}\right] \tag{3.5}$$

where $\sigma_1 = \sqrt{\frac{\Omega_1}{\nu}}$

In a similar manner, the $\hat{z}$ component of the velocity is,

$$w(R,t) = \mathrm{Re}\left[q_0 \sin\beta \frac{I_0(\sqrt{i}\sigma_2 R)}{I_0(\sqrt{i}\sigma_2 a)} e^{i\Omega_2 t}\right] \tag{3.6}$$

where $\sigma_2 = \sqrt{\frac{\Omega_2}{\nu}}$

The tangential stresses on the wall of the cylinder are given by [10],

$$\tau_{R\theta}|_{R=a} = \mu\left[\frac{\partial \mathrm{v}}{\partial R} - \frac{\mathrm{v}}{R}\right]_{R=a} \tag{3.7}$$

$$\tau_{Rz}|_{R=a} = \mu\left[\frac{\partial w}{\partial R}\right]_{R=a} \tag{3.8}$$

Substituting (3.5) into (3.7) gives for the torsional stress on the cylinder's wall,

$$\tau_{R\theta} = \mathrm{Re}\left[\mu\, q_0 \cos\beta \frac{\sqrt{i}\sigma_1\, I_0(\sqrt{i}\alpha_1) - \frac{2}{a} I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)} e^{i\Omega_1 t}\right] \tag{3.9}$$

where $\alpha_1 = \sigma_1 a$.

Also, substituting (3.6) into (3.8) gives for the longitudinal stress on the cylinder's wall,

$$\tau_{Rz} = \text{Re}\left[\mu\, q_0 \sin\beta \frac{\sqrt{i}\sigma_2\, I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)}\, e^{i\Omega_2 t}\right] \tag{3.10}$$

where $\alpha_2 = \sigma_2 a$.

The tangential drag per unit length acting on the cylinder is given by [3],

$$\underline{D} = -2\pi a\left(\tau_{R\theta}\,\hat{\theta} + \tau_{Rz}\,\hat{z}\right)\Big|_{R=a} \tag{3.11}$$

which on using (3.9) and (3.10) gives,

$$\underline{D} = -2\pi a\mu q_0 \text{Re}\left[\cos\beta \frac{\sqrt{i}\sigma_1\, I_0(\sqrt{i}\alpha_1) - \frac{2}{a} I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)} e^{i\Omega_1 t}\,\hat{\theta} + \sin\beta \frac{\sqrt{i}\sigma_2\, I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)} e^{i\Omega_2 t}\,\hat{z}\right] \tag{3.12}$$

It is noted that in the case of the same frequencies of oscillations, (3.5), (3.6), (3.9), (3.10) and (3.12) reduce to that obtained in [3].

4. Alternative Expressions for the Velocity and Stress Components

Using that given in [11], one gets in terms of the real-valued Kelvin functions $ber_\omega x$ and $bei_\omega x$,

$$e^{i\frac{3\omega\pi}{2}} I_\omega\left(\sqrt{i}\sigma R\right) = ber_\omega(-\sigma R) + i\, bei_\omega(-\sigma R) \tag{4.1}$$

From [11],

$$ber_n(-x) = (-)^n ber_n x, \quad bei_n(-x) = (-)^n bei_n x \tag{4.2}$$

which on using in (4.1), gives,

$$I_0\left(\sqrt{i}\sigma R\right) = ber_0(\sigma R) + i\, bei_0\,(\sigma R) \tag{4.3}$$

and

$$i\, I_1\left(\sqrt{i}\sigma R\right) = ber_1(\sigma R) + i\, bei_1(\sigma R) \tag{4.4}$$

