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Nonparametric Tests for the Umbrella Alternative in a Mixed Design for Location

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Abstract

This paper further investigates existing test statistics proposed by Magel et al. (2010) for detecting umbrella alternatives when the peak is known, and the underlying design consists of a completely randomized design (CRD) and randomized complete block design (RCBD). Magel et al. (2010) assumed equal variance between the CRD and the RCBD portions for the power estimates that they conducted. We investigate the powers of the tests compared to each other when testing for location in this design when the variance of the CRD portion is 2, 4, and 9 times larger than the variance of the RCBD portion. Underlying normal, t, and exponential distributions are considered as well as a variety of location shifts, and different ratios between the sample size in the CRD portion compared to the number of blocks in the RCBD portion.

Keywords: CRD design; RCBD design; known turning point; power estimation.

1. Introduction

Nonparametric methods are generally used in many fields, including biostatistics, business, pharmaceutical statistics, psychology, and social sciences. Nonparametric tests are often more suitable due to weaker assumptions about underlying populations and requirements for the measurement scales. Researchers may want to consider the umbrella alternative when they find themselves in a situation, for example, of testing a drug's effectiveness where increasing dosage levels may, at first, have more favorable outcomes but, after a certain dosage level, may have less favorable outcomes. The null and alternative hypotheses for umbrella alternative testing for location only are given in (1):

$$\boldsymbol{H_0: \mu_1 = \mu_2 = \cdots = \mu_k} \qquad \textbf{(1)}$$

$$\boldsymbol{H_a: \mu_1 \le \mu_2 \le \cdots \le \mu_{p-1} \le \mu_p \ge \mu_{p+1} \ge \cdots \ge \mu_k}$$

with at least one strict inequality, where $\mu_1, \dots, \mu_k$ are the location parameters of the populations.
The value, p, is called the turning point or the peak of the umbrella. It is believed that on one side of the peak, the parameters are nondecreasing, and on the other side of the peak, the parameters are nonincreasing. The researcher wants to test whether the location parameters differ at all. It may also be possible that an experiment was started by blocking subjects and applying various levels of the treatment to subjects in each block. However, after beginning

the experiment, not enough blocks were found, or subjects within blocks dropped out. The remaining observations were in the form of a completely randomized design, CRD.

We extended the work of Magel et al. (2010) who proposed two tests to test for the umbrella alternative with respect to location parameters in a mixed design consisting of an RCBD and a CRD. The test statistics were combinations of the Mack-Wolfe test statistic for the CRD (Mack and Wolfe, 1981) and the Kim-Kim test statistic for the RCBD (Kim and Kim, 1992). In the case of Magel et al. (2010), all power estimates between the two test statistics were made when the error variance in the RCBD portion was equal to the error variance in the CRD portion. In our research, we wanted to examine the performance of each test when the error variance for the CRD was larger than the error variance for the RCBD. We considered cases when the ratio of CRD error variance to the RCBD error variance, referred to as CR ratio, was two, four, and nine. Powers were estimated for both tests when the sample size ratio in the CRD portion compared to the number of blocks in the RCBD portion, referred to as SB Ratio, was 1/8, 1/4, 1/3, 1/2, 1, 2, 3, 4, and 8. In all cases in the RCBD portion, we assumed that there was one observation per treatment per block. We assumed equal sample sizes for all treatments in the CRD portion.

2. Review of Literature

2.1 Mack-Wolfe

The Mack-Wolfe test statistic was designed for umbrella alternatives based on a CRD design (Mack and Wolfe, 1981). Their test is an extension of the Jonckheere-Terpstra test (Jonckheere (1954) and Terpstra (1952)) to test for the umbrella alternative. The umbrella alternative hypothesis with known, p, is given in **(1).** The test statistic, A_p for the case of known peak p, is the sum of Mann-Whitney counts to the left of the peak (Mann and Whitney, 1947) and the reverse Mann-Whitney counts to the right of the peak. Therefore, the test statistic A_p, has the form in (2)

$$A_p = \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{uv} + \sum_{u=p}^{v-1} \sum_{v=p+1}^{k} U_{vu} \tag{2}$$

