2018

An Yb3+-Ho3+ codoped glass microsphere laser in the 2.0µm wavelength regions

Angzhen Li

Jibo Yu

Meng Zhang

See next page for additional authors

Authors
Angzhen Li, Jibo Yu, Meng Zhang, Xiaosong Lu, Jiquan Zhang, Elfed Lewis, Gerald Farrell, and Pengfei Wang

An Yb^{3+}-Ho^{3+} Codoped Glass Microsphere Laser in the 2.0 μm Wavelength Regions

Angzhen Li, Jibo Yu, Meng Zhang, Xiaosong Lu, Jiquan Zhang, Elfed Lewis, Gerald Farrell, and Pengfei Wang

***Abstract*— In this letter, an Yb^{3+}-Ho^{3+} codoped sol-gel silica microsphere lasing at around 2.0 μm is reported. The gain microsphere is fabricated by overlaying the 1.0 mol% Yb^{3+}-0.2 mol% Ho^{3+} codoped sol-gel solution on the surface of a pure silica microsphere and is then heated using a CO_2 laser. Using a traditional fiber taper-microsphere coupling method, we observe the single- and multi-mode microsphere laser outputs around 2.0 μm using a 980 nm laser diode as a pump source, with a low threshold pumping power of 14.7 mW. The ability to fabricate sol-gel codoped silica glass microlasers represents a new generation of low-threshold and compact mid-infrared laser sources for use as miniaturized photonic components for a wide range of applications including gas sensing and medical surgery.**

***Index Terms*— Laser, laser excitation, whispering gallery modes.**

I. INTRODUCTION

WHISPERING-GALLERY mode (WGM) optical microcavities including microdroplets, microdisks, microspheres and microtoroids have established themselves as attractive photonic building blocks for use in optoelectronic systems [1]–[5]. Surface-tension-induced microcavities such as microspheres with a near atomic-scale surface finish are known to be superior to all other dielectric miroresonator structures when comparing the quality factor (up to 10^{10}) or the mode volume (of the order of 100 μm^3) [6]–[8]. To date, a large number of laser oscillations have been investigated based on rare-earth-ion doped glass microspheres as it is possible to achieve high quality factors and ultralow pump thresholds using these devices. Yb^{3+}-Er^{3+} codoped visible microlasers have been widely reported based on various glass microsphere resonators [9]–[11]. Nd^{3+}-doped and Er^{3+}-doped microsphere resonators have been demonstrated predominantly for emission in the near-infrared range [12], [13]. Furthermore, Tm^{3+}-doped microsphere laser cavities with wavelengths around 2.0 μm have attracted much recent attention owing to their extensive potential applications in laser surgery, military, remote chemical sensing and monitoring of atmospheric pollution [14], [15].

Ho^{3+} is also an appropriate active ion for the generation of 2.0 μm laser output due to the Ho^{3+}: $^5I_7 \rightarrow {}^5I_8$ transition. Comparatively, the emission cross section of Ho^{3+} is nearly five times greater than that of the Tm^{3+} ion, which indicates more efficient operation of these lasers [16]. However, Ho^{3+} ions cannot be directly pumped by existing commercial 808, 980 or 1550 nm laser diodes without a suitable ground absorption. Yb^{3+} exhibits a large absorption cross section around 980 nm and its single 4f-4f electronic transition $^2F_{5/2}$-$^2F_{7/2}$ matches the transitions among intermediate states of Ho^{3+}, which favors the energy transfer processes from Yb^{3+} to Ho^{3+}. Thus, Yb^{3+}-Ho^{3+} codoped glasses are excellent gain materials for 2.0 μm operation and allow pumping by a conventional 980 nm laser diode and hence many investigations about Yb^{3+}-Ho^{3+} codoped fiber lasers have been reported [17]–[19]. Owing to the excited-state absorption and energy-transfer up-conversion losses, the threshold pump powers of most Yb^{3+}-Ho^{3+} codoped fiber lasers are very high (~10W). As mentioned above, an Yb^{3+}-Ho^{3+} codoped microsphere may be an excellent candidate as a resonant cavity for a low-pump-threshold laser with an output at 2.0 μm due to its unique ultra-high electromagnetic energy density. To the best of our knowledge, no investigations addressing an Yb^{3+}-Ho^{3+} codoped gain microsphere lasers have been reported in the literature to date.

