LOCAL ORTHOTROPIC, PLANAR ELASTICITY COMPUTER PROGRAM

journal or publication title	国立防災科学技術センター研究速報
number	59
page range	81-137
year	1984-03-30
URL	http://id.nii.ac.jp/1625/00002793/

LOCAL ORTHOTROPIC, PLANAR ELASTICITY COMPUTER PROGRAM

USER MANUAL

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SEPTEMBER 1983

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ABSTRACT

Analysis of elasticity and related type problems is facilitated through the development of a finite element (LOPE, Local Orthotropic, Planar Elasticity) code. Direct application is to two-dimensional elasticity problems, that may include material orthotropy, surface and body force loading, and constraint boundary conditions. In addition, the algorithm is applicable to steady state (potential) problems such as in ideal fluid flow, tertiary viscoelastic creep, flow through porous media, and heat transfer. The basic finite element of the code is the constant strain triangular element, which, by superposition, may be used to define quadrilateral shaped elements. The program uses Gauss elimination to solve the set of algebraic equations, which are handled by matrix partitioning in order to accommodate large order problems on medium sized computer systems. The program is operational on the Shinjo Branch (NRCDP) Melcom 70 computer system, and has been used to solve problems where core limits are exceeded. Listing of the computer instructions is given in this reporting, together with input format descriptions, and test problems that check the various options of the program.

Introduction

A computer code entitled Local Orthotropic, Planar Elasticity (LOPE) has been developed to solve planar elasticity problems by application of the finite element method on the Melcom 70 (Mitsubishi Electric) computer system. Program LOPE is a modified version of program PALOS, originally developed by Goudreau, Nickell and Dunham (1967). Program LOPE is both a simplified and performance improved version of program PALOS. Simplification includes deletion of polar planar geometries, axisymmetric shell geometries, and reduction of options on material and thermal environment representations. Performance is improved by changes in algorithm executions consistent with the functions of the Melcom 70 computer system. Original documentation of program PALOS entitled, "Plane and Axisymmetric Finite Element Analysis of Orthotropic Elastic Solids and Orthotropic Shells", will be considered in this reporting as the primary sourse on finite element theory pertaining to computer program LOPE. This users manual is intented as a supplement to indicate how input data is formatted for porgram LOPE, and to provide test problems and statement listings of the code. Anyone using program LOPE is advised to become familiar with the Goudreau, etal. document as may be needed.

Program LOPE may be used to solve planar elasticity problems either in plane-strain or plane-stress, based upon a finite element representation in the plane. Discretization of this plane, designated the x, z plane, into elements may be either triangular or quadrilateral, based upon definition of a set of nodal points interconnected by strainght lines. The layout of the element array is an important consideration in making efficient use of the program. One limiting consideration is the spread or difference between node numbers within the individual elements of the problem. The largest difference between node numbers of any element may not exceed 26 in the listed code. Node numbers should be assigned in a regular pattern, and nodes should be numbered across the narrower width of the region. Examples of efficient and inefficient numbering schemes are shown in Figure 1. After element layout, and nodal numbering, then the maximum node difference in the member can be determined from inspection. Designating this by, D_N, then the bandwidth of the problem can be defined as

$$MBAND = 2D_N + 2$$

For the LOPE code, as listed herein, $D_N \leq 26$, so that MBAND $\leq$ 54. Later, use will be made of MBAND in additional calculations. The LOPE code, as listed in the Appendix of this report, has limits expressed in DIMENSION and COMMON statements which are to be considered as a guide, and may be changed as the capacity of the computer permits.

Plane Stress-Plane Strain

The general 3-D constitutive law used by the LOPE program allows up to seven material coefficients, namely.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{12} & C_{22} & C_{23} & 0 \\ C_{13} & C_{23} & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

To express the C's physically for an isotropic material having material modulus, E, and Poissons ratio, ν, we have.

$$C_{11} = C_{22} = C_{33} = \lambda + 2\mu$$

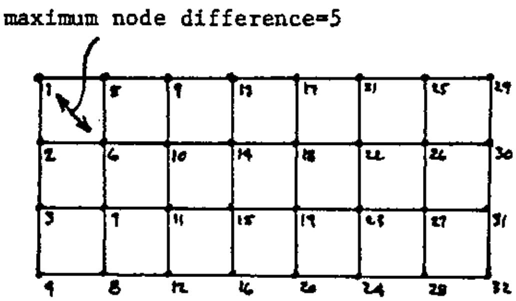
$$C_{12} = C_{13} = C_{23} = \lambda$$

$$C_{44} = \mu$$

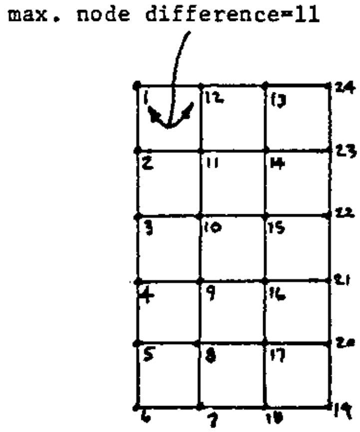
where

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

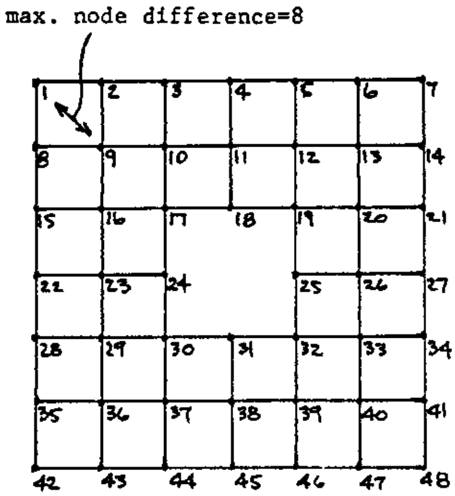
For plane stress in the x, z plane, we set $\sigma_y = 0$, which yields $\varepsilon_y = -\frac{C_{23}}{C_{22}}\varepsilon_z - \frac{C_{12}}{C_{22}}\varepsilon_x$, which when substituted into the constitutive equations yields



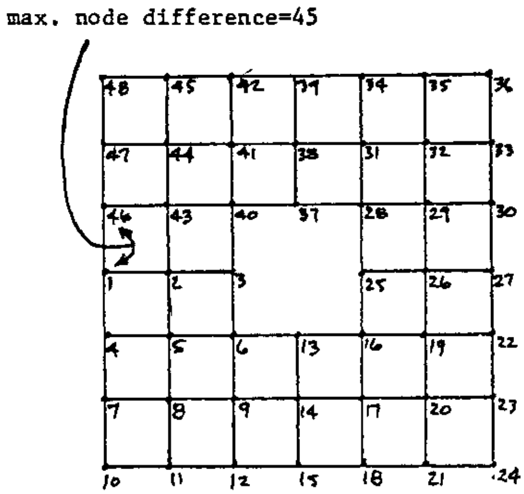
(a) rectangular array-
efficient numbering



(b) rectangular array-
inefficient numbering



(c) ring array-
efficient numbering



(d) ring array-
inefficient numbering

Figure 1 : Node numbering and maximum
element node difference

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11}^* & C_{13}^* & 0 \\ C_{13}^* & C_{33}^* & 0 \\ 0 & 0 & C_{44}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

where

$$C_{11}^* = C_{11} - \frac{C_{12}^2}{C_{22}} \qquad C_{33}^* = C_{33} - \frac{C_{23}^2}{C_{22}}$$

$$C_{13}^* = C_{13} - \frac{C_{12}C_{23}}{C_{22}} \qquad C_{44}^* = C_{44}$$

The transformation to the C*'s is done by the program if C_{22} is non-zero in the original input array and plane stress is indicated in the input data. If $C_{22}=0$ then reduction to plane stress is assumed to have been done already with the coefficients that are input. If the program computes the C*, then these are output and the plane stress condition is noted.

For plane strain, $\varepsilon_y=0$ and the constitutive law reduces to

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{12} & C_{23} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

where no transformation of the C's is required.

If the material is orthotropic in the x, z plane then the seven material constants have different values, and the starting 3D constitutive equation, assuming shear in the x, z plane, is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 \\ 0 & 0 & 0 & \bar{C}_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

The inverse of this equation, namely

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{12} & A_{22} & A_{23} & 0 \\ A_{13} & A_{23} & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix}$$

has the physical meaning $A_{11} = \frac{1}{E_{11}}$, $A_{22} = \frac{1}{E_{22}}$, $A_{33} = \frac{1}{E_{33}}$

$$A_{12} = -\frac{\nu_{21}}{E_{11}} = -\frac{\nu_{12}}{E_{22}} \qquad A_{23} = -\frac{\nu_{32}}{E_{22}} = -\frac{\nu_{23}}{E_{33}}$$

$$A_{13} = -\frac{\nu_{13}}{E_{33}} = -\frac{\nu_{31}}{E_{11}} \qquad A_{44} = \frac{1}{G_{13}}$$

Here E_{ii} is the elastic modulus of the material along the i axis, $\nu_{ij} = -\varepsilon_{ii}/\varepsilon_{jj}$ for stress applied along the i axis, and G_{ij} is the shear modulus corresponding to shear stress in the i, j plane.

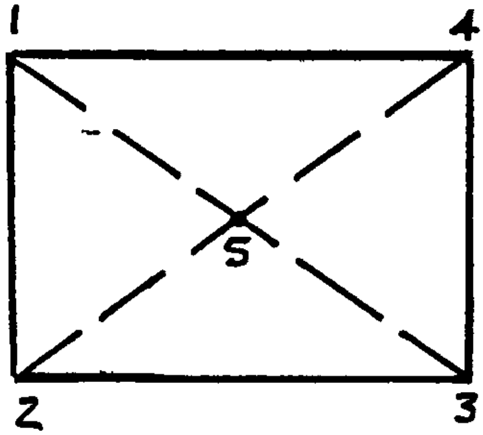
Other interpretations of the C's are required if the computer program is used for steady-state viscoelastic analysis, for steady-state fluid potential flow, or for other planar mechanics problems.

Triangular and Quadrilateral Elements

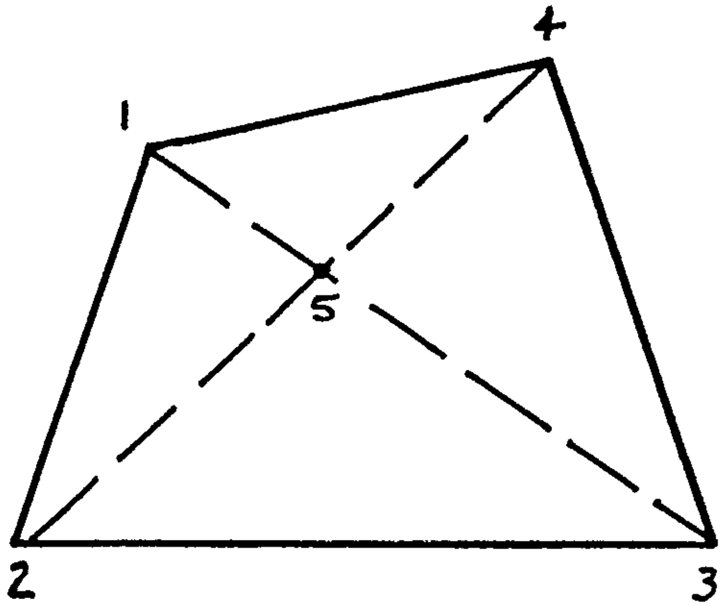
The triangular element used in LOPE is the planar elasticity element commonly reported in finite element texts. The element has 3 nodes, with two displacement components at each node. Thus for the triangular element the stiffness matrix is 6x6. A quadrilateral element, when specified in the input data, is made up from 4 triangular elements by creating a fifth node, which is later deleted from the stiffness matrix. Thus, the final quadrilateral element has 4 nodes, with u, v displacement components at each node, so that the stiffness matrix is 8x8. The quadrilateral element may be trapezoidal, if necessary (Figure 2).

Element Stresses

For each element of a finite element problem one set of stresses is computed, which should be considered as acting at the centroid of the element. In program LOPE several sets of stress components are computed. One set is referenced to the global X, Z coordinate system. These are stresses σ_{11}, σ_{33} and σ_{13} in Figure 3. A second set of stresses are the principal stresses, which are σ_{max}, σ_{min} at angle θ measured from the X axis (Figure 3). A



(a) regular



(b) trapezoidal

Figure 2 : Types of quadrilateral elements

third set of stresses designated σ_{nn} and σ_{nt} are also computed, which are the stresses normal and tangential to the J, K boundary of the element.

In Figure 3 is also shown the correct sequencing of node numbering consistent with the global coordinate system. If the rotation from the X to Z coordinate axis is counterclockwise, as shown in Figure 3, then the listing of nodes in the element data should also be counterclockwise. Thus, three choices of listing the node points are:

1 - 2 - 3

2 - 3 - 1

3 - 1 - 2

Which one to use depends upon which stress σ_{JK} is desired, and is the choice of the user.