At a significance level of α, we reject H_0 if $A_p \geq A_{p,\alpha}$, where the Mann-Whitney test statistic is U_{vu} . The value, U_{vu}, counts the number of times when the observation in sample v less then observation in the sample u when all sets of paired observation are compared with the first entry coming from v sample and second entry from u sample. The test statistic of Mack-Wolfe (A_p) is approximately normally distributed under H_0 as the number of observations increase. The expected value and variance of A_p are given in Equation. (3):

$$\mathrm{E_0}\left(\mathrm{A_p}\right) = \frac{\mathrm{N_1^2+N_2^2-\sum_{i=1}^{k} n_i^2-n_p^2}}{4} \tag{3}$$

$$var_0(A_p) = \frac{1}{72}\left\{2(N_1^3 + N_2^3) + 3(N_1^2 + N_2^2) - \sum_{i=1}^{k} n_i^2(n_i + 3) - n_p^2\left(2n_p + 3\right)\right) + 12n_pN_1N_2 - 12n_p^2N\right\}$$

where $N_1 = \sum_{i=1}^{p} n_i$, $N_2 = \sum_{i=p}^{k} n_i$, and $N = N_1 + N_2 - n_p$, and n_p = the peak sample size.

Mack and Wolfe (1981) used the standardized test statistic A_p^* of the form

$$A_p^* = \frac{A_p - E(A_p)}{\sqrt{Var\,(A_p)}} \tag{4}$$

The null hypothesis is rejected if $A_p^* \geq z_\propto$ where $z_\propto$ is the upper tail value of the standard normal distribution with α probability above this value.

2.2 Kim-Kim

The Kim-Kim statistic was proposed in 1992 to test for the umbrella alternative in the RCBD layout with known peak. This statistic is an extension of the Mack-Wolfe test for the CRD. The Kim-Kim test statistic is used on a RCBD with b as blocks and k treatments and assumes no interaction between blocks and treatments. The Kim-Kim test statistic (A) is the sum of the Mack-Wolfe test statistics calculated for each block, and it is given by (5)

$$A = \sum_{j=1}^{b} A_{jp} \tag{5}$$

where, A_{jp} is the Mack-Wolfe test statistic for the j^{th} block. The value b is the number of blocks in the RCBD, p and k are the known treatment peak level and the number of treatments, respectively. Also, A_{ip} can be calculated using Equation (2) for each block with i= 1, 2, ……., b.

The Kim-Kim (1992) test statistic follows an asymptotic normal distribution when H_0 is true. The mean and variance are given in (6)

$$E_0(A) = \sum_{j=1}^{b} \left\{ \frac{\left\{N_{j1}^2 + N_{j2}^2 - \sum_{i=1}^{k} n_{ji}^2 - n_{jp}^2\right\}}{4} \right\} \tag{6}$$

$$var_0(A) = \frac{1}{72} \sum_{j=1}^{b} \left\{ 2\left(N_{j1}^3 + N_{j2}^3\right) + 3\left(N_{j1}^2 + N_{j2}^2\right) - \sum_{i=1}^{k} n_{ji}^2 (2n_{ji} + 3) - n_{jp}^2 (2n_{jp} + 3) + 12 n_{jp} N_{j1} N_{j2} - 12 n_{jp}^2 N_j \right\}$$

where, n_{ji} = samples size for i^{th} treatment in j^{th} block, n_{jp} = samples size for p^{th} treatment in j^{th} block, $N_{j1} = \sum_{i=1}^{p} n_{ji}$, $N_{j2} = \sum_{i=p}^{k} n_{ji}$, $N_j = N_{j1} + N_{j2} - n_{jp}$, b is equal to the number of blocks, k is equal to the number of treatments, and $p =$ the known peak. In this research, we consider the case when n_{ji} = 1. The expected value and variance of A when n_{ji} = 1 are given in (7).

$$E_0(A) = \frac{b\left(p^2 + (k-p+1)^2 - k - 1\right)}{4}$$

and

$$var_0(A) = \frac{b}{72} \left[\begin{matrix} 2\left(p^3 + (k-p+1)^3\right) + 3\left(p^2 + (k-p+1)^2\right) - 5k \\ -5 + 12p(k-p+1) - 12k \end{matrix} \right] \tag{7}$$

When H_0 is true, the standardized version of the Kim and Kim test, given in (8) has an asymptotic standard normal distribution

$$A^* = \frac{A - E_0(A)}{\sqrt{var_0(A)}} \tag{8}$$

The null hypothesis is rejected when $A^* \geq Z_\alpha$, where Z_α is the upper tail value of a standard normal distribution with α probability above this value.