In this work, Yb^{3+}-Ho^{3+} codoped microspheres were prepared using the well-known sol-gel method, which has been verified as a low-cost, efficient and flexible method for fabricating many active-ion-doped gain microspheres [20], [21]. The mid-infrared (mid-IR) lasing around 2.0 μm was generated and observed by optically coupling the Yb^{3+}-Ho^{3+} codoped microsphere to a silica taper fiber and using a 980 nm laser diode as the pump source. The lasing characteristics

Manuscript received March 21, 2018; revised July 10, 2018; accepted July 11, 2018. Date of publication July 23, 2018; date of current version August 13, 2018. This work was supported in part by the National Key R&D Program of China under Grant 2016YFE0126500, in part by the National Natural Science Foundation of China under Grant 61575050, in part by the Key Program for Natural Science Foundation of Heilongjiang Province of China under Grant ZD2016012, in part by the Open Fund of the State Key Laboratory on Integrated Optoelectronics under Grant IOSKL2016KF03, in part by the 111 Project at the Harbin Engineering University under Grant B13015, in part by the Fundamental Research Funds of the Central University, and in part by the Harbin Engineering University. *(Corresponding author: Pengfei Wang.)*

A. Li, J. Yu, M. Zhang, X. Lu, and J. Zhang are with the Key Laboratory of In-fiber Integrated Optics, Ministry of Education of China, Harbin Engineering University, Harbin 150001, China (e-mail: angzhenli@163.com; yu20131164@hrbeu.edu.cn; mengzhang@hrbeu.edu.cn; xingwadyx@163.com; 1037706223@qq.com).

E. Lewis is with the Optical Fiber Sensors Research Centre, Department of Electronic and Computer Engineering, University of Limerick, Limerick, V94 T9PX, Ireland (e-mail: elfed.lewis@ul.ie).

G. Farrell is with the Photonics Research Centre, Dublin Institute of Technology, Dublin 8, Ireland (e-mail: gerald.farrell@dit.ie).

P. Wang is with the Key Laboratory of In-fiber Integrated Optics, Ministry of Education of China, Harbin Engineering University, Harbin 150001, China, and also with the Key Laboratory of Optoelectronic Devices and Systems of Ministry of Education and Guangdong Province, College of Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China (e-mail: pengfei.wang@dit.ie).

Color versions of one or more of the figures in this letter are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LPT.2018.2859053

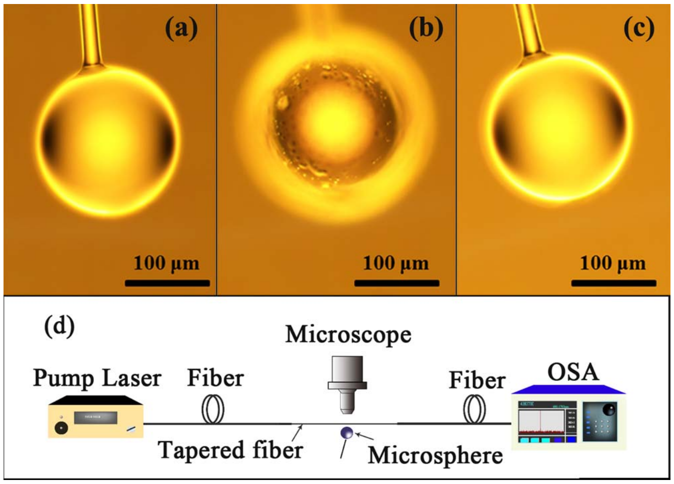

Fig. 1. Optical images of (a) an undoped microsphere, (b) a solution-coated microsphere and (c) an Yb^{3+}-Ho^{3+} codoped microsphere after the heating treatment. (d) Experimental setup for characterizing Yb^{3+}-Ho^{3+} codoped microsphere lasers.

were studied in detail by adjusting the coupling position and distance between the taper fiber and the microsphere.