Program LOPE Structure

Program LOPE is made up of one MAIN program and four subroutines, named LOCATE, TRISTF, MODULI and MODIFY. Subroutine LOCATE has the purpose of reading and writing program input and control data. Also, several calculations are performed in this subroutine including the interpolation for intermediate nodes and elements that are not listed in the input data, and conversion of pressure boundary conditions to equivalent nodal forces. Number of nodes, elements and matrix bandwidth are also checked relative to limiting values in LOCATE. Subroutine TRISTF is the element stiffness and force vector generation code. It is called once if the element is triangular and four times if the element is quadrilateral. Subroutine MODULI transforms the material coefficient matrix from local to global coordinate reference. Subroutine MODIFY is used to impose displacement boundary conditions on the total structure stiffness matrix. All remaining finite element functions are performed in program MAIN. This includes the following:

1. Element stiffness matrix overlay to form the total structure stiffness

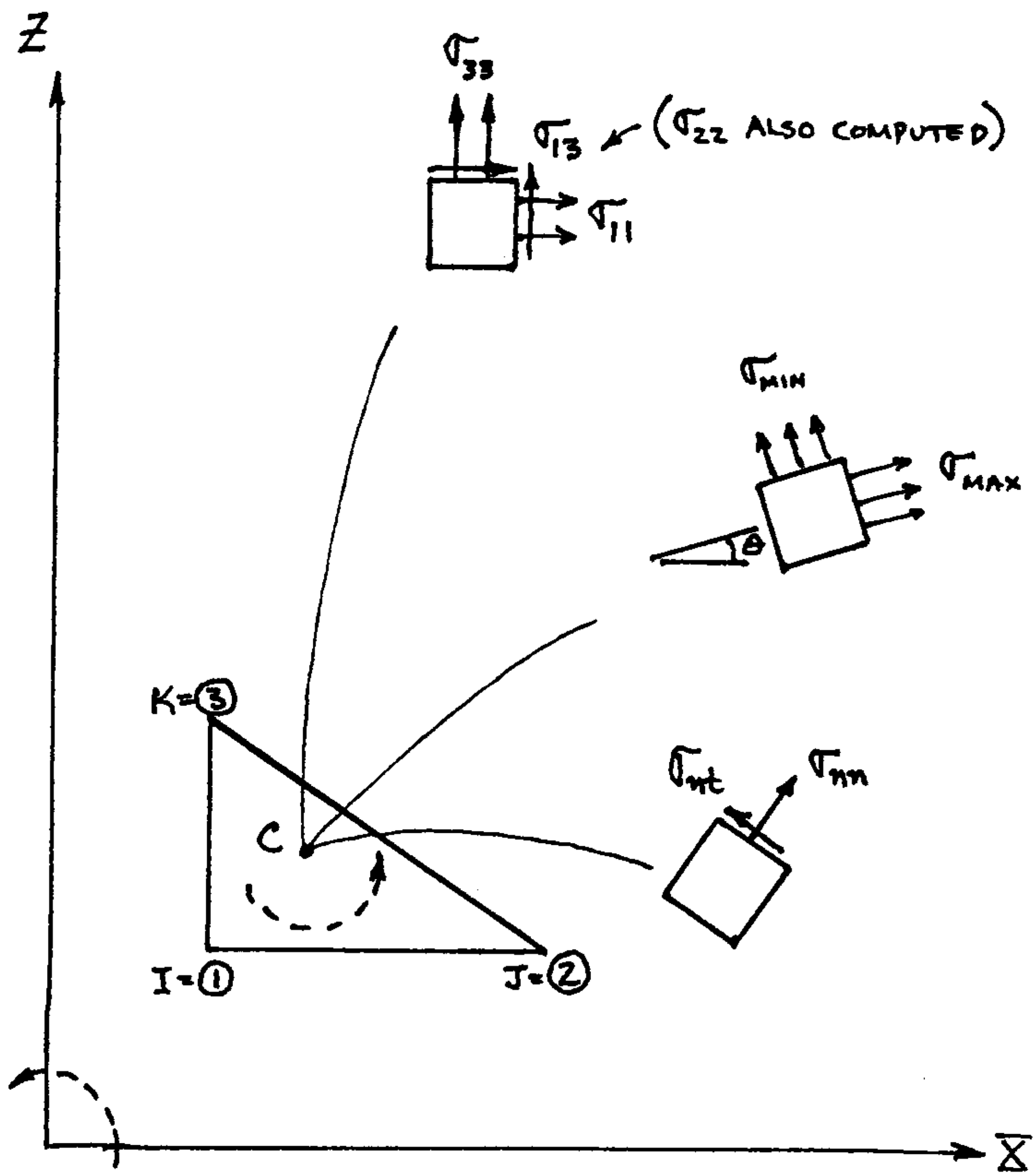


Figure 3-1 : Element Stresses

matrix.

2. Imposition of directional gridpoint displacement conditions.
3. Reduction of the stiffness matrix to semi-diagonal form and back substitution for the displacement solution by Gaussian elimination.
4. Printout of displacements.
5. Calculation and printout of element stresses.

The original PALOS program has proven to be an efficient and useful program to solve 2D elasticity, fluid, and viscoelasticity problems. It if expected that the simplified version, LOPE, has the essential characteristics and utility of the original program.

INPUT DATA DESCRIPTION FOR PROGRAM LOPE

Line 1 : Title data : FORMAT (40A2)

Columns 1 - 80 : Name and identification information

Line 2 : Control Data : FORMAT (5I5, 2F10.0)

Columns 1 - 5 : Number of nodal points (600 maximum)

Columns 6 - 10 : Number of elements (500 maximum)

Columns 11 - 15 : Number of different materials (10 maximum)

Columns 16 - 20 : Number of boundary pressure conditions (50 maximum)

Column 25 : Program option:
1 plane strain analysis
2 plane stress analysis

Columns 26 - 35 : Acceleration in the X direction

Columns 36 - 45 : Acceleration in the Z direction

Line 3 : Material Data : FORMAT (I5, F10.0, 25A2)

Columns 1 - 5 : Material number (number from 1 to 10)

Columns 6 - 15 : Mass density of the material (this is used only if accelerations are specified, to compute equivalent static force)

Columns 16 - 70 : Material title and identification

Line 4 : Material Coefficients : FORMAT (7E10.4)

	ELASTICITY COEF	ISOTROPIC VALUE
Columns 1 - 10 :	C_{11}	$\lambda + 2\mu$
Columns 11 - 20 :	C_{12}	λ
Columns 21 - 30 :	C_{13}	λ
Columns 31 - 40 :	C_{22}	$\lambda + 2\mu$
Columns 41 - 50 :	C_{23}	λ
Columns 51 - 60 :	C_{33}	$\lambda + 2\mu$
Columns 61 - 70 :	C_{44}	μ

All orthotropic material properties are assumed to be defined with respect to the local axes of orthotropy. The Lame constants λ and μ for an isotropic material are related to the engineering constants E and ν by

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

For the case of plane stress (σ_{yy} =0) the material coefficients will be modified by the program unless coefficient C_{22} in columns 31 - 40 is zero, indicating that the coefficients have already been modified.

Line 5 : Nodal Point Data : FORMAT (I5, 3X, 2I1, 5F10.0)

One line is needed for each nodal point (see exception below).

Columns 1 - 5 : Nodal point number

Column 9 : Put 1 if Z displacement is to be specified

Column 10 : Put 1 if X displacement is to be specified

(If columns 9 or 10 are zero or blank, then the corresponding force will be specified).

Columns 11 - 20 : X coordinate of node (in global X, Z)

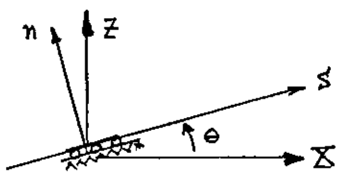
Columns 21 - 30 : Z coordinate of node (in global X, Z)

Columns 31 - 40 : X force or displacement component

Columns 41 - 50 : Z force or displacement component

Columns 51 - 60 : boundary angles (degress from X axis)

(If a boundary point is constrained to move in a direction S, input the angle, in degrees, from the X axis, positive as shown.



$0 \le \theta \le 360°$

Fig.3-2

In this case leave columns 9 and 10 blank. Columns 31-40 are then the S force, and columns 41-50 are the displacement).

Nodal points must be listed in numerical sequence. If values are omitted then nodal points are generated at equal intervals along a straight line between the defined nodal points. If the end point boundary codes (9 and 10) are the same at each end of the interval, then that code will be assigned to the generated nodes. If they are different, then the codes will be assigned zero. The generated nodal point forces or displacements will be zero.

Line 6 : Element Data : FORMAT (6I5, F10.0)

One line is needed for each element (see exception below).

Columns 1 - 5 : Element number
Columns 6 - 10 : Nodal point I
Columns 11 - 15 : Nodal point J
Columns 16 - 20 : Nodal point K
Columns 21 - 25 : Nodal point L
Columns 26 - 30 : Material number
Columns 31 - 40 : Local angle of orthotropy (degrees from X axis)

An element may be triangular or quadrilateral, depending upon whether it is defined by three or four nodes, respectively. If the global axes transform from the X to Z axis by a counterclockwise rotation, then the nodal sequence I-J-K-L must be a counterclockwise rotation around the element. If the element is triangular, set L=K.

The maximum difference between any pair of I, J, K, L may not exceed 26 (the maximum node numbering difference of the problem).

If the principal material axes $\bar{}$, $\bar{}$ of an orthotropic material element are different from the global X, Z axes, then the angle (in degrees) from the X axis is input positive as shown.

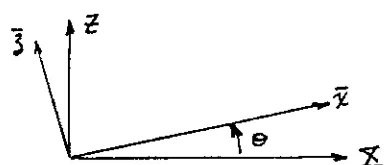


Fig.3-3

Elements must be listed in numerical sequence. If element numbers are omitted, then elements are generated by incrementing I, J, K and L of the previous element by one. The material identification code and the local angle of orthotropy are set equal to that of the previous line. The last element of the problem must always be input.

Line 7 : Boundary Loading Data : FORMAT (2I5, 2F10.0)

One line is needed for each boundary segment which is subjected to a normal or tangential pressure.

Columns 1 - 5 : Nodal point II
Columns 6 - 10 : Nodal point JJ
Columns 11 - 20 : Normal Pressure
Columns 21 - 30 : Tangential Pressure

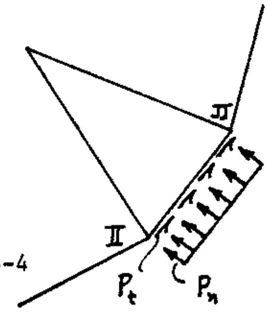


Fig.3-4

The boundary element must be on the left as one progresses from from II to JJ. Surface tensile force is input as a negative pressure. Positive shear pressure is in the direction from II to JJ.

OUTPUT DATA

The following information is output from the computer.

1. Printout of the input data. If, in a plane stress problem the material coefficients are modified, then the modified set are output. The program also outputs whether the plane stress or plane strain option is being used.
2. Printout of the computed node point displacements.
3. Printout of the centroidal stresses and strains in the following coordinate systems:
   A. global
   B. principal (with separate angle for stresses and strains)
   C. Normal and tangential to J-K face

(see Figure 3 for further definition of the stress components)

Program Operation on Melcom 70

The original main program and three levels of eight subroutines comprised in program PALOS, were replaced by a main program and one level of four subroutines in program LOPE. The main program in LOPE was expanded to include three of the longest subroutines in program PALOS. The remaining 5 subroutines were reduced to 4 and accessed at one level of subroutine entry, as required by the Melcom 70 system. One data storage and access option in program PALOS that provides efficient data handling, was simulated in porgram LOPE in order to retain the same efficiency. The option takes into account the array symmetry and band limiting characteristics of the finite element stiffness matrix. In stiffness matrix generation and overlay only the lower dragonal and off-diagonal elements are retained, and these data are generated, stored and re-read from computer memory, to disk, and back into memory by blocks. This process of data handling is efficient relative to total storage allocation needed in problem execution on the Melcom 70 system. A user of the program, in taking advantage of this data handling feature must make some simple preliminary calculations prior to assigning disk storage for program execution.

A BLOCK of data is defined, using the bondwidth (MBAND) defined earlier, as

$$\text{BLOCK OF DATA} = \text{DATA LENGTH} = (\text{MBAND} + 1) \times 54$$

There may be several BLOCKS OF DATA, the number of which may be computed as follows

$$\text{NO. OF BLOCKS} = \frac{\text{MAXIMUM NODE NUMBER OF GRID}}{27}$$

rounded off to the next larger whole number (integer). On the Melcom 70 the record length on the computer disks is defined as

$$\text{RECORD LENGTH (2D DISK)} = 256 \text{ bytes} = 128 \text{ words}$$

for an unformatted R/W statement. However, two words are needed to store each REAL NUMBER of a data set. Thus, the number of RECORDS needed for each BLOCK

OF DATA may be computed based upon the value of the bandwidth (MBAND). The result of this computation is summarized in Table 1.

TABLE 1 : Number of RECORDS necessary for storage of array of bandwidth MBAND

MBAND	DATA LENGTH	NO. OF RECORDS/BLOCK
4	270	5
6	378	6
8	486	8
10	594	10
12	702	11
14	810	13
16	918	15
18	1026	17
20	1134	18
22	1242	20
24	1350	22
26	1458	23
28	1566	25
30	1674	27
32	1782	28
34	1890	30
36	1998	32
38	2106	33
40	2214	35
42	2322	37
44	2430	38
46	2538	40
48	2646	42
50	2754	44
52	2862	45
54	2970	47

In summary, once the bandwidth (MBAND) is known, then from Table 1 the NO. OF RECORDS/BLOCK can be determined, finally multiplying by the NO. OF BLOCKS, as computed above, yields the total NO. OF RECORDS needed for the problem being solved. This number is used to define disk storage, and if designated STORE, the format of the statement is

```
ØØ// DEFI UD, STORE, NO. OF RECORDS, U
```

Following this, DEVICE no. 11 is specified in program LOPE so an assign statement (ASGN) is needed of the form

@@// ASGN U11 = STØRE, UD

Assuming that a problem data file has been built, designated DATA, then the sequence of computer instructions needed to execute (EXEC) a problem is:

@@// ASGN U7 = DATA, UD

@@// DEFI UD, STØRE, NO. OF RECORDS, U

@@// ASGN U11 = STØRE, UD

@@// EXEC LØPE, UD

These instructions assume that data area UD is used and that the LØPE program has been compiled and placed in disk area UD.

LOPE Test Problem #7

This test problem is designed to check the basic operations in program LOPE, including INPUT-OUTPUT, DISPLACEMENT COMPUTATION and ELEMENT STRESS COMPUTATION. In addition, the problem is large enough that three BLOCKS of data are generated, of which two are stored on disk by the WRITE-READ-BACKSPACE function on the Melcom 70 computer.