2.3 Magel, Tersptra, and Canonizado

Magel et al. (2010) proposed two test statistics for the umbrella alternative mixed design, which consists of a completely randomized design and randomized complete block design. (Magel et al. (2009) had previously proposed a test for a mixed design, but for the nondecreasing alternative). The null and alternative hypotheses are as in (1). The first test, A_p^{**} , given by Magel et al. (2010), consists of the standardized version of the Mack-Wolfe test statistic A_p^* for CRD given in Eq. (4), and the Kim-Kim standardized version test statistic A^* for RCBD given in Eq. (8). A_p^{**} is in (9)

$$A_p^{**} = A_p^* + A^* \tag{9}$$

Under H_0, A_p^{**} will have an asymptotic normal distribution. The expected value and variance of A_p^{**} are given in (10) and (11)

$$E_0\left(A_p^{**}\right) = E_0\left(A_p^*\right) + E_0(A^*) \tag{10}$$

and

$$var_0\left(A_p^{**}\right) = var_0\left(A_p^*\right) + var_0(A^*) \tag{11}$$

The standardized version of the first proposed test is given by

$$A^{**} = \frac{A_p^{**} - E_0\left(A_p^{**}\right)}{\sqrt{var_0\left(A_p^{**}\right)}} = \frac{A_p^{**} - 0}{\sqrt{2}} \tag{12}$$

Under H_0, A^{**}has asymptotic standard normal distribution. The null is rejected for $A^{**} \geq Z_\alpha$

The second test, A_p^{***} , given in Magel et al. (2010) consists of the unstandardized version of the Mack-Wolfe test statistic for a CRD given in Eq. (2), and the Kim-Kim unstandardized version test statistic for a RCBD given in Eq. (5). A_p^{***} is given in (13)

$$A_p^{***} = A_P + A \tag{13}$$

Under H_0, the expected value and variance of A_p^{***} are given by (14) and (15)

$$E_0\left(A_p^{***}\right) = E_0(A_P) + E_0(A) \tag{14}$$

and

$$var_0\left(A_p^{***}\right) = var_0(A_P) + var_0(A) \tag{15}$$

where, $E_0(A_P)$, $E_0(A), var_0(A_P), and\ var_0(A)$ are the expected values and variances of the Mack-Wolfe (1981) and Kim-Kim (1992) respectively. The standardized version of the second test in Magel et al. (2010) is given in (16)

$$A^{***} = \frac{A_p^{***} - E_0\left(A_p^{***}\right)}{\sqrt{var_0\left(A_p^{***}\right)}} \tag{16}$$

Under H_0, A^{***} has an asymptotic standard normal distribution. The null hypothesis is rejected for large values. Magel et al. (2010) found the first test statistic A^{**} generally had higher estimated powers. Magel et al. (2010) only considered cases in which the CRD portion was equal to or less than the RCBD portion and when the variance of the CRD portion was equal to the variance of the RCBD portion.

3.Methodology

A simulation study was conducted to compare the two tests based on estimated powers when the variance of the CRD portion was greater than the variance of the RCBD portion. The simulation study was implemented in SAS version 9.4. Powers were estimated when the observations followed three different underlying distributions: normal, exponential and t-distribution with three degrees of freedom. It is assumed that the peak, p, is known and the design used is a mixed design consisting of a CRD and an RCBD portion. Powers were estimated for three, four, and five populations. For three populations, the peak was assumed to be 2. For four populations, the peaks considered were at the second and third populations. In the case of five populations, the peaks considered were at the second, third and fourth populations.