If $M_\omega(x)$ and $\theta_\omega(x)$ are the modulus and argument respectively of $I_\omega(x)$ then [11],

$$M_\omega(x) = \sqrt{ber_\omega^2(x) + bei_\omega^2(x)} \tag{4.5}$$

$$\theta_{\omega}(x) = \tan^{-1}\left(\frac{bei_{\omega}x}{ber_{\omega}x}\right) \tag{4.6}$$

which gives,

$$I_0\left(\sqrt{i}\sigma R\right) = M_0(\sigma R)e^{i\theta_0(\sigma R)} \tag{4.7}$$

and

$$I_1\left(\sqrt{i}\sigma R\right) = -i\, M_1(\sigma R)e^{i\theta_1(\sigma R)} \tag{4.8}$$

resulting in,

$$\mathrm{v}(R,t) = \mathrm{Re}\left[\frac{M_1(\sigma_1 R)}{M_1(\alpha_1)}e^{i[\theta_1(\sigma_1 R)-\theta_1(\alpha_1)]}q_0\cos\beta\, e^{i\Omega_1 t}\right] \tag{4.9}$$

and

$$w(R,t) = \mathrm{Re}\left[\frac{M_0(\sigma_2 R)}{M_0(\alpha_2)}e^{i[\theta_0(\sigma_2 R)-\theta_0(\alpha_2)]}q_0\sin\beta\, e^{i\Omega_2 t}\right] \tag{4.10}$$

It is observed again that for the same frequencies of oscillations, (4.9) and (4.10) reduce to that obtained in [3].

Similarly, from (3.9),

$$\tau_{R\theta}|_{R=a} = \frac{\mu\alpha_1}{a}q_0\cos\beta\frac{M_0(\alpha_1)}{M_1(\alpha_1)}L\cos(\Omega_1 t+\delta) \tag{4.11}$$

where

$$L^2 = \left(\cos\eta - \frac{2}{\alpha_1}\frac{M_1(\alpha_1)}{M_0(\alpha_1)}\right)^2 + \sin^2\eta \tag{4.12}$$

$$\tan\delta = \frac{\sin\eta}{\cos\eta - \frac{2}{\alpha_1}\frac{M_1(\alpha_1)}{M_0(\alpha_1)}} \tag{4.13}$$

$$\eta = \theta_0(\alpha_1) - \theta_1(\alpha_1) + \frac{3\pi}{4} \tag{4.14}$$

Also, from (3.10),

$$\tau_{Rz}|_{R=a} = \frac{\mu\alpha_2}{a}\, q_0\sin\beta\frac{M_1(\alpha_2)}{M_0(\alpha_2)}\cos(\Omega_2 t+\xi) \tag{4.15}$$

where

$$\xi = \theta_1(\alpha_2) - \theta_0(\alpha_2) - \frac{\pi}{4} \tag{4.16}$$

5. Work Done and the Drag Coefficient

(3.11) can be written as,

$$\underline{D} = -2\pi a(T\cos\phi\,\hat{\theta} + T\sin\phi\,\hat{z}) \tag{5.1}$$

where

$$\tau_{R\theta}|_{R=a} = T\cos\phi \tag{5.2}$$

and

$$\tau_{Rz}|_{R=a} = T\sin\phi \tag{5.3}$$

Using (3.9) and (3.10), it follows that,

$$T^2 = \left(\mathrm{Re}\left[\frac{\sqrt{i}\sigma_1 I_0(\sqrt{i}\alpha_1) - \frac{2}{a}I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)}\mu q_0\cos\beta\, e^{i\Omega_1 t}\right]\right)^2 + \left(\mathrm{Re}\left[\frac{\sqrt{i}\sigma_2 I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)}\mu q_0\sin\beta\, e^{i\Omega_2 t}\right]\right)^2 \tag{5.4}$$

and, with the use of (5.2) and (5.3),

$$\tan\phi = \mathrm{Re}\left[\frac{\sqrt{i}\sigma_2 I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)}\mu q_0\sin\beta\, e^{i\Omega_2 t}\right] \Big/ \mathrm{Re}\left[\frac{\sqrt{i}\sigma_1 I_0(\sqrt{i}\alpha_1) - \frac{2}{a}I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)}\mu q_0\cos\beta\, e^{i\Omega_1 t}\right] \tag{5.5}$$

The work done on the fluid per half-cycle of motion is [3],

$$W_j = -\int_0^{\frac{\pi}{\Omega_j}} \underline{D}\cdot\underline{q}(a,t)\mathrm{d}t \tag{5.6}$$

where $j = 1, 2$ refers to the torsional and longitudinal motions respectively.