The test problem is of an axial rod of mild steel supported at one end (X=0) and loaded by a concentrated force of 10,000 LB at the other end (X=10"). The finite element representation of the probelm uses 40 square elements that are 1" by 1", with corresponding 55 nodal points(Figure 4). The 10,000 LB load acts in the positive X direction, and is applied at node point #53 (Figure 4). The axial rod is assumed 1" in the third dimension, otherwise the 10,000 LB force may require redefinition. At the upper (supported) end of the axial rod displacements U_1 , U_2 , U_3 , U_4 , U_5 in the X direction are set to zero. However in the Z direction the rod is allowed to change width, as only V_3=0. These displacement constraints at the supported end of the rod are sufficient to prevent any rigid body translation

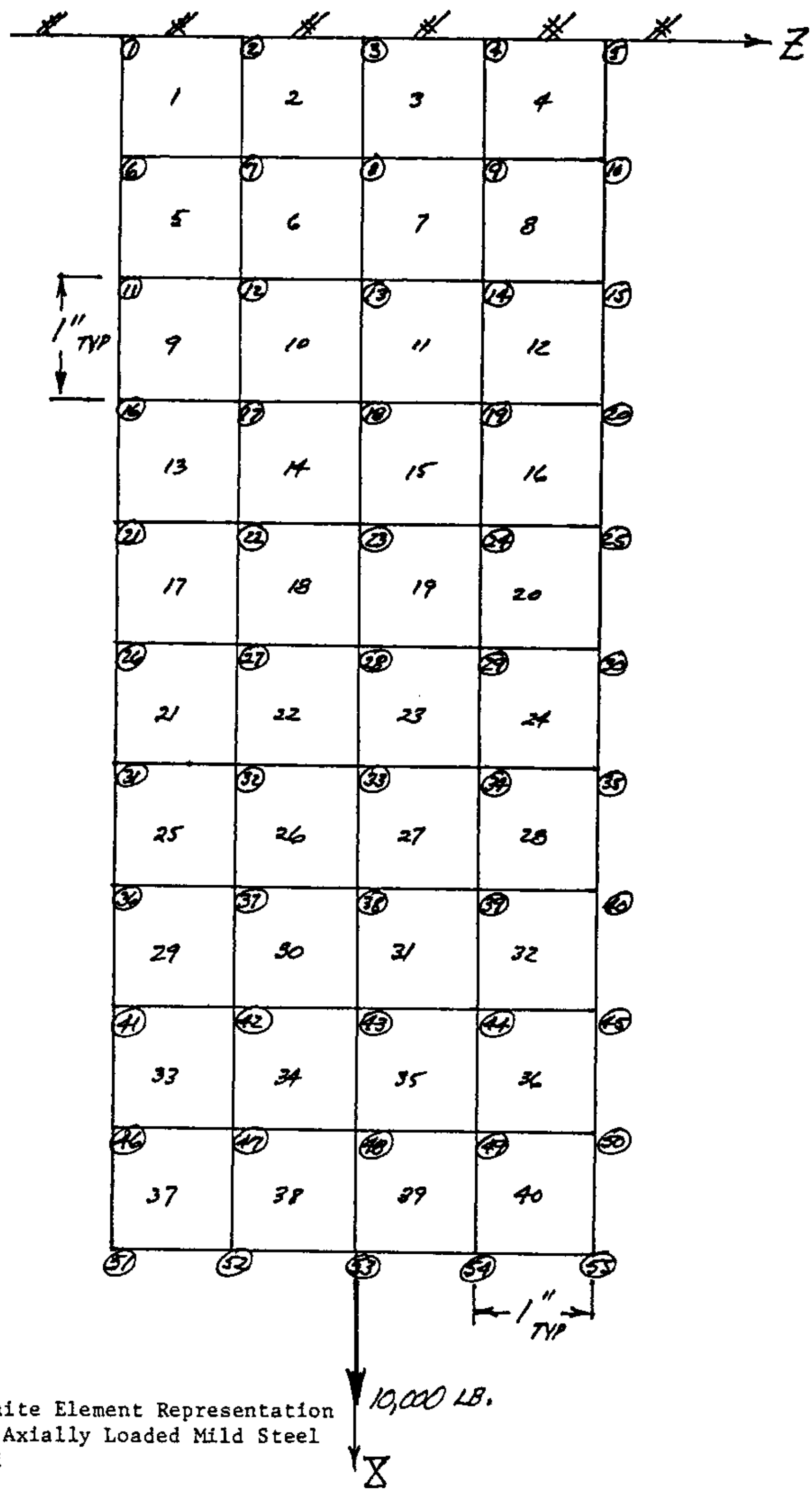


Figure 4 : Finite Element Representation of Axially Loaded Mild Steel Rod

in the X-Z plane, or rotation about the Y axis, which always must be checked. If rigid body motions are not prevented, then the solution returned by LOPE will not be well-defined.

Another reason for selection of the test problem above, is that a closed form solution for the stresses in the rod are given by Timoshenko (1970), which can be compared with the program computed values. The comparison is shown in Figure 5 at three stations along the rod, X=6.0", 8.0" and 9.0". The solid line distribution is that reported by Timoshenko, and the X values are the values obtained from the computer solution. A close correspondance between the two sets of values is evident, and by making the finite element grid finer, more finite element stresses can be computed across the width of the rod.

The input data for this problem is listed in Table 2, and the computer output is listed in Table 3.

Figure 5 : STRESS DISTRIBUTION, TEST PROBLEM #7

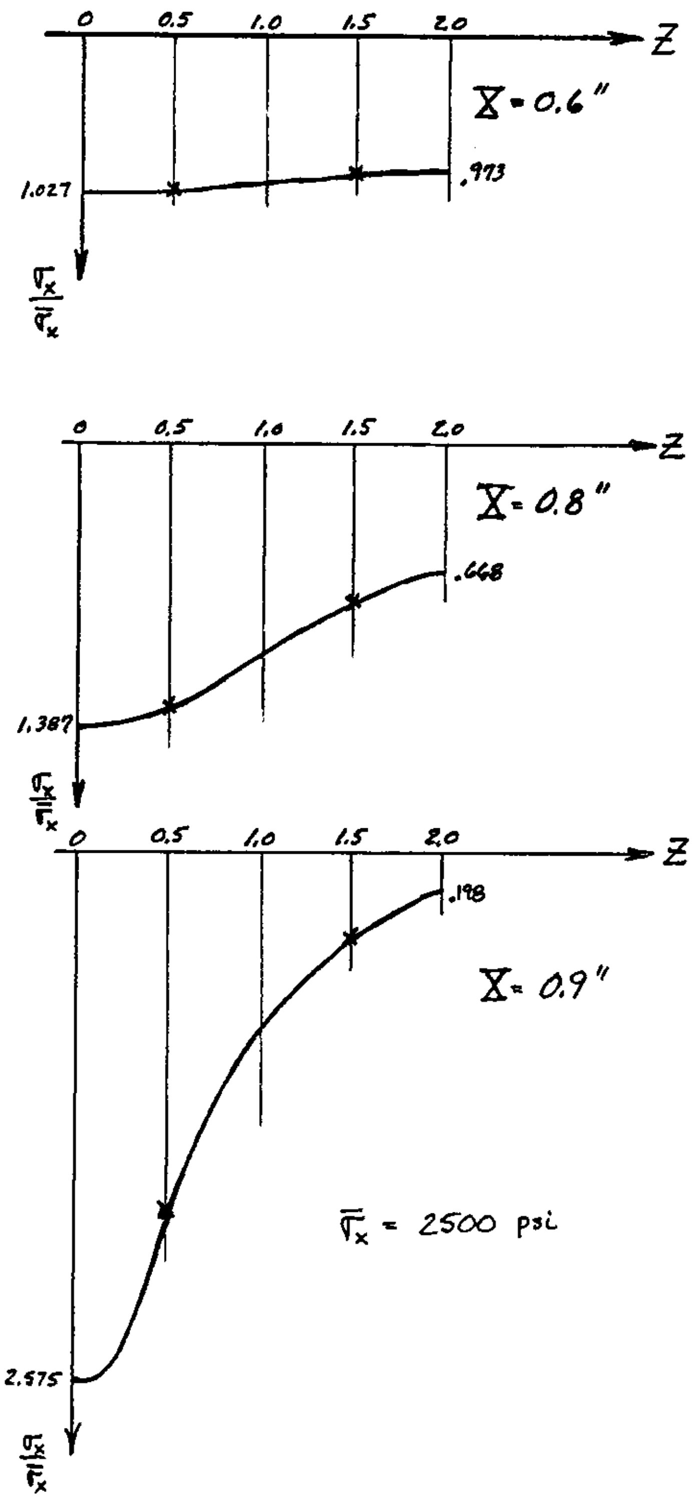


TABLE 2 : INPUT DATA FOR TEST PROBLEM #7

```
 1        LOPE TEST PROBLEM #7
 2     55   40    1    0    2       0.0        0.0
 3      1        0.0    MILD STEEL
 4   .403 E08  .173 E08  .173 E08  .403 E08  .173 E08  .403 E08  .115 E08
 5      1    1       0.0      -2.0      0.0
 6      2    1       0.0      -1.0      0.0
 7      3   11       0.0       0.0      0.0      0.0
 8      4    1       0.0       1.0      0.0
 9      5    1       0.0       2.0      0.0
10      6            1.0      -2.0
11     10            1.0       2.0
12     11            2.0      -2.0
13     15            2.0       2.0
14     16            3.0      -2.0
15     20            3.0       2.0
16     21            4.0      -2.0
17     25            4.0       2.0
18     26            5.0      -2.0
19     30            5.0       2.0
20     31            6.0      -2.0
21     35            6.0       2.0
22     36            7.0      -2.0
23     40            7.0       2.0
24     41            8.0      -2.0
25     45            8.0       2.0
26     46            9.0      -2.0
27     50            9.0       2.0
28     51           10.0      -2.0
29     52           10.0      -1.0
30     53           10.0       0.0    10000
31     54           10.0       1.0
32     55           10.0       2.0
33      1    1    6    7    2    1
34      5    6   11   12    7    1
35      9   11   16   17   12    1
36     13   16   21   22   17    1
37     17   21   26   27   22    1
38     21   26   31   32   27    1
39     25   31   36   37   32    1
40     29   36   41   42   37    1
41     33   41   46   47   42    1
42     37   46   51   52   47    1
43     40   49   54   55   50    1
```

TABLE 3 : COMPUTER OUTPUT FOR TEST PROBLEM #7

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```
           LOPE TEST PROBLEM #7
                        PLANE STRESS PROBLEM

      NUMBER OF NODAL POINTS----------   55
      NUMBER OF ELEMENTS--------------   40
      NUMBER OF DIFFERENT MATERIALS---    1
      NUMBER OF PRES/SHEAR CONDITIONS-    0
      X1 ACCELERATION-----------------   0.0000E-01
      X3 ACCELERATION-----------------   0.0000E-01
```

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```
                 MATERIAL DATA
1               MILD STEEL
   C11=  4.03E+07  C12=  1.73E+07  C13=  1.73E+07  C22=  4.03E+07
   C23=  1.73E+07  C33=  4.03E+07  C44=  1.15E+07  DENSITY=  0.00E-01
                        PLANE STRESS REDUCTION OF MATR. COEF.
   C11=  3.29E+07  C12=  0.00E-01  C13=  9.87E+06  C22=  0.00E-01
   C23=  0.00E-01  C33=  3.29E+07  C44=  1.15E+07  DENSITY=  0.00E-01
```

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NODE PNT	CODE	X1-COORD	X3-COORD	X1-LD OR DISP	X3-LD OR DISP	ANGLE
1	1.	0.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
2	1.	0.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
3	11.	0.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
4	1.	0.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
5	1.	0.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
6	0.	1.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
7	0.	1.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
8	0.	1.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
9	0.	1.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
10	0.	1.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
11	0.	2.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
12	0.	2.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
13	0.	2.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
14	0.	2.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
15	0.	2.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
16	0.	3.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
17	0.	3.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
18	0.	3.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
19	0.	3.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
20	0.	3.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
21	0.	4.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
22	0.	4.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
23	0.	4.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
24	0.	4.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
25	0.	4.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
26	0.	5.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
27	0.	5.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
28	0.	5.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
29	0.	5.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
30	0.	5.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
31	0.	6.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
32	0.	6.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
33	0.	6.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
34	0.	6.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
35	0.	6.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
36	0.	7.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
37	0.	7.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
38	0.	7.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
39	0.	7.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
40	0.	7.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
41	0.	8.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
42	0.	8.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
43	0.	8.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
44	0.	8.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
45	0.	8.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
46	0.	9.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
47	0.	9.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
48	0.	9.0000	0.0000	0.0000E-01	0.0000E-01	0.0000E-01
49	0.	9.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
50	0.	9.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01
51	0.	10.0000	-2.0000	0.0000E-01	0.0000E-01	0.0000E-01
52	0.	10.0000	-1.0000	0.0000E-01	0.0000E-01	0.0000E-01
53	0.	10.0000	0.0000	1.0000E+04	0.0000E-01	0.0000E-01
54	0.	10.0000	1.0000	0.0000E-01	0.0000E-01	0.0000E-01
55	0.	10.0000	2.0000	0.0000E-01	0.0000E-01	0.0000E-01

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ELEMENT	I	J	K	L	MATERIAL	ANGLE
1	1	6	7	2	1	0.0000E-01
2	2	7	8	3	1	0.0000E-01
3	3	8	9	4	1	0.0000E-01
4	4	9	10	5	1	0.0000E-01
5	6	11	12	7	1	0.0000E-01
6	7	12	13	8	1	0.0000E-01
7	8	13	14	9	1	0.0000E-01
8	9	14	15	10	1	0.0000E-01
9	11	16	17	12	1	0.0000E-01
10	12	17	18	13	1	0.0000E-01
11	13	18	19	14	1	0.0000E-01
12	14	19	20	15	1	0.0000E-01
13	16	21	22	17	1	0.0000E-01
14	17	22	23	18	1	0.0000E-01
15	18	23	24	19	1	0.0000E-01
16	19	24	25	20	1	0.0000E-01
17	21	26	27	22	1	0.0000E-01
18	22	27	28	23	1	0.0000E-01
19	23	28	29	24	1	0.0000E-01
20	24	29	30	25	1	0.0000E-01
21	26	31	32	27	1	0.0000E-01
22	27	32	33	28	1	0.0000E-01
23	28	33	34	29	1	0.0000E-01
24	29	34	35	30	1	0.0000E-01
25	31	36	37	32	1	0.0000E-01
26	32	37	38	33	1	0.0000E-01
27	33	38	39	34	1	0.0000E-01
28	34	39	40	35	1	0.0000E-01
29	36	41	42	37	1	0.0000E-01
30	37	42	43	38	1	0.0000E-01
31	38	43	44	39	1	0.0000E-01
32	39	44	45	40	1	0.0000E-01
33	41	46	47	42	1	0.0000E-01
34	42	47	48	43	1	0.0000E-01
35	43	48	49	44	1	0.0000E-01
36	44	49	50	45	1	0.0000E-01
37	46	51	52	47	1	0.0000E-01
38	47	52	53	48	1	0.0000E-01
39	48	53	54	49	1	0.0000E-01
40	49	54	55	50	1	0.0000E-01

