FEDERAL UNIVERSITY OF ITAJUBÁ
GRADUATE PROGRAM IN
INDUSTRIAL ENGINEERING

# Multivariate chance-constrained method applied in multi-objective optimization problems of manufacturing processes

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**Itajubá-MG (Brazil), December 2020**

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**Doctoral dissertation submitted to the Graduate Program in Industrial Engineering as part of the requirements for obtaining the title of Ph.D. of Sciences in Industrial Engineering**

**Research area:** Industrial Engineering

**Advisor:** Dr. Pedro Paulo Balestrassi
**Co-advisor:** Dr. Antonio Fernando Branco Costa

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# ACKNOWLEDGEMENTS

I truly believe I am one of the luckiest people in the world. I might even have enough statistical evidence to prove it. In the next few paragraphs, I present some witnesses and facts as an attempt to prove my hypothesis. But don't worry, that is not my thesis. Perhaps an idea for future research.

My parents, Vera and Reinaldo, my sister, Amanda, my girlfriend, Danúbia, my friends and relatives, you are my priority and my priceless treasure. In particular, my grandmother, Terezinha, is our leader. She raised me until I was fourteen and taught me the most important lesson in life: the meaning of love.

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# ABSTRACT

In the multi-objective optimization problems of manufacturing processes, the responses of interest are often significantly correlated. In addition to the multivariate nature of the problems, product demands, productive capacities, cycle times, the costs of labor, machines, and tools are just some of the many random variables involved in the optimization model. In particular, when using Design of Experiments (DoE) techniques and regression methods, the estimated coefficients for the empirical models - such as response surface models - are also stochastic. However, it has been observed that most of the articles published in this research area are limited to represent the stochastic variables in a deterministic way. Within this context, the present study aimed to propose the use of stochastic programming techniques combined with multivariate statistical methods including some process capability indices widely used in the industry, such as the $C_{pk}$ capacity index and the Parts Per Million ($PPM$) index. The use of the methods combined used resulted in the proposal of the Multivariate Chance-Constrained Programming (MCCP). To test the applicability of the MCCP method, a multi-objective optimization problem of the AISI 52100 hardened steel turning process was selected as a case study given its widespread use and relevance to the industry nowadays. As a starting point for this study, a set of experimental results obtained from a central composite design was used. The decision variables were the cutting speed ($V_c$), the feed rate ($f$) and the depth of cut ($a_p$). The responses of interest selected for this work were the total machining cost per part ($K_p$), the material removal rate ($MRR$), the tool life ($T$), the average roughness ($R_a$) and the total roughness ($R_t$). After analyzing the data and building the mathematical models for the responses of interest, three approaches were carried out. In the first approach, the $C_{pk}$ index included the calculation of the variance of the response surface model of $R_a$. In the second approach, the probability that $K_p$ is less than or equal to a predefined value was modelled as a stochastic objective function. Finally, the third approach described the application of the proposed MCCP method. In this approach, the $PPM$ index was calculated using a normal bivariate distribution for both $R_a$ and $R_t$. The main results of this research were: a) the demonstration and validation of an equation used to calculate the variance of a continuous, derivable and dependent function of stochastic variables; b) the analysis of the impact of seven stochastic industrial variables (setup time, lot size, machine and labor costs, insert changing time, tool holder price, tool holder life and insert price) on the cost of the process; c) finding that maximizing tool life may reduce cost in some cases – for example when using Wiper tools – but the change of the cutting conditions alone does not necessarily reduce the cost of the process, as in what occurred in the case study analyzed.

# LIST OF ACRONYMS

AISI – *American Iron and Steel Institute*

DOE – *Design of Experiments*

GRG – *Generalized Reduced Gradient*

ISO – *International Organization for Standardization*

MCCP – *Multivariate Chance-Constrained Programming*

MOP – *Multi-objective Optimization Programming*

OLS – *Ordinary Least Squares*

RSM – *Response Surface Methodology*

SP – *Stochastic Programming*

# LIST OF FIGURES

# LIST OF SYMBOLS

| | |
|---|---|
| $\mathbf{x}$ | Decision variable vector |
| $\mathbf{X}$ | Experimental matrix of inputs (with all terms of the response surface model) |
| $x$ | Decision Variable (scalar) |
| $i$ | Index |
| $j$ | Index |
| $\mathbf{z}$ | Incontrollable variable vector (noises) |
| $z$ | Incontrollable variable (noise) |
| $\mathbf{Y}$ | Interest results vector |
| $\mathbf{y}$ | Vector of the responses measured for each experimental test for the same interest result |
| $f(\mathbf{x})$ | Mathematical model of interest result (objective function or constraint) |
| $\mathbf{F}(\mathbf{x})$ | Vector of mathematical model of interest result |
| $\mathbf{f}(\mathbf{x})$ | Vector of mathematical model values that represent the same interest result |
| $\boldsymbol{\beta}$ | Vector of coefficients of response surface model |
| $\widehat{\boldsymbol{\beta}}$ | Estimator of $\boldsymbol{\beta}$ |
| $\mathbf{a}(\mathbf{x})$ | Vector containing decision variables and other terms that multiply $\boldsymbol{\beta}$ to reform the response surface model |
| $\mathbf{a}'(\mathbf{x})$ | Transposed Vector |
| $\sum$ | Sum |
| $g(\mathbf{x})$ | Inequality constraint |
| $h(\mathbf{x})$ | Equality constraint |
| $w$ | Weight assigned to objective function |
| $f^U(\mathbf{x})$ | Utopia value of objective function |
| $f^N(\mathbf{x})$ | Nadir value of objective function |
| $\bar{f}(\mathbf{x})$ | Stepped objective function |
| $E(\,)$ | Expected value |
| $\mu$ | Expected value |
| $Var(\,)$ | Variance |
| $\sigma^2$ | Variance |

| | |
|---|---|
| $\sigma$ | Standard Deviation |
| $Cov(\,)$ | Covariance |
| $\sigma_{x_1 x_2}$ | Covariance between $x_1$ and $x_2$ |
| $\boldsymbol{\nabla}$ | Gradient vector |
| $\boldsymbol{\Sigma}$ | Variance and covariance matrix |
| $\boldsymbol{\varepsilon}$ | Waste Vector |
| $\varepsilon$ | Residue (scalar) |
| $\mathbf{I}$ | Identity matrix |
| $\mathbf{X}^{-1}$ | Inverse **X** Matrix |
| $\theta(x)$ | Probability density function |
| $q(x)$ | Component of the normal probability density function |
| $\theta_p(\mathbf{x})$ | Multivariate probability density function |
| $C_p$ | Process capacity index (without displacement from the mean) |
| $C_p$ | Process capacity index (considering the average shift) |
| $USL$ | Upper specification limit |
| $LSL$ | Lower specification limit |
| $PPM$ | Parts Per Million Index |
| $V_c$ | Cutting speed |
| $f$ | Advance |
| $a_p$ | Cutting depth |
| $H$ | Surface part hardness |
| $V_B$ | Tool wear |
| $K_p$ | Total cost per part of a machining process |
| $MRR$ | Material removal rate |
| $T$ | Tool life |
| $R_a$ | Medium roughness |
| $R_t$ | Total roughness |
| $l_f$ | Workpiece length |
| $d$ | Workpiece diameter |
| $t_t$ | Total cycle time |
| $t_c$ | Cutting time |
| $t_s$ | Secondary time (part placement and inspection) |

| | |
|---|---|
| $t_a$ | Tool approach and retraction time |
| $t_p$ | Machine setup or setup time |
| $Z$ | Lot size (in piece units) |
| $N_t$ | Number of tool changes |
| $t_i$ | Insert change time |
| $K_{us}$ | Machining labor cost |
| $K_{um}$ | Machine cost |
| $K_{uf}$ | Tooling cost |
| $S_h$ | Hourly labor cost |
| $S_m$ | Machine cost per hour |
| $K_{th}$ | Tool holder cost |
| $N_{th}$ | Toolholder life |
| $K_i$ | Tool or insert cost |
| $N_i$ | Tool or insert life |

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# 1. INTRODUCTION

Uncertainty can naturally exist in almost any type of real problem (NOCEDAL; WRIGHT, 2006). In industry, product demands, production capacities, process times, labor costs, raw materials costs, and tool costs are just a few of the many random variables involved in industrial processes. Thus, in practical cases of optimization of industrial processes, the assumption that the problem inputs are deterministic data is rarely supported (KALL; MAYER, 2011). In particular, the standard approach of replacing significantly random variables with their expected values can be justified only under specific conditions. In many applications, it can be demonstrated that such an approach is inadequate since the interest results are affected by the whole variability present in the input factors. Therefore, it is necessary to consider the random aspect of the practical cases of optimization in the formulation of the problem. As will be presented in the following chapters, this can be done with the use of stochastic programming techniques (DÍAZ-GARCÍA; RAMOS-QUIROGA; CABRERA-VICENCIO, 2005; TORRES *et al.*, 2019a).

In addition to this random aspect, optimization problems in industrial processes often include multiple results of interest (DÍAZ-GARCÍA; BASHIRI, 2014), which are often conflicting (GOMES *et al.*, 2013). Cost reduction, productivity increase and quality assurance are some of the main objectives pursued simultaneously by companies. Therefore, multi-objective optimization of processes, or Multi-objective Optimization Programming (MOP), stands out as one of the most used techniques in this research area.

The present study focuses on the consideration of the stochastic nature of real MOP problems in manufacturing processes. In general, a process can be defined as a combination of activities that transform inputs (material, energy, information) into outputs (product, energy, information) (MONTGOMERY, 2017). In addition to these elements, there are input variables (controllable and uncontrollable) that can influence the results of interest (or output variables). The decision variables, also known as control variables or input factors, are represented by the vector $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$, and their values can be defined in order to optimize the process. The noises in vector $\mathbf{z} = \{z_1, z_2, \dots, z_p\}$, in turn, are uncontrollable variables that can also influence the process. The results of interest are symbolized by $\mathbf{y} = \{y_1, y_2, \dots, y_k\}$. Figure 1.1 presents the general elements of a process.

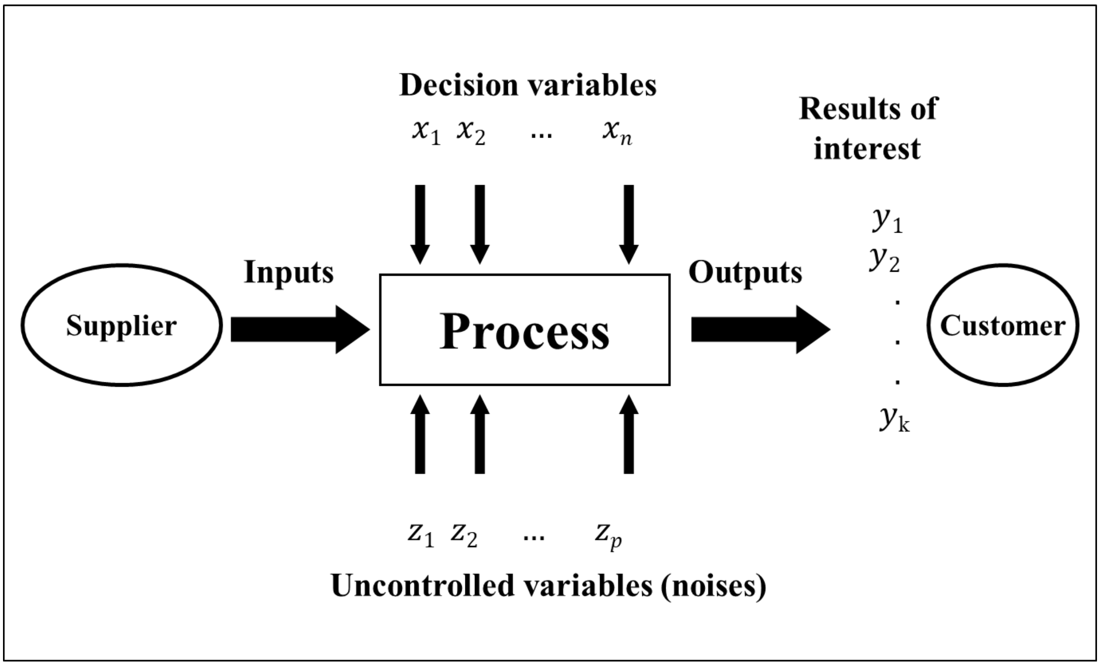


Figure 1.1 - General representation of a process and its elements

Source: adapted from Montgomery (2017)

Therefore, MOP aims to answer the following question: how is it possible to obtain optimal values for vector **y** by changing the values of **x** in view of the influence of **z**?

MOP starts with the definition of the results of interest and the identification of the control variables that can potentially impact such results in a significant manner (MONTGOMERY, 2017). Thus, it seeks to define the relationships between the decision variables and the outputs (or interest results). These relations are represented by the vector of objective functions $\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})\}$. Each function can be either a mechanistic or an empirical mathematical model. A mechanistic model is a definition - for example, the total cost of a process (DINIZ; MARCONDES; COPPINI, 2014), or a physical law, such as Newton's second law. An empirical model is built based on experimental data and using regression methods (MONTGOMERY, 2017).

Empirical models are generally obtained using design of experiments (DoE) and mathematical modelling (OLIVEIRA *et al.*, 2019). The DoE allows the analysis of the impact of the input variables and their interactions in the results of interest by defining a reduced number of experiments, which reduces the experimental costs (ANTONY, 2014). Among the DoE methods most applied to industrial processes, there is the response surface methodology (RSM), which consists of using statistical and mathematical techniques to model objective functions that depend on multiple input variables (MYERS; MONTGOMERY; ANDERSON-COOK, 2016).

After the execution of the experiment, the coefficients of the models are estimated via regression methods such as the ordinary least squares (OLS) (MONTGOMERY; RUNGER, 2018). If the models present adequate adjustments and residuals, they can be used in the formulation of the optimization problem, which are solved by search algorithms such as the generalized reduced gradient (GRG) (RAO, 2009).

## 1.1. Research justification

Many recently published articles present applications of optimization in manufacturing processes (ALOK; DAS, 2019; GAUDÊNCIO et al., 2019; JUNAID MIR; WANI, 2018; KUMAR et al., 2018; MEDDOUR et al., 2018; MIA *et al*., 2018; ROCHA *et al*., 2017; WANG *et al*., 2020). Oliveira *et al*. (2019) present a systematic literature review on optimization using RSM. The research included 49 articles published in the International Journal of Advanced Manufacturing Technology (IJAMT), one of the leading magazines in the field.

However, it was observed that these studies generally disregard the stochastic nature of the input variables and the results of interest to the optimization problem. Therefore, even with a large number of recent publications in this area of research, it is still possible to identify some opportunities for scientific contribution to the area.

In fact, the presence of uncertainty in results is shared by many economic and financial models, which may depend on future interest rates, product demands, and commodity prices (NOCEDAL; WRIGHT, 2006). For example, in the specific case of the total cost per piece of a turning process, setup times, lot sizes, and labor costs are some of the variables that influence the cost of the process (CAUCHICK-MIGUEL; COPPINI, 1996). Contrary to what many recently published studies consider, the parameters present in the cost function and in many others are random, and their variability influences propagates to the objective function.

## 1.2. Research question

Given the previously presented context, the main research question defined for the present research is:

- how is it possible to include the stochastic and multivariate nature of the variables present in the optimization problems of manufacturing processes using the process capability indices most used by the industry?

To answer this question, it is essential to investigate how the randomness present in the problem variables can be modelled. Such randomness that propagates to the variance of mathematical models is present not only in decision variables and in the noise factors, but also in the coefficients of functions and constraints - in the case of empirical models. Therefore, the answer to the research question in this dissertation implies the study of more specific questions, which are detailed in sections 1.2.1 to 1.2.3.

### 1.2.1. Variance of response surface models

The second order polynomial model, represented by Equation (1.1), is one of the most used functions for modeling the results of interest in studies of RSM and optimization of manufacturing processes (OLIVEIRA *et al.*, 2019).

$$y \sim f(\mathbf{x}) = \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{i<j} \beta_{ij} x_i x_j + \epsilon = \mathbf{a}'(\mathbf{x})\boldsymbol{\beta} + \varepsilon \qquad (1.1)$$

Where:

y = result of interest;

$f(\mathbf{x})$ = objective function that represents the result of interest;

$\mathbf{x} = [x_1, x_2, \dots, x_n]$ = vector of decision variables;

$\beta_i$ = $i^{\text{th}}$ coefficient of the objective function estimated by regression methods;

$x_i$ = $i^{\text{th}}$ decision variable;

$\mathbf{a}(\mathbf{x}) = \{1, x_1, x_2, \dots, x_i x_j\}$ = vector consisting of all r terms (a constant 1, linear, quadratic terms and interactions) of the response surface model $f(\mathbf{x})$;

$\boldsymbol{\beta} = \{\beta_0, \beta_1, \beta_2, \dots, \beta_{ij}\}$ = vector of objective function coefficients;

$\varepsilon$ = error between the true value of the y answer and the value predicted by the function f (x).

The coefficients in $\boldsymbol{\beta}$, commonly estimated by the OLS method, are normally distributed by definition and can still be correlated (TORRES *et al.*, 2019b). Equation (1.1) results only in the expected value of $f(\mathbf{x})$ only if the expected values of $\boldsymbol{\beta}$ are used. As previously mentioned, the vast majority of articles in this research area use only the expected values of the coefficients and therefore disregard the stochastic nature of $\boldsymbol{\beta}$.

Díaz-García, Ramos-Quiroga and Cabrera-Vicencio (2005) had already proposed several methods of stochastic programming (SP) for the formulation of multi-objective optimization problems involving response surface models. Abdelaziz (2012) also presented several approaches to multi-objective stochastic problems in which random variables can be present not only in objective functions, but also in constraints. However, even with several techniques already available in the literature, no studies were found that applied such methods in mechanical manufacturing processes before the present research (TORRES *et al.*, 2019a; TORRES *et al.*, 2019b).

Therefore, this dissertation also aims to answer a more specific research question in a particular case study: what is the impact of representing the variance of a response surface model $f(\mathbf{x})$ taking into account the variance of its $\boldsymbol{\beta}$ coefficients?

### 1.2.2. Multivariate nature of responses of interest

The multi-objective character of most real optimization problems implies conflicts of interest among the responses of interest (GOMES *et al*., 2013). That is, improving the level of some responses results in the loss of another interest result. From a statistical point of view, this means that it is common for the responses in $\mathbf{y}$ to be significantly correlated. In these situations, multivariate statistical methods should be used (JOHNSON; WICHERN, 2007). In fact, the use of multivariate statistics is particularly important when there are stochastic constraints involving random and correlated variables.

### 1.2.3. The modeling of process capability indices as stochastic constraints

Another aspect not yet seen in publications in this research area is the combination of stochastic programming methods and process capability indicators widely used by the industry. Among these indices, process capacity and the Parts Per Million (PPM) index (COSTA; EPPRECHT; CARPINETTI, 2018) stand out. Process capacity can be defined as the ratio between the tolerance of a certain quality characteristic and its natural process variation. In particular, the $C_{pk}$ index considers the minimum capacity within the process, taking into consideration its performance relative to both specification limits separately. The Parts Per Million (PPM) index, on the other hand, consists of the number of non-conforming pieces produced per 1 million pieces. Multivariate statistical techniques have already been used to estimate process capacity levels (PERUCHI *et al.*, 2018). However, no studies have yet been found that integrate stochastic programming methods, multivariate statistical techniques and quality indexes widely used in industry, such as $C_{pk}$ and PPM.

## 1.3. Objectives

Given the relevance of the topic of multi-objective optimization for industry and academy, the general objective of this research is to propose alternatives for the use of stochastic programming techniques with multivariate statistical techniques and process capability indices in the formulation of multi-objective optimization problems in manufacturing processes. Specific objectives include:

- Propose the use of a general expression that represents the variance of a continuous objective function dependent on stochastic variables;
- Propose the use of an expression that represents the variance of a response surface model;
- Propose the inclusion of the process capacity and the Parts Per Million (PPM) indices as multivariate stochastic constraints in problems of multi-objective optimization of manufacturing processes considering the variability of the coefficients of the response surface model of quality characteristics;
- In particular, this research also aims to test the hypothesis that, in the case of optimization of turning processes, the maximization of tool life only from the choice of machine parameters (cutting speed, feed, and cutting depth) does not necessarily contribute to reducing the cost of the process.

## 1.4. Research classification

Research can be classified from the perspective of its nature, its objectives, its approach and its method (MIGUEL *et al.*, 2014).

As for its nature, the present research is characterized as applied since the multi-objective stochastic optimization method is specifically proposed for the approach of manufacturing processes. In addition, the method was applied to a specific case study of experimental results that followed experimental design techniques and an application of the RSM.

From the point of view of its objectives, the research is classified as explanatory. The reason is that this research aims to explain and quantify the influence of the variability present in the random variables and the coefficients of the mathematical models in the results of optimization problems.

The quantitative approach is evidenced by multi-objective optimization techniques, stochastic programming and probabilities, multivariate statistics, and calculations of industrial parameters such as process capacity index, PPM index and the calculation of production costs.

In its first stage, this research is strongly related to the experimental method. However, there was no execution of a new experiment. The experimental results used in this work to test and validate the proposed method was carried out by Campos *et al.* (2017). The purpose of this work consists of using mathematical modelling techniques that can complement each other. The main methods used in this research were multi-objective optimization, stochastic programming, and multivariate statistics. According to the classification by Miguel *et al.*

(2014), such methods are inserted in the modelling and simulation methodology. In addition, Monte Carlo simulation was used to validate the results obtained using the stochastic programming method proposed in this dissertation.

## 1.5. Structure of this dissertation

With regards to the remaining content of the present research, chapter 2 consists of a theoretical foundation on topics related to the research, such as multi-objective optimization, stochastic programming, design of experiments, among other mathematical and statistical techniques used in this work. The theoretical foundation also includes practical concepts about the hardened steel turning process and some of the most used indicators in the industry: the $C_{pk}$ and PPM indices.

Based on the content presented in chapter 2, the method proposed and applied in this work, called the Multivariate Probabilistic Constraint Method, or *Multivariate Chance-Constrained Programming* (MCCP) described in chapter 3, is presented.

In order to apply and validate the proposed method, a real case study of the multi-objective optimization of the AISI 52100 hardened steel turning process is presented in chapter 4. This chapter also includes the discussions regarding the results obtained in three different approaches carried out in this research.

Finally, chapter 5 presents the conclusions of this dissertation, also listing its scientific contributions, research limitations, and suggestions for future work.

# 2. THEORETICAL BACKGROUND

## 2.1. Multi-objective optimization

Multi-objective optimization (MOP) can be defined as the formulation of problems whose objective is to optimize at least two results of interest in a process or system (HUANG; GU; DU, 2006). As presented at the beginning of chapter 1, in multi-objective optimization, the results of interest are represented by mathematical functions, denoted as $f_i(\mathbf{x})$. Thus, the objective is to optimize vector $\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})\}$, $k \geq 2$, where $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ is a vector composed of decision variables (RAO, 2009). Therefore, the general formulation of a multi-objective optimization problem can be written according to Equation (2.1) (NOCEDAL; WRIGHT, 2006):

$$\begin{gathered} Min\ \mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})\} \\ subject\ to: \\ h_i(\mathbf{x}) = 0, \qquad i = 1,2,\dots,p \\ g_j(\mathbf{x}) \leq 0, \qquad j = 1,2,\dots,q \\ \mathbf{x}_{\text{mín}} \leq \mathbf{x} \leq \mathbf{x}_{\text{máx}} \end{gathered} \tag{2.1}$$

Where:

$h_i(\mathbf{x})$ = equality constraints of the problem;

$g_j(\mathbf{x})$ = inequality constraints of the problem;

$\mathbf{x}_{\text{mín}}$, $\mathbf{x}_{\text{máx}}$ = vectors composed, respectively, by the lower and upper limits for the decision variables in $\mathbf{x}$, thus defining the solution space.

