7-2020

A Modified SIR Model Equivalent to a Generalized Logistic Model, with Standard Logistic Approximations

David E Clark
Maine Medical Center

Jordan S Peck
Maine Medical Center

Gavin Welch
Maine Medical Center

A modified SIR model equivalent to a generalized logistic model, with standard logistic approximations

David E. Clark, Gavin Welch, Jordan S. Peck -- MMCRI/CORE/MaineHealth

- Biologically-based epidemic models (for COVID etc) often use systems of differential equations
- Population-based models often fit data to logistic or similar curves, and are more familiar to epidemiologists and engineers
- We intended to show that one modeling approach can be approximated by the other

Classic Susceptible-Infectious-Removed (SIR, Kermack/McKendrick 1927)

$$X'(t) = -\beta XY$$

$$Y'(t) = \beta XY - \gamma Y$$

$$Z'(t) = \gamma Y$$

Modified SIR (note X+Y+Z = 1)

$$X'(t) = -\beta XY / (X+Y)$$

$$Y'(t) = \beta XY / (X+Y) - \gamma Y$$

$$Z'(t) = \gamma Y$$

-- Modified SIR solved exactly in terms of Generalized Logistic functions, e.g.,

$$X(t) = \lambda \frac{1}{(1+e^{(t-\mu)/\sigma})^{\beta\sigma}} \quad ,$$

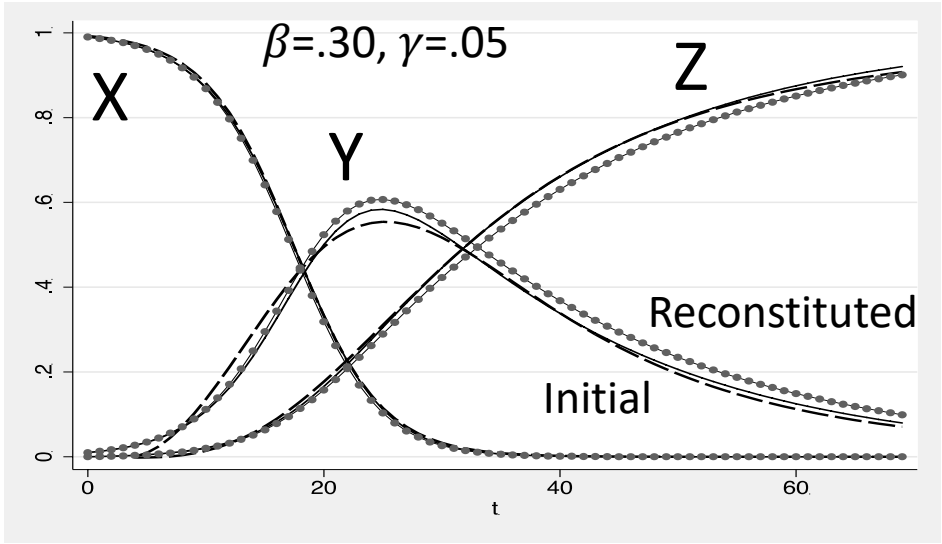
$\sigma=1/(\beta-\gamma)$, $\mu= \sigma \log(x_0/y_0)$, $\lambda = \left(\frac{x_0+y_0}{x_0}\right)^{\beta\sigma}$.

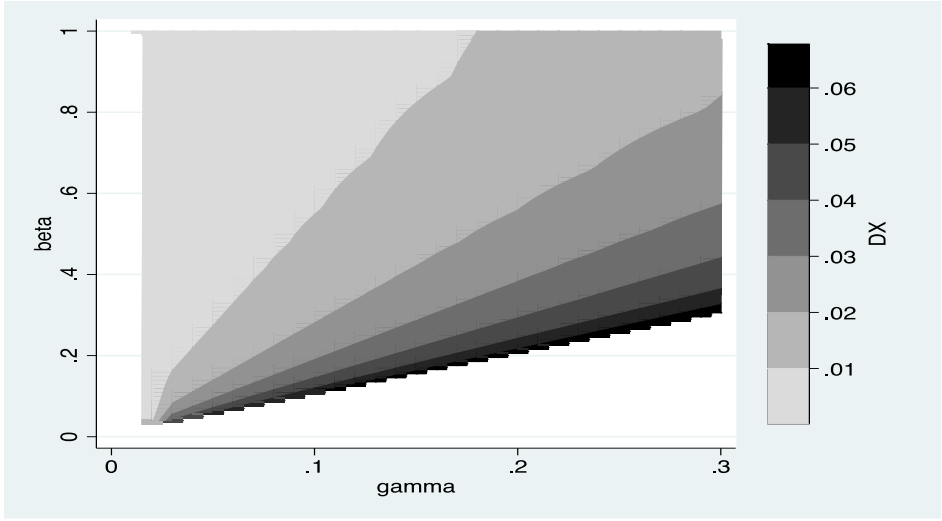
-- Generalized Logistic functions approximated with Standard Logistic (or Log-Logistic) functions by equating quantiles.

-- Evaluated error of approximation (Kolmogorov Distance) for different values of β and γ .

-- Devised algebraic or numeric methods to estimate β and γ from population statistics.

medRχiv July 2020, clarkd@mmc.org





Bio-based and pop-based methods interchangeable