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NP	X1	X3	U1	U3
1	0.00	-2.00	0.0000E-01	5.0207E-05
2	0.00	-1.00	0.0000E-01	2.5101E-05
3	0.00	0.00	0.0000E-01	0.0000E-01
4	0.00	1.00	0.0000E-01	-2.5099E-05
5	0.00	2.00	0.0000E-01	-5.0204E-05
6	1.00	-2.00	8.3599E-05	5.0186E-05
7	1.00	-1.00	8.3609E-05	2.5083E-05
8	1.00	0.00	8.3600E-05	2.1109E-09
9	1.00	1.00	8.3609E-05	-2.5079E-05
10	1.00	2.00	8.3597E-05	-5.0182E-05
11	2.00	-2.00	1.6724E-04	5.0119E-05
12	2.00	-1.00	1.6721E-04	2.5024E-05
13	2.00	0.00	1.6715E-04	3.6198E-09
14	2.00	1.00	1.6721E-04	-2.5017E-05
15	2.00	2.00	1.6724E-04	-5.0111E-05
16	3.00	-2.00	2.5103E-04	5.0037E-05
17	3.00	-1.00	2.5077E-04	2.4936E-05
18	3.00	0.00	2.5057E-04	5.3124E-09
19	3.00	1.00	2.5077E-04	-2.4926E-05
20	3.00	2.00	2.5102E-04	-5.0027E-05
21	4.00	-2.00	3.3518E-04	5.0195E-05
22	4.00	-1.00	3.3426E-04	2.5006E-05
23	4.00	0.00	3.3380E-04	6.9440E-09
24	4.00	1.00	3.3426E-04	-2.4992E-05
25	4.00	2.00	3.3517E-04	-5.0181E-05
26	5.00	-2.00	4.1984E-04	5.1502E-05
27	5.00	-1.00	4.1767E-04	2.5971E-05
28	5.00	0.00	4.1711E-04	8.3810E-09
29	5.00	1.00	4.1767E-04	-2.5954E-05
30	5.00	2.00	4.1983E-04	-5.1485E-05
31	6.00	-2.00	5.0406E-04	5.6130E-05
32	6.00	-1.00	5.0129E-04	2.9730E-05
33	6.00	0.00	5.0223E-04	9.4224E-09
34	6.00	1.00	5.0128E-04	-2.9711E-05
35	6.00	2.00	5.0406E-04	-5.6112E-05
36	7.00	-2.00	5.8258E-04	6.6783E-05
37	7.00	-1.00	5.8625E-04	3.9316E-05
38	7.00	0.00	5.9494E-04	1.0053E-08
39	7.00	1.00	5.8625E-04	-3.9296E-05
40	7.00	2.00	5.8258E-04	-6.6763E-05
41	8.00	-2.00	6.3950E-04	8.0572E-05
42	8.00	-1.00	6.7532E-04	5.3509E-05
43	8.00	0.00	7.0836E-04	1.0617E-08
44	8.00	1.00	6.7532E-04	-5.3488E-05
45	8.00	2.00	6.3950E-04	-8.0551E-05
46	9.00	-2.00	6.4661E-04	4.9492E-05
47	9.00	-1.00	7.5989E-04	5.8812E-05
48	9.00	0.00	8.7538E-04	1.1197E-08
49	9.00	1.00	7.5989E-04	-5.8790E-05
50	9.00	2.00	6.4661E-04	-4.9470E-05
51	10.00	-2.00	6.4642E-04	-5.5701E-05
52	10.00	-1.00	7.5892E-04	-5.7425E-05
53	10.00	0.00	1.1799E-03	1.1411E-08
54	10.00	1.00	7.5892E-04	5.7448E-05
55	10.00	2.00	6.4642E-04	5.5724E-05

ELM	X1 MATERIAL	X3	1-STRESS 1-STRAIN	2-STRESS 2-STRAIN	3-STRESS 3-STRAIN	13-STRESS 13-STRAIN	MAX-STRESS MAX-STRAIN	MIN-STRESS MIN-STRAIN	ANGLE ANGLE	JK-STRESS JK-SHEAR
1	0.50	-1.50	2.500E+03	0.000E-01	1.943E-01	-1.607E-01	2.500E+03	1.948E-01	-0.04	2.500E+03
	1		8.360E-05	0.000E-01	-2.510E-05	-1.398E-08	8.360E-05	-2.510E-05	-0.04	-1.599E-01
2	0.50	-0.50	2.501E+03	0.000E-01	6.345E-01	-1.451E-01	2.501E+03	6.355E-01	-0.03	2.501E+03
	1		8.360E-05	0.000E-01	-2.509E-05	-1.262E-08	8.360E-05	-2.509E-05	-0.03	-1.443E-01
3	0.50	0.50	2.501E+03	0.000E-01	6.650E-01	1.739E-01	2.501E+03	6.655E-01	0.04	2.501E+03
	1		8.360E-05	0.000E-01	-2.509E-05	1.512E-08	8.360E-05	-2.509E-05	0.04	1.747E-01
4	0.50	1.50	2.500E+03	0.000E-01	1.978E-01	1.713E-01	2.500E+03	1.987E-01	0.04	2.500E+03
	1		8.360E-05	0.000E-01	-2.510E-05	1.489E-08	8.360E-05	-2.510E-05	0.04	1.721E-01
5	1.50	-1.50	2.501E+03	0.000E-01	5.308E-01	-8.478E-01	2.501E+03	5.317E-01	-0.19	2.501E+03
	1		8.362E-05	0.000E-01	-2.510E-05	-7.372E-08	8.362E-05	-2.510E-05	-0.19	-8.470E-01
6	1.50	-0.50	2.500E+03	0.000E-01	1.670E+00	-7.189E-01	2.500E+03	1.671E+00	-0.16	2.500E+03
	1		8.357E-05	0.000E-01	-2.505E-05	-6.251E-08	8.357E-05	-2.505E-05	-0.16	-7.181E-01
7	1.50	0.50	2.500E+03	0.000E-01	1.665E+00	7.413E-01	2.500E+03	1.666E+00	0.17	2.500E+03
	1		8.357E-05	0.000E-01	-2.505E-05	6.446E-08	8.357E-05	-2.505E-05	0.17	7.422E-01
8	1.50	1.50	2.501E+03	0.000E-01	5.261E-01	8.623E-01	2.501E+03	5.266E-01	0.20	2.501E+03
	1		8.362E-05	0.000E-01	-2.510E-05	7.499E-08	8.362E-05	-2.510E-05	0.20	8.632E-01
9	2.50	-1.50	2.503E+03	0.000E-01	1.117E+00	-2.615E+00	2.503E+03	1.116E+00	-0.60	2.503E+03
	1		8.368E-05	0.000E-01	-2.510E-05	-2.274E-07	8.368E-05	-2.510E-05	-0.60	-2.614E+00
10	2.50	-0.50	2.498E+03	0.000E-01	3.339E+00	-1.952E+00	2.498E+03	3.338E+00	-0.45	2.498E+03
	1		8.349E-05	0.000E-01	-2.498E-05	-1.698E-07	8.349E-05	-2.498E-05	-0.45	-1.951E+00
11	2.50	0.50	2.498E+03	0.000E-01	3.334E+00	1.979E+00	2.498E+03	3.334E+00	0.45	2.498E+03
	1		8.349E-05	0.000E-01	-2.498E-05	1.721E-07	8.349E-05	-2.498E-05	0.45	1.980E+00
12	2.50	1.50	2.503E+03	0.000E-01	1.112E+00	2.623E+00	2.503E+03	1.110E+00	0.60	2.503E+03
	1		8.368E-05	0.000E-01	-2.510E-05	2.281E-07	8.368E-05	-2.510E-05	0.60	2.624E+00
13	3.50	-1.50	2.507E+03	0.000E-01	9.927E-01	-5.427E+00	2.507E+03	9.814E-01	-1.24	2.507E+03
	1		8.382E-05	0.000E-01	-2.514E-05	-4.720E-07	8.382E-05	-2.515E-05	-1.24	-5.427E+00
14	3.50	-0.50	2.494E+03	0.000E-01	2.355E+00	-3.358E+00	2.494E+03	2.351E+00	-0.77	2.494E+03
	1		8.336E-05	0.000E-01	-2.497E-05	-2.920E-07	8.336E-05	-2.497E-05	-0.77	-3.357E+00
15	3.50	0.50	2.494E+03	0.000E-01	2.353E+00	3.389E+00	2.494E+03	2.349E+00	0.78	2.494E+03
	1		8.336E-05	0.000E-01	-2.497E-05	2.947E-07	8.336E-05	-2.497E-05	0.78	3.389E+00
16	3.50	1.50	2.507E+03	0.000E-01	9.929E-01	5.431E+00	2.507E+03	9.822E-01	1.24	2.507E+03
	1		8.382E-05	0.000E-01	-2.514E-05	4.723E-07	8.382E-05	-2.515E-05	1.24	5.432E+00

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ELM	X1 / MATERIAL	X3	1-STRESS / 1-STRAIN	2-STRESS / 2-STRAIN	3-STRESS / 3-STRAIN	13-STRESS / 13-STRAIN	MAX-STRESS / MAX-STRAIN	MIN-STRESS / MIN-STRAIN	ANGLE / ANGLE	JK-STRESS / JK-SHEAR
17	4.50	-1.50	2.512E+03	0.000E-01	-3.942E+00	-4.649E+00	2.512E+03	-3.949E+00	-1.06	2.512E+03
	1		8.404E-05	0.000E-01	-2.536E-05	-4.043E-07	8.404E-05	-2.536E-05	-1.06	-4.648E+00
18	4.50	-0.50	2.489E+03	0.000E-01	-1.459E+01	-3.114E-01	2.489E+03	-1.459E+01	-0.07	2.489E+03
	1		8.336E-05	0.000E-01	-2.548E-05	-2.708E-08	8.336E-05	-2.548E-05	-0.07	-3.106E-01
19	4.50	0.50	2.489E+03	0.000E-01	-1.458E+01	3.333E-01	2.489E+03	-1.458E+01	0.08	2.489E+03
	1		8.336E-05	0.000E-01	-2.548E-05	2.898E-08	8.336E-05	-2.548E-05	0.08	3.341E-01
20	4.50	1.50	2.512E+03	0.000E-01	-3.941E+00	4.654E+00	2.512E+03	-3.949E+00	1.06	2.512E+03
	1		8.404E-05	0.000E-01	-2.536E-05	4.047E-07	8.404E-05	-2.536E-05	1.06	4.655E+00
21	5.50	-1.50	2.502E+03	0.000E-01	-2.501E+01	1.981E+01	2.502E+03	-2.517E+01	4.49	2.502E+03
	1		8.392E-05	0.000E-01	-2.597E-05	1.723E-06	8.393E-05	-2.597E-05	4.49	1.981E+01
22	5.50	-0.50	2.498E+03	0.000E-01	-8.227E+01	2.381E+01	2.499E+03	-8.249E+01	5.29	2.498E+03
	1		8.436E-05	0.000E-01	-2.784E-05	2.071E-06	8.437E-05	-2.785E-05	5.29	2.381E+01
23	5.50	0.50	2.498E+03	0.000E-01	-8.227E+01	-2.380E+01	2.499E+03	-8.249E+01	-5.28	2.498E+03
	1		8.437E-05	0.000E-01	-2.784E-05	-2.070E-06	8.437E-05	-2.785E-05	-5.28	-2.380E+01
24	5.50	1.50	2.502E+03	0.000E-01	-2.501E+01	-1.980E+01	2.503E+03	-2.516E+01	-4.49	2.502E+03
	1		8.392E-05	0.000E-01	-2.597E-05	-1.722E-06	8.393E-05	-2.597E-05	-4.49	-1.980E+01
25	6.50	-1.50	2.421E+03	0.000E-01	-7.836E+01	1.215E+02	2.427E+03	-8.425E+01	27.77	2.421E+03
	1		8.174E-05	0.000E-01	-2.693E-05	1.057E-05	8.199E-05	-2.719E-05	27.77	1.215E+02
26	6.50	-0.50	2.580E+03	0.000E-01	-2.574E+02	1.105E+02	2.584E+03	-2.617E+02	22.27	2.580E+03
	1		8.884E-05	0.000E-01	-3.451E-05	9.609E-06	8.902E-05	-3.470E-05	22.27	1.105E+02
27	6.50	0.50	2.580E+03	0.000E-01	-2.574E+02	-1.105E+02	2.584E+03	-2.617E+02	-22.27	2.580E+03
	1		8.884E-05	0.000E-01	-3.451E-05	-9.609E-06	8.902E-05	-3.470E-05	-22.27	-1.105E+02
28	6.50	1.50	2.421E+03	0.000E-01	-7.837E+01	-1.215E+02	2.427E+03	-8.426E+01	-27.76	2.421E+03
	1		8.174E-05	0.000E-01	-2.693E-05	-1.056E-05	8.199E-05	-2.719E-05	-27.76	-1.215E+02
29	7.50	-1.50	2.130E+03	0.000E-01	-1.756E+02	3.880E+02	2.194E+03	-2.391E+02	92.99	2.130E+03
	1		7.300E-05	0.000E-01	-2.727E-05	3.374E-05	7.576E-05	-3.003E-05	92.99	3.880E+02
30	7.50	-0.50	2.870E+03	0.000E-01	-5.258E+02	3.215E+02	2.900E+03	-5.559E+02	53.62	2.870E+03
	1		1.012E-04	0.000E-01	-4.640E-05	2.796E-05	1.026E-04	-4.771E-05	53.62	3.215E+02
31	7.50	0.50	2.870E+03	0.000E-01	-5.258E+02	-3.215E+02	2.900E+03	-5.559E+02	-53.62	2.870E+03
	1		1.012E-04	0.000E-01	-4.640E-05	-2.796E-05	1.026E-04	-4.771E-05	-53.62	-3.215E+02
32	7.50	1.50	2.130E+03	0.000E-01	-1.756E+02	-3.880E+02	2.194E+03	-2.391E+02	-92.99	2.130E+03
	1		7.300E-05	0.000E-01	-2.727E-05	-3.374E-05	7.576E-05	-3.003E-05	-92.99	-3.880E+02