The literature usually defines optimization problems as minimization problems in general formulations and for the search for optimal solutions using algorithms. Maximization functions can be multiplied by (-1) and then minimized. Using this definition, we have that a solution $\mathbf{x}^*$ is called Pareto-optimal if there is no other solution $\mathbf{u}$ that reduces the value of some objective function without causing an increase in at least another function. That is, if $\mathbf{x}^*$ is a Pareto-optimal solution, then there is no other solution $\mathbf{u}$ such that $f_i(\mathbf{u}) \leq f_i(\mathbf{x}^*), i = 1,2,\dots,k$, with $f_i(\mathbf{u}) \leq f_i(\mathbf{x}^*)$ in at least one objective $i$. The concept of a Pareto-optimal solution is important because conflicts between the results of interest are common in multi-objective optimization problems (GOMES *et al.*, 2013). Consequently, there is not only an optimal solution in multi-objective optimization problems, but a set of Pareto-optimal solutions which define the so-called Pareto boundaries (KULTUREL-KONAK; SMITH; NORMAN, 2006). Figure 2.1 below represents a Pareto frontier for the case of a bi-objective problem.

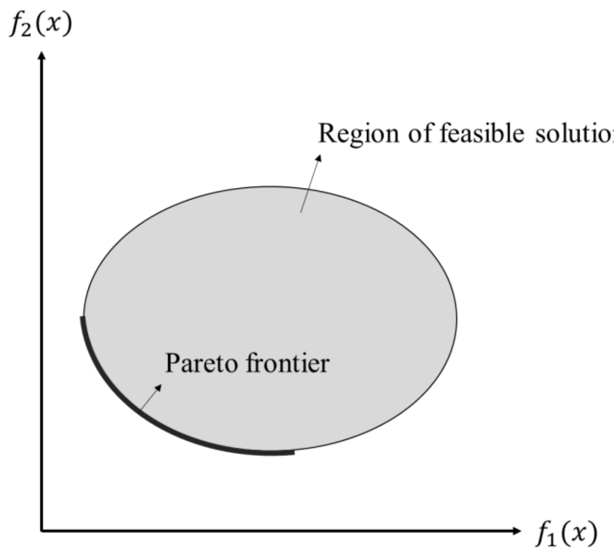


Figure 2.1 – Representation of a Pareto frontier for a bi-objective problem
Source: adapted from Vahidinasab and Jadid (2010)

Equation (2.1) can be defined in several ways, depending on the formulation strategy used. Rao (2009) defines two types of strategies: prioritization and agglutination. The prioritization strategy defines one of the functions as the objective function while the other functions become constraints on the problem. The agglutination strategy consists of modelling a global function that includes all the functions to be optimized. According to Hwang E Masud (1979), the methods used to formulate optimization problems can still be classified as follows:

- Methods without preference: methods in which there is no articulation of information regarding the preference between the results of interest (ROCHA *et al.*, 2015a). The multi-objective optimization problem is solved in a relatively simple way and the solution obtained is presented to the decision maker who can accept or reject it. Non-preferred methods include the Global Criterion method (MIETTINEN, 1998).
- *A priori* methods: in these, the information of preference regarding the results of interest is articulated by the decision maker before solving the optimization problem (ROCHA *et al.*, 2015b). Goal programming (DAUER; KRUEGER, 1977) and lexicographic programming (RAO, 2009) are examples of *a priori* methods.
- Interactive methods: these are methods in which the decision maker actively participates in the solution process, presenting their preferences gradually in each iteration. The

Tchebycheff method (BARIL; YACOUT; CLÉMENT, 2011), the NIMBUS method (ESKELINEN; MIETTINEN, 2012) and the interactive algorithms are examples of interactive methods.

- *A posteriori* methods: are methods in which the preference information is given by the decision maker after the optimization process. After generating a set of Pareto-optimal solutions that make up the Pareto frontier, this set is presented to the decision maker who selects the most preferred solution (ROCHA, 2017). Some examples of methods include weighted sum, the $\varepsilon$-constraint method, and the weighted metrics method (GOMES, 2013).

Section 2.1.1 presents the method of weighted sums (WS), which was used in this research.

### 2.1.1. The weighted sum method

In the weighted sum method, the objective function is a linear combination between the vector $\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})\}$ composed of individual objective functions and the vector $\mathbf{w} = \{w_1, w_2, \dots, w_k\}$ composed by the weights of each function, as presented in Equation (2.2) (RAO, 2009):

$$
\begin{gathered}
Min\ F(\mathbf{x}) = \mathbf{w}'\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{k} w_i\, f_i(\mathbf{x}) \\
subject\ to: \\
h_i(\mathbf{x}) = 0, \qquad i = 1,2,\dots,p \\
g_j(\mathbf{x}) \leq 0, \qquad j = 1,2,\dots,q \\
\mathbf{x}_{\text{mín}} \leq \mathbf{x} \leq \mathbf{x}_{\text{máx}}
\end{gathered}
\tag{2.2}
$$

Equation (2.2) is hardly used in its original format, as the objective functions often have drastically different scales (*e.g.*, efficiency between 0 and 1, and costs between 100 and 200 thousand dollars). In these cases, when the original scales are maintained, the solution of the problem prioritizes the functions of greater magnitude, since they more significantly impact the global function. To avoid this unplanned prioritization, one of the alternatives is to equalize the scale of the objective functions. Equation (2.3) presents an alternative for it (TORRES *et al.*, 2019b):

$$\bar{f}_i(\mathbf{x}) = \frac{f_i(\mathbf{x}) - f_i^U(\mathbf{x})}{f_i^N(\mathbf{x}) - f_i^U(\mathbf{x})} \tag{2.3}$$

Where:

$f_i(\mathbf{x})$ = the current value of function $i$;

$f_i^U(\mathbf{x})$ = the utopia value of function $i$, obtained by the individual optimization of the function;

$f_i^N(\mathbf{x})$ = the Nadir value of function $i$, corresponding to the worst value among all individual optimizations;

When all objective functions are scaled, the objective becomes the minimization of the distances between the current values of each function and its ideal value obtained by the individual optimization of the same function.

Section 2.2 presents the concepts and techniques of stochastic programming used in this study.

## 2.2. Stochastic programming

Stochastic Programming (SP) is a strategy for formulating optimization problems used to build objective functions and constraints whose coefficients or decision variables are described by random variables (BIRGE; LOUVEAUX, 2011). SP provides an important approach to linear programming under uncertainty that began to be developed in the 1950s and continues to be widely used until today (HILLIER; LIEBERMAN, 2014). The objective is to transform a stochastic problem into a deterministic problem, and this transformation depends on the probability distributions used to represent the stochastic variables and parameters. In its general formulation, a stochastic programming problem can be represented by Equation (2.4). The coefficients $c_j$, $A_{ij}$ and $b_j$ are random variables with a known probability distribution.

$$\begin{gathered} Min\ f(\mathbf{x}) = \mathbf{c}'\mathbf{x} \\ subject\ to: \\ \mathbf{g}(\mathbf{x}) = \mathbf{A}'\mathbf{x} \leq \mathbf{b} \\ x_j \geq 0 \end{gathered} \tag{2.4}$$

SP methods transform stochastic formulations into deterministic ones, by determining probability functions to incorporate the random nature of the variables. There are already several methods of SP proposed in the literature (CHARNES; COOPER, 1959; DÍAZ-GARCÍA; BASHIRI, 2014; DÍAZ-GARCÍA; RAMOS-QUIROGA; CABRERA-VICENCIO, 2005; KALL; MAYER, 2011). The present study uses the following SP techniques:

a) The chance-constrained programming approach (CHARNES; COOPER, 1959);
b) An approach used for mechanistic models, linear or non-linear (TORRES *et al.*, 2019a);
c) An approach used in conjunction with the response surface methodology (DÍAZ-GARCÍA; RAMOS-QUIROGA; CABRERA-VICENCIO, 2005).

To demonstrate these propositions, it is important to present some statistical concepts and basic definitions about the expected value and the variance of random variables. Such concepts and definitions are described in Appendix A. Approaches (a) and (b) are presented in sections 2.2.1 and 2.2.2 respectively. Although approach (c) is a particular case of (b), to describe it with sufficient clarity, it is necessary to first present some concepts about DoE, RSM and OLS, which is done in sections 2.3 and 2.4 respectively.

### 2.2.1. Stochastic programming and the chance-constrained programming approach

Consider the general problem presented in Equation (2.2). Suppose that the objective functions and the constraints of the problem are stochastic, that is, their coefficients are random variables. It is possible to transform the stochastic problem into a deterministic problem as long as the expected values and variances of the mathematical models can be calculated. Thus, Equation (2.2) can be rewritten, among other ways, as follows:

$$\begin{gathered} Min\; F(\mathbf{x}) = wE[f_1(\mathbf{x})] + (1-w)Var[f_1(\mathbf{x})] \\ subject\ to: \\ P[f_2(\mathbf{x}) \leq u] \geq p_0 \\ \mathbf{x}_{\text{mín}} \leq \mathbf{x} \leq \mathbf{x}_{\text{máx}} \end{gathered} \tag{2.5}$$

In this case, one of the results of interest is symbolized by $f_1(\mathbf{x})$. The objective is to simultaneously minimize the expected value – assuming it is a minimization function – and the variance of $f_1(\mathbf{x})$. A second result of interest is represented by $f_2(\mathbf{x})$. However, the constraint of the problem consists of a minimum probability that the value of $f_2(\mathbf{x})$ does not exceed the limit $u$. For this reason, such an approach is called the chance-constrained programming technique.

The expected value of a function $f(\mathbf{x})$ can be estimated using the expected values of its variables. The calculation of the variance of $f(\mathbf{x})$ requires other mathematical calculations, as described in section 2.2.2 below.

### 2.2.2. The variance of a continuous model dependent on random variables

This section demonstrates the calculation of the variance of a function dependent on random variables (TORRES *et al.*, 2019a).

Suppose that $f(\mathbf{x})$ is a function dependent on vector $\mathbf{x} = \{x_1, x_2\}$, and that $x_1$ and $x_2$ are random variables. Expanding $f(\mathbf{x})$ in a Taylor series, we have:

$$f(\mathbf{x}) = f(x_1, x_2) = f\left(\mu_{x_1}, \mu_{x_2}\right) + \sum_{i=1}^{n=2} \left(x_i - \mu_{x_i}\right) \frac{\partial f(x_1, x_2)}{\partial x_i} |\mu_{x_i} \tag{2.6}$$

or

$$f(x_1, x_2) - f\left(\mu_{x_1}, \mu_{x_2}\right) = \left(x_1 - \mu_{x_1}\right) \frac{\partial f(x_1, x_2)}{\partial x_1} |\mu_{x_1} + \left(x_2 - \mu_{x_2}\right) \frac{\partial f(x_1, x_2)}{\partial x_2} |\mu_{x_2} \tag{2.7}$$

Raising both terms in Equation (2.7) squared and applying the expected value operator, we have:

$$E[f(\mathbf{x}) - f(\boldsymbol{\mu})]^2 = E\left[\left(x_1 - \mu_{x_1}\right) \frac{\partial f(\mathbf{x})}{\partial x_1} |\mu_{x_1} + \left(x_2 - \mu_{x_2}\right) \frac{\partial f(\mathbf{x})}{\partial x_2} |\mu_{x_2}\right]^2 \tag{2.8}$$

Or

$$\begin{aligned} Var[f(x_1, x_2)] = E\left\{\left[\left(x_1 - \mu_{x_1}\right) \frac{\partial f(\mathbf{x})}{\partial x_1} |\mu_{x_1}\right]^2 + \left[\left(x_2 - \mu_{x_2}\right) \frac{\partial f(\mathbf{x})}{\partial x_2} |\mu_{x_2}\right]^2\right\} \\ + E\left\{2\left[\left(x_1 - \mu_{x_1}\right)\left(x_2 - \mu_{x_2}\right) \frac{\partial f(\mathbf{x})}{\partial x_1} |\mu_{x_1} \frac{\partial f(\mathbf{x})}{\partial x_2} |\mu_{x_2}\right]\right\} \end{aligned} \tag{2.9}$$

By definition, $E\left(x_1 - \mu_{x_1}\right)^2 = \sigma_{x_1}^2$ and $E\left[\left(x_1 - \mu_{x_1}\right)\left(x_2 - \mu_{x_2}\right)\right] = \sigma_{x_1 x_2}$. Thus, Equation (2.9) can be rewritten as follows:

$$Var[f(\mathbf{x})] = \left[\frac{\partial f(\mathbf{x})}{\partial x_1}\right]^2 \sigma_{x_1}^2 + \left[\frac{\partial f(\mathbf{x})}{\partial x_2}\right]^2 \sigma_{x_2}^2 + 2\left[\frac{\partial f(\mathbf{x})}{\partial x_1}\right]\left[\frac{\partial f(\mathbf{x})}{\partial x_2}\right] \sigma_{x_1 x_2} \tag{2.10}$$

It is also possible to write Equation (210) in matrix format as follows,

$$Var[f(\mathbf{x})] = \begin{bmatrix} \dfrac{\partial f(\mathbf{x})}{\partial x_1} & \dfrac{\partial f(\mathbf{x})}{\partial x_2} \end{bmatrix} \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} \\ \sigma_{x_1 x_2} & \sigma_{x_2}^2 \end{bmatrix}^2 \begin{bmatrix} \dfrac{\partial f(\mathbf{x})}{\partial x_1} \\ \dfrac{\partial f(\mathbf{x})}{\partial x_2} \end{bmatrix} = \mathbf{\nabla}' f(\mathbf{x}) \mathbf{\Sigma_x} \mathbf{\nabla} f(\mathbf{x}) \tag{2.11}$$

Where $\mathbf{\nabla} f(\mathbf{x})$ is the gradient vector of $f(\mathbf{x})$ $\mathbf{\Sigma_x}$ is the variance and covariance matrix of the random variables in **x**. Finally, generalizing Equation (2.11) for the case of functions of **n** random variables, we have:

$$Var[f(\mathbf{x})] = \sum_{i=1}^{n} \left[\frac{\partial f(\mathbf{x})}{\partial x_\mathrm{i}}\right]^2 \sigma_{x_i}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\frac{\partial f(\mathbf{x})}{\partial x_\mathrm{i}}\right] \left[\frac{\partial f(\mathbf{x})}{\partial x_\mathrm{j}}\right] \sigma_{x_i x_j} \tag{2.12}$$

Therefore, the variance of a function dependent on random variables can be calculated by Equation (2.13):

$$Var[f(\mathbf{x})] = \mathbf{\nabla}' f(\mathbf{x}) \mathbf{\Sigma_x} \mathbf{\nabla} f(\mathbf{x}) \tag{2.13}$$

As will be presented in section 2.4.3, the calculation of the variance of a response surface model is a particular case of Equation (2.13). The difference is that, in the case of response surface models, the variance is considered to be present in the coefficients $\boldsymbol{\beta}$ of the response surface models, whereas **x** is a vector composed of deterministic decision variables. However, before demonstrating how the $\boldsymbol{\beta}$ variance and covariance matrix is estimated, it is necessary to present some concepts related to design of experiments (DoE) and multiple linear regression, in particular, the method of ordinary least squares (OLS). Such issues are covered in sections 2.3 and 2.4 respectively.

## 2.3. Design of experiments (DoE)

As presented in chapter 1, the relationships between control variables and some of the results of interest in manufacturing processes are typically unknown. Therefore, there are often no mechanistic mathematical models that satisfactorily represent the objective functions or constraints in optimization problems. For this reason, researchers often use the experimental method so that empirical models can be built. Montgomery (2017) defines an experiment as a series of tests in which purposeful changes are made to the input variables of a process or system so that it is possible to identify the reasons for the changes observed in the response variables.

However, conducting experiments in the manufacturing industry requires a high investment of resources, and thus, it is important that the tests are planned in order to reduce experimental costs and provide an adequate level of information about the investigated process. Faced with this need, different planned experimental strategies have been developed over time. This set of strategies came to be known as design of experiments (DoE). Gomes (2013) lists several applications of DoE techniques including the increase in process yield, the reduction of variability with consequent improvement in product quality, cost reduction, and reduction in the time of development of products and processes. DoE is considered fundamental and crucial in increasing the understanding of products and processes (ANTONY *et al*., 2010).

According to Dean, Voss and Dragulic (2017), DoE has three basic principles: replication, blocking and randomization. Replication refers to the repetition of the same experimental conditions on similar objects. Blocking consists of dividing experimental tests into blocks so that the tests in each block can be compared under relatively similar experimental conditions. Finally, randomization is the random selection of objects or materials to prevent intended or unplanned bias from being introduced to the experiment. The first two principles are used to increase the accuracy of experiments, while the third has the function of reducing bias.

Montgomery (2017) establishes the following sequence of steps for an experiment project:

1. Recognition and definition of the problem;
2. Selection of response variables;
3. Definition of input factors or variables and their respective levels;
4. Choice of experimental arrangement;
5. Execution of the experiment;
6. Statistical analysis of the data;
7. Conclusions and recommendations.

According to Tanco, Viles and Pozueta (2008), the main approaches to the implementation of DoE can be classified into three categories: classic DoE, Taguchi, and Shaining. The classic DoE is defined as the first strategies created to overcome the popular tests known as one-factor-at-a-time (OFAT) (ANTONY, 2014). The classic approach includes not only complete factorial and fractional factorial arrangements (GAITONDE *et al*., 2009), but also central composite arrangements (Central Composite Designs - CCD) and Box-Benken arrangements, which are used by the Response Surface Methodology (RSM) (ALOK; DAS, 2019). The Taguchi approach emphasizes the reduction of variability and proposes the use of orthogonal arrangements and the signal/noise ratio (MIA *et al*., 2018). The Shaining approach, in turn, consists of a set of methods used to solve problems progressively, that is, from a sequence of steps (SHAININ; SHAININ, 1988).

This research uses the classic DoE approach, in particular, the response surface methodology described in section 2.3.1.

### 2.3.1. Response Surface Methodology

Response Surface Methodology (RSM) is defined by Myers, Montgomery and Anderson-Cook (MYERS; MONTGOMERY; ANDERSON-COOK, 2016) as a collection of statistical and mathematical techniques used to develop, improve and optimize processes. Also, according to the authors, the main applications of RSM are in the industrial sector, particularly where many input variables potentially influence responses of interest such as performance measures or quality characteristics. Even with the existence of different methods found in the literature, RSM is shown as one of the most effective ways to carry out process optimizations (OLIVEIRA *et al*., 2019).

Oliveira *et al*. (2019) reviewed 49 articles on RSM in the magazine with the highest incidence of publications on the topic. The survey was limited to articles between the years 2014 and 2017. The objective was to establish practical guidelines and critical analyses of applications of the method. After identifying some common failures in several RSM applications, the authors proposed a sequence of steps so that the method is conducted efficiently, as shown in Figure 2.2.

Some steps described in Figure 2.2 coincide with the seven steps proposed by Montgomery (2017). Step 3 mentions the factorial arrangement and central points, while step 6A refers to axial points. All of these points are part of the Central Composite Design (CCD). The CCD is one of the arrangements that allows the identification of possible curvatures in the response surfaces, as it considers more than two different levels for each decision variable (MYERS; MONTGOMERY; ANDERSON-COOK, 2016). Figure 2.3 represents a CCD arrangement for the case of three decision variables. The cube containing the eight blue points consists of the factorial arrangement, the green point in the center of the cube represents the center points, and the axial points are the red points outside the cube.

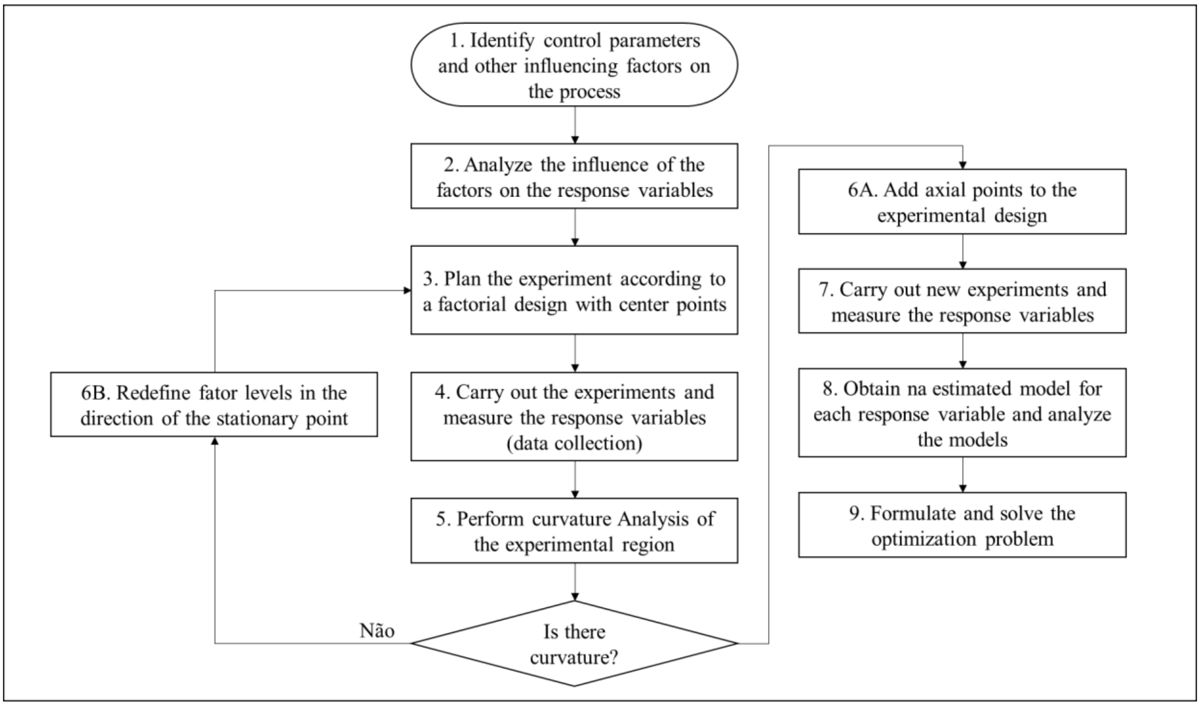


Figure 2.2 - Roadmap for conducting the response surface methodology

Source: adapted from Oliveira et al. (2019)

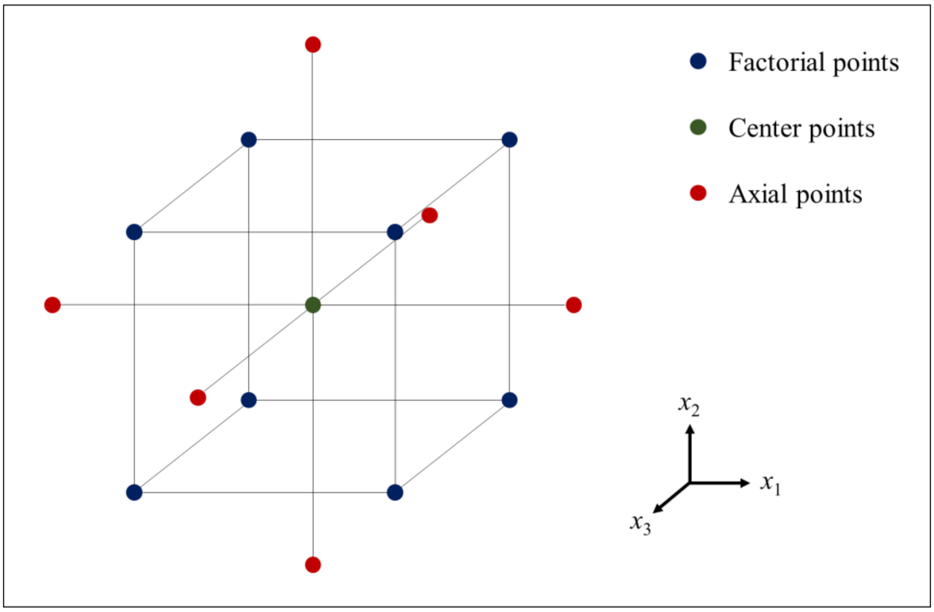


Figure 2.3 - Central composite arrangement (CCD) for three input variables

Source: adapted from Myers, Montgomery and Anderson-Cook (2016)

With the results obtained from CCD arrangements, it is possible to build models that present curvature. As presented in section 1.2.1, Oliveira *et al.* (2019) found that the second order (or quadratic) polynomial model is widely used in studies of manufacturing processes optimization. From quadratic models, it is possible to identify possible curvatures on the response surface. This model, already described in Equation (1.1), is presented again in Equation (2.14) because it is related to the concepts in the next sections.

$$y \sim f(\mathbf{x}) = \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{i<j} \beta_{ij} x_i x_j + \epsilon = \mathbf{a}'(\mathbf{x})\boldsymbol{\beta} + \varepsilon \qquad (2.14)$$

Once constructed, the mathematical models of Equation (2.14) can be represented in the form of surface graphs in a given region of the solution space. Consider, for example, the following theoretical model composed of two decision variables:

$$f(\mathbf{x}) = 0{,}0653 + 1{,}0178x_1 + 0{,}9855x_2 + 0{,}9737x_1^2 + 0{,}9578x_2^2 + 0{,}9555x_1x_2 \qquad (2.15)$$

The surface graph for the model described in Equation (2.15) corresponds to Figure 2.4.

