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Black Holes of Research on Instructional Practice: The Case of Number Talks

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Introduction

When, where, and how do research and instructional practice intersect? As researchers and teachers of mathematics, we often find ourselves engaged in conversations about this question with our colleagues. As we critically examine the question, we realize that the answer is dependent on other questions, such as, "What counts as research, and how do we determine which instructional practices, with all their nuances, have been studied and need to be studied?" Over the past several years, we have observed the use of Number Talks proliferate among teachers, schools, districts, within professional development communities, and on social media. Number Talks have been described as a "short, mathematical, whole-class discussions during which students solve problems and share their ways of seeing and reasoning about mathematics" (Gerstenschlager & Strayer, 2019, p. 363). We employ the practice in our own teaching and fervently think about its efficacy. When asked by a local district to share the strongest published research directly noting the outcomes of Number Talks with K-12 students, we agreed to produce a list of articles for them. However, as we searched for blind-peer-reviewed research about the efficacy of Number Talks with K-12 students, we encountered real difficulty. We initially sought information from books, research and practitioner journals, and electronic resources hoping to find a set of research studies that directly researched this instructional practice. Instead, we came up empty, only finding anecdotal evidence from practitioner articles and non-blinded forms of research such as master's and PhD theses. The difficulty of finding blind-peer-reviewed research on the instructional practice of Number Talks to share with the district was perplexing and became the impetus for the current study. In what follows, we share our broad and deep search for blind-peer-reviewed research literature about the efficacy of Number Talks with K-12 students in order to answer the question: *What does rigorous research* *reveal about the efficacy of the Number Talks instructional practice?* For the purposes of this search, rigorous research was defined as studies that went through a blind-peer-review process and were published in an education journal. We share our literature search process, the analysis of articles, and the thematic finding that we relate metaphorically to the astronomical phenomenon of a black hole. A Black Hole is an instructional practice for which there is a scarcity of blind-peer-reviewed research evidence supporting its efficacy, yet has attained critical gravity in the teaching field. We elaborate on the meaning of a *Black Hole* of research on instructional practice and its implications for the field of mathematics education.

Creating Professional Dialogue within the Field

Through this paper, we seek to open critical dialogue about the intersection of research and instructional practice in mathematics education. We do this to challenge ourselves and other researchers to carefully and directly consider the efficacy of widely used instructional practices, such as Number Talks. Number Talks act as a current exemplar for what we define as Black Holes of instructional practice within research. We will utilize the case of Number Talks to lay the grounding and definition of Black Holes in research. In doing so, it is our hope to create dialogue about instructional practices that appear more as Black Holes than as visible stars in a constellation of understanding between the research field and the teaching field. We believe such dialogue is critical to drawing instructional practice and research closer together to provide well-constructed knowledge about these teaching practices, the professional knowledge and abilities needed to enact them, and the effects of their nuanced variations on K-12 student learning.

The Case of Number Talks

A Number Talk is a mathematical instructional practice that has risen to prominence during the last decade (Flick & Kuchey, 2015) and is used by many teachers (Gerstenschlager &

Strayer, 2019, p. 363). A Number Talk is often described as a “five- to fifteen-minute classroom conversation around purposefully crafted problems that are solved mentally” (Parrish & Dominick, 2016, p. 13). Though the instructional practice varies across elements such as the problem type given to students and the uniqueness of the teacher, in general, it begins with the students being given a mental mathematical computation problem to solve, such as 32 x 5 (see Figure 1). Students are given time to think through possible solutions. They use hand signals to quietly and discretely indicate the number of solutions, or solution strategies, they have derived. This is followed by a time in which the teacher requests all possible answers and writes them visibly for all to see. Next, the teacher asks students to share their ideas and listen to the solution strategies of others. During this time, the teacher and students work to “defend an answer” (Sun et al., 2018, p. 49) and justify their reasoning about the viability of the proposed solution strategy. Lastly, the teacher draws attention to connections that promote the mathematical goal. Those advocating for Number Talks see it as a way to improve two very important elements in the mathematics learning of students: number sense and procedural fluency. Postlewait, Adams, and Shih (2003) state that, “The development of number sense and computational fluency should be an integral part of the mathematics curriculum” (p. 354). In order to foster computational fluency, teachers must work to create learning environments and techniques that help students “make sense of and organize number relations” (Matney, 2014, p. 27). For some teachers, the purpose of incorporating Number Talks into the mathematics classroom is to improve students’ computational fluency (Flick & Kutchey, 2015). When one considers the instructional practice of Number Talks in the light of computational fluency, it is easy to see why teachers are intuitively drawn to Number Talks. Number Talks provide a space for students to demonstrate flexibility in computational methods, finding multiple solution methods when possible. Teachers enacting

Number Talks maintain whole class discussion for the sharing and explanation of student-derived solutions and the investigation of procedural efficiency. As teachers sought instructional techniques to develop their students' number sense, Number Talks became a sensible option due to their flexibility to be crafted for any age and their support of discourse, mental mathematics, and calculations (Flick & Kuchey, 2015).