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ELM	X1 / MATERIAL	X3	1-STRESS / 1-STRAIN	2-STRESS / 2-STRAIN	3-STRESS / 3-STRAIN	13-STRESS / 13-STRAIN	MAX-STRESS / MAX-STRAIN	MIN-STRESS / MIN-STRAIN	ANGLE / ANGLE	JK-STRESS / JK-SHEAR
33	8.50	-1.50	1.419E+03	0.000E-01	1.609E+02	7.091E+02	1.738E+03	-1.579E+02	242.09	1.419E+03
	1		4.584E-05	0.000E-01	-8.872E-06	6.166E-05	5.970E-05	-2.273E-05	242.09	7.091E+02
34	8.50	-0.50	3.581E+03	0.000E-01	-6.038E+02	8.845E+02	3.760E+03	-7.830E+02	114.58	3.581E+03
	1		1.258E-04	0.000E-01	-5.615E-05	7.692E-05	1.336E-04	-6.394E-05	114.58	8.845E+02
35	8.50	0.50	3.581E+03	0.000E-01	-6.038E+02	-8.845E+02	3.760E+03	-7.830E+02	-114.58	3.581E+03
	1		1.258E-04	0.000E-01	-5.615E-05	-7.691E-05	1.336E-04	-6.394E-05	-114.58	-8.845E+02
36	8.50	1.50	1.419E+03	0.000E-01	1.609E+02	-7.091E+02	1.738E+03	-1.579E+02	-242.09	1.419E+03
	1		4.584E-05	0.000E-01	-8.872E-06	-6.166E-05	5.970E-05	-2.273E-05	-242.09	-7.091E+02
37	9.50	-1.50	1.839E+01	0.000E-01	1.191E+02	2.503E+01	1.250E+02	1.251E+01	767.87	1.839E+01
	1		-5.814E-07	0.000E-01	3.798E-06	2.176E-06	4.053E-06	-8.369E-07	767.87	2.503E+01
38	9.50	-0.50	4.982E+03	0.000E-01	1.476E+03	2.416E+03	6.214E+03	2.438E+02	270.19	4.982E+03
	1		1.517E-04	0.000E-01	-6.823E-07	2.101E-04	2.053E-04	-5.425E-05	270.19	2.416E+03
39	9.50	0.50	4.982E+03	0.000E-01	1.476E+03	-2.416E+03	6.214E+03	2.438E+02	-270.19	4.982E+03
	1		1.517E-04	0.000E-01	-6.823E-07	-2.101E-04	2.053E-04	-5.425E-05	-270.19	-2.416E+03
40	9.50	1.50	1.839E+01	0.000E-01	1.191E+02	-2.503E+01	1.250E+02	1.251E+01	-767.88	1.839E+01
	1		-5.814E-07	0.000E-01	3.798E-06	-2.176E-06	4.053E-06	-8.368E-07	-767.88	-2.503E+01

COMPUTATIONS COMPLETED

Additional test problems were run to checkout other functions and options of the computer code. Results of these tests are summarized briefly below. For each problem the input data is listed for reference purposes. All test problems use the same grid system as problem # 7, with only loading or constraint conditions changed.

LOPE Test Problem #8

In this test the concentrated force was replaced by a compressive pressure loading along the unsupported bottom edge of the rod. Thus, the rod is loaded uniformly and the stress in all elements should be $\sigma_x = 2500$ psi, with all other components zero. This result was obtained from the computer solution, but is not listed; however the input data for this problem is listed in Table 4. The nodal displacements were also constant across the rod and became progressively larger with distance X along the rod. The displacement state is shown by the dashed line grid in Figure 6. The theoretically computed displacement of the loaded end of the rod is

$$\delta = -\frac{PL}{AE} = -\frac{2500(4\times 10)}{4(30\times 10^6)} = -8.3333\times 10^{-4}\ in.$$

which compares well with the computer predicted value $\delta = -8.3604 \times 10^{-4}$ in.

A second version of the uniformly loaded rod was run to checkout the gridpoint constraint function of the program. The uniform pressure was applied as before, but gridpoint #51 was constrained to move at a 45° angle to the axis of loading. This caused the distortion in displacement of the rod, as noted in Figure 7. No quantitative analytical results are known for this test case, only that the displacement state conforms to the constraint condition that was imposed. The input data for this problem is listed in Table 5.

LOPE Test Problem #9

This problem tests the application of a shear boundary stress, similar to

TABLE 4 : COMPUTER INPUT DATA FOR TEST PROBLEM #8

1	LOPE TEST PROBLEM #8									
2	55	40	1	4	2		0.0	0.0		
3	1		0.0	MILD STEEL						
4	.403 E08		.173 E08		.173 E08		.403 E08	.173 E08	.403 E08	.115 E08
5	1	1		0.0		-2.0	0.0			
6	2	1		0.0		-1.0	0.0			
7	3	11		0.0		0.0	0.0	0.0		
8	4	1		0.0		1.0	0.0			
9	5	1		0.0		2.0	0.0			
10	6			1.0		-2.0				
11	10			1.0		2.0				
12	11			2.0		-2.0				
13	15			2.0		2.0				
14	16			3.0		-2.0				
15	20			3.0		2.0				
16	21			4.0		-2.0				
17	25			4.0		2.0				
18	26			5.0		-2.0				
19	30			5.0		2.0				
20	31			6.0		-2.0				
21	35			6.0		2.0				
22	36			7.0		-2.0				
23	40			7.0		2.0				
24	41			8.0		-2.0				
25	45			8.0		2.0				
26	46			9.0		-2.0				
27	50			9.0		2.0				
28	51			10.0		-2.0				
29	52			10.0		-1.0				
30	53			10.0		0.0				
31	54			10.0		1.0				
32	55			10.0		2.0				
33	1	1	6	7	2	1				
34	5	6	11	12	7	1				
35	9	11	16	17	12	1				
36	13	16	21	22	17	1				
37	17	21	26	27	22	1				
38	21	26	31	32	27	1				
39	25	31	36	37	32	1				
40	29	36	41	42	37	1				
41	33	41	46	47	42	1				
42	37	46	51	52	47	1				
43	40	49	54	55	50	1				
44	51	52		2500						
45	52	53		2500						
46	53	54		2500						
47	54	55		2500						

Figure 6 : DISPLACEMENT STATE OF UNIFORMLY LOADED ROD

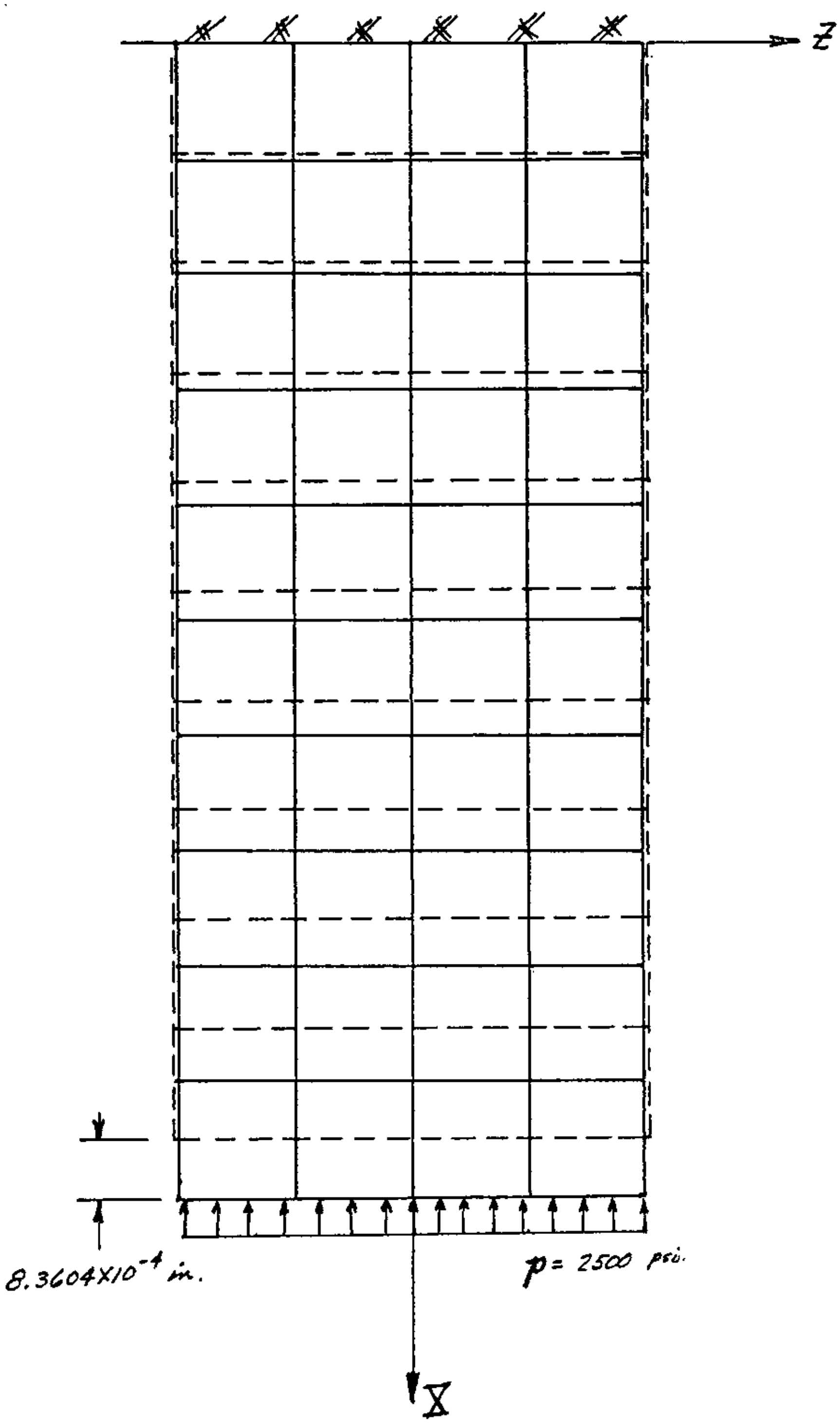

Figure 7 : UNIFORMLY LOADED ROD WITH DISPLACEMENT CONSTRAINT CONDITION

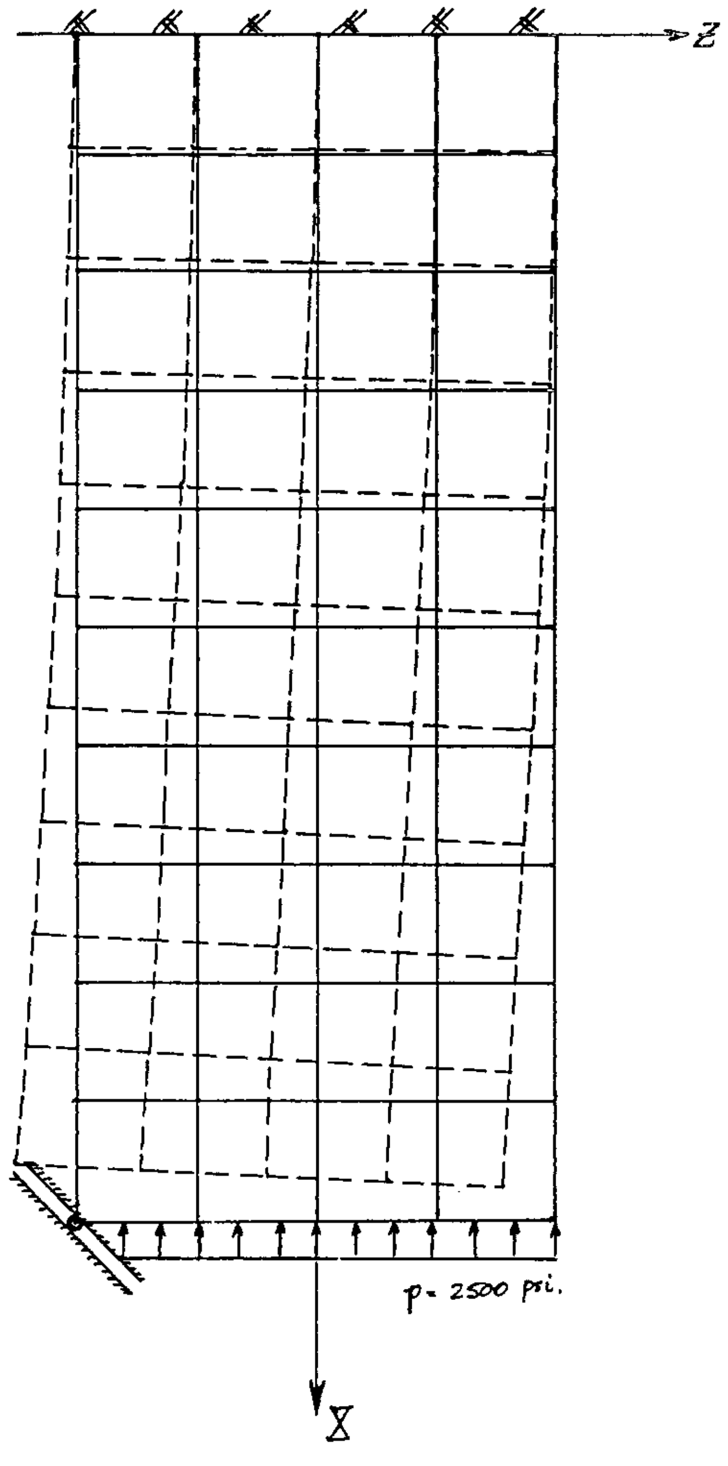

TABLE 5 : LISTING OF INPUT DATA FOR TEST PROBLEM
#8, WITH A CONSTRAINED NODAL POINT

```
                 LOPE TEST PROBLEM #8 - WITH CONSTRAINED NODE
               55   40    1     4    2       0.0       0.0
                1         0.0      MILD STEEL
             .403 E08  .173 E08  .173 E08  .403 E08  .173 E08  .403 E08  .115 E08
                1    1        0.0    -2.0      0.0
                2    1        0.0    -1.0      0.0
                3   11        0.0     0.0      0.0       0.0
                4    1        0.0     1.0      0.0
                5    1        0.0     2.0      0.0
                6             1.0    -2.0
               10             1.0     2.0
               11             2.0    -2.0
               15             2.0     2.0
               16             3.0    -2.0
               20             3.0     2.0
               21             4.0    -2.0
               25             4.0     2.0
               26             5.0    -2.0
               30             5.0     2.0
               31             6.0    -2.0
               35             6.0     2.0
               36             7.0    -2.0
               40             7.0     2.0
               41             8.0    -2.0
               45             8.0     2.0
               46             9.0    -2.0
               50             9.0     2.0
               51            10.0    -2.0                               45
               52            10.0    -1.0
               53            10.0     0.0
               54            10.0     1.0
               55            10.0     2.0
                1    1    6     7    2    1
                5    6   11    12    7    1
                9   11   16    17   12    1
               13   16   21    22   17    1
               17   21   26    27   22    1
               21   26   31    32   27    1
               25   31   36    37   32    1
               29   36   41    42   37    1
               33   41   46    47   42    1
               37   46   51    52   47    1
               40   49   54    55   50    1
               51   52       2500
               52   53       2500
               53   54       2500
               54   55       2500
```

the normal boundary pressure of problem #8. The grid layout, loading, and displacement state are shown in Figure 8. The problem is basically a cantilever beam subjected to a 10,000LB force at its free end. For this configuration the theoretical analytical maximum deflection at the end of the beam is

$$\delta = \frac{PL^3}{3EI} = \frac{10{,}000\,(10)^3}{3(30\times10^6)\frac{(1)(4)^3}{12}} = 2.08\times10^{-2}\ in.$$

compared to the computer determined value of $S = 2.2743 \times 10^{-2}$ in., an error of 9.3%. Part of this error is likely due to the type of cantilever support condition assumed, in which the beam is free to expand in the Z direction except at the centerline gridpoint (#3). Input data for this problem is listed in Table 6.