For problems with $n > 2$ decision variables, each 3D surface graph will present one response of interest at a time and their corresponding values of a pair of decision variables.

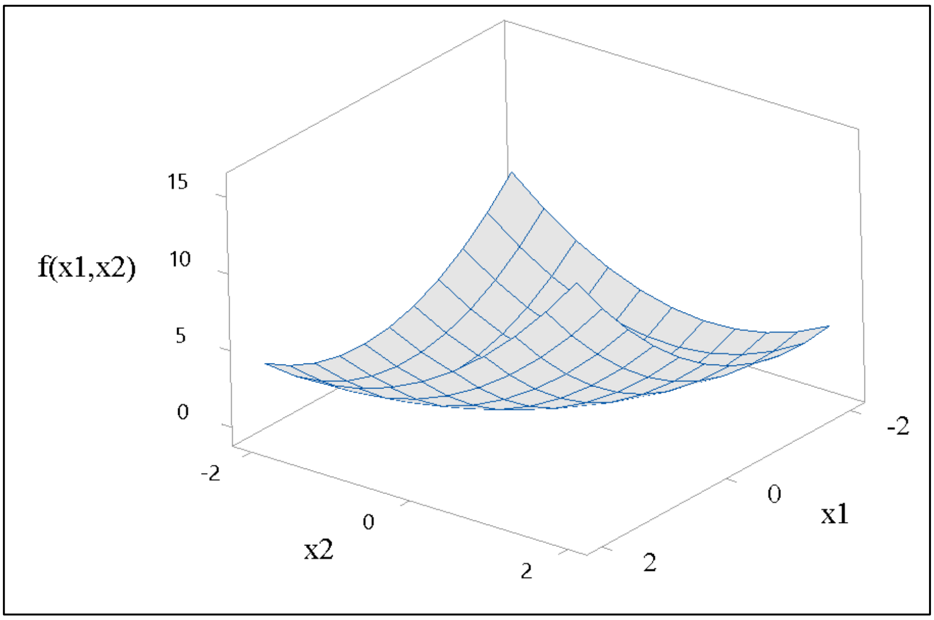


Figure 2.4 - Surface plot for Equation (2.15)

Source: the author

The objective of RSM is, therefore, to represent a vector composed of all the $m$ results for a giving response of interest y using a vector $f(\mathbf{x})$ as a mathematical model. A representation frequently used in RSM applications is Equation (2.16) (MYERS; MONTGOMERY; ANDERSON-COOK, 2016),

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) + \boldsymbol{\varepsilon} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{2.16}$$

Where:

**y** = vector composed of the experimental results of the response of interest;

**ε** = vector $m\ x\ 1$ of residues, where $m$ is the number of experiments performed;

**X** = experimental matrix or design matrix, with dimension $m\ x\ r\ +\ 1$, where $r$ is the total terms of the response surface model;

**β** = vector $r\ x\ 1$ of coefficients.

It is important to highlight the difference between the vector **x** of decision variables and the experimental matrix **X**. The first column of the matrix **X** is composed of numbers "1", so that the multiplication $\mathbf{X}\boldsymbol{\beta}$ results in the vector $\mathbf{f}(\mathbf{x})$ of $m$ estimated values the same answer of interest in each test. The vector $\mathbf{F}(\mathbf{x})$, in turn, is the vector composed of the objective functions, which represent *k* responses of interest, as presented in section 2.1.

Equation (2.17) can be written in more detail as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1r} \\ 1 & x_{21} & \vdots & x_{2r} \\ \vdots & \cdots & \ddots & \vdots \\ 1 & x_{m1} & \cdots & x_{mr} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{ij} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix} \tag{2.17}$$

The estimation of the coefficients in **β** is done using regression methods. Among them, there is the method of ordinary least squares, described in section 2.4.

## 2.4. The Ordinary Least Squares Method

The ordinary least squares (OLS) method consists of estimating the **β** coefficients in Equation (2.16) in order to minimize the sum of the squared residuals (ROCHA *et al.*, 2017). More specifically, Montgomery and Runger (2018) define that the objective of the OLS method is to determine values for the vector $\widehat{\boldsymbol{\beta}}$ of least squares estimators which minimizes the sum of the squared residuals (L), defined as follows:

$$L = \sum_{i=1}^{m} {\varepsilon_i}^2 = SS_\varepsilon = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) = \mathbf{y}'\mathbf{y} - \mathbf{2}\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} \tag{2.18}$$

Where:

ε = residuals between the real result of experiment *i* and its corresponding result by the regression model;

**ε** = vector *m x* 1 of residues, for the *m* experiments performed.

The value of $L$ as a function of the choice of $\widehat{\boldsymbol{\beta}}$ results in a second order convex equation. Therefore, there will be only one point of minimum: the global minimum. Thus, the least squares estimator $\widehat{\boldsymbol{\beta}}$ is the solution of Equation (2.19),

$$\frac{\partial L}{\partial \boldsymbol{\beta}} = -2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\widehat{\boldsymbol{\beta}} = 0 \tag{2.19}$$

which can be simplified to

$$\mathbf{X}'\mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{X}'\mathbf{y} \tag{2.20}$$

According to Rocha *et al*. (2017), Equation (2.19) refers to normal least square equations in their matrix form. Multiplying both sides of Equation (2.20) by the inverse of the matrix $\mathbf{X}'\mathbf{X}$, equation (2.21) is obtained.

$$\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{y} \tag{2.21}$$

Once a response surface model is built, some important features must be assessed. The significance of the coefficients in $\boldsymbol{\beta}$ can be verified through an Analysis of Variance (ANOVA) (CHRISTENSEN, 2016). ANOVA allows for verifying which terms are significant and which can be disregarded from the statistical point of view. The adjustment of the model is represented by the coefficient of determination $R^2$, which represents the percentage of data observed in the response that the mathematical model (GOMES, 2013) explains. Adjusted $R^2$ is a correction of $R^2$ that takes into account the inclusion of little explanatory terms. Predicted $R^2$ is related to the quality of the model's forecasts. More detailed information on the procedures for checking the quality of the models is found in Paiva (2006).

The following sections focus on detailing the stochastic nature of multi-objective optimization problems. The random nature of the coefficients in $\boldsymbol{\beta}$ is demonstrated in section 2.4.1.

### 2.4.1. RSM model coefficient randomness

According to Díaz-garcía and Bashiri (2014), assuming that the residuals in $\boldsymbol{\varepsilon}$ are normally distributed, the coefficient vector also follows a normal probability distribution. The calculations of the vector of expected values $E(\hat{\boldsymbol{\beta}})$ and the variance and covariance matrix $Cov(\hat{\boldsymbol{\beta}})$ are presented as follows: using Equation (2.21), the expected value of estimator ($\hat{\boldsymbol{\beta}}$) can be written as:

$$E(\hat{\boldsymbol{\beta}}) = E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{y}] \tag{2.22}$$

Substituting $\mathbf{y}$ in Equation (2.22) according to Equation (2.16), we have:

$$E(\hat{\boldsymbol{\beta}}) = E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon})] \tag{2.23}$$

Multiplying and rearranging the terms of Equation (2.23), then:

$$E(\hat{\boldsymbol{\beta}}) = E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X}\boldsymbol{\beta} + \mathbf{X}'\boldsymbol{\varepsilon})] = E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X})\boldsymbol{\beta}] + E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})] \tag{2.24}$$

Since $(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X}) = \mathbf{I}$ and $E[\boldsymbol{\varepsilon}] = 0$, we have:

$$E(\hat{\boldsymbol{\beta}}) = E(\boldsymbol{\beta}) + E[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})] = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}')E(\boldsymbol{\varepsilon}) = \boldsymbol{\beta} \tag{2.25}$$

Therefore, the expected values for the coefficients correspond to the vector $\boldsymbol{\beta}$ itself, which is calculated by Equation (2.21). In addition, it is concluded that the estimators in $\hat{\boldsymbol{\beta}}$ are non-biased (or non-biased), that is, $E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$.

In turn, the variance and covariance matrix $Cov(\hat{\boldsymbol{\beta}})$ is defined by Equation (2.26):

$$Cov(\widehat{\boldsymbol{\beta}}) = E\left\{[\widehat{\boldsymbol{\beta}} - E(\widehat{\boldsymbol{\beta}})]'[\widehat{\boldsymbol{\beta}} - E(\widehat{\boldsymbol{\beta}})]\right\} \tag{2.26}$$

As $E(\widehat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$, as shown in Equation (2.25), we have:

$$Cov(\widehat{\boldsymbol{\beta}}) = E[\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}]'[\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}] \tag{2.27}$$

Using Equations (2.16) and (2.21), then:

$$\begin{aligned}\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{y} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}[\mathbf{X}'(\mathbf{X}'\boldsymbol{\beta} + \boldsymbol{\varepsilon})] = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}[(\mathbf{X}'\mathbf{X})\boldsymbol{\beta} + \mathbf{X}'\boldsymbol{\varepsilon}] \\ = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X})\boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})\end{aligned} \tag{2.28}$$

Again, as $(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X}) = \mathbf{I}$, we have:

$$\widehat{\boldsymbol{\beta}} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon}) \tag{2.29}$$

Therefore, the difference $\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}$ results in:

$$\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon}) \tag{2.30}$$

Using Equation (2.30) in Equation (2.27):

$$Cov(\widehat{\boldsymbol{\beta}}) = E\{[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})]'[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})]\} \tag{2.31}$$

Rewriting $(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})$ in the form of $(\boldsymbol{\varepsilon}'\mathbf{X})(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}$:

$$\begin{aligned}Cov(\widehat{\boldsymbol{\beta}}) = E\{[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\boldsymbol{\varepsilon})]'[(\boldsymbol{\varepsilon}'\mathbf{X})(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}]\} \\ = E\{[(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X})][(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon})(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}]\}\end{aligned} \tag{2.32}$$

Again, as $(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}(\mathbf{X}'\mathbf{X}) = \mathbf{I}$, then:

$$Cov(\widehat{\boldsymbol{\beta}}) = E[(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon})(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}] = E(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon})(\mathbf{X}'\mathbf{X})^{-\mathbf{1}} \tag{2.33}$$

The scalar $\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}$ corresponds to the estimated variance of the residuals, commonly represented by $\hat{\sigma}^2$ and calculated by Equation (2.34) (ROCHA et al., 2017):

$$E(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}) = \hat{\sigma}^2 = \frac{SS_\varepsilon}{(m-r)} = \frac{\mathbf{y}'\mathbf{y} - \boldsymbol{\beta}'(\mathbf{X}'\mathbf{y})}{(m-r)} \tag{2.34}$$

Where:

$SS_\varepsilon$ = sum of squares of errors, $SS_\varepsilon = \sum_{i=1}^{m} \varepsilon_i^2$

Therefore, the matrix $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$ of variance and covariance of the coefficients of a response surface model is calculated by Equation (2.35),

$$Cov(\widehat{\boldsymbol{\beta}}) = \boldsymbol{\Sigma}_{\boldsymbol{\beta}} = \sigma^2(\mathbf{X}'\mathbf{X})^{-\mathbf{1}} \tag{2.35}$$

where $\sigma^2 = SS_\varepsilon/(m-r)$ is the estimated variance of the residuals.

Still according to Rocha et al. (2017), the confidence interval for each $\beta_j$ coefficient is given by Equation (2.36), where $C_{jj}$ is the jj$^{th}$ element of matrix $(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}$.

$$\hat{\beta}_j - t_{\alpha/2,m-r}\sqrt{\hat{\sigma}^2 C_{jj}} \le \hat{\beta}_j \le \hat{\beta}_j + t_{\alpha/2,m-r}\sqrt{\hat{\sigma}^2 C_{jj}} \tag{2.36}$$

Thus, it is noted that the response surface models built based on experiments have a random nature since the variance present in the coefficients propagates to the objective function. Section 2.4.2 below describes how the variance of a response surface model is calculated considering the $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$ matrix of variance and covariance of the coefficients.

## 2.4.2. Response surface model variance

According to what was demonstrated in section 2.4.1, the estimated coefficients for a response surface model are normally distributed with a vector of means $\widehat{\boldsymbol{\beta}}$ estimated according to equation (2.21) and a matrix $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$ $r\ x\ r$ of variance and covariance, as presented in Equation (2.35).

It is important to remember that $\mathbf{a}(\mathbf{x})$ is a vector composed of all $r$ terms (the constant 1, linear, quadratic terms and second order interactions) of the response surface model $f(\mathbf{x})$, as presented in Equation (2.14). In this case, the variables in $\mathbf{x}$ are considered deterministic and the random variables are the coefficients in $\boldsymbol{\beta}$. Thus, it is necessary to rewrite Equation (2.13) by replacing $\mathbf{x}$ with $\boldsymbol{\beta}$ in all elements, according to Equation (2.37):

$$Var[f(\mathbf{x})] = \boldsymbol{\nabla}' f(\boldsymbol{\beta})\boldsymbol{\Sigma}_{\boldsymbol{\beta}}\boldsymbol{\nabla} f(\boldsymbol{\beta}) \tag{2.37}$$

Equation (2.37) can be rewritten in more detail as follows:

$$Var[f(\mathbf{x})] = \begin{bmatrix} \frac{\partial f(\boldsymbol{\beta})}{\beta_0} & \frac{\partial f(\boldsymbol{\beta})}{\beta_1} & \cdots & \frac{\partial f(\boldsymbol{\beta})}{\beta_{ij}} \end{bmatrix} \begin{bmatrix} \sigma^2_{\beta_0} & \sigma_{\beta_0\beta_1} & \cdots & \sigma_{\beta_0\beta_{ij}} \\ \sigma_{\beta_0\beta_1} & \sigma^2_{\beta_1} & \cdots & \sigma_{\beta_1\beta_{ij}} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\beta_0\beta_{ij}} & \sigma_{\beta_1\beta_{ij}} & \cdots & \sigma^2_{\beta_{ij}} \end{bmatrix} \begin{bmatrix} \frac{\partial f(\boldsymbol{\beta})}{\beta_0} \\ \frac{\partial f(\boldsymbol{\beta})}{\beta_1} \\ \cdots \\ \frac{\partial f(\boldsymbol{\beta})}{\beta_{ij}} \end{bmatrix} \tag{2.38}$$

Note that $\partial f(\boldsymbol{\beta})/\beta_0 = 1$, $\partial f(\boldsymbol{\beta})/\beta_1 = x_1$ and so on. Note that the partial derivatives of the gradient of $\boldsymbol{\nabla} f(\boldsymbol{\beta})$ are equal to the terms of the vector of responses to $\mathbf{a}(\mathbf{x})$. Thus, the variance of response surface models can be calculated from Equation (2.39):

$$Var[f(\mathbf{x})] = \sigma^2 \mathbf{a}'(\mathbf{x})(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}(\mathbf{x}) \tag{2.39}$$

As an example, consider the response surface model presented in Equation (2.15) and whose expected value is illustrated in Figure 2.4. Suppose the mean square error $\sigma^2$ is 0.3694. Then, evaluating Equation (2.38) in space $(-2,-2) \leq \mathbf{x} \leq (2,2)$, the following surface graph is obtained for the standard deviation of the function represented here by $SD[f(\mathbf{x})]$ and illustrated in Figure 2.5.

As mentioned in chapter one, in many multi-objective optimization problems, the results of interest are significantly correlated in addition to stochastic. Therefore, the calculation of the probabilities of the constraints of stochastic problems must be carried out taking into account the multivariate nature of the responses of interest (Johnson; Wichern, 2007). In these cases, one of the most used alternatives is the use of multivariate probability distributions. Among them, the multivariate normal distribution, which is presented in section 2.5, stands out. Appendix A includes concepts and formulas for calculating the correlation between a pair of random variables.

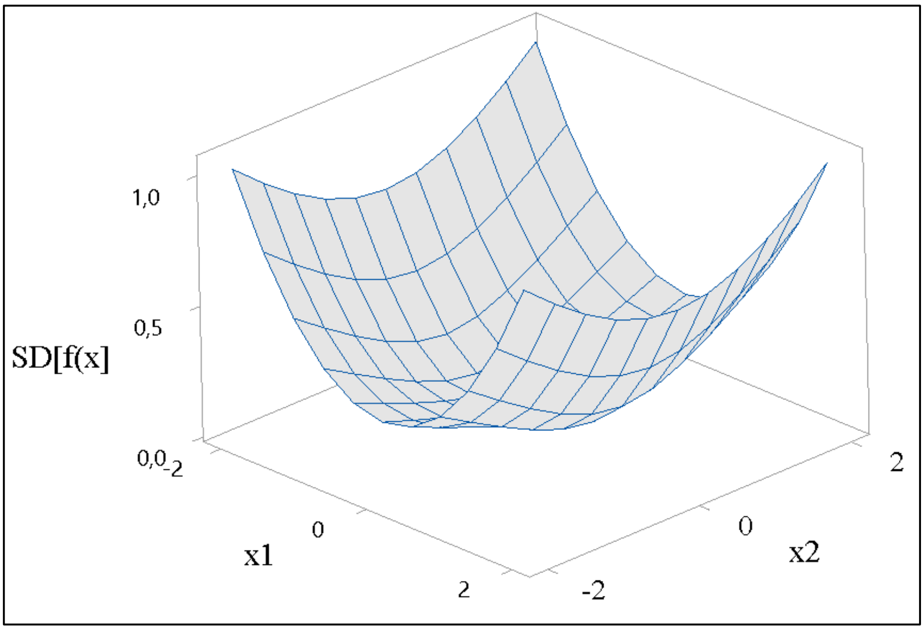


Figure 2.5 - Surface graph for the standard deviation of Equation (2.15)

Source: the author

## 2.5. Multivariate normal distribution

The normal probability distribution is one of the most used for representing continuous random variables. In fact, whenever a random experiment is replicated, the random variable corresponding to the average (or total) result of the replicates tends to be distributed according to a normal probability distribution (MONTGOMERY; RUNGER, 2018). According to the authors, for the univariate case with $\mu = E[x]$ e $\sigma^2 = Var(x)$, the normal probability density function, also known as Gaussian distribution and here represented by $\theta(x)$, is defined by Equation (2.40):

$$\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}q(x)}, x \in ]-\infty, \infty[ \tag{2.40}$$

Where $q(x) = (x - \mu)/\sigma^2$. Note that $q(x)$ determines the absolute distance between $x$ and μ on the scale of a standard deviation. Also note that it is possible to rewrite $q(x)$ as follows:

$$q(x) = (x - \mu)(\sigma^2)^{-1}(x - \mu) \tag{2.41}$$

For multivariate problems, that is, problems whose responses of interest have significant correlations and are joined, it is necessary to use multivariate distributions. According to

Johnson and Wichern (2007), the definition of $q(x)$ according to Equation (2.41) can be extended to the case where $\mathbf{x}$ is a vector of $n \geq 2$ decision variables, as defined in the beginning section 2.1. Thus, $\boldsymbol{\mu} = \{\mu_1, \mu_2, \dots, \mu_n\}$ is the vector of means for each variable in $\mathbf{x}$, and $\boldsymbol{\Sigma}_{\mathbf{x}}$ is the variance and covariance matrix of the variables in $\mathbf{x}$. Thus, we have:

$$Q_p(\mathbf{x}) = (\mathbf{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}_p^{-1} (\mathbf{x} - \boldsymbol{\mu}) \tag{2.42}$$

Thus, Equation (2.43) below refers to the calculation of the multivariate normal probability density function,

$$\theta_p(\mathbf{x}) = \frac{1}{(2\pi)^{p/2} |\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2} Q_p(\mathbf{x})} \tag{2.43}$$

where $x_i \in ]-\infty, \infty[$ for $i = 1,2, \dots, n$.

Therefore, the probability $\varphi_p(\mathbf{x})$ of the vector x being within a range delimited by $\mathbf{x}_l$ and $\mathbf{x}_u$ (lower and upper limits respectively) can be calculated by Equation (2.44):

$$\varphi_p(\mathbf{x}) = \int_{\mathbf{x}^{\mathrm{L}}}^{\mathbf{x}^{\mathrm{U}}} \frac{1}{(2\pi)^{p/2} |\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2} Q_p(\mathbf{x})} d\mathbf{x} \tag{2.44}$$

From the beginning of chapter 2 to the present section, statistical techniques were presented of the formulation of a multi-objective optimization problem. Once formulated, the problem is solved using search algorithms whose function is to identify the optimal solution within the set of viable solutions. Among such algorithms, there is the generalized reduced gradient (GRG), described in section 2.6.

## 2.6. The generalized reduced gradient algorithm

There are several optimization algorithms available to search for the optimal solution in optimization problems. The Generalized Reduced Gradient (GRG) is considered one of the most robust and efficient gradient algorithms (KÖKSOY; DOGANAKSOY, 2003). GRG can be used to solve several restricted and unrestricted nonlinear optimization problems. In addition, GRG can easily be accessed (KÖKSOY, 2008) and is already available for use in some commercial software, as in the *Microsoft Excel® Solver* supplement. The name “reduced gradient” is related to its functioning: at each iteration, the constraints replace the objective function, decreasing the number of variables and, consequently, reducing the number of gradients (NASH; SOFER, 1996).

GRG has three advantages over other gradient methods:

1. If the search for the optimal solution ends before its confirmation, the last point found is viable since each point generated is viable and, generally, close to the optimum point;
2. If the algorithm has a convergent sequence, the limit point guarantees at least a locally optimal solution;
3. Most primal methods such as GRG are generally absolute; that is, they do not depend on a specific structure, such as convexity.

Consider, for instance, the general formulation for a nonlinear optimization problem of an objective function described in Equation (2.45) (LASDON *et al.*, 1978). First, if there are inequality constraints $g_i(\mathbf{x}) \leq 0$, they should be transformed into equality constraints $h_i(\mathbf{x}) = 0$, adding slack variables to each constraint $h_i$, $i = 1, 2, \dots, k$.

$$\begin{gathered} Min\ f(\mathbf{x}) \\ subject\ to\colon g_i(\mathbf{x}) \leq 0 \\ h_i(\mathbf{x}) = 0 \end{gathered} \tag{2.45}$$

The GRG algorithm is based on the transformation of constraints into an unrestricted one through direct substitution (GOMES, 2013). Thus, vector $\mathbf{x}$ of decision variables is divided into two sub-vectors $\mathbf{x_B}$ and $\mathbf{x_N}$, with $\mathbf{x_B}$ being the vector of basic (or dependent) variables and $\mathbf{x_N}$ being the vector of non-basic (or independent) variables. Rewriting Equation (2.45) distinguishing $\mathbf{x}$ in basic and non-basic variables, there is (CHEN; FAN, 2002):

$$\begin{gathered} Min\ f(\mathbf{x}) = f[\,\mathbf{x_B}\,(\,\mathbf{x_N}\,), \mathbf{x_N}\,] \\ subject\ to\colon \mathbf{l}_N \leq \mathbf{x_N} \leq\ \mathbf{u_N} \end{gathered} \tag{2.46}$$

Where:

$\mathbf{l}_N$ = lower limit for $\mathbf{x_N}$;

$\mathbf{u_N}$ = upper limit for $\mathbf{x_N}$.