These aspects make Number Talks a sensible teaching technique in mathematics classrooms, and it stands to reason why so many teachers have been drawn to it. Advocates of the instructional practice have described its benefits related to sense making (Sun et al., 2018) and computational fluency. One example of this was Jo Boaler's indication that students' number sense improves from the experience of mental computation through Number Talks (Humphreys & Parker, 2015, p. viii). Parrish's (2011) book that indicates how Number Talks help students develop "…more accurate, efficient, and flexible strategies" (p. 199), all of which are aspects of computational fluency. We note that these seem to be reasonable statements. On the other hand, we wonder what rigorous evidence exists demonstrating that Number Talks generate these benefits, and if so, for which students and to what degree are the benefits present.

Five Essential Components of Number Talks

The proponents of Number Talks have carefully considered the elements of the routine and delineated the typical sequence (Humphreys & Parker, 2015). There are five essential components: classroom environment and community, classroom discourse, the teacher's role in questioning and supporting student reasoning, role of mental math, and purposeful computation problems (Parrish, 2011). In what follows, we concisely state the components of a typical Number Talk. Firstly, a Number Talk should occur in a classroom with a safe learning environment. Within such an environment, teachers and students practice appropriate sociomathematical norms (Yackel & Cobb, 1996) to ensure students feel comfortable sharing their mathematical strategies without fear of ridicule or shame (Flick & Kuchey, 2015). A safe learning environment affords teachers the opportunity to discover misconceptions in students' thinking because students are comfortable sharing incorrect solutions. Secondly, during a Number Talk, students indicate their thinking through the use of hand signals. As an example, while holding a fist to their chest, students can indicate that they are thinking by gradually extending fingers from their fist to reveal to the teacher how many strategies they have used to solve the given computation (Humphreys & Parker, 2015; Parrish, 2011). Following this time of thinking, students engage in classroom discussions about number relationships, the meaning of operations, which solutions make sense, and the efficiency of the proposed solution strategies. Thirdly, the teacher's role throughout the Number Talk is to act as a facilitator that reinforces the socio-mathematical norms and poses purposeful, strategic questions. Teachers should facilitate mathematical discussions through "graduated pressing" (Humphreys & Parker, 2015, p. 18). An example of this could be a teacher asking students "How did you solve this problem?" rather than "What answer did you get?" (Parrish, 2011, p. 204). Fourthly, during a Number Talk, students are encouraged to use mental mathematics to arrive at a solution. The mental mathematics encouraged during Number Talks is meant to develop understanding of number relationships rather than memorizing procedures. Lastly, purposeful computation problems are an essential component of Number Talks. Computational problems are strategically chosen by the teacher to achieve mathematical goals and to draw mathematical connections between concepts or procedures. The five components are considered important to the successful implementation of Number Talks as an instructional practice. In our quest to find rigorous research about the efficacy of the Number Talks instructional practice, we sought blind-peer-reviewed studies in accordance with the above definition and five components.

Methodology

Search Procedure

The goal of this literature review was to find rigorous research on the instructional practice of Number Talks with K-12 students. In order to find viable studies, we searched 15 education and teaching databases: Education Full Text (H.W. Wilson), Education Research Complete, Education Resources Information Center (ERIC), Chronicle of Higher Education, EBSCOhost Curriculum Standards, Educational & Instructional Videos (OhioLINK), Explora Primary Schools, Explora Secondary Schools, Higher Education Abstracts, Mental Measurements Yearbook with Tests in Print, National Center for Education Statistics (NCES), Professional Development Collection, TeachingBooks.net, Tests and Measures in the Social Sciences: Tests Available in Compilation Volumes, and TOPICsearch. Additionally, we included searches of JSTOR and Dissertation Abstracts International for a total of 17 databases. The literature reviews of relevant dissertations were also analyzed for viable references of blind-peer-reviewed research articles in the K-12 setting. The 17 databases were searched using the descriptors found in the practitioner books about Number Talks (Humphreys & Parker, 2015; Parrish, 2011; Parrish & Dominick, 2016). The descriptors used were *number talk*, *math talk*, and *number string*. These descriptors were chosen for their common use in the practitioner literature, hence lending to their viability of connecting research about Number Talks to those most interested in the efficacy of the instructional practice. Our search methods involved the use of a Boolean AND to find articles that contain both words within the manuscript. Furthermore, we conducted a manual search of influential mathematics education journals using these same descriptors. The journals searched manually included *Educational Studies in Mathematics, Investigations in Mathematics Learning, Mathematics Teacher Educator*, *Journal for Research in Mathematics Education, Journal of Mathematics Teacher Educatio*n, and *School Science and Mathematics*. We recognized that some practitioner pieces were written with research in mind. Ninety-one practitioner articles were examined for references to research publications. We cross-referenced these with our search results to form a coherent set of possible research about the efficacy of Number Talks. All searches were done at the university library of Bowling Green State University. All articles found through the searches were electronically catalogued.