LOPE Test Problem #10

This test problem involved the evaluation of the axial rod of test problem #7 under gravitational loading along the X axis. For this calculation acceleration along the X axis was input as ACEL1 = 384.4 in/sec^2, and density of the material was set at $\rho = 0.001$ lb-sec^2/in^4.

Only the input data listing is given for this problem (Table 7). The results obtained by the computer calculation, agree well with the exact solution to the problem. The axial stress computed at the centroid of the finite elements next to the upper support was $\sigma_x = 3.62$ psi. compared to the exact value of $\sigma_x = 3.65$ psi. Displacement at the lower, free end of the rod by computer solution was $\delta = 6.380 \times 10^{-7}$in, compared to the exact solution of $\delta = 6.40 \times 10^{-7}$ in.

References

Goudreau, G.L., Nickeu, R.E., Dunham, R.S., 1967.'Plane and axisymmetric finite element analysis of locally orthotropic elastic solids and orthotropic shells', University of California, Berkeley, California, Report 67-15.

Figure 8 : LOADING AND DISPLACEMENT STATE TEST PROBLEM #9

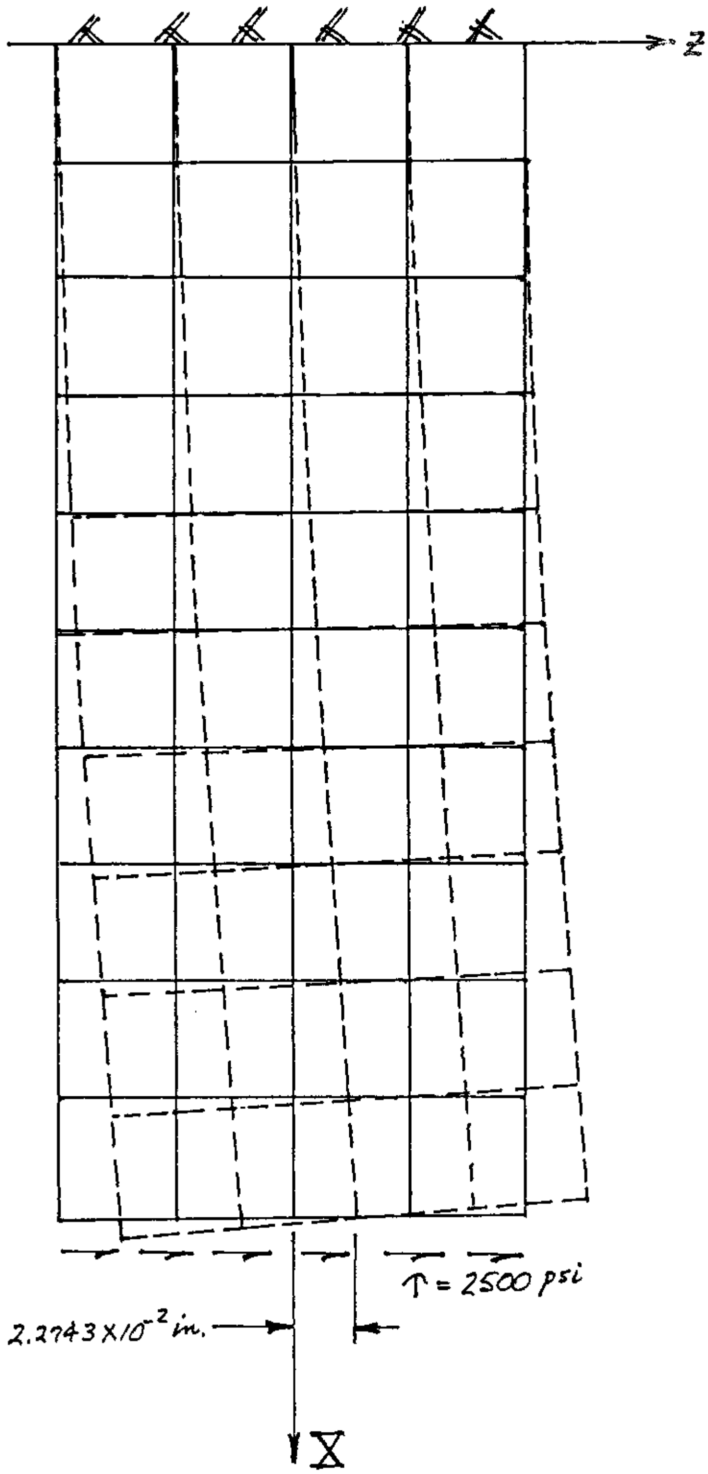

TABLE 6 : INPUT DATA FOR TEST PROBLEM #9

```
          LOPE TEST PROBLEM #9
        55   40    1    4    2       0.0        0.0
         1         0.0     MILD STEEL
      .403 E08  .173 E08  .173 E08  .403 E08  .173 E08  .403 E08  .115 E08
         1    1        0.0      -2.0       0.0
         2    1        0.0      -1.0       0.0
         3   11        0.0       0.0       0.0        0.0
         4    1        0.0       1.0       0.0
         5    1        0.0       2.0       0.0
         6             1.0      -2.0
        10             1.0       2.0
        11             2.0      -2.0
        15             2.0       2.0
        16             3.0      -2.0
        20             3.0       2.0
        21             4.0      -2.0
        25             4.0       2.0
        26             5.0      -2.0
        30             5.0       2.0
        31             6.0      -2.0
        35             6.0       2.0
        36             7.0      -2.0
        40             7.0       2.0
        41             8.0      -2.0
        45             8.0       2.0
        46             9.0      -2.0
        50             9.0       2.0
        51            10.0      -2.0
        52            10.0      -1.0
        53            10.0       0.0
        54            10.0       1.0
        55            10.0       2.0
         1    1    6    7    2       1
         5    6   11   12    7       1
         9   11   16   17   12       1
        13   16   21   22   17       1
        17   21   26   27   22       1
        21   26   31   32   27       1
        25   31   36   37   32       1
        29   36   41   42   37       1
        33   41   46   47   42       1
        37   46   51   52   47       1
        40   49   54   55   50       1
        51   52                   2500
        52   53                   2500
        53   54                   2500
        54   55                   2500
```

TABLE 7 : INPUT DATA FOR TEST PROBLEM #10

```
   LOPE TEST PROBLEM #10
 55   40    1     0    2    3
 1-     0.001     MILD STEEL
.403 E08  .173 E08  .173 E08
  1    1        0.0      -2.0
  2    1        0.0      -1.0
  3   11        0.0       0.0
  4    1        0.0       1.0
  5    1        0.0       2.0
  6             1.0      -2.0
 10             1.0       2.0
 11             2.0      -2.0
 15             2.0       2.0
 16             3.0      -2.0
 20             3.0       2.0
 21             4.0      -2.0
 25             4.0       2.0
 26             5.0      -2.0
 30             5.0       2.0
 31             6.0      -2.0
 35             6.0       2.0
 36             7.0      -2.0
 40             7.0       2.0
 41             8.0      -2.0
 45             8.0       2.0
 46             9.0      -2.0
 50             9.0       2.0
 51            10.0      -2.0
 52            10.0      -1.0
 53            10.0       0.0
 54            10.0       1.0
 55            10.0       2.0
  1    1    6     7    2    1
  5    6   11    12    7    1
  9   11   16    17   12    1
 13   16   21    22   17    1
 17   21   26    27   22    1
 21   26   31    32   27    1
 25   31   36    37   32    1
 29   36   41    42   37    1
 33   41   46    47   42    1
 37   46   51    52   47    1
 40   49   54    55   50    1
```

Timoshenko, S.P. and Goodier, J.N., 1970. Theory of elasticity, third edition, M Graw-Hill, N.Y., New York, p.59.

APPENDIX

MAIN PROGRAM AND SUBROUTINE LISTING

```
REAL TIME FORTRAN VER.E00  PAGE  1        DATE 83  08  23

C     *  LOCAL ORTHOTROPIC PLANAR ELASTICITY PROGRAM
      DIMENSION LM(4),EPS(8),SIG(8)
      COMMON A(54,108),B(108)
      COMMON/NPELD/E(7,10),RO(10),CODE(600),X(600),Y(600),
     *UX(1200),IX(500,5),ANGLE(500),NUMNP,NUMEL,NUMPC
      COMMON/QADTRI/XXX(5),YYY(5),C(4,4),S(10,10),P(10)
      DATA PII/28.648/
C     *     *     *     *     PARAMETER INITIALIZATION
      NB=27
      ISTOP=0
      NUMBLK=0
      IJK=0
      NEL=0
      CALL LOCATE(NPP,ISTOP,ACEL1,ACEL3,NB,MBAND)
      IF(ISTOP.NE.0) GO TO 999
C     *     *     *     *     *  OTHER INITIALIZATION
      ND=2*NB
      ND2=4*NB
      DO 5 N=1,ND2
      B(N)=0.0
      DO 5 M=1,ND
    5 A(M,N)=0.0
C     *     *     *    FORM STIFFNESS MATRIX IN BLOCKS
    6 NUMBLK=NUMBLK+1
      NM=NB*NUMBLK
      NL=NM-NB+1
      NT=NM+MBAND
      NEL=NEL+1
      KSHIFT=2*NL-2
      IF(NM.GT.NUMNP) NM=NUMNP
      IF(NEL.GE.NUMEL)GO TO 210
C     *     *     *     *     * SELECT ELEMENT IN BLOCK
      DO 210 N=NEL,NUMEL
      MT=IX(N,5)
      IF(MT.GT.0) GO TO 10
      WRITE(6,3001) N
      GO TO 210
   10 AM1=ACEL1*RO(MT)
      AM3=ACEL3*RO(MT)
      DO 7 I=1,4
      IF(IX(N,I).LT.NL) GO TO 7
      IF(IX(N,I).LE.NT) GO TO 8
    7 CONTINUE
      GO TO 210
    8 IF(N.GT.NEL) NEL=N
      DO 9 I=1,10
      P(I)=0.0
      DO 9 J=1,10
    9 S(I,J)=0.0
      IF(ISTOP.NE.0) GO TO 90
C     *     *     *     *    FORM STRESS-STRAIN RELATIONSHIP
      CALL MODULI(N,I,J,K,L)
      DO 80 M=1,4
      C(M,2)=0.0
```

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```
   80 C(2,M)=0.0
C     *     *     *     *   FORM QUADRILATERAL STIFFNESS MATRIX
   90 DO 100 M=1,4
      MM=IX(N,M)
      XXX(M)=X(MM)
  100 YYY(M)=Y(MM)
      IF(K.NE.L) GO TO 110
      CALL TRISTF(1,2,3,NPP,ISTOP,COMM,AM1,AM3)
      IF(COMM.LE.0.0) GO TO 190
      GO TO 120
  110 XXX(5)=0.25*(XXX(1)+XXX(2)+XXX(3)+XXX(4))
      YYY(5)=0.25*(YYY(1)+YYY(2)+YYY(3)+YYY(4))
      CALL TRISTF(1,2,5,NPP,ISTOP,COMM,AM1,AM3)
      IF(COMM.LE.0.0) GO TO 190
      CALL TRISTF(2,3,5,NPP,ISTOP,COMM,AM1,AM3)
      IF(COMM.LE.0.0) GO TO 190
      CALL TRISTF(3,4,5,NPP,ISTOP,COMM,AM1,AM3)
      IF(COMM.LE.0.0) GO TO 190
      CALL TRISTF(4,1,5,NPP,ISTOP,COMM,AM1,AM3)
      IF(COMM.LE.0.0) GO TO 190
  120 IF(ISTOP.NE.0) GO TO 210
C     *     *   SOLVE FOR MIDDLE NODAL POINT DISPLACEMENTS
      IF(K.EQ.L) GO TO 170
      DO 150 I=1,9
      CC=S(I,10)/S(10,10)
      P(I)=P(I)-CC*P(10)
      DO 150 J=1,9
  150 S(I,J)=S(I,J)-CC*S(10,J)
      DO 160 I=1,8
      CC=S(I,9)/S(9,9)
      P(I)=P(I)-CC*P(9)
      DO 160 J=1,8
  160 S(I,J)=S(I,J)-CC*S(9,J)
  170 CONTINUE
      GO TO 200
  190 WRITE(6,3000) N
      ISTOP=1
      IF(ISTOP.NE.0) GO TO 210
C     *     *     *   MODIFY FOR SLOPING BOUNDARY CONDITIONS
  200 DO 230 K=1,4
      M=IX(N,K)
      IF(CODE(M).GE.0.0) GO TO 230
      CC=-CODE(M)
      DX=COS(CC)
      DY=SIN(CC)
      L=2*K-1
      DEN=P(L)*DX+P(L+1)*DY
      P(L+1)=P(L+1)*DX-P(L)*DY
      P(L)=DEN
      DO 220 J=1,8,2
      IF(J.EQ.L) GO TO 220
      DEN=S(L,J)*DX+S(L+1,J)*DY
      S(L+1,J)=S(L+1,J)*DX-S(L,J)*DY
      S(L,J)=DEN
```