Starting from a viable starting point $\mathbf{x}^k$, the GRG algorithm defines a direction of movement to search for a better value for the objective function at each iteration. Such a direction of movement is defined by calculating the reduced gradient. This procedure is repeated with each iteration and the search for an optimal solution ends when the value of the reduced gradient reaches a pre-established minimum error (convergence criterion).

The GRG algorithm was used in this study to obtain Pareto-optimal solutions for the multi-objective optimization problems addressed. Further details on the operation of the algorithm can be found in Rocha *et al*. (2017).

Section 2.7 presents the concepts related to the $C_{pk}$ and $PPM$ indexes, which are indicators widely used by the industry and which have already been included in multi-objective optimization problems, also combined with multivariate statistical techniques (PERUCHI et al., 2018; WANG ; CHEN, 1998).

## 2.7. Process capability indices

Capability analysis is defined as a set of activities to quantify the variability of a process, to analyze the variability related to the requirements and specifications of the product and to support the reduction of this variability (MONTGOMERY, 2013). The calculation of the process capability is an important part of the DMAIC method (from English, Define, Measure, Analyze, Improve and Control). DMAIC is the sequence of phases in Lean Six Sigma projects (ANTONY, 2014), and the process capability refers to its uniformity. The variability of critical to quality (CTQ) characteristics is, therefore, a measure of process uniformity (COSTA; EPPRECHT; CARPINETTI, 2018).

Process capability can be measured using different indices. Kotz and Johnson (2002) performed a literature review on process capacity indices and found 170 publications between 1992 and 2000. One of the most used capacity indices is $C_p$, calculated by Equation (2.47),

$$C_p = \frac{USL - LSL}{6\sigma} \tag{2.47}$$

Where:

$USL$ = Upper Specification Limit;

$LSL$ = Lower Specification Limit.

The USL-LSL difference consists of the tolerance for the CTQ, which may also refer to the response of interest $y$. In the denominator of the result of $C_p$, there is the so-called natural variation range of the process, assuming that the data are normally distributed. More specifically, the probability of obtaining a value belonging to the range corresponding to $6\sigma$ (six population standard deviations) centred on the mean is 99.73% for normally distributed data (COSTA; EPPRECHT; CARPINETTI, 2018).

In practical applications, the population standard deviation σ is almost always unknown. For this reason, it must be replaced by an estimated $\hat{\sigma}$, which can be obtained by using different strategies.

Costa, Epprecht and Carpinetti (2018) present four different ways to estimate $\hat{\sigma}$, as presented in Equation (2.48). Suppose we have a data set composed by $m$ samples of $n$ units each, and that $N = mn$. Then, $X_{ij}$ is the value for the critical-to-quality characteristic of the j^th^ observation in the i^th^ subgroup. In Equation (2.48), $S_A$ represents the global unbiased estimator of $\hat{\sigma}$. $S_B$ refers to another unbiased estimator that used the expected values of each subgroup in order to estimate $\hat{\sigma}$. $S_C$ and $S_D$ are unbiased estimators that takes into account the variability present within the subgroups, using either the mean standard deviation $\bar{S}$, as for $S_C$, or the mean range $\bar{R}$, as for $S_D$. The constants $c_4$ and $d_2$ are dependent on the sample size $n$ for $S_C$ and $S_D$, $m$ for $S_B$ or $mn$ for $S_A$. When using $S_D$, and there is only one observation per subgroup, then the moving range is used to estimate $\hat{\sigma}$, such as

$$\hat{\sigma} \sim \begin{cases} S_A = \dfrac{1}{c_4}\sqrt{\dfrac{\sum_{i=1}^{m}\sum_{i=1}^{n}\left(X_{ij} - \bar{\bar{X}}\right)^2}{mn-1}} \\ S_B = \left[\dfrac{1}{c_4}\sqrt{\dfrac{\sum_{i=1}^{m}(\bar{X}_i - X_i)^2}{m-1}}\right]\sqrt{n} \\ S_C = \dfrac{\bar{S}}{c_4} \\ S_D = \dfrac{\bar{R}}{d_2} \end{cases} \tag{2.48}$$

For the use of Equation (2.47), in addition to the usual assumption of normality, it is assumed that the process is under statistical control and that the population average μ is centered between the specification limits, that is, $\mu = (USL - LSL)/2$.

The $C_p$ index does not allow for identifying the displacement between the average resulting from the process and its nominal value, also called the target value (or target) – note that the mean $\mu$ of the process is not present in Equation (2.47). In order to consider the displacement of the process average in relation to its target value, the $C_{pk}$, which the index calculated by Equation (2.49), is used in which the $C_{pu}$ and $C_{pl}$ are unilateral capacity indices.

$$C_{pk} = Mín\left(C_{pu} = \frac{USL - \mu}{3\sigma}, \qquad C_{pl} = \frac{\mu - LSL}{3\sigma}\right) \tag{2.49}$$

One limitation of the $C_{pk}$ is that it only considers the worst case scenario for the process capability. The sigma level, also know as z benchmark, is an alternative to solve this problem, and its calculation is related to the parts per million (PPM) index, which is described in section 2.7.1.

### 2.7.1. Parts per million (PPM)

The $C_{pk}$ index is widely disseminated in the manufacturing industry as it is a key result in many Lean Six Sigma projects (ANTONY, 2014). The term six sigma is strongly related to the capacity index. According to Montgomery and Runger (2018), a process classified as "six sigma" is a process whose $C_{pk}$ is greater than or equal to 2. At this level of process capacity, if there is a change in the process average of up to 1.5 standard deviations in relation to its target value, only 3.4 pieces will fail on average for every one million produced (ANTONY, 2014). This indicator is known as parts per million, or Parts Per Million (PPM). In addition to the $C_{pk}$ index, PPM is an indicator widely used in industry as it considers the expected value and the variance of the critical quality characteristic. The observed PPM is calculated by the simple division between the failed pieces observed in the study by the total of analyzed pieces, multiplying the result of this division by 106. The estimated PPM, in turn, assumes that data follow a certain distribution and can be calculated by Equation (2.50).

$$PPM = 10^6 \left[1 - \int_{LSL}^{USL} \theta(x)dx\right] \tag{2.50}$$

Where:

$\theta(x)$ = probability density function that represents the quality characteristic.

Section 2.8 presents some general concepts of Monte Carlo Simulation, which was used in this research to validate Equation (2.13).

## 2.8. Monte Carlo simulation

The term "Monte Carlo" is typically associated with the modelling and simulation process influenced by randomness (BRANDIMARTE, 2014). In this type of simulation, several random scenarios are generated, and the relevant statistics are computed to measure the performance of a decision policy or an asset value (LAW, 2015). The Monte Carlo simulation is classified as static as it is a representation of a system at a specific time (CHWIF; MEDINA, 2010).

To carry out a Monte Carlo simulation, it is necessary to perform the following sequence of steps (BANKS *et al.*, 2010):

1. Definition of stochastic variables;
2. Construction of the mathematical model of the response of interest, involving deterministic and stochastic variables;
3. Generation of random scenarios;
4. Measurement of statistical interest parameters;
5. Analysis of decision-making performance.

In this research, Monte Carlo Simulation was used to validate the proposed method of stochastic programming for data related to the selected case study. Among the manufacturing processes addressed by multi-objective optimization studies, the turning process of hardened steels stands out, which was the object of study in this research and is presented in section 2.9.

## 2.9. The hardened steel turning process

Turning is a process of removing material from a part's revolutionary movement in relation to its own axis and from its contact with a cutting tool (KLOCKE, 2011). The turning process of hardened steels is a particular case of turning, in which the parts have a hardness greater than 45 HRC (CAMPOS *et al.*, 2017), more typically varying between 52 and 65 HRC (FERREIRA *et al.*, 2016; GRZESIK, 2009).

Traditionally, hardened steels were machined in grinding processes (BOUACHA *et al.*, 2014). However, advances in the manufacture of hardened steel - such as the hard-turning process - have contributed significantly to the quality of products (KUMAR *et al.*, 2018; PAIVA *et al.*, 2009; REVEL *et al.*, 2016). For this reason, hard turning has been widely used in the industry. Many of the main mechanical components, such as gears, shafts and bearings, can be manufactured using hard-turning (ALOK; DAS, 2019). In fact, compared to grinding, hard turning can have a similar, or even better surface finish (LIMA *et al.*, 2005) and with a higher material removal rate (BARTARYA; CHOUDHURY, 2012). Other advantages of hard turning include reducing or eliminating the use of a coolant cutting fluid, reducing the process cost, increasing productivity, improving material properties and reducing energy consumption (GAITONDE *et al.*, 2009; HUANG ; CHOU; LIANG, 2007; PERUCHI *et al.*, 2018).

Several types of hardened steels are machined in the turning process nowadays. Among them, AISI 52100 steel stands out as it is often used in the manufacturing of bearings, shafts and joints due to its mechanical and corrosion resistance (ALOK; DAS, 2019). Chinchanikar and Choudhury (2015) carried out a literature review on the machining of hardened steels in which AISI 52100 steel is highlighted among the most studied hardened steels. One of the possible reasons for this is that AISI 52100 steel is part of a set of alloys classified as “hard-to-cut steel alloys” (ALOK; DAS, 2018). Such difficulty is mainly related to the availability and adequate selection of materials for the cutting tools and to the economic viability of the process (TORRES *et al.*, 2019a).

Figure 2.6 illustrates some of the main variables (input and output) of a hardened steel turning process.

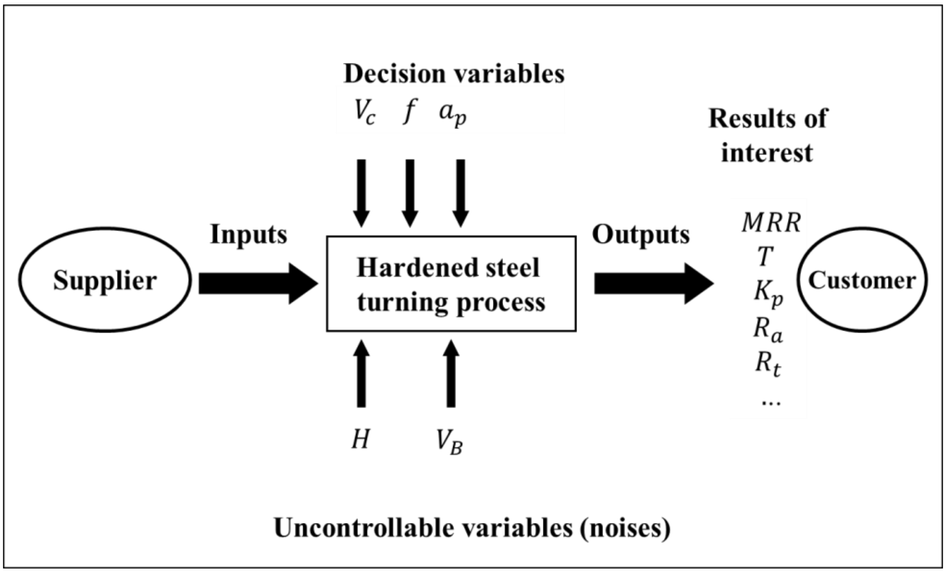


Figure 2.6 - Representation of the variables of the hard-turning process
Source: the author

The main decision variables (or machine parameters) considered in studies of optimization of hard turning processes are (CAMPOS et al., 2014):

a) Cutting speed ($V_c$): usually measured in meters per minute (m/min), it is the displacement of the uncut surface of the part in relation to the cutting edge of the tool (TORRES *et al.*, 2016).

b) Feed rate ($f$): it measures the displacement between the part and the tool at each revolution, commonly measured in millimeters per revolution (mm/rev.);

c) Depth of cut ($a_p$): it is the difference between the diameter of the cut part and the diameter of the uncut part, usually measured in millimeters (mm).

Figure 2.7 illustrates the turning process and the three main decision variables previously described. Uncontrollable variables (noises), often include the level of tool wear $V_B$ (or flank wear), measured in millimeters, and the surface hardness $H$ of the work piece (PAIVA *et al.*, 2012), measured in HRC. However, there are studies that do not include noise analysis separately from the experiment responses, as in the case of the object of study selected for this research. In these cases, the variability arising from the noise is not separated from the decision variables and thus remains included in the mean square error of the model ($\hat{\sigma}^2$), calculated by Equation (2.34).

The main responses of interest in a turning process can be classified into three categories: results of productivity, costs and quality. The results of interest considered in the present research are presented in sections 2.8.1 to 2.8.3 respectively.

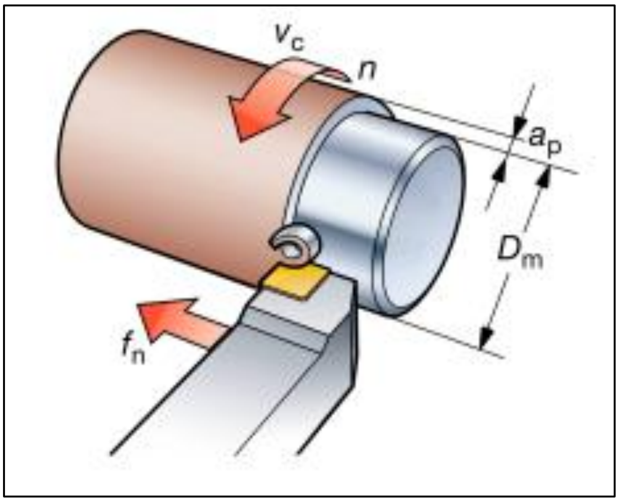


Figure 2.7 - Representation of the main decision variables of the turning process

Source: Sandvick (2017)

### 2.9.1. Productivity measures and turning process times

For machining processes in general, one of the main productivity metrics is the Material Removal Rate ($MRR$). In the case of the turning process, $MRR$ can be calculated from Equation (2.51) (DINIZ; MARCONDES; COPPINI, 2014):

$$MRR = V_c . f . a_p \tag{2.51}$$

Another measure related to productivity is the Cutting Time ($C_t$), usually measured in minutes and calculated by Equation (2.52):

$$C_t = \frac{l_f . \pi . d}{100 f . V_c} \tag{2.52}$$

Where:

$l_f$ = length of the part in millimeters;

$d$ = diameter of the part in millimeters.

The cutting time is only a small part of all machining times. Considering the production of a batch of $Z$ parts, the machining cycle is directly composed of the following activities (DINIZ; MARCONDES; COPPINI, 2014):

1. Placement and fixation of the piece;
2. Approach and positioning of the tool;
3. Cut;
4. Tool spacing;
5. Inspection (if necessary) and removal of the part;

   In addition to these five direct activities, three indirect activities are also included:
6. Preparation of the machine;
7. Removal of the tool for its replacement;
8. Replacement and adjustment of the new tool.

Therefore, the total cycle time ($T_t$) includes other times necessary for the execution of the process (all in minutes) in addition to the cut-off time (activity 3), as shown in Equation (2.53) (TORRES *et al*., 2019a):

$$T_t = C_t + t_s + t_a + \frac{t_p}{Z} + \frac{N_t}{Z} t_i \tag{2.53}$$

Where:

$t_s$ = secondary time (activities 1 and 5);

$t_a$ = time of approach and removal of the tool (activities 2 and 4);

$t_p$ = time of setup or preparation of the machine (activity 6);

$Z$ = lot size (in units of part);

$N_t$ = number of tool changes;

$t_i$ = insert change time (activities 7 and 8).

Equation (2.54) is used to calculate $N_t$, $T$ is tool life, measured in minutes (TORRES *et al.*, 2019b):

$$N_t = Máx\left[0, I\left(Z\frac{C_t}{T} - 1\right)\right] \tag{2.54}$$

Where:

$T$ = tool life (min).

It is important to highlight that the tool life ($T$) corresponds to the total time (in minutes) and is used in the process as it is related to both productivity and the cost of the process. As will be shown in chapter 4, maximizing tool life through the choice of decision variables does not necessarily reduce the process costs (TORRES *et al.*, 2019a). In the sequence, section 2.8.2 presents the main quality characteristics of the hard-turning process.

## 2.9.2. Total cost per part of the turning process

The total cost of machining a part is considered one of the most important aspects in manufacturing a product (KUMAR *et al.*, 2018). The total cost includes manufacturing costs directly related to the process (costs of machines, labor and tools) and other indirect costs (costs of quality control, raw materials, indirect labor, *etc.*) (CAUCHICK-MIGUEL; COPPINI, 1996). The cost of manufacturing a part is also defined by the sum of operating costs, tooling and tool changes per part (GAUDÊNCIO *et al.*, 2019).

Diniz, Marcondes and Coppini (2014) define the costs directly involved in the production of a piece according to Equation (2.55):

$$K_p = K_{us} + K_{um} + K_{uf} \tag{2.55}$$

Where:

$K_p$ = production cost per piece;

$K_{us}$ = machining labor cost;

$K_{um}$ = machine cost;

$K_{uf}$= tooling cost.

Equations (2.56) to (2.58) show the calculations of $K_{us}$, $K_{um}$ and $K_{uf}$, respectively:

$$K_{us} = T_t \frac{S_h}{60} \tag{2.56}$$

$$K_{um} = T_t \frac{S_m}{60} \tag{2.57}$$

$$\frac{C_t}{T}\left(\frac{K_{th}}{N_{th}} + \frac{K_i}{N_i}\right) \tag{2.58}$$

Where:

$S_h$ = hourly cost of labor, measured in dollars per hour (U \$/h);

$S_m$ = cost per machine hour, in dollars per hour (U \$/h);

$K_{th}$ = tool holder cost (U\$);

$N_{th}$ = tool holder life (in number of edges);

$K_i$ = cost of the tool or insert (U\$);

$N_i$ = tool or insert life (in units).

Therefore, the total cost of production per piece ($K_p$) is calculated by Equation (2.59):

$$K_p = T_t \frac{(S_h + S_m)}{60} + \frac{C_t}{T}\left(\frac{K_{th}}{N_{th}} + \frac{K_i}{N_i}\right) \tag{2.59}$$

### 2.9.3. The stochastic nature of industrial variables related to the process cost

Some variables related to the total cost of production per piece ($K_p$) have a random nature and, for this reason, can be considered as noises in the process. This means that its representation should not be given in a deterministic way, but rather use probability distributions or other stochastic models. Such variables, here called industrial variables, include setup time ($t_p$), tool or insert change time ($t_i$), production batch size ($Z$), labor and machine costs ($S_h + S_m$), the tool holder cost ($K_{th}$), the tool holder life and the tool or insert cost ($K_i$). Figure 2.8 presents the relationship between the decision variables, which are deterministic, the stochastic industrial variables related to the cost and the total cost itself.

Samaddar (2001) presents some evidence of how the setup time variance ($t_p$) may affect a production system. Tas *et al.* (2019), in turn, argue that setup times are stochastic in practice; therefore, deterministic model solutions can compromise the quality of the solution if applied to real problems. The same authors also state that there is always an inherent variability in the execution time of a specific activity. Since the tool change time ($t_i$) is also a setup activity, this time must be represented stochastically in the optimization problem.

Additionally, an increasing number of companies have adopted the just-in-time production philosophy (LINN; BENJAMIN; WEI, 2000). As a result, the size of the production lots ($Z$) varies depending on the demand of each client, which is random (TEMPELMEIER; HILGER, 2015).

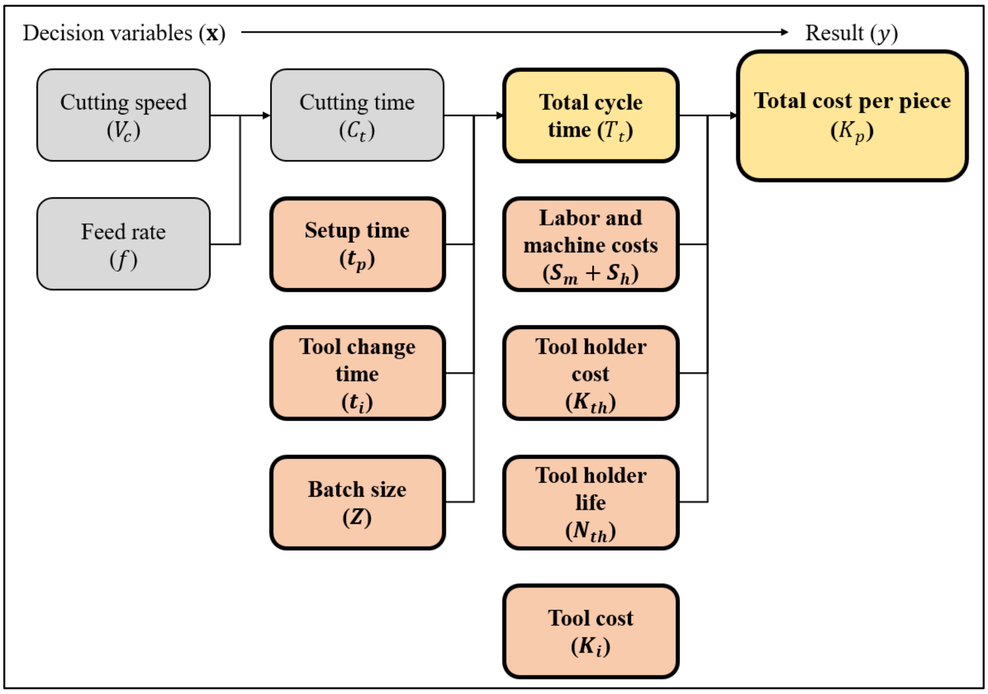


Figure 2.8 – Cause and effect relationships between decision variables, stochastic variables and the cost

Source: the author

Canyakmaz, Özekici and Karaesmen (2019) studied the impact of stochastic item prices on the optimal inventory configuration. According to these authors, price uncertainty is one of the most critical challenges that manufacturing companies face. Such uncertainties can be caused by unstable economies, crises and changes in exchange rates, among other factors. Thus, the cost of the tool holder ($K_{th}$) and the cost of the tool ($K_i$) can be included in the group of stochastic variables related to the total cost of production ($K_p$). Finally, in real applications, predicting the service life of the $N_{th}$ tool holder is extremely difficult. Nevertheless, this variable should be represented by a probability distribution (TORRES *et al*., 2019a).

Other industrial variables present in the calculation of the total cost of production can also be considered random. However, the variability present in these variables can be considered negligible compared to the aforementioned variables. This is the case with the time of approach and removal of the tool in relation to the part, movement made automatically by the machine, and the secondary time (placement, removal of the part and inspection), activities that are commonly done very fast and in following a standardized procedure.

### 2.9.4. Quality characteristics of the hard-turning process

Lean manufacturing is a production system whose goal is to eliminate or at least reduce different kinds of wastes, such as transport, inventory, motion, waiting, overproduction, over-processing and defects (OHNO, 1997). The present study is focused on the waste due to defects in the products, and such waste is strongly connected to the variability present in the process. Such variability can be included in an optimization problem and thus reduced by using stochastic programming.

The quality characteristics most frequently assessed in studies of hardened steel turning processes are related to the machined surface. In fact, surface roughness measures stand out among the main results of interest in optimization studies in this research area (PAIVA *et al*., 2012; PERUCHI *et al*., 2018). Next, the roughness measures used in this work are described: the average surface roughness ($R_a$) and the maximum height of the irregularities ($R_t$).

- Average roughness ($R_a$): also known as arithmetic mean deviation, it is one of the most used roughness measures in works related to the quality of turned parts. In addition, the average roughness is adopted by the Brazilian Standard as a measurement method. The result $R_a$ consists of the arithmetic mean of the absolute vertical distances of the measured (effective) profile in relation to a median line drawn in a sampling length (AGOSTINHO; RODRIGUES; LIRANI, 1990). More specifically, $R_a$ corresponds to the mean of the absolute values of the $n$ measures of $Y$ ($Y_1, Y_2, \dots, Y_n$) as illustrated in Figure 2.8.