Inclusion and Exclusion Criteria

The following inclusion conditions were used to ascertain the viability of an article for review: (1) empirical studies published between 2000 and August 2019, (2) published in English, (3) available in full text, (4) published in a blind-peer-reviewed journal, (5) research about students in a K-12 setting. The two-decade span of the search represents the period following the initial creation of Number Talks (Humphreys & Parker, 2015, p. vii) throughout its rise in use in K-12 classrooms. Next, we considered criteria through which research about the efficacy of Number Talks as an instructional practice would occur. These criteria were developed in connection with the practitioner literature to ensure the research was about the efficacy of the instructional practice. Articles were considered for further analysis if they met at least one of the following criteria because research projects may exhibit different foci and still reveal something of relevance about the efficacy of Number Talks. Articles were included if they:

1) Investigated an instructional practice meeting the definition of Number Talks. For the purposes of this search, we adapted Parrish and Dominick’s (2016, p.13) definition of number talk to - a five- to fifteen-minute classroom conversation about number

relationships and calculations stemming from purposefully-crafted mathematics tasks that are solved mentally.

2) Pertained to research of instruction using at least one of the five essential components of a Number Talk: classroom environment and community, classroom discussions, teacher's role in promoting number sense and/or conversation, role of mental mathematics, and purposeful mental computation problems (Humphreys & Parker, 2015; Parrish, 2011).
3) Directly and exclusively examined the relationship or outcome of the instructional practice of Number Talks with another research construct (e.g. classroom discourse).

Articles meeting both the five conditions and at least one of the three criteria for efficacy research were considered viable studies for inclusion in our full-read thematic analysis (Hatch, 2002) on the instructional practice of Number Talks.

Analysis of the Number Talks Research Articles

Following the cataloguing of articles found through the database search, the abstract of each article was read to determine if it met the five conditions and three categories of the inclusion and exclusion criteria. When there was evidence in the abstract that any part of the criteria was met, the article was categorized as "relevant", and it received a full read as part of our thematic analysis (Hatch, 2002). When an article did not meet the five conditions, or there was no evidence related to the criteria for Number Talks research, the article was categorized as "not relevant." Articles that were categorized as relevant were divided among the authors and read twice, with the first author reading all of the articles and the second and third authors each reading approximately half of the articles. Authors examined the relevant articles for evidence of research of instructional practices that (1) adhered to the definition of Number Talks, (2) included the five essential components of Number Talks, and/or (3) directly connected Number Talks to another research construct. While reading the articles independently, the authors wrote paragraphs noting how each of the research articles satisfied, or did not satisfy, the criteria for research about the instructional practice of Number Talks.

Additionally, the authors coded which of the five essential components of Number Talks were found in the research manuscripts. The independent codes were then analyzed and revealed 98.5% exact agreement. The authors met and discussed the article in which there were discrepancies with two codes and reached full agreement about all codes. The agreement level here, thus, exceeds the minimum threshold of 90% needed to conduct reliability analyses (James, Demaree, & Wolf, 1993). Lastly, thematic analysis (Creswell & Guetterman, 2019; Hatch, 2002) was applied to the articles and paragraph write-ups to find patterns of relation. We used the five essential components of Number Talks to frame the analysis. Through this frame, we sought to understand the possible themes among the articles related to the research question: What does rigorous research reveal about the efficacy of the Number Talks instructional practice?

Findings

Altogether, our searches yielded 793 potential articles across all databases. These articles were examined for duplicates revealing 576 unique articles. Next, the abstracts were analyzed to determine which of the 576 articles were relevant to research about the instructional practice of Number Talks. Through this search procedure, we found 26 blind-peer-reviewed articles having potential to be research on the efficacy of the instructional practice of Number Talks. These 26 articles received a full read and were included in our thematic analysis. From the analysis of these 26 articles the Black Hole theme was developed. Close examination of these articles revealed that, although they had related research on components of Number Talks, such as classroom discourse or promoting number sense, only one of the articles could be classified as research on the efficacy of Number Talks in K-12 settings.