II. Experiments

To fabricate the gain microspheres, silica microspheres with diameters ranging from 40 μm to 160 μm were prepared, using single-mode optical fibers (SMF-28, Thorlabs). These silica microspheres were made in the standard manner using a circular ZnSe-lens-focused CO_2 laser beam (power ~3 W) which was directed onto a section of the silica single-mode optical fiber. A small weight attached to the bottom of the silica fiber upon heating facilitated the formation of a very thin tapered region (diameter circa 3 μm), which acts as the stem of the microsphere. The CO_2 laser was then used to cut the fiber and the remaining glass at the tip was reheated. The surface tension of the molten silica at the fiber tip when subjected to high temperature (circa 1800 °C) causes the fiber tip to assume a spherical morphology under the effect of gravity. Finally, a further broad-focus laser heating process was conducted to reduce geometric irregularities on the microsphere surface.

A silica microsphere sample with a diameter of 159.5 μm was prepared and its micrograph image is shown in Fig. 1(a). The Yb^{3+}-Ho^{3+} codoped solution was prepared using the well-established sol-gel method [20]. The sol-gel solution was prepared using a 6.5-ml tetraethoxysilane in a 0.7-ml water and a 0.5-ml hydrochloric acid, where a 8.3-ml isopropanol acts as the co-solvent. $YbCl_3$ and $HoCl_3$ were added to the solution to introduce Yb^{3+} and Ho^{3+} ions. The concentrations of Yb^{3+} and Ho^{3+} were 1.0 mol% and 0.2 mol%, respectively. The solution was mixed in a silica beaker and placed on a hotplate, stirred at 70 °C and 400 rpm for 2 hours. This ensured that the Yb^{3+} and Ho^{3+} ions were evenly dispersed in the solution. Following aging at room temperature for a further 10 h, then the solution was ready to use. The undoped microsphere was immersed in the solution for 5 min and the microsphere was coated using the Yb^{3+}-Ho^{3+} codoped solution. The resulting coated sphere is shown in the micrograph image of Fig. 1 (b). Finally, the coated microsphere was heated using a CO_2 laser beam in order to reach the melting temperature of silica glass (~1650 ± 50 °C) and hence

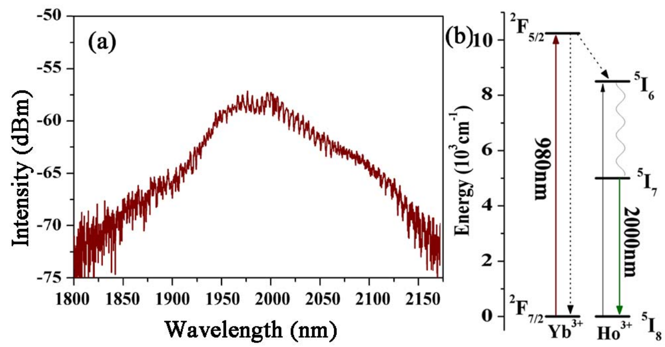

Fig. 2. (a) Output spectrum from the Yb^{3+}-Ho^{3+} codoped microsphere excited using a 980 nm laser with a pumping power of 10.8 mW. (b) The energy-level diagrams of Yb^{3+} and Ho^{3+} ions and relative transitions.

fabricate the Yb^{3+}-Ho^{3+} codoped microsphere. When heated to this temperature, the organic solvents were completely evaporated and the rare-earth ions dissolved onto and then absorbed below the surface of the silica microsphere. This process cycle was repeated three times in order to ensure sufficient active ions were introduced into the microsphere. The image of the Yb^{3+}-Ho^{3+} codoped microsphere is shown in Fig. 1 (c). The fabricated microsphere exhibits an excellent surface finish similar to the undoped microsphere and the resulting diameter was 160.2 μm.