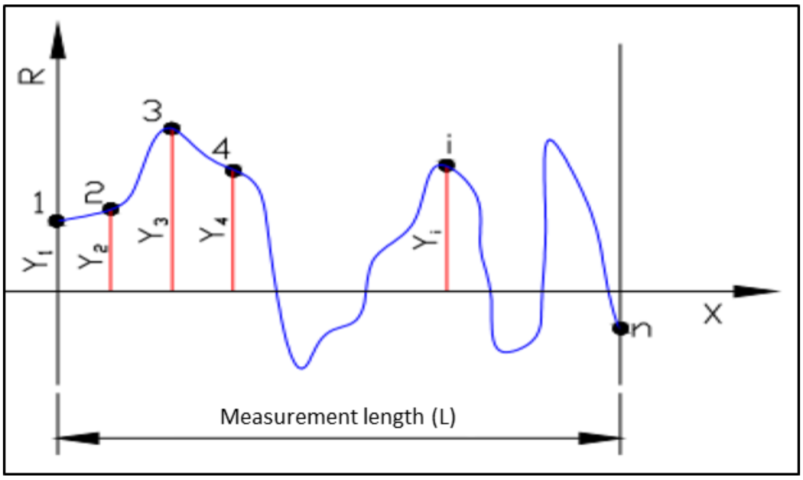


Figure 2.9 - Measurement of average roughness $\boldsymbol{R_a}$

Source: adapted from Agostinho, Rodrigues and Lirani (1990)

- Total roughness, or maximum height of the deviations ($R_t$): symbolized as $R_{\max}$ in some studies, it is defined as the distance between the highest and lowest points of the irregularity measured in the sample length. Figure 2.9 shows how $R_t$ is measured.

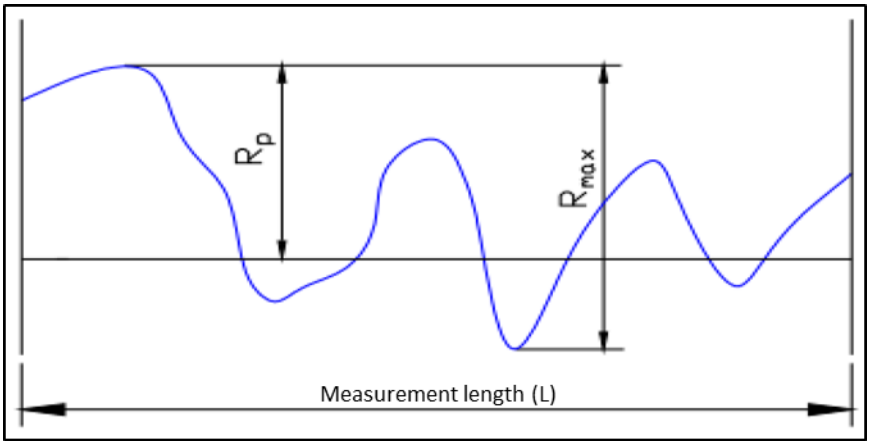


Figure 2.9 - Measurement of the maximum height of irregularities ($R_{\max}$)

Source: adapted from Agostinho, Rodrigues, Lirani (1990)

# 3. MULTIVARIATE CHANCE-CONSTRAINT PROGRAMMING

Based on the concepts and techniques presented in chapter 2, a Multivariate Chance-Constrained Programming (MCCP) method is proposed. The sequence of MCCP steps is shown in Figure 3.1 and the method steps are detailed below.

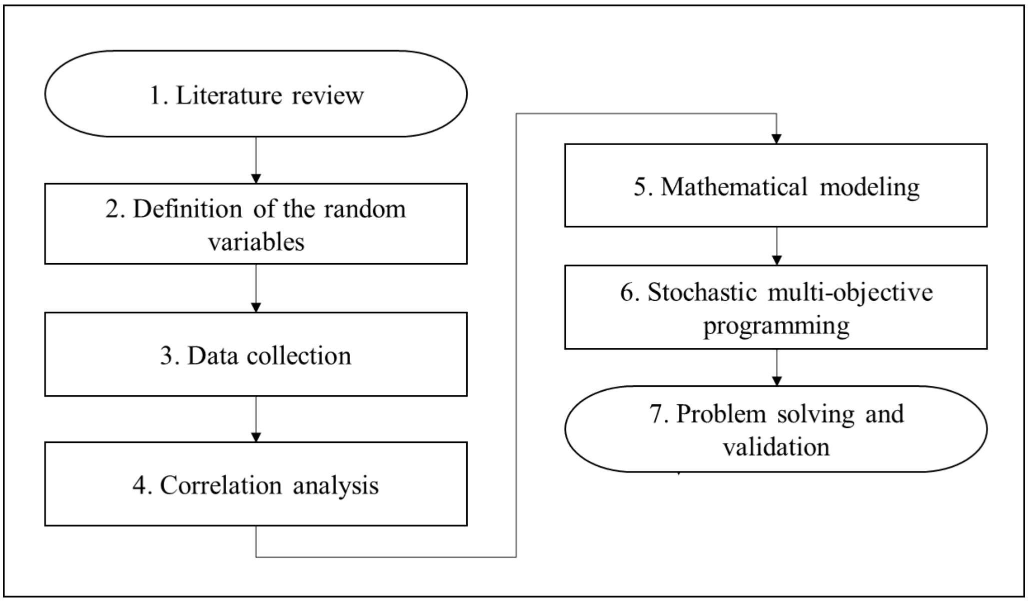


Figure 3.1 – Steps of the multivariate chance-constrained programming method ($\boldsymbol{R}_{\mathbf{max}}$)

Source: the author

**1. Literature review**

Studies published on the same process or system can provide important information for researchers or professionals who aim to improve or optimize their object of study. Therefore, the literature review on the main decision variables, uncontrollable variables, and results of interest, as well as the most used experimental levels for decision variables, is the first step in the method. It is also possible that an optimal or at least viable solution to the problem had already been published.

**2. Definition of the random variables**

Some stochastic variables may have a relevant impact on the results of interest and, for this reason, they should have their random nature considered in the optimization problem. Step 1 (literature review) can assist in the definition of which variables should be stochastically modelled in the problem.

**3. Data collection**

In the case of specific problems or problems that have not yet been addressed, it is necessary to obtain experimental results. In this step, the researcher has the option to conduct their own experiments. However, if the object of study has already been investigated through experimental procedures, one can start from the results of the experiment, which corresponds to the case of this research.

**4. Correlation analysis**

When there are multiple results of interest, it may be the case that such responses are significantly correlated. If so, it is necessary to use multivariate statistical techniques to calculate the probabilities for ranges of values of the responses of interest, either for objective functions or for constraints. Therefore, correlation analyses are necessary.

**5. Mathematical modeling**

In this phase, empirical models are constructed, which are mathematical models built based on experimental results and using regression methods. Some responses of interest, on the other hand, are already computed using mechanistic models. Such models are also selected at this stage. In the case of studies of optimization of manufacturing processes, the responses of interest related to the quality of the process can be represented by their $C_{pk}$ capability indices.

**6. Stochastic multi-objective programming**

Once the input variables (deterministic and random) and the results of interest have been determined, the multi-objective optimization problem is formulated. Among the various possibilities, the alternative presented by Equation (3.1) is used in the present study:

$$
\begin{gathered}
Máx \int_{f_1^l}^{f_1^u} \emptyset \left\{ E[f_1(\mathbf{x})], \sqrt{Var[f_1(\mathbf{x})]} \right\} \\
subject\ to: \\
1 - \int_{f_2^l}^{f_2^u} \emptyset \left\{ E[f_2(\mathbf{x})] \sqrt{Var[f_2(\mathbf{x})]} \right\} \leq P_{mín} \\
\text{and/or:} \\
C_{pk}[f_3(\mathbf{x})] = \frac{USL_{f_3(\mathbf{x})} - E[f_3(\mathbf{x})]}{3\sqrt{Var[f_3(\mathbf{x})]}} \geq C_{pkf3}^{mín} \\
\mathbf{x}_{\text{mín}} \leq \mathbf{x} \leq \mathbf{x}_{\text{máx}}
\end{gathered}
\tag{3.1}
$$

## 7. Problem solving and validation

To solve the problem, search algorithms must be used, such as the GRG algorithm. In sequence, the solution found must be validated in some way, either by carrying out confirmation experiments, using simulation techniques such as Monte Carlo, or by comparing the solution obtained and solutions of similar problems.

Chapter 4 presents the materials, experimental data, and three different approaches for the same case study: the AISI52100 hardened steel turning process. These approaches are variations of the MCCP strategy.

# 4. CASE STUDY – AISI 52100 HARDENED STEEL TURNING PROCESS

The choice of the turning process in step 1 of the MCCP method was made based on the relevance of the process to the industry and the interest of the academy in research on this process, as presented in section 2.9.

## 4.1. Materials, machines and tools

The experimental results used in this research were obtained by Campos *et al*. (2017). In this experiment, a CNC lathe (Computer Numerical Control) of the Nardini brand, model Logic 175, was used with a maximum rotation of 4000 rpm (revolutions per minute) and a cutting power of 5.5 Kw. The machined parts were made of AISI 52100 steel and prepared with an initial diameter $d$ = 49 mm and length $l_f$ = 50 mm. The chemical composition of steel is shown in Table 4.1.

Table 4.1 - Chemical composition of parts (AISI 52100 steel)

| C | Si | Mn | Cr | Mo | Ni | S | P |
|---|---|---|---|---|---|---|---|
| 1.03% | 0.23% | 0.35% | 1.40% | 0.04% | 0.11% | 0.00% | 0.01% |

Source: Campos (CAMPOS *et al.*, 2017)

The parts were previously tempered and, after such heat treatments, the hardness of the $H$ parts was 49 to 52 HRC, to a depth of 3 mm below the surface. The tool holder was used in conjunction with a negative geometry with ISO code DCLNL 1616H12 in addition to an entry angle of 95 degrees. Figure 4.1 shows the turning process covered in this case study.

Ceramic straightening tools, also called wiper tools $Al_2O_3$ + TiC, with ISO CNGA 120408 S01525WH geometry, were coated with a layer of titanium nitride (TiN). Such tool is recommended for finishing operations in hardened steels and in hardened cast iron in which the combination of wear resistance and good thermal properties are needed (CAMPOS, 2012). This geometry presents a better smoothing effect, which significantly improves the surface quality when compared to conventional tools and enables a productivity increase simultaneously with a higher feed rate.

To measure the tool life ($T$), wiper tools were used until their flank wear ($V_B$) reached 0.3 mm. This was the criterion adopted to determine the end of the tool's life. The ($V_B$) result was measured using a microscope. The $T$ values corresponded to the time required for $(V_B)$ to move from 0 to 0.3 mm in each test.

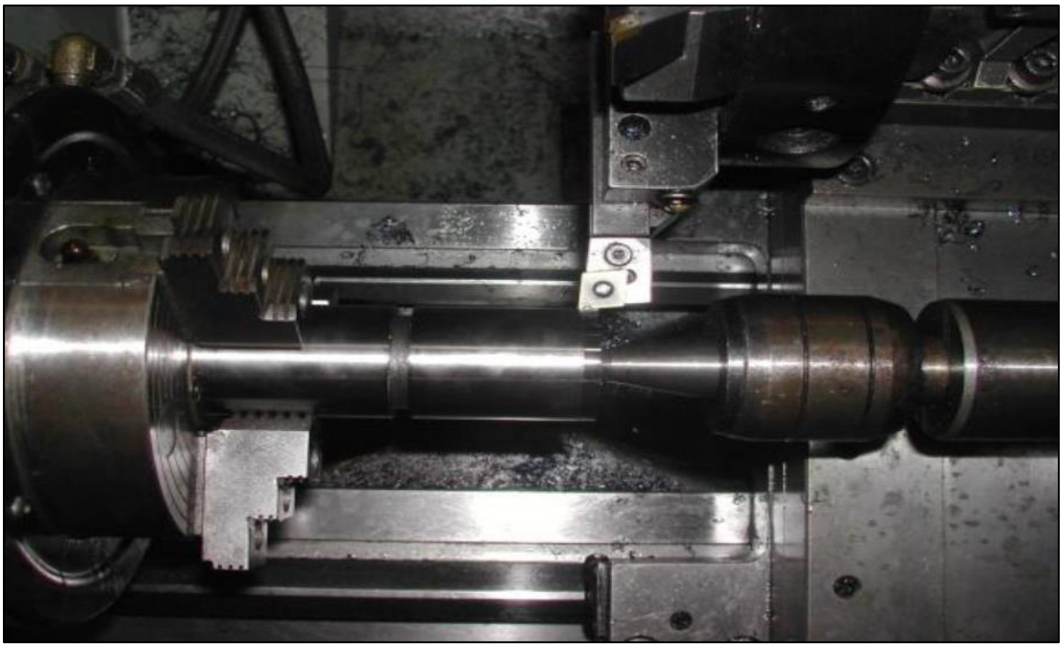

Figure 4.1 -Turning process covered in this work
Source: Campos (2011)

The average $(R_a)$ and total $(R_t)$ surface roughness values were measured four times on each of the three lines A, B and C illustrated by points in Figure 4.2. The three lines loop the surface of the part around its axis. The four measurements were separated by 90 degrees on the same line. A portable Mitutoyo rugosimeter and model Surftest SJ-201P was used to measure $R_a$ and $R_t$ in micrometers (µm).

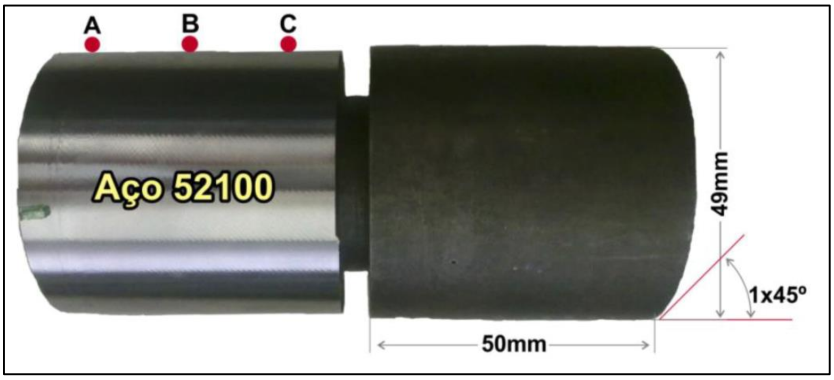


Figure 4.2 - Specimen (part) and measurements of the roughness $\boldsymbol{R_a}$ and $\boldsymbol{R_t}$
Source: Campos (2011)

## 4.2. Experimental planning and obtained responses

The decision variables chosen were the cutting speed ($V_c$), the feed ($f$) and the cutting depth ($a_p$). Therefore, vector $\mathbf{x} = \{x_1, x_2, x_3\}$, is such that $x_1 = V_c$, $x_2 = f$ and $x_3 = a_p$. In this specific experimental data set, Campos *et al*. (2017) did not include the analysis of the impact of noise variables such as tool wear and the surface hardness of the parts. However, the variability from these noise variables is included in the mean square error ($\hat{\sigma}^2$) in this case study, as explained in section 2.9. The Central Composite Design (CCD), was used for the design of the experiment.

The three decision variables were defined at two factor levels. Thus, 19 experimental conditions were necessary: eight factorial points ($2^n = 2^3 = 8$), six axial points ($2k = 2 * 3 = 6$), and a center point that was executed five times. The coded and decoded levels of the decision variables are shown in Table 4.2. The distance from the central point to each of the axial points was established as $\rho = 1.682$ ($\sqrt[4]{2^n} = \sqrt[4]{2^3} = 1.682$).

Table 4.2 - Decision variables and their respective levels (encoded and decoded)

| **Decision Variable** | **Unit** | **Notation** | **Levels** | | | | |
|---|---|---|---|---|---|---|---|
| | | | -1.682 | -1 | 0 | 1 | 1.682 |
| Cutting Speed | m/min | $V_c$ | 186 | 200 | 220 | 240 | 254 |
| Feed rate | mm/rev. | $f$ | 0.13 | 0.20 | 0.30 | 0.40 | 0.47 |
| Cutting depth | mm | $a_p$ | 0.01 | 0.15 | 0.23 | 0.30 | 0.35 |

Source: Campos *et al.* (CAMPOS *et al.*, 2017)

Table 4.3 shows the decision variables at their decoded levels and the results for the responses of interest.

The values of $T$, $R_a$ and $R_t$ were measured after each test and, therefore, their mathematical models are empirical. The material removal rate (MRR) and the production cost per part ($K_p$) were calculated based on their mechanistic models. It is important to highlight that the decision variables were considered deterministic in this study. The random variables considered were the $\boldsymbol{\beta}$ coefficients of the response surface models (empirical) and some variables related to the cost of the process.

Table 4.3 - Decision variables and results of interest - CCD arrangement

| Test | Decision Variables | | | Measured interest results | | | | |
|---|---|---|---|---|---|---|---|---|
| | $V_c$ | $f$ | $a_p$ | $T$ | $R_a$ | $R_t$ | $MRR$ | $K_p$ |
| 1 | 200 | 0.20 | 0.15 | 17.21 | 0.25 | 1.41 | 6.00 | 0.809 |
| 2 | 240 | 0.20 | 0.15 | 11.37 | 0.27 | 1.72 | 7.20 | 0.807 |
| 3 | 200 | 0.40 | 0.15 | 5.96 | 0.31 | 2.12 | 12.00 | 0.771 |
| 4 | 240 | 0.40 | 0.15 | 4.48 | 0.30 | 2.15 | 14.40 | 0.771 |
| 5 | 200 | 0.20 | 0.30 | 9.42 | 0.25 | 1.45 | 12.00 | 0.888 |
| 6 | 240 | 0.20 | 0.30 | 7.37 | 0.25 | 1.58 | 14.40 | 0.871 |
| 7 | 200 | 0.40 | 0.30 | 4.03 | 0.34 | 2.01 | 24.00 | 0.834 |
| 8 | 240 | 0.40 | 0.30 | 6.10 | 0.29 | 1.99 | 28.80 | 0.727 |
| 9 | 186 | 0.30 | 0.22 | 9.51 | 0.29 | 1.69 | 12.55 | 0.790 |
| 10 | 254 | 0.30 | 0.22 | 6.86 | 0.26 | 1.81 | 17.14 | 0.767 |
| 11 | 220 | 0.13 | 0.22 | 14.18 | 0.21 | 1.54 | 6.43 | 0.934 |
| 12 | 220 | 0.47 | 0.22 | 4.12 | 0.31 | 2.54 | 23.26 | 0.777 |
| 13 | 220 | 0.30 | 0.10 | 9.42 | 0.31 | 1.94 | 6.60 | 0.754 |
| 14 | 220 | 0.30 | 0.35 | 4.92 | 0.31 | 1.74 | 23.10 | 0.864 |
| 15 | 220 | 0.30 | 0.22 | 4.89 | 0.26 | 1.81 | 14.85 | 0.852 |
| 16 | 220 | 0.30 | 0.22 | 5.00 | 0.26 | 1.71 | 14.85 | 0.852 |
| 17 | 220 | 0.30 | 0.22 | 4.77 | 0.26 | 1.71 | 14.85 | 0.852 |
| 18 | 220 | 0.30 | 0.22 | 5.01 | 0.26 | 1.71 | 14.85 | 0.852 |
| 19 | 220 | 0.30 | 0.22 | 5.12 | 0.26 | 1.71 | 14.85 | 0.852 |

Source: adapted from Campos *et al.* (2017)

## 4.3. Correlation analysis

From the values presented in Table 4.3, correlations analyses between the pairs of the five responses of interest ($T$, $R_a$ e $R_t$, $MRR$ e $K_p$) were carried out. Using a 95% confidence level, it was possible to identify some significant correlations between the responses of interest as shown in Table 4.4.

Table 4.4 - Analyses of correlations $\boldsymbol{r_{y_1 y_2}}$ between the responses of interest

| | | $T$ | $R_a$ | $R_t$ | $MRR$ |
|---|---|---|---|---|---|
| $R_a$ | Correlation | -0.497 | | | |
| | $p$-value | 0.031 | | | |
| $R_t$ | Correlation | -0.585 | 0.720 | | |
| | $p$-value | 0.009 | 0.001 | | |
| $MRR$ | Correlation | -0.706 | 0.507 | 0.510 | |
| | $p$-value | 0.001 | 0.027 | 0.026 | |
| $K_p$ | Correlation | 0.146 | -0.595 | -0.615 | -0.246 |
| | $p$-value | 0.508 | 0.007 | 0.005 | 0.309 |

Source: the author

Tool life ($T$) has a negative correlation with the average roughness $R_a$ ($r$ = -0.497) and the total roughness $R_t$ ($r$ = -0.585). This means that the increase in $T$ tends to cause a decrease in both $R_a$ and $R_t$ . In fact, the lowest roughness values are generally obtained when the cut parameters - or decision variables ($V_c$, $f$, e $a_p$) - are set at their lowest levels. As a result, the cutting conditions are less aggressive, and the tool wear tends to be lower, which results in a longer $T$ life. However, another consequence of choosing low levels for cutting parameters is the decrease in process productivity, which is represented by the material removal rate ($MRR$). This tradeoff between tool life and productivity is evidenced by the negative correlation between $T$ and $MRR$ ($r$ = -0.706), and the positive correlations between $MRR$ and $R_a$ ($r$ = 0.507) and between $MRR$ and $R_t$ ($r$ = 0.510).

Therefore, the decision to use low levels for cutting parameter results in an increase in tool life and a decrease in roughness. On the other hand, this decision compromises process productivity and, as will be presented in section 4.6.5, can increase the total production cost of the process.

# 4.4. Mathematical modeling of the responses of interest

## 4.4.1. Construction of response surface models

The mathematical models for $T$, $R_a$ and $R_t$ were built based on the results in Table 4.3 and starting from the second order polynomial model, whose general formulation is presented again in Equation (4.1). Matrix $\mathbf{X}$ and vectors $\mathbf{a}'(\mathbf{x})$, $\boldsymbol{\beta}$, and $\mathbf{y}$ specific to this case study are presented in Appendix B.

$$y \sim f(\mathbf{x}) = \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{i<j} \beta_{ij} x_i x_j + \epsilon = \mathbf{a}'(\mathbf{x})\boldsymbol{\beta} + \varepsilon \tag{4.1}$$

The material removal rate ($MRR$) and the total production cost per piece ($K_p$) were calculated from their mechanistic models. Therefore, there is no need to build response surface models for such results.

The method of Ordinary Least Squares (OLS) was used to obtain the expected values of the response surface models. The variances these mathematical models were determined using Equation (2.39) presented in chapter 2.

- **Expected values of response surface models**

Equation (4.2) was used to obtain the expected values of the coefficients, that is, $\widehat{\boldsymbol{\beta}} = E[\boldsymbol{\beta}]$.

$$\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \tag{4.2}$$

Table 4.5 shows the values obtained for the coefficients. Note that the models' adjustments, estimated by the adjusted R square values ($R^2_{adj.}$), were all greater than 94%, which means that the models are satisfactory representations of the expected values of the responses of interest, symbolized as $E[f_i(\mathbf{x})]$. The substitutions of the terms of $\boldsymbol{\beta}$ in Equation (4.1) results in Equations (4.3) to (4.5).

Table 4.5 - Coefficients of the response surface models

| **Notation** | **Values** | | |
|---|---|---|---|
| | $T$ | $R_a$ | $R_t$ |
| $\beta_0$ | 4.963 | 0.260 | 1.733 |
| $\beta_1$ | -0.861 | -0.007 | 0.048 |
| $\beta_2$ | -3.055 | 0.028 | 0.278 |
| $\beta_3$ | -1.440 | 0.000 | -0.052 |
| $\beta_{11}$ | 1.115 | 0.005 | -0.010 |
| $\beta_{22}$ | 1.456 | 0.000 | 0.092 |
| $\beta_{33}$ | 0.756 | 0.018 | 0.021 |
| $\beta_{12}$ | 1.060 | -0.010 | -0.054 |
| $\beta_{13}$ | 0.918 | -0.008 | -0.029 |
| $\beta_{23}$ | 1.435 | 0.005 | -0.021 |
| $R^2_{adj.}$ | 99.74% | 98.66% | 94.35% |
| $\hat{\sigma}^2$ | $3.51(10^{-2})$ | $1.31(10^{-5})$ | $4.32(10^{-3})$ |

Source: the author

$$E[T(\mathbf{x})] = 4.963 - 0.861x_1 - 3.055x_2 - 1.440x_3 + 1.115x_1^2 + 1.456x_2^2 + 0.756x_3^2 + 1.060x_1x_2 + 0.918x_1x_3 + 1.435x_2x_3 \tag{4.3}$$

$$E[R_a(\mathbf{x})] = 0.260 - 0.007x_1 + 0.028x_2 + 0.000x_3 + 0.005x_1^2 + 0.000x_2^2 + 0.018x_3^2 - 0.010x_1x_2 - 0.008x_1x_3 + 0.005x_2x_3 \tag{4.4}$$

$$E[R_t(\mathbf{x})] = 1.773 + 0.048x_1 + 0.278x_2 - 0.052x_3 - 0.010x_1^2 + 0.092x_2^2 + 0.021x_3^2 - 0.054x_1x_2 - 0.029x_1x_3 - 0.021x_2x_3 \quad (4.5)$$

Figures 4.3 to 4.11 show the surface graphs of the responses (expected values) as a function of the decision variables (in pairs).