Murata et al. (2017) was identified as research that revealed findings about the efficacy of the Number Talks instructional practice. The research was an in-depth small-scale study focused on two first-grade teachers and their 46 students. The study focused on understanding the connections between math-talk and the strategy trajectories of students. The article clearly states the pedagogical process of the teachers engaging the students in math-talk, referencing that the teachers' process for these lessons was modified from Parish (2011). Additionally, the data that Murata et al. (2017) used to analyze mathematics discourse, students' strategies, and teacher math-talk practices originates from instructional practices that align with the five essential components of Number Talks. The authors' findings suggest that, "students' strategy development is supported by talk moves coordinating a wide range of student strategies, representations and discussions that are at a process-level and maintain student ownership of ideas" (Murata et al., 2017, p. 290). Furthermore, it is discernable from the study that the efficacy of the enacted Number Talks, for the strategy development of the students, depended on the teachers' own talk-moves. Important to the findings here, Murata et al.'s (2017) research demonstrates that the efficacy of the Number Talks instructional practice contains potential nuances that may be better conceptualized and understood by research directly focused on its various practical instantiations.

Number Talks have risen in prominence over the last 20 years among K-12 teachers. Despite the gravity with which teachers are drawn to this instructional practice, only one blind-peer-reviewed research article on the efficacy of the practice with K-12 students was found. Our thematic analysis of the 26 articles revealed the theme of what we call, a *Black Hole of research* *on instructional practice.* The meaning of this finding can be concisely stated as: A Black Hole is an instructional practice for which there is a paucity of blind-peer-reviewed research evidence supporting its efficacy, yet there is existing research pointing to its potential. To explicate this theme further, we will first elaborate on the meaning of *Black Hole* in the context of the instructional practice of Number Talks. Next, we will share the ways in which the remaining 25 research articles included components related to Number Talks (see accretion disk below) yet did not yield evidence of efficacy for the instructional practice. Following these findings, we will discuss implications of Black Holes for the field of mathematics education and the intersection of instructional practice and research endeavors.

Black Hole of Research on Instructional Practice

In our analysis of rigorous research about Number Talks, a theme emerged that metaphorically invoked the astronomical phenomenon of a black hole. In astrophysics, a *black hole* is “a region in space-time in which the gravitational field is so strong that it precludes even light from escaping to infinity” (Frolov & Novikov, 1998, p. 3). Though nothing that goes into a black hole may come out, astronomers have been able to learn about black holes, in part, because of the activity of particles whirling around the black hole called the *accretion disk*. The accretion disk has been vastly important for human learning about black holes. It is also immensely important for our findings here. What has been found is not simply a “hole” containing nothing, within the research. Rather we have found something, which is in practice quite dense and theoretically more complex than a hole; we have found a Black Hole.

The efficacy of the instructional practice of Number Talks has a dearth of research knowledge. From a research perspective, this is the unknown and unseen *Black Hole of research on instructional practice* (see Figure 2). When we speak of Number Talks as a Black Hole within research, we are not implying that Number Talks are void of any substance, but rather, quite the opposite. It is an instructional practice dense with probability and richness--found to be valuable by many teachers. Number Talks have increased in gravity in the teaching field, yet what is believed to be known about its particularities, benefits, and limitations lacks critical and rigorous examination from the research field. Another important aspect of the Black Hole of Number Talks was what constitutes its accretion disk. The five essential components of Number Talks intersect with several research domains. The research field's knowledge of these components creates a large accretion disk whirling around the Black Hole of Number Talks. Within the accretion disk of this Black Hole, there exists rigorous knowledge that many researchers and practitioners believe, in theory and/or practice, is connected to, and perhaps validates, the instructional practice of Number Talks. In the case of Number Talks, some of the connected areas of research include topics about the development of students' number sense and number reasoning, teacher questioning, mental mathematics, classroom discourse, and environment. Each of these areas contribute to the accretion disk of research knowledge that whirls around the Black Hole of Number Talks (see Figure 2). It is the research knowledge making up the accretion disk, intertwined with the anecdotal evidence among practitioners, which generates the belief that Number Talks are an effective instructional practice despite a paucity of blind-peer-reviewed research evidence about the efficacy of the practice itself. Under these conditions, the instructional practice of Number Talks developed into a Black Hole at the intersection of the mathematics education research and teaching fields.