The experimental setup for measuring the codoped microsphere laser characteristics is shown in Fig. 1(d). The tapered fiber used for light coupling was fabricated by heating a strand of 1060XP single-mode fiber using a ceramic microheater (CMH-7-19, NTT-AT) and simultaneously stretching it at both ends [22]. In this work, the waist diameter of the tapered silica fiber used was controlled from 1.0 to 1.5 μm. Light from a 980 nm laser pump diode (OFLD1000, Ovlink, China) was transmitted into one end of the taper to couple light in and out of the sol-gel gain microsphere. The transmitted spectrum was acquired using an optical spectrum analyzer (OSA) (AQ-6375, Yokogawa, Japan). The coupling positions between the taper and the microsphere were monitored from two orthogonal directions using two 20X microscope eyepieces attached to charge-coupled device (CCD) cameras.

As the position of the microsphere was adjusted and it was aligned with the fiber taper, the pump laser light was coupled into the Yb^{3+}-Ho^{3+} codoped microsphere and the resulting 2 μm emission from the microsphere was guided and coupled out of it through the fiber taper. Fig. 2 shows the output optical spectrum from the Yb^{3+}-Ho^{3+} codoped microsphere when the pump power is below the laser threshold. The spectrum was recorded using the OSA with a spectral resolution of 1 nm. When the 980 nm pump laser power reached 10.8 mW, a relatively broad mid-IR emission in the range from 1800 nm to 2200 nm was observed (Fig. 2 (a)). This is the characteristic emission attributed to $^5I_7 \rightarrow {}^5I_8$ transitions of Ho^{3+} ions as previously reported [23]–[25]. It is well known that Ho^{3+} ions cannot directly absorb light at 980 nm, while Yb^{3+} exhibits a large absorption cross section around 980 nm. The emission centered at 2.0 μm originates from the energy transfer from Yb^{3+} to Ho^{3+} ions. As shown in the schematic of the energy-level diagrams in Fig. 2 (b), Yb^{3+} ions are excited to the $^2F_{5/2}$ energy level through the absorption of the 980-nm-wavelength