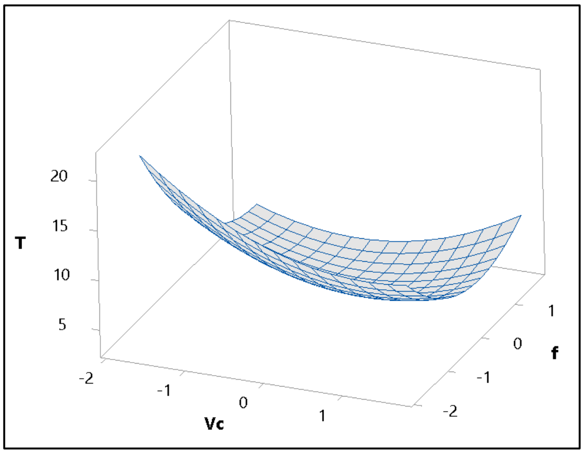


Figure 4.3 – $T$ as a function of $V_c$ and $f$

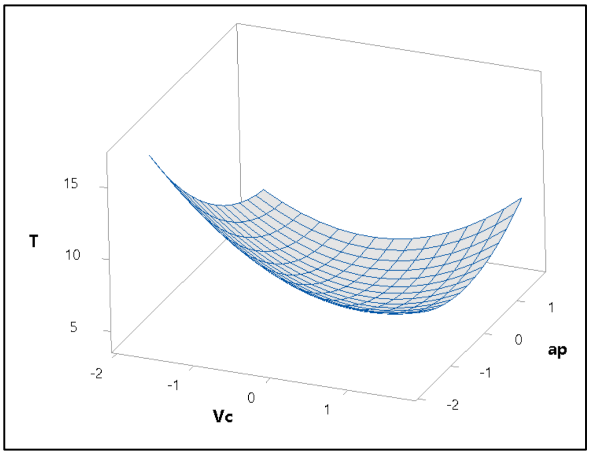


Figure 4.4 – $T$ as a function of $V_c$ and $a_p$

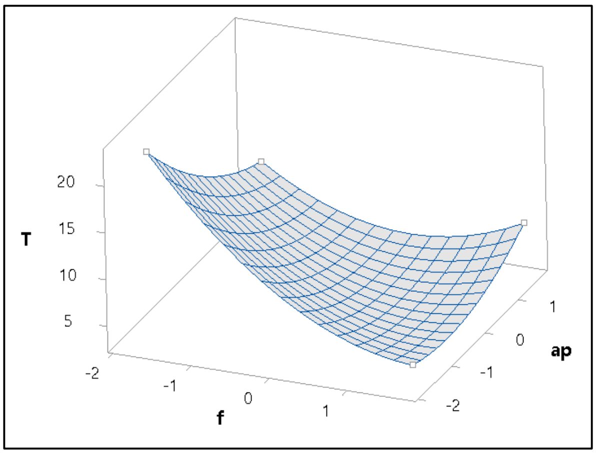


Figure 4.5 – $T$ as a function of $f$ and $a_p$

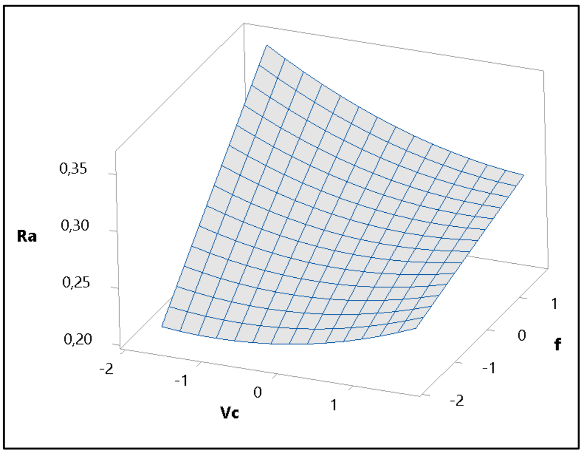


Figure 4.6 – $R_a$ as a function of $V_c$ and $f$

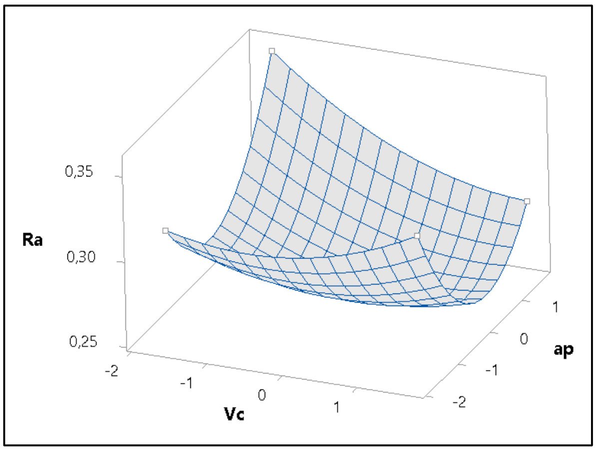


Figure 4.7 – $R_a$ as a function of $V_c$ and $a_p$

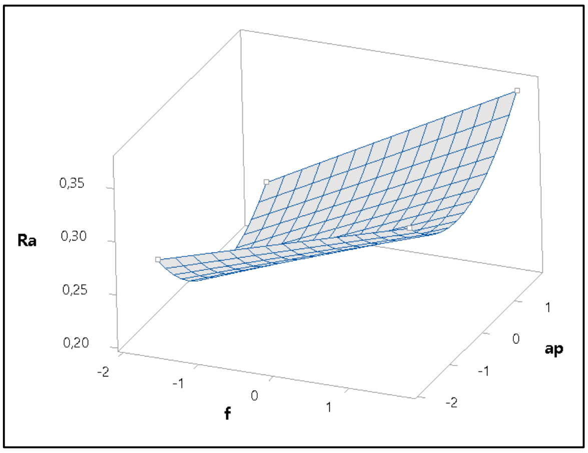


Figure 4.8 – $R_a$ as a function of $f$ and $a_p$

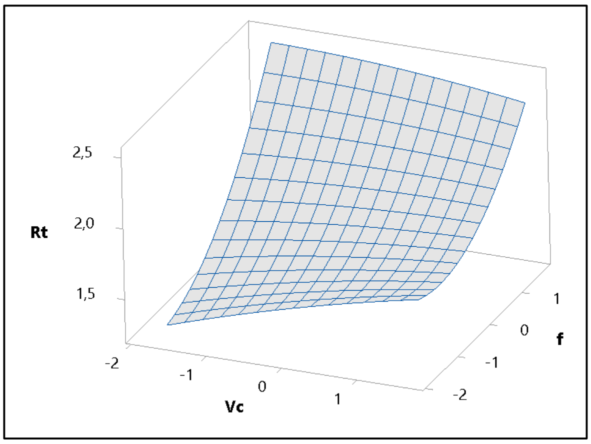


Figure 4.9 – $R_t$ as a function of $V_c$ and $f$

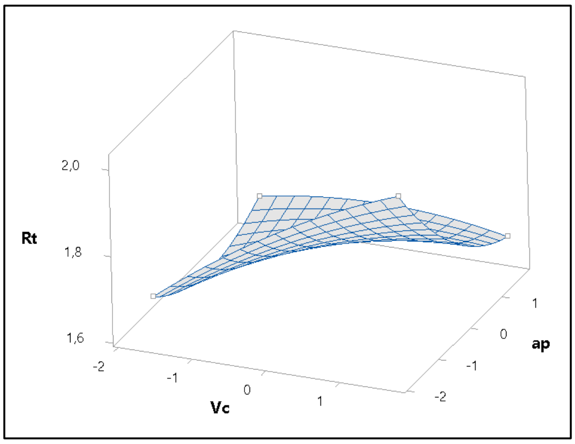


Figure 4.10 – $\boldsymbol{R_t}$ as a function of $\boldsymbol{V_c}$ and $\boldsymbol{a_p}$

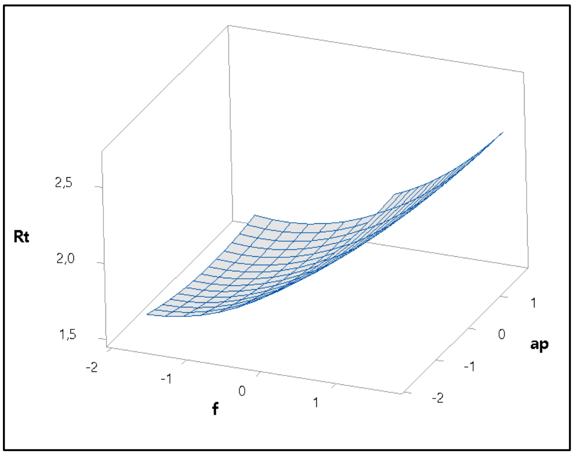


Figure 4.11 – $\boldsymbol{R_t}$ as a function of $\boldsymbol{f}$ and $\boldsymbol{a_p}$

- **Response surface model variances**

Table 4.5 also shows the estimated variances of the residuals, or mean square of errors (MSE), represented by $\hat{\sigma}^2$ and calculated by Equation (4.6).

$$\hat{\sigma}^2 = \frac{SS_\varepsilon}{(m - r)} \tag{4.6}$$

Where:

$SS_\varepsilon$ = sum of squares of the residuals, $SS_\varepsilon = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}$;

$m$ = number of observations (in this case, number of tests in the experiment);

$r$ = number of terms in the mathematical model.

In this case study, $m$ = 19 tests and $r$ = 10 terms (including the constant). After calculating $\hat{\sigma}^2$ for each response surface model, Equation (4.7) was used to calculate the variance of the response surface models.

$$Var[f(\mathbf{x})] = \sigma^2\mathbf{a}'(\mathbf{x})(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}(\mathbf{x}) \tag{4.7}$$

The values of the variances of the objective functions were calculated using the Microsoft® Excel software.

## 4.4.2. Construction of mechanistic models

- **Expected values of mechanistic models**

The results of material removal rate ($MRR$) and the total cost of production per part ($K_P$) were calculated from mechanistic models. $MRR$ values were calculated by Equation (4.8).

$$MRR = V_c . f . a_p \tag{4.8}$$

It is worth mentioning that Equation (4.8) results in the expected value of $MRR$, and the variance of this answer is assumed to be insignificant as the decision variables are considered deterministic in this case study.

The expected value for the production cost ($K_P$) was calculated from Equation (4.9).

$$E[K_p(\mathbf{x})] = t_t \frac{(S_h + S_m)}{60} + \frac{t_c}{T}\left(\frac{K_{th}}{N_{th}} + \frac{K_i}{N_i}\right) \tag{4.9}$$

Where:

$t_t$ = total cycle time (in minutes);

$S_h + S_m$ = hourly labor and machine costs, measured in dollars per hour (U \$ / h);

$K_{th}$ = tool holder cost (U \$);

$N_{th}$ = toolholder life (in number of edges);

$K_i$ = cost of the tool or insert (U \$);

$N_i$ = number of cutting edges of the insert.

Equation (4.10) shows the calculation of the total cycle time ($T_t$):

$$t_t = t_c + t_s + t_a + \frac{t_p}{Z} + \frac{N_t}{Z} t_i \tag{4.10}$$

Where:

$t_s$ = secondary time;

$t_a$ = time of approach and removal of the tool;

$t_p$ = setup time;

$Z$ = lot size (in units);

$N_t$ = number of tool changes;

$t_i$ = insert change time.

Equation (4.11) was then used to calculate the value of $N_t$. The cut time ($t_c$), in turn, was calculated by Equation (4.12):

$$N_t = Máx\left[0, I\left(Z\frac{t_c}{T} - 1\right)\right] \tag{4.11}$$

$$t_c = \frac{l_f \pi d}{100 f V_c} \tag{4.12}$$

- **Variance of mechanistic model of production cost**

As previously described in section 2.9.2, the cost of a turning process depends on a number of other variables, which can be called industrial variables. Some of these variables have a considerable random character, while others have a variability that can be neglected.

Table 4.6 presents the values of the industrial variables that were considered in a stochastic or deterministic way in this case study.

Some assumptions were necessary to conduct this case study. At first, all industrial variables were included in the problem in a deterministic way (CAMPOS *et al*., 2017). Historical data about industrial variables were not collected in order to estimate their variances and adjust the probability distributions. Therefore, the choices of the probability distributions and the magnitude of the variances were attributed by the author in order to present a first analysis regarding the impact of the randomness of the stochastic variables present in the cost.

Table 4.6 - Industrial variables (deterministic and stochastic)

| **N** | **Stochastic Variables** | **Unit** | **Symbol** | **Mean** | **Standard Deviation** |
|---|---|---|---|---|---|
| 1 | Setup time | Min | $t_p$ | 60 | 6 |
| 2 | Tool change time | Min | $t_i$ | 1 | 0.1 |
| 3 | Batch size | Pieces | $Z$ | 1000 | 100 |
| 4 | Labor and machine costs | US$ | $S_h + S_m$ | R$ 50.00 | R$ 5.00 |
| 5 | Tool holder cost | US$ | $K_{th}$ | R$ 125.00 | R$ 12.50 |
| 6 | Tool holder life | Edges | $N_{th}$ | 1000 | 100 |
| 7 | Tool cost (or insert) | US$ | $K_i$ | R$ 31.25 | R$ 3.13 |
| | **Deterministic Variables** | | | | |
| 8 | Secondary time | Min | $t_s$ | 0.50 | - |
| 9 | Approach and departure time | Min | $t_a$ | 0.10 | - |
| 10 | Number of insert cutting edges | Units | $N_i$ | 4.00 | - |
| 11 | Workpiece length | Mm | $l_f$ | 50.00 | - |
| 12 | Workpiece diameter | Mm | $d$ | 49.00 | - |

Different probability distributions could be used to represent the stochastic variables. Lot size $(Z)$ is the only discrete variable among the seven random ones. A probability distribution often used to represent discrete variables is the Poisson distribution. However, it is possible to approximate a Poisson distribution to a normal distribution using $z = (x - \lambda)/\sqrt{\lambda}$ (MONTGOMERY; RUNGER, 2018). For the other seven stochastic variables presented in Table 4.6, the normal probability distribution was used. The reason was that, in the real cases of application in the industry, it is common to represent the parameters as the weekly or daily averages of a longer period of time, a quarter or a semester. Thus, according to the Central Limit Theorem, samples composed of averages tend to follow a normal probability distribution

(MONTGOMERY; RUNGER, 2018). A variation coefficient was defined as 10% for each stochastic variable, as shown in Table 4.6. *A priori*, it was admitted that the correlations between these stochastic variables were not significant, as there was no evidence that such correlations exist in this study. Equation (4.13) was used to calculate the $K_p$ variance.

$$Var[f(\mathbf{x})] = \sum_{i=1}^{n} \left[\frac{\partial f(\mathbf{x})}{\partial x_{\mathrm{i}}}\right]^2 \sigma_{x_1}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\frac{\partial f(\mathbf{x})}{\partial x_{\mathrm{i}}}\right]\left[\frac{\partial f(\mathbf{x})}{\partial x_{\mathrm{j}}}\right]\sigma_{x_i x_j} \qquad (4.13)$$

Therefore, the variance of $K_p$ - a particular case of Equation (4.14) - results in:

$$\begin{aligned} Var[K_p(\boldsymbol{\mu})] &= \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_{\mathrm{p}}}\right]^2 \sigma_{t_p}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_i}\right]^2 \sigma_{t_i}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial \mathrm{Z}}\right]^2 \sigma_Z^2 \\ &+ \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial (S_h + S_m)}\right]^2 \sigma_{(S_h+S_m)}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial K_{th}}\right]^2 \sigma_{K_{th}}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial N_{th}}\right]^2 \sigma_{N_{th}}^2 \\ &+ \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial K_i}\right]^2 \sigma_{K_i}^2 \end{aligned} \qquad (4.14)$$

Table 4.7 shows the equations used to calculate partial derivatives. The response surfaces for the expected cost value $E[K_p(\mathbf{x})]$ and its SD standard deviation $SD[K_p(\mathbf{x})] = \sqrt{Var[K_p(\mathbf{x})]}$ are shown in Figures 4.12 to 4.17. Figures 4.12, 4.14 and 4.16 show that $E[K_p(\mathbf{x})]$ can vary between U \$ 0.66 to U \$ 1.00 depending on the levels of the decision variables. It is also observed that a low expected value of the cost can be obtained from the choice of high levels for the decision variables - also called, in this case study, cutting conditions or machine parameters. However, the SD values $SD[K_p(\mathbf{x})]$ varied only between U \$ 0.06 and U \$ 0.08, as shown in Figures 4.13, 4.15 and 4.17

After the mathematical modeling of the responses of interest, three different approaches were analyzed in this research. These approaches are presented in sections 4.5 to 4.7 below.

Table 4.7 - Partial $\boldsymbol{K_p}$ derivatives in relation to industrial variables

| Variables | Partial derivative |
|---|---|
| $t_p$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_p} = \frac{(S_h + S_m)}{60Z}$ |
| $t_i$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_i} = \frac{N_t^*(S_h + S_m)}{60Z}$ |
| $Z$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial Z} = \frac{-(t_p + N_t^* t_i)(S_h + S_m)}{60Z^2}$ |
| $S_h + S_m$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial (S_h + S_m)} = \frac{t_t}{60}$ |
| $K_{th}$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial K_{th}} = \frac{C_t}{T.N_{th}}$ |
| $N_{th}$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial N_{th}} = -\frac{C_t.K_{th}}{T.{N_{th}}^2}$ |
| $K_i$ | $\frac{\partial K_p(\boldsymbol{\mu})}{\partial K_i} = \frac{C_t}{T.N_i}$ |

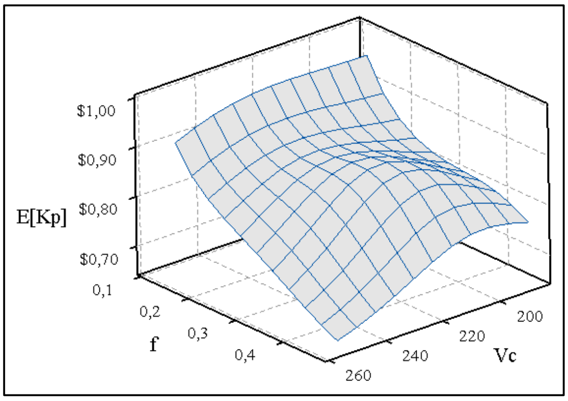


Figure 4.12 – $E\left[K_p(\mathbf{x})\right]$ as a function of $V_c$ and $f$

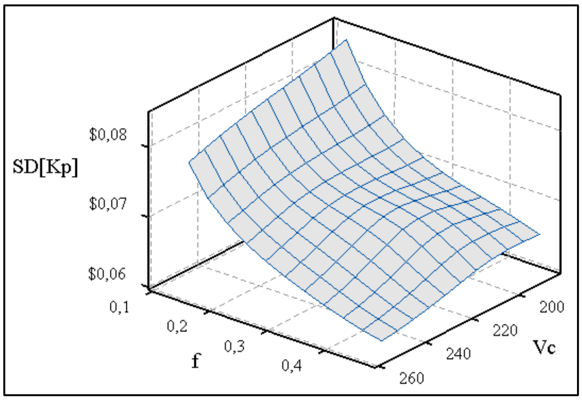


Figure 4.13 – $SD\left[K_p(\mathbf{x})\right]$ versus $V_c$ and $f$

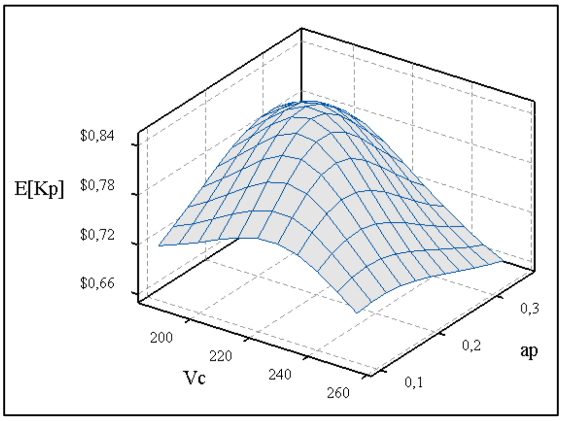


Figure 4.14 – $E\left[K_p(\mathbf{x})\right]$ as a function of $V_c$ and $a_p$

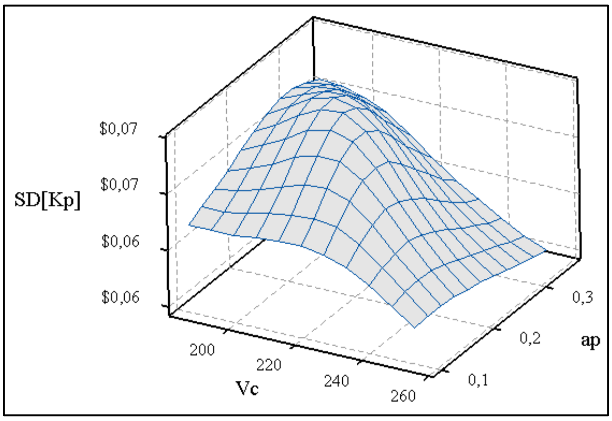


Figure 4.15 – $SD\left[K_p(\mathbf{x})\right]$ versus $V_c$ and $a_p$

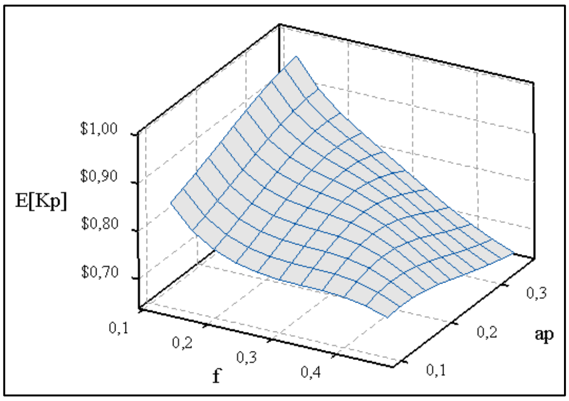


Figure 4.16 – $\boldsymbol{E}\left[\boldsymbol{K_p}(\mathbf{x})\right]$ as a function of $\boldsymbol{f}$ and $\boldsymbol{a_p}$

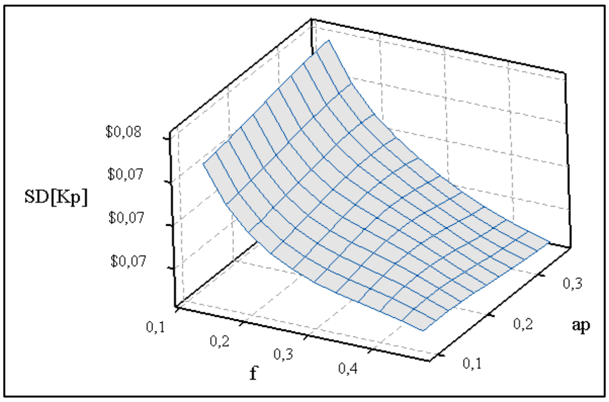


Figure 4.17 – $\boldsymbol{SD}\left[\boldsymbol{K_p}(\mathbf{x})\right]$ versus $\boldsymbol{f}$ and $\boldsymbol{a_p}$

## 4.5. Approach 1: Multi-objective optimization problem subject to a stochastic process capacity constraint

The first approach of this research consisted of optimizing three results of interest simultaneously (TORRES *et al*., 2019b):

- Process productivity, represented by $MRR$;
- The cost of production per piece, represented by $K_p$;
- The quality, which was represented only by the average roughness $R_a$ in this first approach.