With the use of (2.1) and (3.12), (5.6) gives the work done, W_j, by the drag force, $\underline{D}$, per half cycle of torsional and longitudinal motion as,

$$W_j = -\pi a\mu q_0^2\left[Re\left\{\frac{\sqrt{i}\sigma_1 I_0(\sqrt{i}\alpha_1) - \frac{2}{a}I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)}\cos(\Omega_1 t)\cos^2\beta\,\hat{I}(\Omega_j,\Omega_1)\right\} + Re\left\{\frac{\sqrt{i}\sigma_2 I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)}\cos(\Omega_2 t)\sin^2\beta\,\hat{I}(\Omega_j,\Omega_2)\right\}\right] \tag{5.7}$$

where

$$\hat{I}(\Omega_j,\Omega) = 2\int_0^{\frac{\pi}{\Omega_j}} e^{i\Omega t}\cos(\Omega t)\,dt = \frac{\Omega_j\cos\left(\frac{\pi\Omega}{\Omega_j}\right)\sin\left(\frac{\pi\Omega}{\Omega_j}\right) + \pi\Omega - i\Omega_j\cos^2\left(\frac{\pi\Omega}{\Omega_j}\right) + i\Omega_j}{\Omega\Omega_j}$$

(5.8)

Substituting (2.1) and (5.1) into (5.6) gives an alternative expression for the work done,

$$W_j = 2\pi a q_0 \int_0^{\frac{\pi}{\Omega_j}} T[\cos\phi\cos\beta\cos(\Omega_1 t) + \sin\phi\sin\beta\cos(\Omega_2 t)]\,dt$$

(5.9)

It is noted that for the same frequencies of oscillations, this reduces to that obtained in [3].

The drag coefficient, C, can be obtained by equating the work done on the fluid by a hypothesized drag force, which is given by [3],

$$\underline{D}_H = -Cq_b^n(\cos\beta\,\hat{\theta} + \sin\beta\,\hat{z})$$

Using this in (5.6) and equating it to (5.7) gives on solving,

$$C =$$

$$-\pi a\mu q_0^2\left[Re\left\{\frac{\sqrt{i}\sigma_1\, I_0(\sqrt{i}\alpha_1) - \frac{2}{a}\, I_1(\sqrt{i}\alpha_1)}{I_1(\sqrt{i}\alpha_1)}\cos(\Omega_1 t)\cos^2\beta\,\hat{I}(\Omega_j,\Omega_1)\right\} + Re\left\{\frac{\sqrt{i}\sigma_2 I_1(\sqrt{i}\alpha_2)}{I_0(\sqrt{i}\alpha_2)}\cos(\Omega_2 t)\sin^2\beta\,\hat{I}(\Omega_j,\Omega_2)\right\}\right] \Big/$$

$$\left[\int_0^{\frac{\pi}{\Omega_j}} [q_0^{n+1}\cos^{n+2}\beta\cos^{n+1}(\Omega_1 t) + q_0\cos^2\beta\cos^n(\Omega_2 t)\sin^n\beta\cos(\Omega_1 t)\right.$$

$$\left. + q_0\sin^{n+2}\beta\cos^{n+1}(\Omega_2 t) + q_0^{n+1}\cos^n\beta\cos^n(\Omega_1 t)\sin^2\beta\cos(\Omega_2 t)]\,dt\right]$$

(5.10)