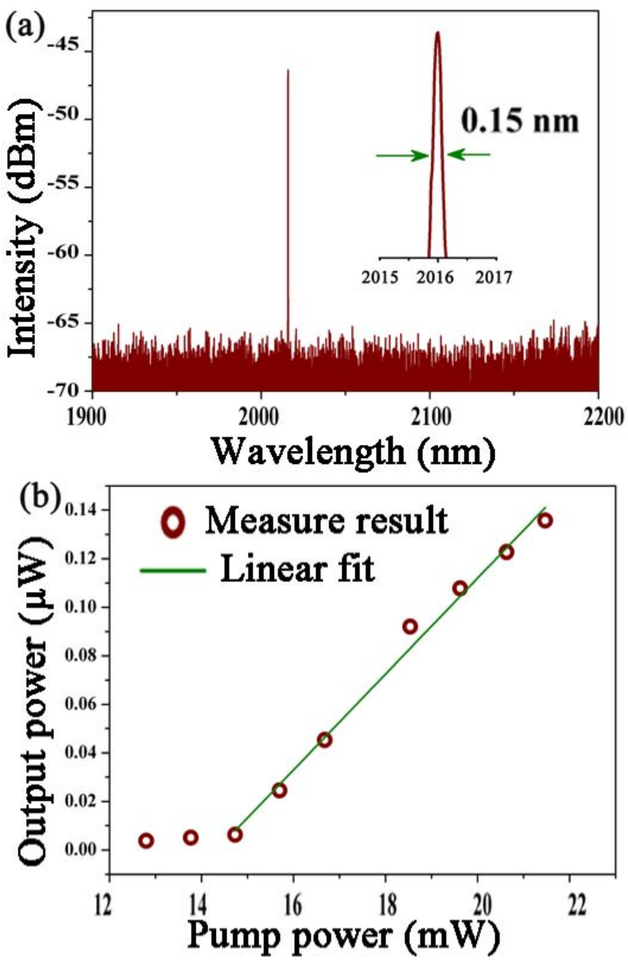

Fig. 3. (a) Single-mode laser emission spectrum from the Yb^{3+}-Ho^{3+} codoped microsphere when the pump power reaches 15.7 mW. The inset is the zoom-in view of the laser peak. (b) Microsphere laser output power as a function of pump power at 2016.3 nm.

light of the pump laser. Since the $^5I_6$ level of the Ho^{3+} ions is close to the $^2F_{5/2}$ level of the Yb^{3+} ions, energy can be efficiently transferred from the Yb^{3+}: $^2F_{5/2}$ level to the Ho^{3+}: $^5I_6$ level, the depopulation process takes place through the rapid non-radiative relaxation from the Ho^{3+}: $^5I_6$ to the $^5I_7$ level and transfer to $^5I_8$ level with the mid-infrared emission occurring at around 2.0 μm.

By optimizing the gap spacing and the coupling position between the Yb^{3+}-Ho^{3+} codoped microsphere and the silica fiber taper, we observed the single-mode lasing emission from the microsphere and the output spectrum is shown in Fig. 3(a). The spectrum was recorded on the OSA with a spectral resolution of 0.05 nm. The wavelength of the single-mode lasing peak is centered at 2016.3 nm, the peak power is 0.023 μW and the linewidth of the single-mode laser emission is 0.15 nm when the pump power reaches 15.7 mW.

The output power of the single-mode laser at 2016.3 nm collected from the fiber taper as a function of the pump power is presented in Fig. 3 (b). Below the threshold pump power at 14.7 mW, the emission of the microsphere corresponds to spontaneous radiation. The microsphere laser operates in single-mode until the output power reaches 0.14 μW (Fig. 3(b)). The mid-IR microsphere laser of this investigation with an output emission at around 2.0 μm and a low threshold level is ideal as a miniaturized optical source for a large number of applications including gas sensing and surgery in medicine. To obtain a high power transfer to the cavity from the fiber taper, light coupling into the microsphere requires improvements or optimizations. In this investigation, we obtained an improvement through enhancing the coupling coefficient between the taper and the microsphere, which is as a result of the phase-matching condition, i.e., ensuring size matching between the microsphere resonator and the fiber taper, as discussed in [26] and [27].

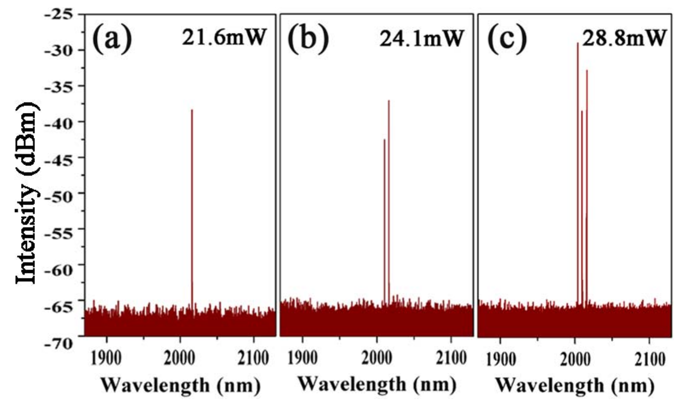

Fig. 4. Laser emission spectra from the Yb^{3+}-Ho^{3+} codoped microsphere when the pump power was set to (a) 21.6 mW, (b) 24.1 mW and (c) 28.8 mW.