More specifically, the aim was to answer the following question: how is it possible to simultaneously optimize the productivity and cost of the process, guaranteeing a minimum acceptable quality? Thus, $K_p$ and $MRR$ were modeled as the objective functions of the problem. The stochastic programming method integrated with the $C_{pk}$ index (one of the proposals in this dissertation) to assess the process capability with regards to the critical-to-quality (CTQ) characteristic $R_a$ as a constraint. In fact, the average roughness ($R_a$) is a CTQ widely used in the industry, and CTQs are often evaluated from the point of view of the process capacity. In this first approach, only the expected values of $K_p$ and $MRR$ were considered in the optimization problem. In addition, these two objective functions have been scaled as their original dimensions are considerably different. That is, while the cost varies between U $ 0.68 and U $ 0.89, MRR is between 6 and 28.8 cm$^3$/min.

### 4.5.1. Scaling the objective functions

For the scheduling, it was necessary to determine the utopian and Nadir values for the objective functions through the individual optimization of each function. Such values were obtained after solving Equations (4.15) and (4.16), which correspond to the individual optimizations of $E\left[K_p(\mathbf{x})\right]$ and $E[MRR(\mathbf{x})]$, respectively. The results are shown in Table 4.8.

$$\begin{gathered}\underset{\mathbf{x}}{Min}\, E\left[K_p(\mathbf{x})\right] \\ subject\ to: \\ \sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1.682\end{gathered} \tag{4.15}$$

$$\begin{gathered}\underset{\mathbf{x}}{Min}\, E[MRR(\mathbf{x})] \\ subject\ to: \\ \sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1.682\end{gathered} \tag{4.16}$$

Table 4.8 - Utopia and Nadir values for objective functions

| Objective function | Utopia value $f_i^U(\mathbf{x})$ | Nadir value $f_i^N(\mathbf{x})$ |
|---|---|---|
| $E\left[K_p(\mathbf{x})\right]$ | \$ 0.700 | \$ 0.748 |
| $E[MRR(\mathbf{x})]$ | 29.52 | 14.85 |

In sequence, the objective functions were scaled using Equation (4.17):

$$\bar{f_i}(\mathbf{x}) = \frac{f_i(\mathbf{x}) - f_i^U(\mathbf{x})}{f_i^N(\mathbf{x}) - f_i^U(\mathbf{x})} \tag{4.17}$$

Therefore, the objective functions resulting from Equation (4.17) and based on the values in Table 4.8 were:

$$E\left[\overline{K_p}(\mathbf{x})\right] = \frac{E\left[K_p(\mathbf{x})\right] - 0.700}{0.748 - 0.700} \tag{4.18}$$

$$E[\overline{MRR}(\mathbf{x})] = \frac{E[MRR(\mathbf{x})] - 29.52}{14.85 - 29.52} \tag{4.19}$$

Once scaled, both functions have a sense of minimization because the lower their scaled value, the closer to their utopian values they are. Then, the objective functions could be combined in the formulation of the optimization problem.

### 4.5.2. Formulation of the optimization problem

The weighted sum method (WS) was used to formulate the global objective function $F(\mathbf{x})$. Thus, Equation (4.20) describes the optimization problem defined by approach 1.

$$
\begin{gathered}
\underset{\mathbf{x}}{Mín}\, F(\mathbf{x}) = wE\left[\overline{K_p}(\mathbf{x})\right] + (1-w)E\left[\overline{MRR}(\mathbf{x})\right] \\
C_{pk}[R_a(\mathbf{x})] = \frac{USL_{R_a} - E[R_a(\mathbf{x})]}{3\sqrt{Var[R_a(\mathbf{x})]}} \geq 1.67 \\
\sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1.682
\end{gathered}
\tag{4.20}
$$

In this approach, the upper specification limit for $R_a$ ($USL_{R_a}$) was set at 0.25 μm, and a minimum value of 1.67 was defined for the $C_{pk}$ of $R_a$ capacity index. The expected value $E[R_a(\mathbf{x})]$ and the variance $Var[R_a(\mathbf{x})]$ correspond to Equations (4.4) and (4.7) respectively.

Approach 1 was compared to an optimization that considered only the expected values of the responses of interest, in particular, the average roughness ($R_a$). This alternative corresponds to Equation (4.21).

$$
\begin{gathered}
\underset{\mathbf{x}}{Mín}\, F(\mathbf{x}) = wE\left[\overline{K_p}(\mathbf{x})\right] + (1-w)E[MRR(\mathbf{x})] \\
E[R_a(\mathbf{x})] \leq 0.25\ \mu m \\
\sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1.682
\end{gathered}
\tag{4.21}
$$

### 4.5.3. Solving the optimization problem

The optimization problems defined in Equations (4.20) and (4.21) were programmed in Microsoft® Excel software and solved using the Solver supplement for the weight $w$ ranging from 0 to 1, in increments of 0.05. The Generalized Reduced Gradient (GRG) algorithm was chosen to obtain the optimal solutions. The Pareto frontiers of approach 1 and the deterministic optimization are illustrated in Figure 4.18.

At first, the results of deterministic optimization may seem better than the results obtained with approach 1. In fact, Pareto-optimal solutions for conventional optimization reflect lower costs and higher material removal rates in terms of expected values. As an example, observe the results of both formulations for $w$ = 0.5 in Table 4.9. However, all solutions obtained in approach 1 have a $C_{pk}$ index of at least 1.67. This means that there is more than 99.99% probability that the result of $R_a$ is less than or equal to 0.25 μm. The solutions provided by deterministic optimization, in turn, guaranteed an index $C_{pk} = 0$ for $R_a$. This was because the deterministic constraint $E[R_a(\mathbf{x})] \leq 0{,}25$ µm for all configurations of $w$. Thus, the probability of having parts that do not conform to the established specification is 50%.

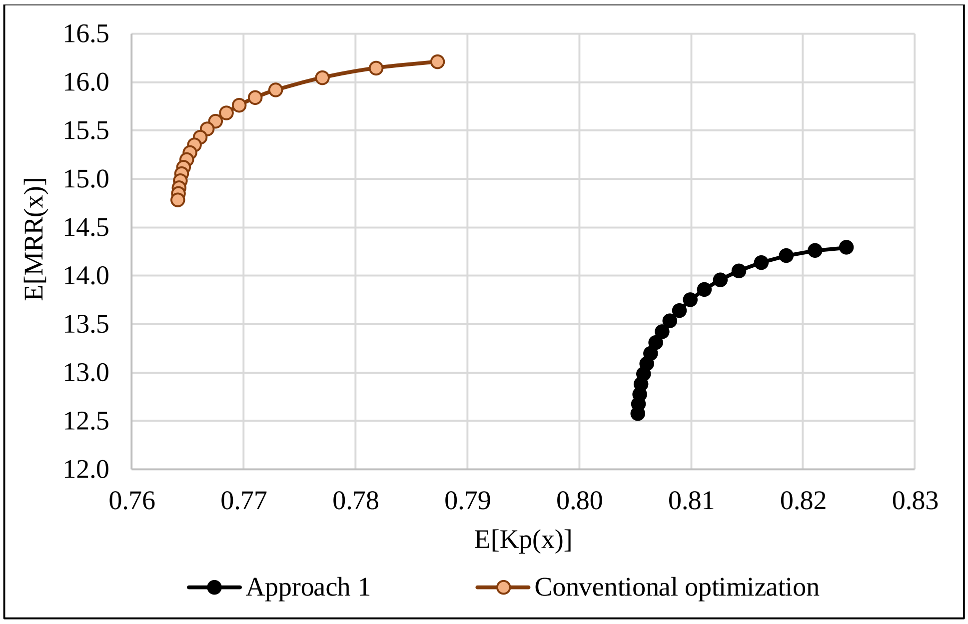


Figure 4.18 - Pareto frontiers for approach 1 and for conventional optimization

Table 4.9 - Comparison between approach 1 and conventional optimization

| **Result** | **Approach 1** | **Conventional optimization** |
|---|---|---|
| $E[K_p(\mathbf{x})]$ | U\$ 0.81 | U\$ 0.77 |
| $E[MRR(\mathbf{x})]$ | 13.5 | 15.4 |
| $E[R_a(\mathbf{x})]$ | 0.243 | 0.250 |
| $C_{pk}[R_a(\mathbf{x})]$ | 1.67 | 0 |

The decision maker could also opt for a conventional optimization and define an average roughness level slightly below tolerance. However, it is possible that the arbitrary choice of value restricts the problem excessively, which would imply worse solutions from economic and productive points of view.

The Pareto-optimal solution of approach 1 for $w$ = 0.5 was compared to the optimal solution obtained by Campos *et al.* (2017), The authors (CAMPOS *et al.*, 2017) used multivariate statistical techniques to optimize seven responses of interest: $T$, $C_t$, $T_t$, $K_p$, $R_a$, $R_t$ and $MRR$. As shown in Table 4.10.

Table 4.10 - Comparison between the results of approach 1 and Campos *et al.* (2017)

| Result | Unit | Campos *et al.* (2017) | Approach 1 |
|---|---|---|---|
| $V_c$ | m/min | 252.0 | 204 |
| $f$ | mm/ver | 0.33 | 0.22 |
| $a_p$ | mm/ver | 0.25 | 0.19 |
| $T$ | min | 6.46 | 12.13 |
| $t_c$ | min | 0.09 | 0.17 |
| $t_t$ | min | 0.77 | 0.85 |
| $K_p$ | U$ | 0.750 | 0.817 |
| $R_a$ | m | 0.266 | 0.242 |
| $R_t$ | m | 1.823 | 1.501 |
| $MRR$ | cm$^3$/min | 21.40 | 8.52 |

The solutions obtained, that is, the cutting conditions or the values of the decision variables, were significantly different between the studies. The cutting speed ($V_c$), the feed ($f$) and the depth of cut ($a_p$) were defined at higher levels in the work of Campos $et\ al$. (2017) compared to approach 1 of the present study. As a result, the material removal rate ($MRR$) - which is the result of multiplying the three decision variables - was much lower in approach 1 of the present study. For the same reason, the cut time ($t_c$) and the total cycle time ($t_t$) were also shorter in relation to the work by Campos $et\ al$. (2017).

Although the tool life ($T$) was significantly shorter in the study by Campos $et\ al$. (2017), this did not have a significant impact on the production cost ($K_p$). In fact, it can be assumed that the $T$ life must be a function of maximization. However, it is important to highlight that by maximizing $T$ from the choice of machine parameters (in this case, the decision variables) is not always an advantageous strategy since this strategy can compromise the productivity and the cost of the process, as will be presented in approach 2.

## 4.6. Approach 2: cost optimization considering stochastic industrial variables

In addition to the optimization strategy presented in approach 1, the problem was approached in another way. This time, the stochastic variables presented in Table 4.6 were considered. The main objective of approach 2 was to assess the impact of the variability present in the stochastic industrial variables on the cost of the process, both in terms of its expected value and its variance.

### 4.6.1. Formulation of the optimization problem

The optimization problem described in approach 2 consisted of maximizing the probability that the cost would be less than or equal to an established maximum value ($K_p^*$), subject to maximum results for the expected roughness values $R_a$ and $R_t$. The value of $K_p^*$ was established to be $ 0.90. In this case study, it is assumed that this value is the maximum allowed by the company in order to guarantee an acceptable margin for its product. The upper specification limits for the medium ($USL_{R_a}$) and total ($USL_{R_t}$) roughness were defined as 0.8 µm and 4 µm respectively. These limits correspond to the N6 classification of surface finish according to ISO 1302 (2002). Equation (4.22) presents the formulation of the multi-objective optimization problem related to approach 2.

$$
\begin{gathered}
\underset{\mathbf{x}}{Max}\, F(\mathbf{x}) = \int_{-\infty}^{K_p^*} \emptyset \left\{ E\left[K_p(\mathbf{x})\right], \sqrt{Var\left[K_p(\mathbf{x})\right]} \right\} \\
subject\ to: \\
E[R_a(\mathbf{x})] \le USL_{R_a} \\
E[R_t(\mathbf{x})] \le USL_{R_t} \\
\sqrt{\mathbf{x}'\mathbf{x}} \le \sqrt[4]{2^n} = 1.682
\end{gathered}
\tag{4.22}
$$

### 4.6.2. Validation of the process cost mathematical models

The results of the mathematical models of the expected cost value of the process $E\left[K_p(\mathbf{x})\right]$, as defined in Equation (4.9), and its standard deviation $\sqrt{Var\left[K_p(\mathbf{x})\right]}$ - such as Equation (4.14) , were validated using Monte Carlo Simulation. The purpose was to validate the demonstration of Equation (2.13) and, consequently, the stochastic programming method proposed in this dissertation. The mathematical models were evaluated at the center points of the cutting conditions - at levels 0 of the decision variables, as shown in Table 4.2. Equation (4.9) resulted in $E\left[K_p(\mathbf{x})\right]$ = U$ 0.853 and Equation (4.14) resulted in $\sqrt{Var\left[K_p(\mathbf{x})\right]}$ =U$ 0.070. A Monte Carlo Simulation with 10,000 replicates were generated and the results were $E\left[K_p(\mathbf{x})\right]$

= U\$ 0.852 and $\sqrt{Var[K_p(\mathbf{x})]}$ = U\$ 0.070.Therefore, the mathematical models for $K_p$ have been validated.

### 4.6.3. Solving the optimization problem

The GRG algorithm was again used to solve the optimization problem of approach 2 in Equation (4.22). The problem was programmed using Microsoft® Excel software and its Solver supplement. The optimal values of the decision variables were:

- $V_c$ =240.9 m/min;
- $f$ = 0.42 mm/rev;
- $a_p$ = 0.26 mm.

Regarding the results of interest, a 95% confidence interval was obtained for the cost of U \$ 0.73 ± 0.12. Therefore, the probability that the cost is less than or equal to U \$ 0.90 is 99.71%. Other results are shown in Table 4.11. It shows that the cut time $C_t$ and the total cycle time $T_t$ obtained are relatively low in relation to other options within the solution space.

Table 4.11 - Results in optimal cutting conditions in approach 2

| $C_t$ (min) | $T_t$ (min) | $T$ (min) | $R_a$ (μm) | $R_t$ (μm) | MRR (cm$^3$/min) |
|---|---|---|---|---|---|
| 0.08 | 0.75 | 5.93 | 0.28 | 2.12 | 26.8 |

It is also possible to notice a low result for the tool life ($T$) in a solution whose cost has been optimized. Section 4.6.5 explains the $T$ result in more detail. The values of $R_a$ and $R_t$ were significantly below their upper specification limits. In fact, these two constraints (defined deterministically in approach 2) were inactive in solving the problem. This means that, for the experimental region of this case study, there is a high probability that the roughness meets its specifications. The material removal rate ($MRR$) obtained was significantly high, which corroborates with short cut and cycle times.

### 4.6.4. Effects of industrial variables on the process cost

The individual results of the partial derivatives in $Var[K_p(\mathbf{x})]$, whose calculations are presented in Table 4.7, did not represent their impact from a practical point of view. In fact, the derivatives represent only the impact of industrial variables per unit. For example, in the optimal conditions of the problem as presented in the previous section (4.6.3), if only the time for insert change ($t_i$) changes from 1 min to 2 min, the cost would increase by \$ 0.01. Under the same conditions, if only labor and machine costs ($S_h + S_m$) increased by \$ 1.00, then $K_p$ would also increase by \$ 0.01. Thus, one could assume that $t_i$ and $S_h + S_m$ have the same impact. However, a 1-minute increase in $t_i$ would actually mean a 100% increase over its expected value. An increase of U \$ 1.00 in $S_h + S_m$ would represent an increase of only 2% compared to its average.

Thus, to properly measure the impact of industrial variables, a complete factorial arrangement composed of $2^7$ = 128 combinations between industrial variables was defined. Since these variables were considered normally distributed in this case study, the levels -1 and +1 were established as being $-3\sigma$ and $-3\sigma$, respectively, which corresponds to the limits of what is considered the natural variation of the process (99.73% of probability). Table 4.12 shows the levels defined for the complete factorial arrangement and Figure 4.18 shows the standardized effects on the variables and their second order interactions.

After calculating $K_p$ for all 128 combinations, it was possible to identify that labor and machine costs ($S_h + S_m$), assessed here together, consist of the most significant industrial variable in the value of the process cost. The second variable with the main effect was the cost of the insert ($K_i$), followed by the setup time ($t_p$) and the insert changing time ($t_i$). As shown in Table 4.12, an increase from U \$ 50.00 to U \$ 65.00 (corresponding to $\mu + 3\sigma$) in $S_h + S_m$ would imply an increase in U \$ 0.19 in $K_p$, if the other industrial variables remained in their expected values. This impact is 10 times greater than the impact of the second most significant variable ($K_i$). The other industrial variables did not have a significant impact individually. The following second interactions were also significant: (i) $t_p$ and $Z$; (ii) $t_p$ and $S_h + S_m$; (iii) $Z$ and $S_h + S_m$.

Table 4.12 - Individual impacts of industrial variables on the cost of the process

| Variable | Level -1 | Level +1 | Impact (+ 3σ) |
|---|---|---|---|
| $t_p$ | 30 | 90 | U\$ 0.02 |
| $t_p$ | 0.7 | 1.3 | U\$ 0.00 |
| $Z$ | 700 | 1300 | - U\$ 0.01 |
| $S_m + S_h$ | 35.00 | 65.00 | U\$ 0.19 |
| $K_{th}$ | 87.50 | 162.5 | U\$ 0.00 |
| $N_{th}$ | 700 | 1300 | - U\$ 0.00 |
| $K_i$ | 21.88 | 40.63 | U\$ 0.03 |

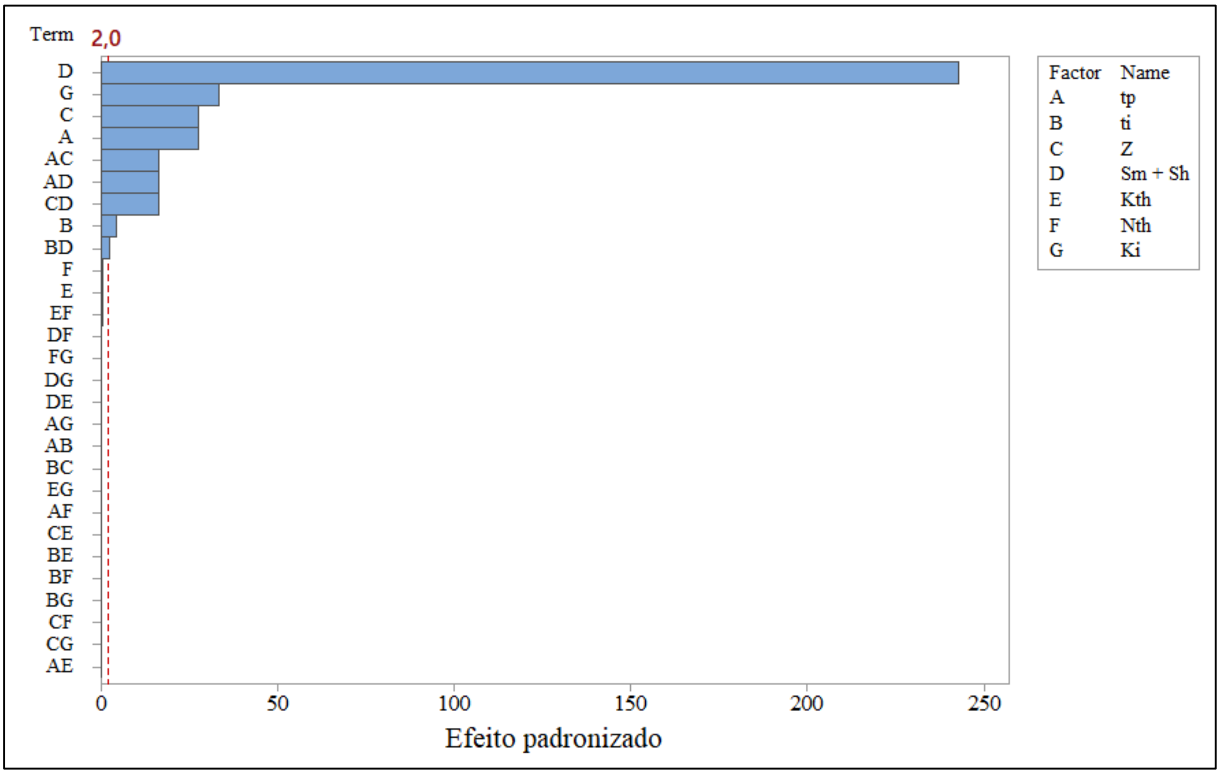


Figure 4.19 - Pareto chart for the standardized effects of stochastic industrial variables

In addition, it is possible to estimate the potential impacts if the values of some industrial variables are modified in improvement projects. As an example of this case study, if the setup time were reduced from 60 min to 9 min, there would be a 6.5% reduction in $K_p$ (from U \$ 0.73 to U \$ 0.68), considering expected values. This reduction in setup to a minute digit is the objective of the SMED - Single Minute Exchange of Die (SHINGO, 1983). Therefore, decision makers can use sensitivity analysis to analyze the feasibility of implementing SMED and other methods in their production system, instead of focusing their studies only on the machine parameters $V_c$, $f$ and $a_p$.

### 4.6.5. Minimum process cost versus maximum tool life

In this section, the solutions to the following optimization problems were compared:

a) The maximization of the probability that the cost is less than U \$ 0.90, a problem defined in Equation (4.22);

b) The maximization of the expected value of the tool life, subject to the experimental space constraint and deterministic constraints for the roughness results ($R_a$) and ($R_t$), as shown in Equation (4.23) as follows.

$$\begin{gathered} Max.\, f(\mathbf{x}) = E[T(\mathbf{x})] \\ subject\ to: \\ E[R_a(\mathbf{x})] \leq USL_{R_a} \\ E[R_t(\mathbf{x})] \leq USL_{R_t} \\ \sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1{,}682 \end{gathered} \tag{4.23}$$

Table 4.13 presents the solutions for Equations (4.22) and (4.23).

Table 4.13 - Solutions to Equations (4.22) and (4.23)

| **Problem** | **Decision variables** | | | **Results** | | | |
|---|---|---|---|---|---|---|---|
| | $V_c$ (m/min) | $f$ (mm/rev) | $a_p$ (mm) | 95% CI for $K_p$ (U$) | $T$ (min) | $R_a$ (μm) | $MRR$ (cm3/min) |
| a) | 240.9 | 0.42 | 0.26 | 0.73 ± 0.12 | 5.93 | 0.284 | 26.8 |
| b) | 206.0 | 0.17 | 0.17 | 0.85 ± 0.15 | 17.18 | 0.232 | 5.7 |

The solution of problem "b" presented in Table 4.13 provided an expected value for tool life $E[T(\mathbf{x})]$ of 17.18 minutes. However, values of the decision variables obtained in this solution provided a cost of U \$ 0.85 ± 0.15, for a 95% confidence level. With this interval, illustrated in Figure 4.20, the probability that the cost would be less than U \$ 0.90 was only 75.47%. More specifically, the maximization of tool life resulted in an increase of 16.9% in the expected value of $K_p$.

It was observed that, in order to maximize the tool life, that is, solving the problem "b", it was necessary to choose low levels for the decision variables ($V_c$, $f$, $a_p$), as shown in Table 4.13.

Tool life increased by 187.5% (from 5.93 min to 17.18 min) when it was maximized individually. However, the cut-off time ($t_c$) increased from 0.08 min to 0.22 min, that is, a 192.6% increase. Therefore, in this case study, increasing the tool life did not imply advantages for the process, as the cutting time increased more than the life. In other words, with $T$ maximization, the tool would in fact last longer, but it would take even longer to machine each part. Consequently, the number of parts machined per tool edge ($T/t_c$) would be 78, solving problem "a" and 77 for problem "b". The number of tool changes ($N_t$) would be the same (12) for a batch of 1000 pieces.