For all simulations, replications of 5,000 sets of samples were used. To begin with, the alpha levels of both tests were estimated for all different situations. The estimated alpha values were compared to the stated alpha value which was always at 0.05 for this study. The alpha values were estimated by counting the number of times the null hypothesis was rejected and then dividing by 5,000. The second part of the simulation study was to compare powers of the test statistics under various conditions. Powers were estimated by counting the number of times the null hypothesis was rejected by each of the tests for a given situation divided by 5,000.

We are interested in investigating testing for location in this mixed design case when the variance of the CRD portion is larger than the variance of the RCBD portion. We defined the CR ratio to be the variance of the CRD portion divided by the variance of the RCBD portion. We considered three different CR proportions of 2, 4, and 9 under three different distributions; standard normal distribution, t-distribution, and exponential distribution. We defined the SB ratio to be the ratio of the sample size in the CRD for each treatment (assuming equal sample sizes) divided by the number of blocks in the RCBD. The SB ratios considered in this study were 1/8,1/4, 1/3, 1/2, 1, 2, 3, 4, 8. The SB ratios were obtained under the following conditions:

1) RCBD portion: Block = 40; CRD portion: n = 5

2) RCBD portion: Block = 40; CRD portion: n = 10

3) RCBD portion: Block = 30; CRD portion: n = 10

4) RCBD portion: Block = 30; CRD portion: n = 15

5) RCBD portion: Block = 10; CRD portion: n = 10

6) RCBD portion: Block = 15; CRD portion: n = 30

7) RCBD portion: Block = 10; CRD portion: n = 30

8) RCBD portion: Block = 10; CRD portion: n = 40

9) RCBD portion: Block = 5; CRD portion: n = 40

In the cases of three, four, five populations with the peak p assumed to be known, the following locations parameter configurations are considered:

Three populations with peak at 2, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between parameters, for example (0.0, 0.5, 0.0).
2. The peak is distinct and there is unequal spacing between parameters, for example (0.8, 1.0, 0.5).
3. One additional parameter equals the peak, for example (0.5, 0.5, 0.0) and (0.0, 0.5, 0.5).

Four populations with peak at 2, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between the first parameters, for example (0.0, 0.5, 0.0, 0.0).
2. The peak is distinct and there is unequal spacing between the first three parameters, for example (0.2, 1.0, 0.8, 0.0).
3. One additional parameter equals the peak, for example (0.5, 0.5, 0.0, 0.0) and (0.0, 0.5, 0.5, 0.0).

Four populations with peak at 3, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between the last three parameters, for example (0.0, 0.0, 0.5, 0.0).
2. The peak is distinct and there is unequal spacing between the last three parameters, for example (0.5, 0.8, 1.0, 0.2).
3. One additional parameter equals the peak, for example (0.0, 0.5, 0.5, 0.0) and (0.0, 0.0, 0.5, 0.5).

Five populations with peak at 2, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between the first parameters, for example (0.0, 0.5, 0.0, 0.0, 0.0).
2. The peak is distinct and there is unequal spacing between the first three parameters, for example (0.4, 1.0, 0.8, 0.5, 0.2).
3. One additional parameter equals the peak, for example (0.5, 0.5, 0.0, 0.0, 0.0) and (0.0, 0.5, 0.5, 0.0, 0.0).

Five populations with peak at 3, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between the middle three parameters, for example (0.0, 0.0, 0.5, 0.0, 0.0).
2. The peak is distinct and there is unequal spacing between the middle three parameters, for example (0.2, 0.4, 1.0, 0.8, 0.5).
3. One additional parameter equals the peak, for example (0.0, 0.5, 0.5, 0.0, 0.0) and (0.0, 0.0, 0.5, 0.5, 0.0).

Five populations with peak at 4, the powers are estimated in the following cases:
1. The peak is distinct and there is equal spacing between the last three parameters, for example (0.0, 0.0, 0.0, 0.5, 0.0).
2. The peak is distinct and there is unequal spacing between the last three parameters, for example (0.2, 0.5, 0.8, 1.0, 0.4).
3. One additional parameter equals the peak, for example (0.0, 0.0, 0.5, 0.5, 0.0) and (0.0, 0.0, 0.0, 0.5, 0.5).