Research in the Number Talks Accretion Disk

Only one of the articles in our search of 576 potential articles revealed knowledge about the efficacy of the instructional practice. The Black Hole of Number Talks holds many potential articles in its accretion disk because the practice invokes several domains that have been researched over a long period. Table 1 shows the alignment of research domains in the accretion disk, examples from research literature, and the five essential components of Number Talks (Parish, 2011). We have shared examples from the research literature that are representative of each research domain's longevity. Each research domain in Table 1 has many studies that span decades of knowledge gathered on those topics. For example, research on teacher questioning began prior to the 1980's, and in their meta-analysis, Redfield and Rousseau (1981) reviewed 20 articles examining the role of higher and lower cognitive questions in relation to student achievement. Though Redfield and Rousseau's research article was not part of our literature search because the studies were done long before Number Talks were invented, we mention it, here, as an example of the articles that demonstrate the significance of the research domains in the accretion disk.

As we were analyzing the 26 articles that warranted full reads, it became apparent that 25 of these articles strongly connected to some components of Number Talks but did not actually investigate the efficacy of the instructional practice itself. The research in these articles did not involve all five essential components but rather focused on particular components. It became apparent that the instruction being investigated in these 25 studies were not Number Talks. Details about these articles can be found at https://bit.ly/BlackHolesNumberTalks. Analysis of these articles revealed how they were exemplars of the research domains in the accretion disk of the Black Hole of Number Talks. Table 2 shows the number of articles containing related research to the five essential components of Number Talks. The majority of articles (17 out of 25) contained research related to classroom discourse. This is not surprising, given the perceived importance among educational researchers that effective instructional practices include student reflective discourse (Drageset, 2014; Mercer & Howe 2012; Walshaw & Anthony 2008) and the prominent role student-talk has in the Number Talks instructional practice (Humphreys & Parker, 2015; Parrish, 2011; Sun et al., 2018). In half of the articles, researchers investigated aspects of classroom environment (13 out of 25) and the teacher's role in questioning and supporting students' reasoning (14 out of 25). To a lesser extent, these articles explored the role of mental computation (1 out of 25) and purposeful computation problems (4 out of 25). In what follows, we describe our findings within each of the five essential components as exemplars of research in the accretion disk of the Black Hole of Number Talks.

Classroom Discourse

Number Talks necessitate student, teacher, and whole class discourse. This component is essential for promoting development of students' own reasoning and justification. As 17 of the 25 articles involved research in this part of the accretion disk, we note that discourse intersects with classroom environment and community, the teacher's role, mental mathematics, and purposeful computation problems. As we share our findings within each of these components, we have included specifics about articles that involve research on discourse. Analysis of these 17 research studies revealed that none of them investigated the kinds of discourse that occurs or results from Number Talks. However, they did reveal that in other types of instruction, nuances in teachers' talk moves result in various learning outcomes from students (Heng & Sudarshan, 2013; Imm & Stylianou, 2012; Webel & DeLeeuw, 2016). These articles provide examples of research that could be conducted on the instructional practice to denote the ways discourse is occurring among those enacting Number Talks, how nuances of discourse within Number Talks affect student learning, and how those findings connect with literature in the accretion disk.

Classroom Environment and Community

An essential component of Number Talks is a classroom environment and community that routinely reinforces mathematical norms and strategies. Many of the research articles analyzed in this study detail the importance of the teacher developing a routine in the classroom that encourages mental math and discourse and the different outcomes that result from the moves teachers make in the moment of practice. More specifically, the role of community and environment in mathematics classrooms is highlighted in a 2012 analysis of discourse communities by Imm and Stylianou (2012). This study investigates five middle school teachers and notes the ways these teachers interact with their students. Imm and Stylianou found there are instances in which teachers use specific language to help students understand procedural and conceptual mathematical ideas (p. 142), yet there is room for improvement for using discourse to promote students' mathematical reasoning and justification (p. 143). This study highlighted that classroom environments which encourage verbal reasoning and verbal justification of mathematical thinking "provided the expectation that students need to establish knowledge and certainty on their own rather by relying on the teacher's expertise" (p. 139). The mathematical thinking environment described here by Imm and Stylianou is one desired by many teachers when instantiating Number Talks (Humphreys & Parker, 2015; Lustgarten & Matney, 2019). Since Number Talks involve important aspects of environment and community, their potential to promote students' mathematical reasoning is viable. Yet the research in all 13 of the studies that investigated environment and community did not use the Number Talk instructional practice and therefore could not reveal evidence describing the related environment and classroom community conditions when teachers enact it.