```
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            S(J,L+1)=S(L+1,J)
            S(J,L)=S(L,J)
            DEN=S(L,J+1)*DX+S(L+1,J+1)*DY
            S(L+1,J+1)=S(L+1,J+1)*DX-S(L,J+1)*DY
            S(L,J+1)=DEN
            S(J+1,L+1)=S(L+1,J+1)
            S(J+1,L)=S(L,J+1)
        220 CONTINUE
            DEN=S(L,L)*DX*DX+2.0*S(L+1,L)*DX*DY+S(L+1,L+1)*DY*DY
            S(L,L+1)=DX*DY*(S(L+1,L+1)-S(L,L))+S(L+1,L)*(DX*DX-DY*DY)
            S(L+1,L+1)=S(L,L)*DY*DY-2.0*S(L+1,L)*DX*DY+S(L+1,L+1)*DX*DX
            S(L,L)=DEN
            S(L+1,L)=S(L,L+1)
        230 CONTINUE
      C     *    *    *  ADD STIFFNESS AND FORCE VECTORS
            DO 240 I=1,4
        240 LM(I)=2*IX(N,I)-2
            DO 250 I=1,4
            DO 250 K=1,2
            II=LM(I)+K-KSHIFT
            IF(II) 250,250,270
        270 KK=2*I+K-2
            B(II)=B(II)+P(KK)
            DO 250 J=1,4
            DO 250 L=1,2
            JJ=LM(J)+L-II-KSHIFT+1
            LL=2*J+L-2
            IF(JJ) 250,250,260
        260 A(JJ,II)=A(JJ,II)+S(KK,LL)
        250 CONTINUE
        210 CONTINUE
            IF(ISTOP.NE.0) GO TO 390
      C     *    *  ADD CONCENTRATED FORCES, MODIFY FOR DISPL B. C.
            DO 350 N=NL,NM
            K=2*N-KSHIFT-1
            CC=CODE(N)
            IF(CC.EQ.0.0) GO TO 340
            IF(CC.EQ.1.0.OR.CC.EQ.11.0) CALL MODIFY(K,UX(2*N-1),NB,MBAND)
            IF(CC.LT.0.0.OR.CC.EQ.10.0) CALL MODIFY(K+1,UX(2*N),NB,MBAND)
            IF(CC.EQ.11.0) CALL MODIFY(K+1,UX(2*N),NB,MBAND)
        340 B(K)=B(K)+UX(2*N-1)
            B(K+1)=B(K+1)+UX(2*N)
        350 CONTINUE
      C     *    *    *    *  REDUCE BLOCK OF EQUATIONS
            DO 370 N=1,ND
            IF(A(1,N).EQ.0.0) GO TO 370
            B(N)=B(N)/A(1,N)
            DO 360 L=2,MBAND
            IF(A(L,N).EQ.0.0) GO TO 360
            CC=A(L,N)/A(1,N)
            I=N+L-1
            J=0
            DO 380 K=L,MBAND
            J=J+1
```

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  380 A(J,I)=A(J,I)-CC*A(K,N)
      B(I)=B(I)-A(L,N)*B(N)
      A(L,N)=CC
  360 CONTINUE
  370 CONTINUE
  390 IF(NM.EQ.NUMNP) GO TO 400
      IF(ISTOP.NE.0) GO TO 6
C     *    *    WRITE BLOCK OF REDUCED EQUATIONS ON DISK
      WRITE(11) (B(J),(A(I,J),I=1,MBAND),J=1,ND)
C     *    *    *  SHIFT BLOCK OF EQUATIONS UP FOR NEXT BLOCK
      DO 410 N=1,ND
      MM=ND+N
      B(N)=B(MM)
      B(MM)=0.0
      DO 410 M=1,MBAND
      A(M,N)=A(M,MM)
  410 A(M,MM)=0.0
      GO TO 6
  400 IF(ISTOP.NE.0) GO TO 999
      IF(IJK.NE.0) GO TO 405
      ENDFILE 11
      CALL F:ABST(@440,IER)
      BACKSPACE 11
  440 CONTINUE
      IJK=1
C     *    *    *    *   BACK SUBSTITUTION
  405 NU=ND*NUMBLK+1
  415 DO 430 M=1,ND
      N=ND-M+1
      CC=B(N)
      DO 420 K=2,MBAND
      L=N+K-1
  420 CC=CC-A(K,N)*B(L)
      B(N)=CC
      NM=N+ND
      B(NM)=CC
      NU=NU-1
  430 UX(NU)=CC
      NUMBLK=NUMBLK-1
      IF(NUMBLK.LE.0) GO TO 500
      BACKSPACE 11
      READ(11) (B(J),(A(I,J),I=1,MBAND),J=1,ND)
      BACKSPACE 11
      GO TO 415
C     *    *    WRITE DISPLACEMENTS OF ELASTICITY NODES
  500 WRITE(6,2000)
C     *    *    *   BOUNDARY ANGLE TRANSFORMATION
      DO 510 N=1,NUMNP
      IF(X(N).LT.0.0) GO TO 510
      CC=-CODE(N)
      IF(CC.LE.0.0) GO TO 520
      DX=COS(CC)
      DY=SIN(CC)
      DEN=UX(2*N-1)*DX-UX(2*N)*DY
```

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```
           UX(2*N)=UX(2*N-1)*DY+UX(2*N)*DX
           UX(2*N-1)=DEN
       520 WRITE(6,2001) N,X(N),Y(N),UX(2*N-1),UX(2*N)
       510 CONTINUE
C          *     *   COMPUTE AND OUTPUT ELASTICITY ELEMENT STRESSES
           MPRINT=0
           DO 600 N=1,NUMEL
           MT=IABS(IX(N,5))
C          *     *     *     FORM STRESS STRAIN RELATIONSHIP
           CALL MODULI(N,I,J,K,L)
C          *     *  CALCULATION OF STRAINS AT CENTER NODAL POINT
           IF(K.EQ.L) GO TO 610
           F=0.25
           NM=4
           UK=F*(UX(2*I-1)+UX(2*J-1)+UX(2*K-1)+UX(2*L-1))
           WK=F*(UX(2*I)+UX(2*J)+UX(2*K)+UX(2*L))
           XK=F*(X(I)+X(J)+X(K)+X(L))
           YK=F*(Y(I)+Y(J)+Y(K)+Y(L))
           GO TO 620
       610 XK=X(K)
           YK=Y(K)
           F=1.0
           NM=1
           K=2*K
           UK=UX(K-1)
           WK=UX(K)
       620 CONTINUE
           DO 630 NN=1,4
       630 EPS(NN)=0.0
           DO 640 NN=1,NM
           M=NN+1
           IF(NN.EQ.4) M=1
           I=IX(N,NN)
           J=IX(N,M)
           AJ=X(J)-X(I)
           AK=XK-X(I)
           AT=AJ-AK
           BJ=Y(J)-Y(I)
           BK=YK-Y(I)
           BT=BJ-BK
           DT=AJ*BK-AK*BJ
           DT=F/DT
           I=2*I
           J=2*J
           EPS(1)=EPS(1)+DT*(BT*UX(I-1)+BK*UX(J-1)-BJ*UK)
           EPS(3)=EPS(3)+DT*(-AT*UX(I)-AK*UX(J)+AJ*WK)
           EPS(4)=EPS(4)+DT*(-AT*UX(I-1)+BT*UX(I)-AK*UX(J-1)+BK*UX(J)+
          *AJ*UK-BJ*WK)
       640 CONTINUE
C          *     *     *     CALCULATION OF STRESSES
           DO 700 L=1,4
           SIG(L)=0.0
           DO 700 LL=1,4
       700 SIG(L)=SIG(L)+C(L,LL)*EPS(LL)
```

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```
      C     *     *     CALCULATE PRINCIPAL STRESSES AND STRAINS
            DD=(EPS(1)+EPS(3))*0.5
            DX=EPS(1)-EPS(3)
            DR=SQRT(DX**2+EPS(4)**2)*0.5
            EPS(5)=DD+DR
            EPS(6)=DD-DR
            IF(EPS(4).EQ.0.0.AND.DX.EQ.0.0) GO TO 800
            EPS(7)=PII*ATAN2(EPS(4),DX)
            CC=(SIG(1)+SIG(3))*0.5
            CX=0.5*(SIG(1)-SIG(3))
            CR =SQRT(CX*CX+SIG(4)**2)
            SIG(5)=CC+CR
            SIG(6)=CC-CR
            SIG(7)=PII*ATAN2(SIG(4),CX)
      C     *     *     *     STRESSES PARALLEL TO LINE J-K
            I=IX(N,2)
            J=IX(N,3)
            ANGL=2.0*ATAN2(Y(J)-Y(I),X(J)-X(I))
            CSA=COS(ANGL)
            SNA=SIN(ANGL)
            SIG(8)=-CX*CSA-SIG(4)*SNA+CC
            EPS(8)=CX*SNA-SIG(4)*CSA
            IF(MPRINT.GT.0) GO TO 810
            MPRINT=16
            WRITE(6,2002)
        810 WRITE(6,2003) N,XK,YK,(SIG(I),I=1,8),MT,(EPS(I),I=1,8)
        600 MPRINT=MPRINT-1
        800 CONTINUE
            WRITE(6,2004)
            WRITE(6,4000)
            WRITE(6,4000)
       2000 FORMAT(1H1,2X,'NP',11X,'X1',11X,'X3',10X,'U1',10X,'U3'/)
       2001 FORMAT(I5,2F13.2,1P2E12.4)
       2002 FORMAT(1H1,5H ELM ,5X,2HX1,6X,2HX3,4X,8H1-STRESS,4X,8H2-STRESS,
           *4X,8H3-STRESS,3X,9H13-STRESS,2X,10HMAX-STRESS,2X,
           *10HMIN-STRESS,2X,5HANGLE,3X,9HJK-STRESS/10X,8HMATERIAL,
           *7X,8H1-STRAIN,4X,8H2-STRAIN,4X,8H3-STRAIN,3X,9H13-STRAIN,
           *2X,10HMAX-STRAIN,2X,10HMIN-STRAIN,2X,5HANGLE,4X,8HJK-SHEAR)
       2003 FORMAT(1H0,I4,2F8.2,1P6E12.3,F7.2,1PE12.3/I15,6X,1P6E12.3,
           *F7.2,1PE12.3)
       2004 FORMAT(//5X,'COMPUTATIONS COMPLETED')
       3000 FORMAT(33X,'ZERO OR NEGATIVE AREA, ELEMENT=',I4)
       3001 FORMAT(10X,'MATERIAL TYPE NOT SPECIFIED FOR ELEMENT=',I4)
       4000 FORMAT(1H1)
        999 STOP
            END
```

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```
      SUBROUTINE LOCATE(NPP,ISTOP,ACEL1,ACEL3,NB,MBAND)
      DIMENSION IBC(50),JBC(50),PN(50),PT(50),XC(2),IN(2),XTYPE(25),
     *NAME(40),ANG(1)
      COMMON A(54,108),B(108)
      COMMON/NPELD/E(7,10),RO(10),CODE(600),X(600),Y(600),
     *UX(1200),IX(500,5),ANGLE(500),NUMNP,NUMEL,NUMPC
      EQUIVALENCE (IBC,A(1,1)),(JBC,A(1,5)),(PN,A(1,7)),
     *(PT,A(1,9)),(ANG,A(1,11))
      DATA PI/0.017453/,MAXNP/600/,MAXEL/500/
C     *    *    *     READ AND PRINT CONTROL INFORMATION
      READ(7,1000) NAME
      WRITE(6,2000) NAME
      READ(7,1001) NUMNP,NUMEL,NUMMAT,NUMPC,NPP,ACEL1,ACEL3
      IF(NPP.EQ.1) GO TO 7
      IF(NPP.EQ.2) GO TO 9
      WRITE(6,2014)
      ISTOP=1
      GO TO 12
    7 WRITE(6,2001)
      GO TO 12
    9 WRITE(6,2002)
   12 IF(NUMNP.LE.MAXNP) GO TO 6
      WRITE(6,3000) MAXNP
      ISTOP=1
    6 IF(NUMEL.LE.MAXEL) GO TO 8
      WRITE(6,3001) MAXEL
      ISTOP=1
      RETURN
    8 CONTINUE
      WRITE(6,2003) NUMNP,NUMEL,NUMMAT,NUMPC,ACEL1,ACEL3
C     *    *    *    *  READ AND PRINT MATERIAL PROPERTIES
      WRITE(6,2004)
      DO 20 M=1,NUMMAT
      READ(7,1002) MT,RO(MT),XTYPE
      WRITE(6,2005) MT,XTYPE
      READ(7,1003)(E(J,MT),J=1,7)
      WRITE(6,2006)(E(J,MT),J=1,7),RO(MT)
C     *    *    *    *  MODIFY FOR PLANE STRESS IN LOCAL COORDINATES
      IF(NPP.NE.2) GO TO 20
      IF(E(4,MT).EQ.0.0) GO TO 20
      EA=1.0/E(4,MT)
      E(1,MT)=E(1,MT)-EA*(E(2,MT)**2)
      E(3,MT)=E(3,MT)-EA*E(2,MT)*E(5,MT)
      E(6,MT)=E(6,MT)-EA*(E(5,MT)**2)
      E(2,MT)=0.0
      E(4,MT)=0.0
      E(5,MT)=0.0
      WRITE(6,2009)
      WRITE(6,2006)(E(J,MT),J=1,7),RO(MT)
   20 CONTINUE
C     *    *    *    * READ AND PRINT NODAL DATA
      N=0
      WRITE(6,2007)
      DO 95 L=1,NUMNP
```

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      IF(L-N) 70,85,55
   55 READ(7,1004) N,CODE(N),X(N),Y(N),UX(2*N-1),UX(2*N),ANG(N)
      CODE(N)=ABS(CODE(N))
      DV=X(N)
      DU=Y(N)
      IF(L-N) 60,85,100
   60 DZ=N-L+1
      DX=(X(N)-X(L-1))/DZ
      DY=(Y(N)-Y(L-1))/DZ
   70 CODE(L)=0.0
      IF(CODE(N).EQ.CODE(L-1)) CODE(L)=CODE(L-1)
      IF(ANG(N).EQ.ANG(L-1)) ANG(L)=ANG(L-1)
      UX(2*L-1)=0.0
      UX(2*L)=0.0
      X(L)=X(L-1)+DX
      Y(L)=Y(L-1)+DY
   85 WRITE(6,2008) L,CODE(L),X(L),Y(L),UX(2*L-1),UX(2*L),ANG(L)
      IF(ANG(L).NE.0.0) CODE(L)=-ANG(L)*PI
   95 CONTINUE
      GO TO 110
  100 WRITE(6,3002) N
      ISTOP=1
  110 CONTINUE
C     *     *     *     * READ AND PRINT ELEMENT PROPERTIES
      N=0
      WRITE(6,2010)
  130 READ(7,1005) M,(IX(M,I),I=1,5),ANGLE(M)
  140 N=N+1
      IF(M.LE.N) GO TO 160
      IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
      ANGLE(N)=ANGLE(N-1)
  160 WRITE(6,2011) N,(IX(N,I),I=1,5),ANGLE(N)
      IF(M-N) 185,180,140
  180 IF(NUMEL-N) 185,190,130
  185 WRITE(6,3003) N
      ISTOP=1
  190 CONTINUE
C     *     *     *     * READ AND PRINT PRESSURE BOUNDARY CONDITIONS
      IF(NUMPC.EQ.0) GO TO 310
      WRITE(6,2012)
      DO 300 L=1,NUMPC
      READ(7,1006) IBC(L),JBC(L),PN(L),PT(L)
  300 WRITE(6,2013) IBC(L),JBC(L),PN(L),PT(L)
  310 CONTINUE
C     *     *   DETERMINE BANDWIDTH AND COMPUTE SURFACE INTEGRALS
      ND=2*NB
      MB=0
      DO 340 N=1,NUMEL
      ANGLE(N)=PI*ANGLE(N)
      DO 340 I=1,4
```