An alternate expression for the drag coefficient is obtained by substituting (5.9) into (5.6) which gives,

$$C = 2\pi a q_0 \int_0^{\frac{\pi}{\Omega_j}} T[\cos\phi \cos\beta \cos(\Omega_1 t) + \sin\phi \sin\beta \cos(\Omega_2 t)]\, dt \Bigg/$$

$$\left[\int_0^{\frac{\pi}{\Omega_j}} [q_0^{n+1} \cos^{n+2}\beta \cos^{n+1}(\Omega_1 t) + q_0 \cos^2\beta \cos^n(\Omega_2 t) \sin^n\beta \cos(\Omega_1 t)\right.$$

$$\left. + q_0 \sin^{n+2}\beta \cos^{n+1}(\Omega_2 t) + q_0^{n+1} \cos^n\beta \cos^n(\Omega_1 t) \sin^2\beta \cos(\Omega_2 t)]\, dt\right]$$

(5.11)

In the case when that frequencies of oscillations are the same, one gets that obtained in [3].

6. Graphical Results

For the following graphs, the effects of having independent oscillating frequencies is examined. Due to some practical oceanographic problems, [2] gives, $1 \leq \frac{\Omega_j}{2\pi} \leq 10$ Hz and $9.30 \times 10^{-7} \leq \nu \leq 1.86 \times 10^{-7}$ m^{2}s^{-1} Hence, based on this, the values Ω_j and ν have been chosen. It should be noted that various values of α_i correspond to different frequencies.

Figures 1 and 2 illustrate the effects of different frequencies of oscillations in the torsional and longitudinal directions of the velocity components respectively. The torsional frequency is taken to be the same as that in [3], which is different to the higher frequency longitudinal oscillation in figure 2. It is observed, when also comparing with figure 4 which has the same longitudinal frequency as in [3], that the magnitude of the higher frequency longitudinal component of the velocity field is smaller closer to the centre of the cylinder. As a result, the overall velocity field in this case would have a greater contribution from the torsional component. The general characteristic shapes of the curves are however similar. Figures 3 and 4 also show the effects of different frequencies of oscillations in the torsional and longitudinal directions of the velocity components respectively.