The relative position between the microsphere and the fiber taper was maintained constant and the output optical spectra were measured for different pump powers. These results are shown in Figs. 4 (a), (b) and (c). In the case of Fig. 4 (a) it is clear that the microsphere laser was operating in single-mode when the pump power of 980 nm laser was set to 21.6 mW. However, as the pump power was increased side modes appeared, indicating multimode lasing from the microsphere. Multimode laser peaks were observed at 2010.2 nm and 2016.3 nm when the pump power was increased to 24.1 mW and when the pump power was further increased to 28.8 mW, output laser peaks appeared at 2004.1 nm, 2010.2 nm and 2016.3 nm. The diameter of the microsphere was 160 μm, the lasing wavelength was around 2 μm, and the refractive index of the silica microsphere was 1.4381. According to the resonance formula in microspheres [28], the wavelength of the modes (azimuthal mode number $(l) = 349$, 348 and 347, radial order $(n) = 1$) can be calculated as 2.0039, 2.0096 and 2.0152 μm respectively, the resulting average free spectral range (FSR) is 5.6 nm. The experimentally measured FSR is 6.1 nm, so there is a 0.5 nm difference in the FSR value between the calculated value and the experimental value. The difference between the experimental and the calculated FSR results is most likely to be a result of measurement errors and other limitations of the experimental setup, for example any errors in the measured value of the microsphere's diameter determined using the optical microscope will mean that the calculated value of the FSR is slightly in error. This multi-wavelength microsphere laser can therefore be considered as a possible candidate for applications for which a tunable laser source is required in the mid-IR wavelength range e.g. for gas sensing.

In order to better investigate the influence from the polarization states of the pump laser, the higher-radial-order modes should be controlled in this experiment, and the number of the modes in the experiment should be as few as possible. In the

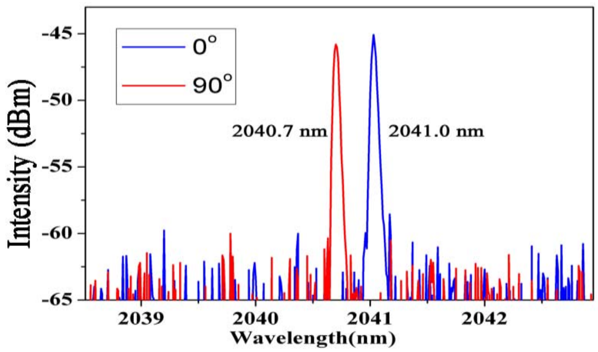

Fig. 5. Variation of the wavelength of the output when adjusting the polarization state (0 and 90 degree) of the pump laser.

numerical simulation, the diameter of the tapered fiber was 2 μm, and the diameter of the microsphere was 80 μm. In this situation, the mode of the microsphere can be relatively easily obtained (l = polar mode number (m) = 160, n = 2) [28]. By adjusting the polarization orientation of the pump laser by 90 degree, we demonstrated the feasibility to shift the wavelength of the output laser from 2041.0 nm to 2040.7 nm, presented in Figure 5.

III. Conclusion

In conclusion, an Yb^{3+}-Ho^{3+} codoped silica microsphere laser with strong emissions around 2.0 μm in a tapered-fiber-coupled system has been successfully fabricated and characterized. A single-mode laser was fabricated using a sol-gel deposition method. Both the pump and the lasing emission were efficiently guided through a taper formed in a standard silica single-mode fiber. The source was pumped using a commercial 980 nm laser diode, and this resulted in a single-mode operation of the microsphere at a wavelength centered on 2016.3 nm. The threshold pump power was as low as 14.7 mW and the microsphere laser continued to operate in single-mode until the output power reached 0.14 μW. When the pump power was increased beyond 21.6 mW, multimode laser lines at 2004.1 nm, 2010.2 nm and 2016.3 nm were observed. By adjusting the polarization orientation of the pump laser by 90 degree, it was possible to shift the wavelength of the output laser from 2041.0 nm to 2040.7 nm. This Yb^{3+}-Ho^{3+} codoped microsphere resonator therefore provides a low-threshold laser source for miniaturized optical devices in the mid-IR wavelength region. The sol-gel-based fabrication method is low-cost, efficient and flexible for fabricating microspheres to be used effectively as laser sources.