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            IF(NUMPC.LE.0) GO TO 240
            J=I+1
            IF (I.EQ.4) J=1
            K=IX(N,I)
            L=IX(N,J)
            DO 230 MM=1,NUMPC
            IF(K.NE.IBC(MM).OR.L.NE.JBC(MM)) GO TO 230
            IBC(MM)=0
            XC(1)=CODE(K)
            XC(2)=CODE(L)
            IN(1)=K
            IN(2)=L
            DX=X(L)-X(K)
            DY=Y(L)-Y(K)
            FX=0.5*(PT(MM)*DX-PN(MM)*DY)
            FY=0.5*(PT(MM)*DY+PN(MM)*DX)
            DO 225 K=1,2
            L=IN(K)
            C=XC(K)
            IF(C.GE.0.0) GO TO 205
            UX(2*L-1)=UX(2*L-1)+FX*COS(C)-FY*SIN(C)
            GO TO 225
        205 IF(C.EQ.1.0.OR.C.EQ.11.0) GO TO 210
            UX(2*L-1)=UX(2*L-1)+FX
        210 IF(C.EQ.10.0.OR.C.EQ.11.0) GO TO 225
            UX(2*L)=UX(2*L)+FY
        225 CONTINUE
        230 CONTINUE
        240 DO 325 J=1,4
            K=IABS(IX(N,I)-IX(N,J))
        325 IF(K.GT.MB) MB=K
        340 CONTINUE
            IF(NUMPC.EQ.0) GO TO 355
            DO 350 MM=1,NUMPC
            IF(IBC(MM).EQ.0) GO TO 350
            WRITE(6,3004) MM
            ISTOP=1
        350 CONTINUE
        355 MBAND=2*MB+2
            WRITE(6,2015) MBAND
            IF(MBAND.LE.ND) GO TO 360
            WRITE(6,3005) MBAND
            ISTOP=1
        360 RETURN
       1000 FORMAT(40A2)
       1001 FORMAT(5I5,2F10.0)
       1002 FORMAT(I5,F10.0,25A2)
       1003 FORMAT(7E10.4)
       1004 FORMAT(I5,F5.0,5F10.0)
       1005 FORMAT(6I5,F10.0)
       1006 FORMAT(2I5,2F10.0)
       2000 FORMAT(1H1,5X,40A2)
       2001 FORMAT(1H0,20X,'PLANE STRAIN PROBLEM')
       2002 FORMAT(1H0,20X,'PLANE STRESS PROBLEM')
```

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      2003 FORMAT(1H0,4X,'NUMBER OF NODAL POINTS-----------',I4/
          *5X,'NUMBER OF ELEMENTS---------------',I4/
          *5X,'NUMBER OF DIFFERENT MATERIALS---',I4/
          *5X,'NUMBER OF PRES/SHEAR CONDITIONS-',I4/
          *5X,'X1 ACCELERATION-----------------',1PE12.4/
          *5X,'X3 ACCELERATION-----------------',1PE12.4/)
      2004 FORMAT(1H1,20X,'MATERIAL DATA')
      2005 FORMAT(1H0,2X,I2,10X,25A2)
      2006 FORMAT(7X,'C11=',1PE10.2,2X,'C12=',E10.2,2X,'C13=',E10.2,2X,
          *'C22=',E10.2/7X,'C23=',E10.2,2X,'C33=',E10.2,2X,'C44=',
          *E10.2,2X,'DENSITY=',E10.2)
      2007 FORMAT(1H1,3X,'NODE PNT',8X,'CODE',4X,'X1-COORD',4X,
          *'X3-COORD',2X,'X1-LD OR DISP',2X,'X3-LD OR DISP',9X,'ANGLE'/)
      2008 FORMAT(I12,F12.0,2F12.4,1P3E15.4)
      2009 FORMAT(30X,'PLANE STRESS REDUCTION OF MATR. COEF.')
      2010 FORMAT(1H1,5X,'ELEMENT',5X,'I',5X,'J',5X,'K',5X,'L',5X,
          *'MATERIAL',7X,'ANGLE'/)
      2011 FORMAT(I13,4I6,I13,1PE12.4)
      2012 FORMAT(1H1,10X,'PRESSURE AND/OR SHEAR BOUNDARY CONDITIONS'/
          *4X,'I',4X,'J',7X,'PRESSURE',10X,'SHEAR')
      2013 FORMAT(2I5,2F15.3)
      2014 FORMAT(10X,'NPP NOT SPECIFIED CORRECTLY')
      2015 FORMAT(//5X,'BANDWIDTH-----------------------',I4)
      3000 FORMAT(1H0,10X,'NUMBER OF NODES EXCEEDS',I5)
      3001 FORMAT(1H0,10X,'NUMBER OF ELEMENTS EXCEEDS',I5)
      3002 FORMAT(1H0,10X,'NODAL POINT DATA ERROR N=',I5)
      3003 FORMAT(1H0,10X,'ELEMENT DATA ERROR N=',I5)
      3004 FORMAT(1H0,10X,'PRESSURE BOUNDARY CONDITION ERROR N=',I5)
      3005 FORMAT(1H0,10X,'BANDWIDTH EXCEEDED MBAND=',I5)
           END
```

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      SUBROUTINE TRISTF(II,JJ,KK,NPP,ISTOP,COMM,AM1,AM3)
      DIMENSION F(6,10),H(6,10),DD(3,3),LM(3),D(6,6),XI(3)
      COMMON/QADTRI/XXX(5),YYY(5),C(4,4),S(10,10),P(10)
      DATA D/36*0.0/
      LM(1)=II
      LM(2)=JJ
      LM(3)=KK
      C11=XXX(II)
      C12=XXX(JJ)
      C13=XXX(KK)
      C21=YYY(II)
      C22=YYY(JJ)
      C23=YYY(KK)
      COMM=C12*(C23-C21)+C11*(C22-C23)+C13*(C21-C22)
      IF(COMM.LE.0.0.OR.ISTOP.NE.0) RETURN
C     *    *    *    *    *     * FORM INTEGRAL GT*C*G
      F(1,4)=(C11+C12)/2.0
      F(1,5)=(C12+C13)/2.0
      F(1,6)=(C13+C11)/2.0
      F(2,4)=(C21+C22)/2.0
      F(2,5)=(C22+C23)/2.0
      F(2,6)=(C23+C21)/2.0
      GO TO(30,10),NPP
   10 DO 20 I=4,6
   20 F(3,I)=1.0
      GO TO 40
   30 DO 35 I=4,6
   35 F(3,I)=F(1,I)
   40 DO 50 I=1,3
   50 XI(I)=0.0
      DO 100 I=4,6
      XI(1)=XI(1)+F(3,I)
      XI(2)=XI(2)+F(3,I)*F(1,I)
      XI(3)=XI(3)+F(3,I)*F(2,I)
  100 CONTINUE
      DO 108 I=1,10
      DO 108 J=1,6
      H(J,I)=0.0
  108 F(J,I)=0.0
      D(2,6)=XI(1)*C(1,3)*COMM/6.0
      D(3,5)=XI(1)*C(4,4)*COMM/6.0
      D(5,5)=XI(1)*C(4,4)*COMM/6.0
      D(5,6)=XI(1)*C(3,4)*COMM/6.0
      D(6,6)=XI(1)*C(3,3)*COMM/6.0
      D(3,6)=XI(1)*C(3,4)*COMM/6.0
      D(2,2)=XI(1)*C(1,1)*COMM/6.0
      D(2,3)=XI(1)*C(1,4)*COMM/6.0
      D(2,5)=XI(1)*C(1,4)*COMM/6.0
      D(3,3)=XI(1)*C(4,4)*COMM/6.0
      DO 110 I=2,6
      DO 110 J=1,I
  110 D(I,J)=D(J,I)
C     *    *    *    * FORM COEF-DISPL TRANSFORMATION MATRIX
      DD(1,1)=(C12*C23-C13*C22)/COMM
```

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      DD(1,2)=(C13*C21-C11*C23)/COMM
      DD(1,3)=(C11*C22-C12*C21)/COMM
      DD(2,1)=(C22-C23)/COMM
      DD(2,2)=(C23-C21)/COMM
      DD(2,3)=(C21-C22)/COMM
      DD(3,1)=(C13-C12)/COMM
      DD(3,2)=(C11-C13)/COMM
      DD(3,3)=(C12-C11)/COMM
      DO 120 I=1,3
      J=2*LM(I)-1
      H(1,J)=DD(1,I)
      H(2,J)=DD(2,I)
      H(3,J)=DD(3,I)
      H(4,J+1)=DD(1,I)
      H(5,J+1)=DD(2,I)
  120 H(6,J+1)=DD(3,I)
C     *     *     *     * FORM ELEMENT STIFFNESS MATRIX HT*D*H
      DO 130 J=1,10
      DO 130 K=1,6
      IF(H(K,J).EQ.0.0) GO TO 130
      DO 125 I=1,6
  125 F(I,J)=F(I,J)+D(I,K)*H(K,J)
  130 CONTINUE
      DO 140 I=1,10
      DO 140 K=1,6
      IF(H(K,I).EQ.0.0) GO TO 140
      DO 135 J=1,10
  135 S(I,J)=S(I,J)+H(K,I)*F(K,J)
  140 CONTINUE
C     *     *     *     * FORM BODY FORCE VECTOR
      F(1,1)=AM1*XI(1)*COMM/6.0
      F(2,1)=AM1*XI(2)*COMM/6.0
      F(3,1)=AM1*XI(3)*COMM/6.0
      F(4,1)=AM3*XI(1)*COMM/6.0
      F(5,1)=AM3*XI(2)*COMM/6.0
      F(6,1)=AM3*XI(3)*COMM/6.0
      DO 160 I=1,10
      DO 160 K=1,6
  160 P(I)=P(I)+H(K,I)*F(K,1)
      RETURN
      END
```

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```
      SUBROUTINE MODULI(N,I,J,K,L)
      COMMON/NPELD/E(7,10),RO(10),CODE(600),X(600),Y(600),
     *UX(1200),IX(500,5),ANGLE(500),NUMNP,NUMEL,NUMPC
      COMMON/QADTRI/XXX(5),YYY(5),C(4,4),S(10,10),P(10)
C     *     *     *     *     * INITIALIZATION
      DO 10 I=1,4
      DO 10 J=1,4
   10 C(I,J)=0.0
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MT=IABS(IX(N,5))
      C11=E(1,MT)
      C12=E(2,MT)
      C13=E(3,MT)
      C22=E(4,MT)
      C23=E(5,MT)
      C33=E(6,MT)
      C44=E(7,MT)
C     *     *     *     * ROTATE TO GLOBAL STIFFNESSES
      CS=COS(ANGLE(N))
      SN=SIN(ANGLE(N))
      CS2=CS**2
      SN2=SN**2
      CS4=CS2**2
      SN4=SN2**2
      DEN=2.0*(C13+2.0*C44)*CS2*SN2
      C(1,1)=C11*CS4+C33*SN4+DEN
      C(3,3)=C33*CS4+C11*SN4+DEN
      C(1,3)=C13*(CS4+SN4)+(C11+C33-4.0*C44)*CS2*SN2
      DEN=(C13+2.0*C44)*(CS2-SN2)
      C(1,4)=(C11*CS2-C33*SN2-DEN)*CS*SN
      C(3,4)=(C11*SN2-C33*CS2+DEN)*CS*SN
      C(4,4)=C44*(CS2-SN2)**2+(C11-2.0*C13+C33)*CS2*SN2
      C(2,2)=C22
      C(1,2)=C12*CS2+C23*SN2
      C(2,3)=C23*CS2+C12*SN2
      C(2,4)=(C12-C23)*CS*SN
      C(2,1)=C(1,2)
      C(3,1)=C(1,3)
      C(4,2)=C(2,4)
      C(3,2)=C(2,3)
      C(4,1)=C(1,4)
      C(4,3)=C(3,4)
      RETURN
      END
```

```
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      SUBROUTINE MODIFY(N,X,NB,MBAND)
      COMMON A(54,108),B(108)
      ND2=4*NB
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K.LE.0) GO TO 240
      B(K)=B(K)-A(M,K)*X
      A(M,K)=0.0
  240 K=N+M-1
      IF(K.GT.ND2) GO TO 250
      B(K)=B(K)-A(M,N)*X
      A(M,N)=0.0
  250 CONTINUE
      A(1,N)=0.0
      B(N)=0.0
      RETURN
      END
```

(Manuscript prepared September 1983, Manuscript received February 13, 1984)

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コンピュータープログラム・LOPE

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弾性問題やそれに類する問題は，有限要素法を用いた本プログラム（ＬＯＰＥ．この略称は局所直交異方性をもつ弾性体の平面問題を意味する）の適用によって，より容易に解くことができる．本プログラムの直接の適用対象は，二次元弾性問題であり，そこでは材料の直交異方性，弾性体の表面や内部への荷重，拘束的境界条件を考慮することができる．さらに，本プログラムのアルゴリズムは定常流（ポテンシャル流）の問題，例えば理想流体の流れ，粘弾性定常クリープ現象，多孔質の媒体中の流れ，熱伝達などの解析にも適用できる．本プログラムにおける基本的要素は，ひずみが一定の三角形であるが，これを重ねることにより四辺形の要素を定義することもできる．本プログラムは一連の代数方程式を解くためにガウスの消去法を用いており，大規模な問題をさほど容量の大きくないコンピューターシステムを用いて解くことを可能にするために，これらの方程式はマトリックス分割の手法によって処理されている．すなわち，マトリックスの分割された部分を補助記憶装置に一時的に格納しつつ順次演算することにより，主記憶容量の制約を回避している．こうした特徴を持つため，本プログラムは国立防災科学技術センター新庄支所のメルコム70コンピューターシステムで稼動することが可能となり，主記憶容量を超過する大規模な問題を解くのに利用されている．本報告には原始プログラムリストのほか，入力フォーマットの記述法と，各種のオプションの働きを確認するためのいくつかの例題を掲載した．

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