4. Results

4.1 Three Treatment Results

Under the three distributions, when the SB ratio is equal to 1/8, 1/4 1/3, 1/2, 1, 2, 3, and 4, and the CR ratio is equal to 2,4, and 9, A^{**} has higher powers than A^{***}. However, when the SB ratio is equal to 8, and the CR ratio is equal to 2 or greater A^{***} has slightly higher powers than A^{***} for the normal and exponential distributions. The test statistics have about the same powers for the t-distribution. Table 1 and Table 2 give estimated power results for the normal and exponential distributions when the SB ratios are 1/2 and the CR ratios are 2. The A^{**} statistic clearly has larger powers than the A^{***} statistic. Table 3 gives the estimated powers for normal distributions for a few cases when the SB ratio is 8 and CR is 2. In these cases, A^{***} has slightly higher powers than A^{**}.

Table 1: Estimated rejection percentages for the two proposed tests for trt=3 p=2 n=15 Blk =30 under N((0,1) and N (0.1) *sqrt (2) for the Mixed design*

MU1	MU2	MU 3	A**	A***
0	0	0	0.0482	0.0482
0	0.5	0	0.7804	0.5338
0	0.5	0.5	0.3212	0.2150
0.5	0.5	0	0.3146	0.2060
0.8	1	0.5	0.5076	0.3164

*SB ratio= 1/2, CR ratio= 2

Table 2: Estimated rejection percentages for the two proposed tests for trt=3 p=2 n=15 Blk =30 under exp(1) and exp (1) *sqrt (2) for the Mixed design*

MU1	MU2	MU 3	A**	A***
0	0	0	0.0440	0.0514
0	0.5	0	0.9364	0.6166
0	0.5	0.5	0.4444	0.2314
0.5	0.5	0	0.4376	0.2330
0.8	1	0.5	0.7188	0.3828

*SB ratio= 1/2, CR ratio= 2

Table 3: Estimated rejection percentages for the two proposed tests for trt=3 p=2 n=40 Blk =5 under N(0,1) and N (0.1) *sqrt (2) for the Mixed design*:

MU1	MU2	MU 3	A**	A***
0	0	0	0.0488	0.0526
0	0.5	0	0.7424	0.7942
0	0.5	0.5	0.3168	0.3396
0.5	0.5	0	0.3040	0.3382
0.8	1	0.5	0.4904	0.5550

*SB ratio= 8, CR ratio= 2

4.2 Four Treatment Results

The four treatment results were similar to the three treatment results regardless of the peak. When the SB ratio is equal to 1/8, 1/4 1/3, 1/2, 1, 2, 3, and 4 the CR ratio is 2 ,4, and 9, A** has greater powers than A^{***}. However, A^{***} is sometimes better than A^{**} when the SB ratio is equal to 8, and the CR ratio is 2, 4, 9. This occurs when the underlying distributions are either normal or exponential. Tables 4-6 give a few results for 4 treatments with peak 2. Both Tables 4 and 5 show the results from underlying normal distributions. The SB and the CR ratios are 2 and

4 in Table 4, and 8 and 2 in Table 5. For all the SB ratios except for 8 in our study, A^{**} had the largest powers. Table 6 gives some results for the exponential distribution when the SB ratio is 2, and the CR ratio is 4.

Table 4: Estimated rejection percentages for the two proposed tests for trt=4 p=2 n=30 Blk =15 under N(0,1) and N (0.1) * (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	A**	A***
0	0	0	0	0.0518	0.0522
0	0.5	0	0	0.8028	0.6990
0	0.5	0.5	0	0.7830	0.6858
0.5	0.5	0	0	0.4848	0.4190
0.2	1	0.8	0.5	0.9008	0.8250

*SB ratio= 2, CR ratio= 4

Table 5: Estimated rejection percentages for the two proposed tests for trt=4 p=2 n=40 Blk =5 under N(0,1) and N (0.1) *sqrt (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	A**	A***
0	0	0	0	0.0468	0.0488
0	0.5	0	0	0.7384	0.8004
0	0.5	0.5	0	0.7308	0.7874
0.5	0.5	0	0	0.4416	0.4834
0.2	1	0.8	0.5	0.8522	0.8988