References

[1] K. Vahala, "Optical microcavities," *Nature*, vol. 424, no. 6950, p. 839, 2003.

[2] L. He, Ş. K. Özdemir, and L. Yang, "Whispering gallery microcavity lasers," *Laser Photon. Rev.*, vol. 7, no. 1, pp. 60–82, Jan. 2013.

[3] V. V. Datsyuk and I. A. Izmailov, "Optics of microdroplets," *J. Mol. Liquids*, vol. 93, no. 1, pp. 159–175, 2001.

[4] Y. Yamamoto and R. E. Slusher, "Optical processes in microcavities," *Phys. Today*, vol. 46, no. 6, pp. 66–73, 1993.

[5] X.-F. Jiang, C.-L. Zou, L. Wang, Q. Gong, and Y.-F. Xiao, "Whispering-gallery microcavities with unidirectional laser emission," (in English), *Laser Photon. Rev., Rev.*, vol. 10, no. 1, pp. 40–61, Jan. 2016.

[6] D. W. Vernooy, V. S. Ilchenko, H. Mabuchi, E. W. Streed, and H. J. Kimble, "High-Q measurements of fused-silica microspheres in the near infrared," (in English), *Opt. Lett.*, vol. 23, no. 4, pp. 247–249, Feb. 1998.

[7] X. Fan, P. Palinginis, S. Lacey, H. Wang, and M. C. Lonergan, "Coupling semiconductor nanocrystals to a fused-silica microsphere: A quantum-dot microcavity with extremely high Q factors," *Opt. Lett.*, vol. 25, no. 21, p. 1600, 2000.

[8] M. L. Gorodetsky, A. D. Pryamikov, and V. S. Ilchenko, "Rayleigh scattering in high-Q microspheres," *J. Opt. Soc. Amer. B*, vol. 17, no. 6, pp. 1051–1057, 2000.

[9] Y.-H. Wang, C.-S. Ma, X. Yan, and D.-M. Zhang, "Analysis for amplifying characteristics of Er^{3+}-Yb^{3+}−co-doped microring resonators," *Opt. Laser Technol.*, vol. 42, no. 2, pp. 336–340, 2010.

[10] J. A. Vallés and R. Gălătus, "Modeling of Yb^{3+}/Er^{3+}−codoped microring resonators," *Opt. Mater.*, vol. 41, pp. 126–130, Mar. 2015.

[11] C. H. Dong *et al.*, "Low-threshold microlaser in Er: Yb phosphate glass coated microsphere," (in English), *IEEE Photon. Technol. Lett.*, vol. 20, no. 5, pp. 342–344, Mar. 1, 2008.

[12] Q. Li, Y. Huang, Y. Lin, J. Wu, J. Huang, and T. Wu, "Ultralow-threshold laser in a Nd^{3+} doped silica microsphere," *Opt. Commun.*, vol. 356, pp. 368–372, Dec. 2015.

[13] X. Peng, F. Song, S. Jiang, N. Peyghambarian, M. Kuwata-Gonokami, and L. Xu, "Fiber-taper-coupled L-band Er^{3+}-doped tellurite glass microsphere laser," *Appl. Phys. Lett.*, vol. 82, no. 10, pp. 1497–1499, 2003.

[14] Y. Chen, Y. Lin, X. Gong, Z. Luo, W. Guo, and Y. Huang, "Efficient 805 nm diode-pumped continuous-wave 1.9 μm Tm^{3+}:$NaLa(MoO_4)_2$ laser," *J. Phys. D, Appl. Phys.*, vol. 42, no. 20, p. 205105, 2009.