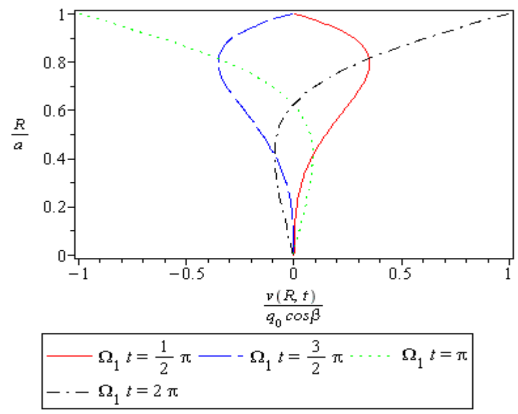

Figure 1: $\frac{R}{a}$ versus $\frac{v(R,t)}{q_0 \cos\beta}, \alpha_1 = 6$

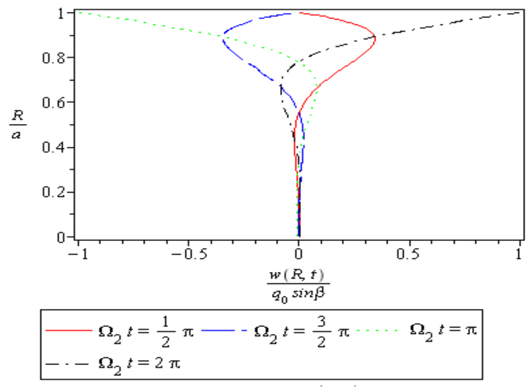

Figure 2: $\frac{R}{a}$ versus $\frac{w(R,t)}{q_0 \sin\beta}, \alpha_2 = 10$

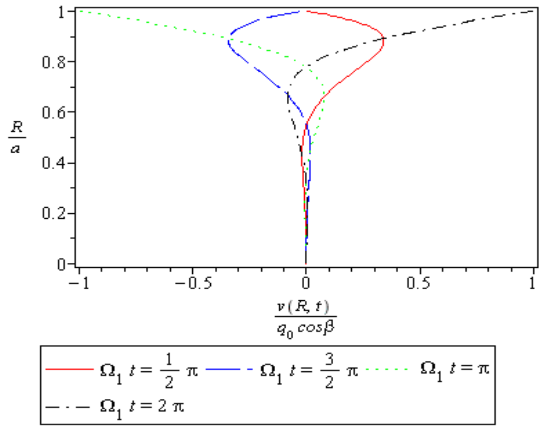

Figure 3: $\frac{R}{a}$ versus $\frac{v(R,t)}{q_0 \cos\beta}, \alpha_1 = 10$

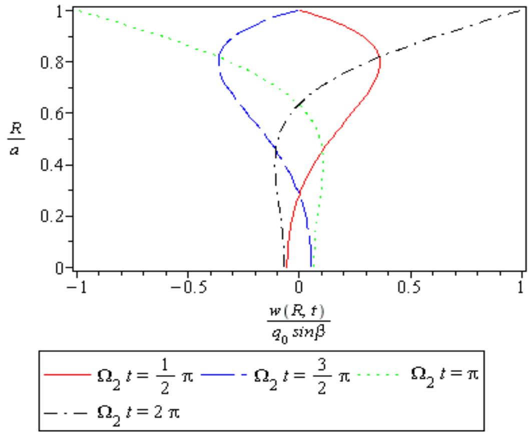

Figure 4: $\frac{R}{a}$ versus $\frac{w(R,t)}{q_0 \sin\beta}, \alpha_2 = 6$

The longitudinal frequency is now taken to be the same as that in [3], which is different to the torsionafrequency in figure 3. It is observed, when also comparing with figure 1, which has the same torsional frequency as in [3], that the magnitude of the higher frequency torsional component of the velocity field is smaller closer to the

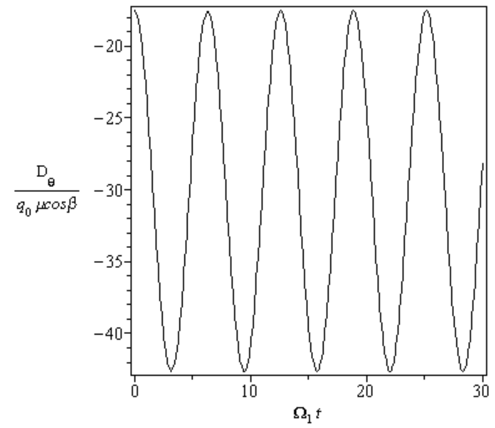

Figure 5: $\frac{D_\theta}{q_0\,\mu\cos\beta}$ versus $\Omega_1 t, \alpha_1 = 6$

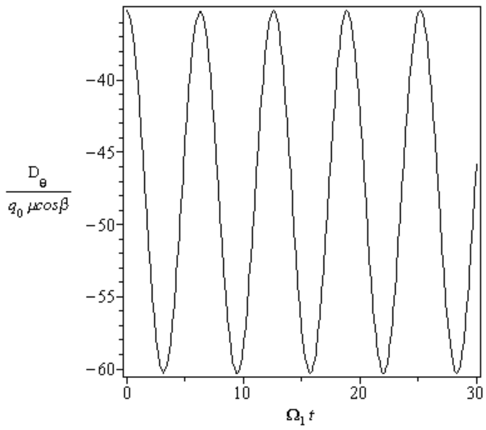

Figure 6: $\frac{D_\theta}{q_0\,\mu\cos\beta}$ versus $\Omega_1 t, \alpha_1 = 10$