*SB ratio= 8, CR ratio= 2

Table 6: Estimated rejection percentages for the two proposed tests for trt=4 p=2 n=30 Blk =15 under exp(1) and exp (1) * (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	A**	A***
0	0	0	0	0.0540	0.0530
0	0.5	0	0	0.8360	0.5350
0	0.5	0.5	0	0.8138	0.5366
0.5	0.5	0	0	0.5202	0.3070
0.2	1	0.8	0.5	0.9240	0.6644

*SB ratio= 2, CR ratio= 4

4.3 Five Treatment Results:

Results were the same for five treatments as with 3 and 4 treatments, regardless of the peak as to which test statistic was better. When the SB ratio was less than 8, and for any CR ratio included in the study, A^{**} had higher powers regardless of the peak. When the SB ratio was 8, A*** sometimes had slightly higher powers when the CR ratio was 2 or more. Tables 7-9 give a few of the results for different peaks, different distributions, and different ratios.

Table 7: Estimated rejection percentages for the two proposed tests for trt=5 p=3 n=15 Blk =30 under N(0,1) and N (0.1) * (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	MU 5	A**	A***
0	0	0	0	0	0.0446	0.0506
0	0	0.5	0	0	0.7792	0.5222
0	0	0.5	0.5	0	0.7902	0.5102
0	0.5	0.5	0	0	0.7824	0.5260
0.2	0.5	1	0.8	0.4	0.9638	0.7732

*SB ratio= 1/2, CR ratio= 4

Table 8: Estimated rejection percentages for the two proposed tests for trt=5 p=2 n=15 Blk =30 under exp(1) and exp (1) *sqrt (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	MU 5	A**	A***
0	0	0	0	0	0.0510	0.0486
0	0.5	0	0	0	0.9184	0.5474
0	0.5	0.5	0	0	0.9836	0.7288
0.5	0.5	0	0	0	0.7018	0.3634
0.4	1	0.8	0.5	0.2	0.9992	0.8976

*SB ratio= 1/2, CR ratio= 2

Table 9: Estimated rejection percentages for the two proposed tests for trt=5 p=4 n=30 Blk =15 under T(3) and T(3)* (2) for the Mixed design*:

MU1	MU2	MU 3	MU 4	MU 5	A**	A***
0	0	0	0	0	0.0416	0.0456
0	0	0	0.5	0	0.2946	0.2032
0	0	0	0.5	0.5	0.1838	0.1488
0	0	0.5	0.5	0	0.4836	0.2680
0.2	0.5	0.8	1	0.4	0.5926	0.4014

*SB ratio= 2, CR ratio= 4

5. Conclusions

Two nonparametric tests for location for the umbrella alternative for a mixed design (CRD and RCBD) were proposed by Magel et al. (2010). Magel et al. (2010) compared powers of the tests when the SB ratios were 1/8,1/4, 1/3, 1/2, and 1, and when the CR ratio was always 1. We further investigated how the two tests compared for when the CR ratios were 2, 4, and 9 and when the SB ratios were 2, 3, 4, and 8 in addition to the SB ratios considered by Magel et al. (2010). As in Magel et al. (2010), all tests maintained their stated alpha value of 0.05 under all conditions considered. The results as to which test had the higher powers were the same as in Magel et al. (2010) even though the variance of the CRD portion went up to 9 times the variance of the RCBD portion. Namely, A^{**} had the highest powers. We did consider SB ratios greater than 1 which were not considered in Magel et al. (2010). We found A^{**} had the highest powers except for when the SB ratio was 8 and the CR ratio was 2 or greater for the normal and exponential distributions. In some of these cases, the powers for A^{***} was slightly higher. We did not notice this for the t-distribution. The powers were close, but A^{**} had slightly higher powers. Overall, it is recommended that A^{**} be used to test for the umbrella alternative for location in a mixed design when the variance ratio of the CRD to the RCBD and the underlying distributions are unknown. The test has worked well for a variety of variance ratios, sample size ratios, and in both symmetric and asymmetric distributions.

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