Teacher's Role in Questioning and Supporting Student Reasoning

Understanding the importance of a teacher's questions, how different types of questions yield different outcomes from students, and teacher support of student reasoning are features of research in the accretion disk. Articles discussing this component focused on improving one of these areas. The methods of these research articles explained that some range of K-12 students were engaged in mathematics problems, as well as, analyzed teacher moves and student learning or behaviors. Heng & Sudarshan (2013) acts as a good example of the research involving this component of Number Talks. They investigated ten teachers and 51 students through analysis of lessons, pre- and post-lesson discussions, mathematics task-based interviews, and pre- and post-interviews with teachers. Among the findings, it was determined that teachers came to realize the instructional moves they thought were supporting students' mathematics learning for simpler problems became inhibitors for student sense-making as problems increased in sophistication (p. 9). Additionally, though teachers reported questioning as a common practice, the study found teachers' questioning to be closed-ended, stating, "teachers typically used questions to prompt students toward correct and appropriate responses that teachers had in mind, instead of as a means to uncover children's everyday knowledge or invented strategies" (p. 10). Heng & Sudarshan (2013) and the other articles researching this component of Number Talks reveal how the nuances of teaching practice, such as key-word gimmicks for problem solving and teacher questioning, can yield varied results.

The kinds of research methods and findings in these studies are examples of what one might expect in a study of Number Talks in which the role of teacher questioning and support of student reasoning is being examined. However, we found that the instructional practices in these articles were not Number Talks. Thus, while these articles do not give us evidence to make claims about Number Talks, their relatedness to a connected component does provide blueprints for future research that could investigate the efficacy of Number Talks.

Role of Mental Math

Dyson et al. (2015) conducted a study to analyze the outcomes of a number-sense intervention on low-achieving kindergarten students. The findings from this study are evidence of research related to the mental math component of Number Talks. This number-sense intervention focused on number, number relations, and number operations. Variables that were analyzed included number competencies, arithmetic fluency, and mathematics calculation achievement. After in-depth analysis, the authors reported, "Careful, targeted instruction combined with speeded but meaningful practice of facts seems to be optimal for kindergartners with the most serious delays" (Dyson et al., 2015, p. 363). While mental math tasks were central to this study, Number Talks were not one of the instructional practices implemented within the carefully-designed number sense intervention. As Number Talks involve mental computation and reflection on those processes, it holds potential for students' development of the same number sense attributes investigated by Dyson et al. (2015). However, we found no blind-peer-reviewed research investigating the efficacy of Number Talks for students' mental mathematics and number sense ability.

Purposeful Computation Problems

Teacher's choices of assigned computation problems are an essential element of Number Talks. Research of this component includes teachers and researchers describing reasons for selecting computation problems. One example of this was in Webel and DeLeeuw's (2016) investigation involving six selected fraction problems and the effect of teacher language on student learning. This study highlights how variations in the way teachers discuss fraction multiplication can greatly affect how students interpret the meaning of this concept. Webel and Deleeuw (2016) analyzed teachers and students from three fifth-grade classrooms. They compared the mathematical language used to create meaning around purposefully selected problems related to fraction multiplication. This study focused on how teachers' use of the words "multiply," "times," and "[fraction] of" to discuss the same task led to inconsistent understandings of fraction multiplication for the students. Webel and DeLeeuw concluded, "…even when the opportunities presented by a written task are the same, linguistic differences in the enactment of the task can create important differences in opportunities to learn" (Webel & Deleeuw, 2016, p. 137). Similar to the findings of Murata et al. (2017) mentioned earlier, the nuances of teacher talk affect student learning. We note that the Number Talks instructional practice was not used in Webel and DeLeeuw's research or any of the other articles in this domain of the accretion disk. Our analysis of research in this domain revealed again that there is a dearth of research on the efficacy of Number Talks and effects of the practice of choosing purposeful computation problems.

Conclusions:

Our literature review sought blind-peer-reviewed research that might potentially give evidence for the efficacy of the Number Talks instructional practice with K-12 students. Among the 576 articles in our search, 26 were found to have potential relevance. Thematic analysis of these articles revealed a Black Hole of research on instructional practice, in that only one of the articles could inform the field about the efficacy of Number Talks. The other 25 articles included one or some of the five essential components, yet none of these were research on the efficacy of Number Talks. Thus, we have drawn three overarching conclusions from the findings of this study. Firstly, when we consider that the instructional practice of Number Talks consists of five components that are carefully woven together, there sparsely exists blind-peer-reviewed literature informing the field of the benefits and limitations. While the paucity of blind-peer-reviewed research evidence supporting its efficacy is the Black Hole, there is also a large accretion disk of research knowledge pointing to Number Talks potential. Therefore, our second conclusion is that the research in the accretion disk shows the individual merit and nuance of each essential component of Number Talks. The accretion disk has a wide berth of research revealing the potential of Number Talks to be a beneficial practice in K-12 mathematics instruction. Thirdly, we conclude that more research on the efficacy of Number Talks is needed because the research in the studies we read and described here give evidence for the effects of nuances within instruction for components related to Number Talks. To have an understanding of the kinds of results occurring through the instantiation of Number Talks with K-12 students, research needs to illuminate the effect of these nuances as they are interwoven within the five components and as they occur within teaching practice. As we shared, research like Murata et al. (2017) and Webel and DeLeeuw's (2016) found that teachers' own talk moves affected student learning, and this is only one of the variations that might take place among components of a Number Talk. Further research on the efficacy of Number Talks is warranted.