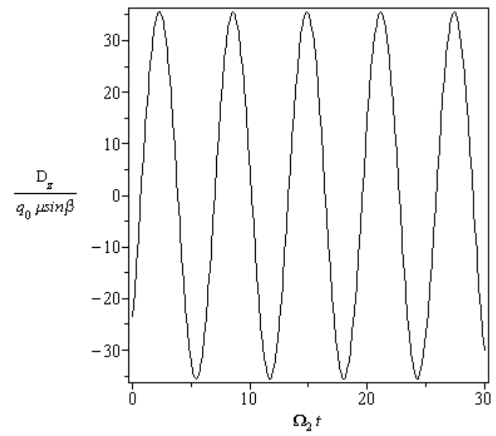

Figure 7: $\frac{D_z}{q_0\,\mu\sin\beta}$ versus $\Omega_2 t,\ \alpha_2 = 6$

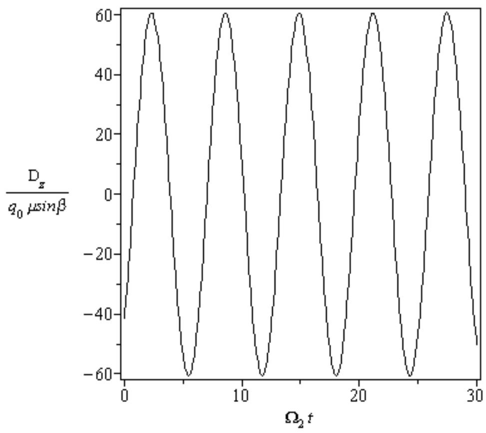

Figure 8: $\frac{D_z}{q_0\,\mu\sin\beta}$ versus $\Omega_2 t,\ \alpha_2 = 10$

centre of the cylinder. As a result, the overall velocity field in this case would have a greater contribution from longitudinal component. The general characteristic shapes of the curves are again similar. As a result, it seems that in each case of different frequencies, that with the higher one seems to suppress the magnitude of that component of the velocity field closer to the centre of the oscillating cylinder.

In figures 5 and 6, the $\hat{\theta}$ component of the drag is depicted graphically for different values of α_1. Similarly, figures 7 and 8 display the $\hat{z}$ component of the drag for different values of α_2. The magnitude of the drag in the $\hat{\theta}$ direction oscillates between negative values, while the magnitude in the $\hat{z}$ direction oscillates between positive and negative values. It is noted that for each component, when the oscillating frequencies increase, such results in an increase in the respective magnitudes.