[15] X. Liu *et al.*, "Realization of 2 μm laser output in Tm^{3+}-doped lead silicate double cladding fiber," *Mater. Lett.*, vol. 125, pp. 12–14, Jun. 2014.

[16] K. Li *et al.*, "Tm^{3+} and Tm^{3+}-Ho^{3+}co-doped tungsten tellurite glass single mode fiber laser," *Opt. Express*, vol. 20, no. 9, pp. 10115–10121, 2012.

[17] A. V. Kir'yanov, V. P. Minkovich, and Y. O. Barmenkov, "All-fiber 2.05-μm continuous-wave Ytterbium–Holmium laser pumped at 1.064 μm," *IEEE Photon. Technol. Lett.*, vol. 26, no. 19, pp. 1924–1927, Oct. 1, 2014.

[18] G. Bai, L. Tao, K. Li, L. Hu, and Y. H. Tsang, "Enhanced ∼2 μm and upconversion emission from Ho–Yb codoped oxyfluoride glass ceramics," *J. Non-Cryst. Solids*, vol. 361, pp. 13–16, Feb. 2013.

[19] J. Zmojda *et al.*, "Investigation of upconversion luminescence in antimonyâŁ"germanate double-clad two cores optical fiber co-doped with Yb^{3+}/Tm^{3+} and Yb^{3+}/Ho^{3+} ions," *J. Lumin.*, vol. 170, pp. 795–800, Feb. 2016.

[20] X. Orignac, D. Barbier, X. M. Du, R. M. Almeida, O. McCarthy, and E. Yeatman, "Sol–gel silica/titania-on-silicon Er/Yb-doped waveguides for optical amplification at 1.5 μm," *Opt. Mater.*, vol. 12, no. 1, pp. 1–18, 1999.

[21] A. E. Danks, S. R. Hall, and Z. Schnepp, "The evolution of 'sol–gel' chemistry as a technique for materials synthesis," (in English), *Mater. Horizons, Rev.*, vol. 3, no. 2, pp. 91–112, Mar. 2016.

[22] J. M. Ward, A. Maimaiti, V. H. Le, and S. N. Chormaic, "Contributed review: Optical micro- and nanofiber pulling rig," *Rev. Sci. Instrum.*, vol. 85, no. 11, p. 111501, Nov. 2014.

[23] N. Rakov, G. S. Maciel, C. B. de Araújo, and Y. Messaddeq, "Energy transfer assisted frequency upconversion in Ho3+ doped fluoroindate glass," *J. Appl. Phys.*, vol. 91, no. 3, pp. 1272–1276, 2002.

[24] R. Chen *et al.*, "2 μm fluorescence of Ho^{3+}:$^5I_7 \rightarrow {}^5I_8$ transition sensitized by Er^{3+} in tellurite germanate glasses," *Opt. Mater.*, vol. 49, pp. 116–122, Nov. 2015.

[25] S. B. Rai, A. K. Singh, and S. K. Singh, "Spectroscopic properties of Ho^{3+} ions doped in tellurite glass," *Spectrochimica Acta A, Mole. Biomole. Spectrosc.*, vol. 59, no. 14, pp. 3221–3226, 2003.

[26] A. F. J. Levi, R. E. Slusher, S. L. McCall, J. L. Glass, S. J. Pearton, and R. A. Logan, "Directional light coupling from microdisk lasers," *Appl. Phys. Lett.*, vol. 62, no. 6, pp. 561–563, 1993.

[27] H. T. Hattori, Z. Li, D. Liu, I. D. Rukhlenko, and M. Premaratne, "Coupling of light from microdisk lasers into plasmonic nano-antennas," *Opt. Express*, vol. 17, no. 23, pp. 20878–20884, 2009.

[28] C. C. Lam, P. T. Leung, and K. Young, "Explicit asymptotic formulas for the positions, widths, and strengths of resonances in Mie scattering," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 9, no. 9, pp. 1585–1592, Sep. 1992.