Discussion

It seems reasonable to assume that instructional practices like Number Talks may proliferate among practitioners and spread by word of mouth, social media, and electronic resource sites. Perhaps, they can spread more quickly than carefully-deliberated research projects can be conducted, analyzed, written up, submitted, peer-reviewed, revised, and published. Number talks were created in the 1990's by Cathy Richardson and Ruth Parker (Humphreys & Parker, 2015, p. vii). As more practitioners tried enacting Number Talks with their students, the ideas spread, and books were published. When we began conducting our initial search, it was January of 2018, about 20 years after the creation of Number Talks and several years after the first book was written. The timeline suggests that research studies could have been conducted and submitted to peer-reviewed journals to inform the practice. We offer our research here with the hope of generating conversations between practitioners and researchers about helpful moves to breach chasms between instructional practices and research-based practices. Furthermore, we encourage the mathematics education research community to identify other Black Holes of research on instructional practice, identify their accretion disks, and systematically work together to illuminate the currently unseen nuances of practice and their related outcomes. After all, Number Talks may be but one of the Black Holes existing in this chasm.

The meaning of "Research demonstrates…"

In the common parlance of professional development, it is common to hear the phrase "research demonstrates…" followed by a statement about a particular instructional practice or beneficial routine that is thought to improve student learning. There is a difficulty involving the precision of language when we speak generally, of what is known from research. In the case of Number Talks, what does it mean for research in the accretion disk to demonstrate knowledge of the efficacy of the instructional practice? With what precision can we know how it improves students learning? With what precision can we know how variations in instruction change the outcomes of student learning? We contend that research focused directly at an instructional practice allows more precision than related research studies residing in the accretion disk.

One of the major research domains existing in the accretion disk of Number Talks is *mathematical discourse*. There is a large swath of blind-peer-reviewed research articles investigating this area (Ryve, 2011). In our own practice of mathematics teaching, we also feel the inherit connection between discourse and Number Talks. As researchers, however, we note that within the mathematics education research literature, a variety of definitions and assumptions guide these studies (Ryve, 2011). The lack of clear conceptualization on the meaning of discourse is one example of why generalizing an instructional practices' efficacy based on non-direct, yet reasonably associated areas of research, is dubious. The one article we found by Murata et al., (2017) that sheds light on the efficacy of Number Talks supports the idea that the efficacy of this instructional practice is nuanced, and as such, any associated generalizations about discourse occurring during a Number Talk from research, not enacting the practice itself, should be thoughtfully questioned. Upon reflection of the findings in this study, we wonder how research and teaching communities might linguistically differentiate between research in the accretion disk of an instructional practice and research directly about the efficacy of that practice. Clear communication between the teaching and research communities about whether or not a study is demonstrating an outcome associated with the efficacy of a particular instructional practice is important for the integrity of research knowledge.

The Importance of Researching Instructional Nuance

Research is vital to help inform instructional practices like Number Talks. Across hundreds of observations of teachers implementing their practice of Number Talks, we have noticed subtle variations and enacted beliefs. Some of these variations may lead to drastically different outcomes in student learning. Other modifications may result in little-to-no difference for mathematical learning but may promote other benefits in addition to mathematical learning. We share two examples in what follows.

Example 1 of Number Talk Variations

One common practice among teachers who enact Number Talks is that once the students are finished thinking, the teacher requests that they share the result of the mental computation (Parish, 2011). The teacher writes these often-varied results on the board or on chart paper and solicits from the class justifications for each of the solutions. Students share their ideas, and the teacher records student thinking visibly and often symbolically in a way that the class can see a mathematical representation of the idea. On the other hand, we have noticed teachers who change this routine element regularly and other teachers who do not use it at all. For example, some teachers have shared with us that in their desire to promote greater discourse through listening and making sense of another person's reasoning, they modify this component of Number Talks. Following the initial think time about the mental computation problem, the teachers ask students to turn and talk to a partner with the goal of understanding that partner's ideas well enough to be able to explain them to others. The teachers listen intently to the partner talk, then select and sequence (Smith & Stein, 2011) students to share their partner's thinking according to a pre-determined mathematical goal. The teachers enacting this version of Number Talk believe it promotes greater safety of student ideas through a lower-risk environment and more beneficial math-talk. These variations in the instructional practice raise several research questions about Number Talks. What are the effects of these variations on students' number sense? What are the beliefs and goals underlying teacher decisions about these modifications? How do these variations affect student identity, equity, and access? How do these variations affect students' sense of safety, mathematical discourse, and community?