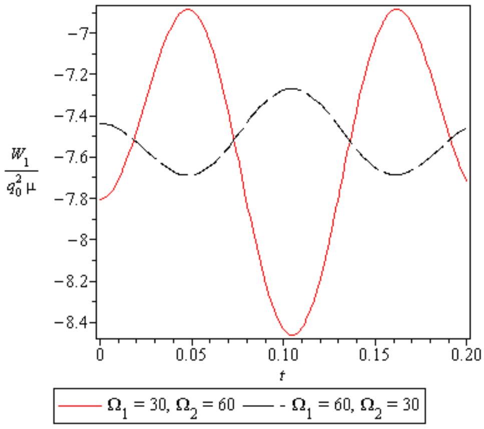

Figure 9: $\frac{W_1}{q_0^2\mu}$ versus t

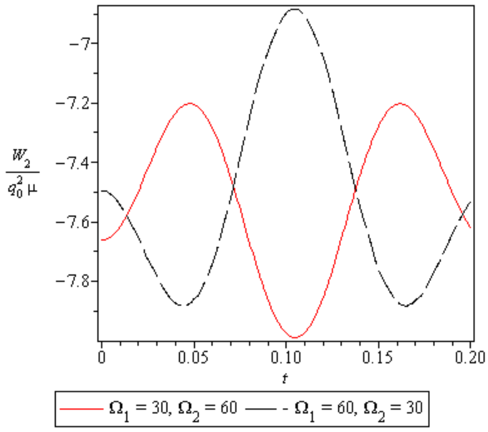

Figure 10: $\frac{W_2}{q_0^2\mu}$ versus t

In Figure 9, the work done in the $\hat{\theta}$ direction is depicted graphically, where two scenarios are considered; a plot when $\Omega_1 = 30$ and $\Omega_2 = 60$, and another when $\Omega_1 = 60$ and $\Omega_2 = 30$. Both graphs are periodic and it is observed that the magnitudes are negative, with the amplitude for $\Omega_1 = 30$ and $\Omega_2 = 60$ being greater than that for $\Omega_1 = 60$ and $\Omega_2 = 30$, which indicates more work is being done. In figure 10, the work done in the $\hat{z}$ direction is examined for two cases, in particular, when $\Omega_1 = 30$ and $\Omega_2 = 60$, and when $\Omega_1 = 60$ and $\Omega_2 = 30$. These plots appear to be periodic, each with a negative magnitude. The amplitude of the work done in this $\hat{z}$ direction when $\Omega_1 = 60$ and $\Omega_2 = 30$ is observed to be larger than when $\Omega_1 = 30$ and $\Omega_2 = 60$, which shows that more work is being done.

6. Conclusion

The velocity of the fluid is affected if the frequencies of the cylinder's oscillations are different, as was observed when compared to when the frequencies were the same. In particular, it was noted that the higher oscillating frequency tended to suppress the magnitude of the corresponding component of the velocity field closer to the centre of the cylinder. The magnitude of the drag increased with an increase in the oscillating frequency. The work done in the $\hat{\theta}$ direction is more when the torsional frequency is less than that of the longitudinal one. In the $\hat{z}$ direction, the work done is less when the longitudinal frequency is more than that of the torsional oscillation.

References

[1]. G. G. Stokes. "On the effect of rotation of cylinders and spheres about their own axes in increasing the logarithmic decrement of the arc of vibration". *Mathematical and Philosophical Papers 5 Cambridge: Cambridge University Press, England*, pp. 207-214, 1886.

[2]. M.J. Casarella, P.A. Laura. "Drag on an Oscillating Rod with Longitudinal and Torsional Motion". *Journal of Hydronautics*, v. 3, n. 4, pp. 180-183, 1969.

[3]. H. Ramkissoon, S.R. Majumdar. "Flow due to the Longitudinal and Torsional Oscillation of a

Cylinder". *Journal of Applied Mathematics and Physics*, v. 41, pp. 598-603, 1990.

[4]. W. Phillips, K. Rahaman. "Motion of a Viscous Fluid contained in a Cylinder of Infinite Length subjected to Longitudinal and Torsional Oscillations of Independent Amplitudes". *American International Journal of Contemporary Research*, v. 2, n. 12, pp. 58-71, 2012.

[5]. C. Calmelet-Eluhu, D. Majumdar. "Flow of a micropolar fluid through a circular cylinder subject to longitudinal and torsional oscillations". *Mathematical and Computer Modelling*, v. 27, n. 8, pp. 69-78, 1998.

[6]. K. Rahaman. "Internal Flow due to the Longitudinal and Torsional Oscillation of a Cylinder". *Asia Journal of Information Technology*, v. 3, n. 10, pp. 985-991, 2004.

[7]. D. Owen, K. Rahaman. "On the flow of an Oldroyd-B liquid through a straight tube performing longitudinal and torsional oscillations of different frequencies". *Math: Ensenanza Univ*., v. 14, pp. 34-43, 2006.

[8]. J. V. R. Murthy, N. K. Bahali. "Micropolar Fluid flow in a Straight Circular Cylinder performing Longitudinal and Torsional Oscillations". *Computational Thermal Sciences: An International Journal*, v. 3, n. 2, pp. 123-131, 2011.

[9]. G.K. Batchelor. *An Introduction to Fluid Dynamics.* Cambridge University Press, pp. 75, 1967.

[10]. W. Hughes. *An Introduction to Viscous Flows*. New York. Hemisphere Publishing Corporation, 1979.

[11]. M. Abramowitz, I.A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables.* U.S. Government Printing Office, pp. 379-382, 1972.