Example 2 of Number Talk Variations

Some teachers strictly enact Number Talks where students cannot use any tools (paper, pencil, white board, calculators, etc.) and all computation must be held in the mind of the student. Other teachers claim that not every Number Talk need to be without an element of writing or drawing. As we observed this instructional nuance, we noted that students were still following the Number Talk routine but that after the individual mental think time, each student was given an opportunity to represent computational ideas on a white board before engaging in class dialogue. These teachers shared with us that when students are allowed to map out their own ideas on white boards, it provides a space for more effective mathematical dialogue. They further claimed that when students map out their computations, they are still "mentally computing" and in addition to improved classroom discourse, the students have an added opportunity to represent an idea in their own way prior to seeing how a teacher would represent it. Again, these variations in the instructional practice raise several research questions about Number Talks. In addition to the questions already identified in example 1, we wonder, what are the mathematical affordances and limitations of having students represent their own mental computations as part of a Number Talk? Due to the modification of writing and drawing, teachers claimed they experienced more effective mathematical dialogue from their students. What are the elements of the resulting dialogue, how are these elements different from the dialogue without the modification of writing and drawing, and in what ways is the dialogue more effective? In the context of Number Talks, are there differences in mathematical learning when students create their own internal representations of mental calculations versus having a teacher externally represent students' ideas as they verbally share?

These examples demonstrate only two modifications to Number Talks that we have seen anecdotally. Each modification elicits many possible research questions. Studies seeking to understand this Black Hole of instructional practice could illuminate the values and limitations of these modifications and help to make stronger connections between teaching and research.

Black Holes at the Intersection of Teaching and Research

We do not know much about the efficacy of the instructional practice of Number Talks. Although presentations and online chatter have become more prominent in the last decade, we have relatively no research evidence on the benefits and limitations of Number Talks in K-12 schools. Furthermore, there has not been an attempt to catalogue and understand the variance in how Number Talks are enacted in K-12 classrooms. It is beyond the scope of this study to determine for the field which modifications should count as a Number Talk or to describe what other categories should be developed for deeper study of the nuanced variations in instructional practice. However, research focused on these instructional nuances could provide helpful knowledge for guiding instructional decisions and strengthen connections between the research and teaching communities. We see Black Holes as prominent areas for connection between the research and teaching fields precisely because of their gravity and large accretion disks.

Specific to the Black Hole of Number Talks, we wonder what kinds of variations of the Number Talks instructional practice exist in the field, to what extent and for what teaching and learning purposes are these variations effective, and for which students are they beneficial? What variations of the instructional practice offer greater equity? What types of professional noticings and teacher moves contribute to making Number Talks highly effective? How do we define, from research evidence, what makes a Number Talk highly effective? Do variations in the Number Talks instructional practice allow for better learning among particular populations such as English Language Learners? From our perspective, these are good exemplars of research questions that need examination to elucidate the Black Hole of Number Talks; moving us from understandings of research we believe to be connected to the instructional practice (accretion disk) to informing us about the instructional practice itself (illuminating the Black Hole).

As mathematics teachers develop and enact an ever-expanding set of new classroom routines comparable to Number Talks, researchers could be working alongside them critically thinking, collecting data, discerning the limitations and affordances, and publishing research about these instructional practices. Without such knowledge, many of the teachers with whom we work feel compelled to use whatever they can find on teaching idea websites. If something as prolific as Number Talks has existed in principal for at least two decades now without research going through the field's blind-peer-reviewed process to thoughtfully examine the practice, it seems doubtful that the instructional inventions being sold on teaching idea websites have been critically examined and thoughtfully discussed. Researchers, teachers, parents, students, and administrators could be working together to explore these instructional strategies in profound ways. Doing so takes time and a clear openness among those involved to examine the complexities of teaching and learning. These efficacy studies would benefit everyone as we seek to improve learning and instruction.

Researchers will need help from teachers, parents, students and administrators to obtain adequate permissions to think alongside educators about the affordances and limitations of these instructional routines. Only by working together and in various classroom settings can we hope to gain clear knowledge of these privileged instructional practices that have developed a critical gravity and have become a *Black Hole of research on instructional practice*. In the end, our hope is for a collapse of the Black Hole metaphor for the most commonly used instructional practices. That is, we hope to see researchers and teachers find ways to do the seemingly impossible, to see inside the black holes of these instructional practices, and to illuminate the possibilities of what is happening with students' mathematical learning, procedural fluency, and other important results to the best of our capabilities.

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