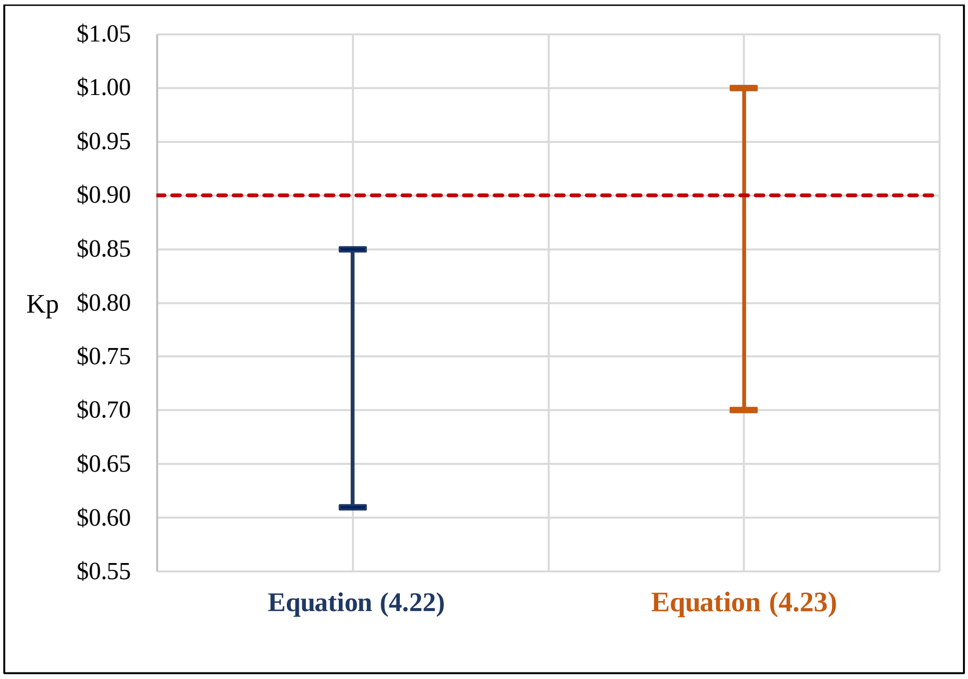


Figure 4.20 - Confidence intervals for $\boldsymbol{K_p}$ for the two solutions obtained

In addition, it was found that, after maximizing tool life, the total cycle time ($t_t$) increased from 0.748 min to 0.894 min. This increase of 19.5% is the main cause of the increase in the expected value of the cost from U $ 0.73 to U $ 0.85, as shown in Table 4.13.

Therefore, the maximization of the tool life obtained from the choice of cutting conditions (or decision variables) does not necessarily reduce the cost of the process. In fact, such a strategy can even increase the cost, as noted in the present case study.

## 4.7. Approach 3: multivariate stochastic constraint (MCCP)

The multivariate chance-constrained programming (MCCP) was applied in the case study as the third and last approach of this work. The multi-objective optimization problem formulated in the present approach consisted in maximizing the probability that the cost ($K_p$) would be less than or equal to its upper limit specified as U $ 0.90, this time subject to a multivariate probabilistic constraint of both $R_a$ and $R_t$ roughness values. No studies were found on multi-objective optimization that have used this type of constraint until the moment of the present research. The multivariate probabilistic constraint was represented by the estimated $PPM$ (parts per million) indicator. Equation (4.24) describes the optimization problem defined for approach 3.

$$\underset{\mathbf{x}}{Max}\, F(\mathbf{x}) = \int_{-\infty}^{K_p^*} \emptyset \left\{ E\left[K_p(\mathbf{x})\right], \sqrt{Var\left[K_p(\mathbf{x})\right]} \right\}$$

$$subject\ to:$$

$$10^6 \left\{ 1 - \int_{-\infty}^{\mathbf{USL}} \emptyset\left[E(\mathbf{R}), \sqrt{Var(\mathbf{R})}\right] \right\} \leq 3.4 \tag{4.24}$$

$$\sqrt{\mathbf{x}'\mathbf{x}} \leq \sqrt[4]{2^n} = 1.682$$

Where:

**USL** = vector composed of the upper specification limits of $R_a$ and $R_t$ (0,8 µm and 4 µm respectively);

$E(\mathbf{R})$ = vector composed by the functions of the expected values of $R_a$ and $R_t$;

$\sqrt{Var(\mathbf{R})}$ = vector composed by the functions of the standard deviations of $R_a$ and $R_t$.

As in approach 2, the solution of Equation (4.24) is unique, as there is no weight configuration to be predefined. Table 4.14 presents the results of approach 3.

Table 4.14 - Solutions and results of approach 3

| **Decision variable** | **Unit** | **Value** |
|---|---|---|
| $V_c$ | m/min | 202.3 |
| $f$ | mm/rev | 0.3 |
| $a_p$ | Mm | 0.12 |
| **Result** | **Unit** | **Value** |
| $K_p$ | U$ | 0.753+-0.132 |
| $P(K_p <= 0.90)$ | - | 98.54% |
| $PPM(\mathbf{R})$ | Parts | 3.4 |
| $R_a$ | µm | 0.297+-0.006 |
| $R_t$ | µm | 1.775+-0.098 |
| $C_{pk}(R_a)$ | - | > 2 |
| $C_{pk}(R_t)$ | - | 1.5 |
| $MRR$ | $cm^3/min$ | 7.22 |
| $T$ | Min | 11.19 |

Using the GRG algorithm, a cost of U \$ 0.753 ± 0.132 was obtained, with 95% confidence. With this confidence interval, the probability that the cost would be less than U \$

0.90 would be 98.54%. In addition, only 3.4 $PPM$ would fail, taking into account the characteristics $R_a$ and $R_t$ simultaneously. These results were obtained for a cutting speed ($V_c$) of 202.3 m / min, a feed rate ($f$) of 0.3 mm/rev, and a depth of cut ($a_p$) of 0.12 mm. To satisfy the constraints of the problem, and at the same time maximize the probabilistic function of the cost, the decision variables were defined at lower levels. As a result, the material removal rate ($MRR$) has been significantly reduced (7.22 cm3/min) and the tool life has increased (11.21 min) compared to other approaches. Finally, it was observed that the confidence intervals of $R_a$ and $R_t$ are below their upper specification limits (0.8 µm and 4 µm respectively). These results implied a $C_{pk}$ greater than 2 for $R_a$ and a $C_{pk}$ equal to 1.50 for $R_t$.

Of the three approaches presented in this study, approach 3 provides the final and recommended solution for this case study since this approach was the most complete one. In fact, approaches 1 and 2 were only the initial tested strategies, and the goal was to analyze issues such as the representation of process capacity as stochastic constraint, the influence of stochastic industrial variables on the mean and variance of the cost and the consequence of maximizing the tool life by changing only the cutting parameters.

# 5. CONCLUSIONS

This work aimed to propose, apply, and analyze the combined use of stochastic programming techniques, process capacity indices and multivariate statistical methods. To this end, a literature review was carried out on the main statistical techniques used in the optimization of manufacturing processes. Based on such techniques, the multivariate stochastic constraint, or multivariate chance-constrained programming (MCCP) method was proposed. This method was applied to a case study of multi-objective optimization of the turning process of hardened steel AISI 52100. The main results and contributions of this research are presented below.

## 5.1. Research contributions

The main contributions of this dissertation can be synthesized as follows:

a) An alternative to include the variance of response surface models - specifically, second order polynomial models - was proposed in the calculation of the $C_{pk}$ process capacity index and the Parts Per Million ($PPM$) index. This index, although widely used in the industry, does not appear as often in modeling optimization problems. In fact, at the time of this research, no studies were found that included stochastic programming methods in the calculation of the $C_{pk}$ index in a real case study of a mechanical manufacturing process. This approach was proposed in Torres *et al.* (2019b). This study showed that the strategy of modelling only the expected values of the critical quality characteristics does not guarantee a satisfactory capacity index for the process. In contrast, 21 Pareto-optimal solutions were obtained via the proposed method, each with a $C_{pk} \geq 1.67$ index for the hardened steel turning process analyzed.

b) The calculation of the variance of a continuous and derivable function (linear or non-linear) was demonstrated, applied and validated using Monte Carlo simulation. The resulting equation could be applied to calculate the variance of both empirical and mechanistic models.

c) An analysis of the impact of some of the main industrial variables that have a random nature and that are included in the turning process cost was carried out. Until then, at the best of the author's knowledge, the works that considered such variables were not related to experimental design techniques and multi-objective optimization of manufacturing processes.

d) The multivariate and the stochastic characteristics of the modeled responses of interest was considered as constraints in the formulation of the optimization problem. The result was the use of a quality index widely used in the industry (the $PPM$) as a multivariate stochastic constraint.

It is important to highlight that many studies published in the area of multi-objective optimization of manufacturing processes are limited to analyzing the main effects and second-order interactions that the decision variables have on the expected values of the responses of interest. The present research, however, also included the influence of some important stochastic industrial variables on the cost of the process.

A practical and specific contribution to the analyzed case study was the realization that maximizing the life of the tool does not always mean cost savings. In this object of study, the maximum tool life actually resulted in an increase in the cost of the process since the cutting parameters were set at such a low level that there was an increase in labor costs and machine without even reducing tooling costs.

## 5.2. Research limitations

Some limitations of this study are worth mentioning. Firstly, the conclusions made in relation to the case study are limited to its characteristics. It would be necessary to apply the proposed method to other study objects to test the applicability and effectiveness of the method more widely.

Another limitation of this research was that the selected case study did not provide information for the analysis of the interactions between noise variables, for they were considered independent. Such information is relevant for works that take into account the variability of the results of interest.

Replications of the experiments within the same arrangement and confirmation experiments after obtaining the final solution (approach 3) would be of great value for the present study, especially with regard to the estimation of the mean square error of each response surface model. However, carrying out replications were unfeasible from an economic point of view.

It is also important to highlight that there are other variables related to the process flow that could also be included in the present study, such as the throughput of the previous process and the next process. However, this research on the hard turning process only. To analyze the total cost to produce a part in a complete way, the other operations must also be analyzed, so the method proposed needs to be extended to the entire flow.

## 5.3. Recommendations for future work

For future research, the application of the proposed method in other manufacturing processes is suggested so that the proposal is validated for processes of different natures. The authors have already published an article applying the MCCP method in a coating flux-cored arc welding process for a stainless steel cladding application (Torres *et al.*, 2020). This time, the study aimed to estimate the natural variability of the process in relation to geometrical characteristics of the welding bead.

In addition, other regression methods can be used in addition to the ordinary least squares (OLS) method. Among them, we have the method of weighted least squares, or weighted least squares (WLS). Another alternative is the Poisson regression. These and other methods can be compared for the same object of study, or for the same set of manufacturing processes.

Finally, aspects related to productivity could and should be included in optimization problems related to the industry. It is possible to consider a stochastic overall equipment effectiveness, for instance. As a final idea, the author suggests the calculation and consideration of the *takt time* of a process to establish the total cycle time and other process parameters. Considering a stochastic demand and a random time available for work, there will also be a random *takt time,* maybe called *takt* interval. In this problem, a potential result of interest would be the maximum probability of meeting the demand *just-in-time,* that is, in the correct time and in the correct quantity of products without committing the waste of overproduction (OHNO, 1997).

# APPENDIX A – Definitions of expected value and variance of linear combinations

The following definitions were made based on statistical references (BIRGE; LOUVEAUX, 2011; MONTGOMERY; RUNGER, 2018). Suppose $x$ is a random variable and has an expected value $E(x)$ and a variance $Var(x)$, respectively, as:

$$E(x) = \mu_x \quad \text{(A.1)}$$

$$Var(x) = E(x - \mu_x)^2 = \sigma_x^2 \quad \text{(A.2)}$$

If $x$ is multiplied by a constant value $c$, then:

$$E(cx) = cE(x) = c\mu_x \quad \text{(A.3)}$$

$$Var(cx) = E(cx - c\mu_x)^2 = E[c^2(x - \mu_x)^2] = c^2E(x - \mu_x)^2 = c^2\sigma_x^2 \quad \text{(A.4)}$$

If $x_1$ and $x_2$ are two random variables, then:

$$E(x_1 + x_2) = E(x_1) + E(x_2) = \mu_{x_1} + \mu_{x_2} \quad \text{(A.5)}$$

$$E(x_1 - x_2) = E(x_1) - E(x_2) = \mu_{x_1} - \mu_{x_2} \quad \text{(A.6)}$$

The covariance between $x_1$ and $x_2$ is defined as follows:

$$Covar(x_1, x_2) = E\big[(x_1 - \mu_{x_1})(x_2 - \mu_{x_2})\big] = \sigma_{x_1x_2} \quad \text{(A.7)}$$

The correlation between $x_1$ and $x_2$, in turn, is given by:

$$\rho_{x_1x_2} = \frac{\sigma_{x_1x_2}}{\sigma_{x_1}^2\sigma_{x_2}^2} \; ou \; r_{x_1x_2} = \frac{s_{x_1x_2}}{s_{x_1}^2 s_{x_2}^2} \quad \text{(A.8)}$$

In Equation (A.8), $\rho_{x_1x_2}$ is the population correlation. In its calculation, the covariance and population variances of $x_1$ and $x_2$are used. The result $r_{x_1x_2}$ refers to the sample correlation, calculated from the sample covariance and variances. The significance of the correlations can be obtained through Pearson's hypothesis test. For more details, see Montgomery and Runger (2018).

Following with the definitions, if $c_1$ and $c_2$ are constant, then we have:

$$\begin{gathered} Covar(c_1x_1, c_2x_2) = E\left[\left(c_1x_1 - c_1\mu_{x_1}\right)\left(c_2x_2 - c_2\mu_{x_2}\right)\right] \\ = c_1c_2E\left[\left(x_1 - \mu_{x_1}\right)\left(x_2 - \mu_{x_2}\right)\right] = c_1c_2Covar(x_1, x_2) = c_1c_2\sigma_{x_1x_2} \end{gathered} \tag{A.8}$$

Now suppose that $f(\mathbf{x})$ is a linear combination of $x_1$ and $x_2$ such that $f(\mathbf{x}) = c_1x_1 + c_2x_2$. So, the expected value of $f(\mathbf{x})$ is:

$$E[f(\mathbf{x})] = E(c_1x_1 + c_2x_2) = c_1E(x_1) + c_2E(x_2) = c_1\mu_{x_1} + c_2\mu_{x_2} \tag{A.9}$$

The calculation of the variance of $f(\mathbf{x})$ is demonstrated as follows:

$$\begin{aligned} &Var[f(\mathbf{x})] = [c_1x_1 + c_2x_2] = E\left[(c_1x_1 + c_2x_2) - \left(c_1\mu_{x_1} + c_2\mu_{x_2}\right)\right]^2 \\ &= E\left[\left(c_1x_1 - c_1\mu_{x_1}\right) + \left(c_2x_2 - c_2\mu_{x_2}\right)\right]^2 \\ &= E\left[{c_1}^2\left(x_1 - \mu_{x_1}\right)^2 + b^2\left(x_2 - \mu_{x_2}\right)^2 + 2c_1c_2\left(c_1x_1 - c_1\mu_{x_1}\right)\left(c_2x_2 - c_2\mu_{x_2}\right)\right] \\ &= {c_1}^2E\left[\left(x_1 - \mu_{x_1}\right)^2\right] + {c_2}^2E\left[\left(x_2 - \mu_{x_2}\right)^2\right] + 2c_1c_2E\left[\left(c_1x_1 - c_1\mu_{x_1}\right)\left(c_2x_2 - c_2\mu_{x_2}\right)\right] \\ &= {c_1}^2Var(x_1) + b^2Var(x_2) + 2c_1bCovar(x_1, x_2) \\ &= {c_1}^2\sigma_{x_1}^2 + {c_2}^2\sigma_{x_2}^2 + 2c_1c_2\sigma_{x_1x_2} \end{aligned} \tag{A.10}$$

Equation (A.10) is related to some of the concepts of Markowitz Theorem (MARKOWITZ, 1952) for the particular case of a portfolio of two stocks, for instance, where $x_1$ is the value of stock 1, $x_2$ is the value of stock 2, $c_1$ and $c_2$ are the number of shares of 1 and 2 respectively. Equation (A.10) can also be written in a matrix form as follows:

$$Var[f(\mathbf{x})] = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} \\ \sigma_{x_1x_2} & \sigma_{x_2}^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \mathbf{c}'\mathbf{\Sigma}_\mathbf{x}\mathbf{c} \tag{A.11}$$

In cases where $x_1$ and $x_2$ are deterministic variables and $c_1$ and $c_2$ are random coefficients, the variance of $f(\mathbf{x})$ is given by:

$$Var[f(\mathbf{x})] = [x_1 \quad x_2] \begin{bmatrix} \sigma_{c_1}^2 & \sigma_{c_1 c_2} \\ \sigma_{c_1 c_2} & \sigma_{c_2}^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{x}'\mathbf{\Sigma_c}\mathbf{x} \tag{A.12}$$

# APPENDIX B – Main vectors and matrices related to the case study

Next, the vectors and matrices used for calculating the expected values and variances of the response surface models are included in the multi-objective optimization problems of this work.

$$\mathbf{a}(\mathbf{x}) = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_1^2 \\ x_2^2 \\ x_3^2 \\ x_1x_2 \\ x_1x_3 \\ x_2x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ V_c \\ f \\ a_p \\ V_c^2 \\ f^2 \\ a_p^2 \\ V_cf \\ V_ca_p \\ fa_p \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_{11} \\ \beta_{22} \\ \beta_{33} \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{bmatrix} \tag{B.1}$$

The Central Composite Design (CCD) arrangement used in this case study resulted in matrix X and vector y as follows:

$$\mathbf{X} = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1{,}68 & 0 & 0 & 2{,}83 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1{,}68 & 0 & 0 & 2{,}83 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1{,}68 & 0 & 0 & 2{,}83 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1{,}68 & 0 & 0 & 2{,}83 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1{,}68 & 0 & 0 & -2{,}83 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1{,}68 & 0 & 0 & 2{,}83 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_4 \\ y_{15} \\ y_{16} \\ y_{17} \\ y_{18} \\ y_{19} \end{bmatrix} \tag{B.2}$$

# APPENDIX C – Excel spreadsheets used in formulations and calculations of multi-objective optimization problems

| | Variáveis de decisão | | |
|---|---|---|---|
| Solução | **Vc** | **f** | **ap** |
| Valor cod. | **0,000** | **0,000** | **0,000** |
| Valor decod. | 220,0 | 0,30 | 0,23 |

| | T | Ra | Rt | Kp | MRR |
|---|---|---|---|---|---|
| E[f(x)] | 4,963 | 0,260 | 1,733 | $ 0,853 | 14,850 |
| Var[f(x)] | 0,007 | 2,62E-06 | 0,001 | 0,0049 | - |
| SD[f(x)] | 0,084 | 0,002 | 0,029 | $ 0,070 | - |
| | USL | 0,400 | 2,000 | $ 0,90 | |
| | Cpk | 28,834 | 3,097 | | |
| | | | E[F(x)] esc. | 5,485 | 0,660732777 |

| | |
|---|---|
| F(x) WS | 3,073009 |
| w | 0,5 |
| | |
| P(Kp<USL) | |
| **75,06%** | |
| MCCP | Corr RaRt |
| **1,00E+00** | 0,720 |
| | |
| **$ 0,725** | **$ 0,748** |
| **7,32** | **29,52** |

| PPM (Ra e Rt) |
|---|
| **0,00000** |
| <= |
| 3,4 |

# APPENDIX D – Complete articles published in journals

1. TORRES, A. F.; MIRANDA, R. P. R.; PAIVA, A. P.; CAMPOS, P. H. S.; BALESTRASSI, P. P.; FERREIRA, J. R. Stochastic optimization of AISI 52100 hard turning with six sigma capability constraint. **IEEE Access**, v. 7, p. 46288-46294, 2019.

IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

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## Stochastic Optimization of AISI 52100 Hard Turning With Six Sigma Capability Constraint

**ALEXANDRE F. TORRES, RENATA P. R. MIRANDA, ANDERSON P. PAIVA, PAULO H. S. CAMPOS, PEDRO PAULO BALESTRASSI, AND JOÃO R. FERREIRA**

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**ABSTRACT** Hard turning optimization problems are usually approached using response surface methodology. By running designed experiments, researchers build analytical models to represent the outputs under interest. However, most studies focus on the expected values of the outputs, and only a few consider the variances of the models, even though there are several stochastic programming (SP) techniques available in the literature. Such variances may have a significant impact on the problem solution. This paper aims to optimize the AISI 52100 hardened steel turning process using SP. The decision variables are cutting speed, feed rate,

2. TORRES, A. F.; ALMEIDA, F. A.; PAIVA, A. P.; FERREIRA, J. R.; BALESTRASSI, P. P.; CAMPOS, P. H. S. Impact of stochastic industrial variables on the cost optimization of AISI 52100 hardened-steel turning process. **The International Journal of Advanced Manufacturing Technology**, v. 104, p. 4331-4340, 2019.

The International Journal of Advanced Manufacturing Technology
https://doi.org/10.1007/s00170-019-04273-1

ORIGINAL ARTICLE

# Impact of stochastic industrial variables on the cost optimization of AISI 52100 hardened-steel turning process

Alexandre Fonseca Torres[1] · Fabrício Alves de Almeida[1] · Anderson Paulo de Paiva[1] · João Roberto Ferreira[1] · Pedro Paulo Balestrassi[1] · Paulo Henrique da Silva Campos[1]

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**Abstract**
An optimization problem of the AISI 52100 hard-steel turning process is examined. A new approach is presented in which not only the machine parameters (cutting speed, feed rate, and depth of cut) but also the stochastic industrial variables of setup time, insert changing time, batch size, machine and labor costs, tool holder price, tool holder life, and insert price are considered. By representing each of these variables by a given probability distribution, the goal was to analyze their impact on the total process cost per piece ($K_p$). Experiments were carried out following a central composite design to model tool life ($T$), average surface roughness ($R_a$), and peak-to-valley surface roughness ($R_t$) using a response surface methodology. Then, stochastic programming was used to model $K_p$'s expected value and standard deviation. The approach to the optimization problem aimed to maximize the probability for the cost to be less than a target value, subject to the experimental space and to maximum values of both $R_a$ and $R_t$. The results were optimal values for the cutting conditions that provide a suitable confidence interval for $K_p$. The most-significant

3. TORRES, A. F.; ROCHA, F. B.; ALMEIDA, F. A.; GOMES, J. H. F.; PAIVA, A. P.; BALESTRASSI, P. P. multivariate stochastic optimization approach applied in a flux-cored arc welding process. **IEEE Access**, v. 8, p. 61267-61276, 2020.

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Multidisciplinary | Rapid Review | Open Access Journal

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# Multivariate Stochastic Optimization Approach Applied in a Flux-Cored Arc Welding Process

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**ABSTRACT** One of the main goals in flux-cored arc welding processes is the optimization of bead geometry, in which multiple geometric characteristics of the welding bead are important; therefore, multiobjective optimization programming is often applied. However, several optimization problems that use stochastic programming do not consider the impact of the correlation between the output variables on their probabilistic constraints. In this context, this paper aims to present a multiobjective optimization method based on multivariate stochastic programming. To demonstrate the applicability of the proposal, we conducted a design of experiments to optimize a flux-cored arc welding process for stainless-steel claddings. The weighting-sums method was applied to formulate the multiobjective optimization problem. It was possible to formulate a multivariate probability distribution for the penetration and dilution. In addition, a 95%

4. COSTA, A. F. B.; TORRES, A. F.; BALESTRASSI, P. P. The monitoring of mean vectors with VCS charts for multivariate processes. **Journal of Statistical Computation and Simulation**, v. 90, n. 10, p. 1897-1920, 2020.

JOURNAL OF STATISTICAL COMPUTATION AND SIMULATION
2020, VOL. 90, NO. 10, 1897–1920
https://doi.org/10.1080/00949655.2020.1751162

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# The monitoring of mean vectors with VCS charts for multivariate processes

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**ABSTRACT**

We propose to control the mean vector by taking larger samples but inspecting fewer quality characteristics if we work with samples of size $n$ to control bivariate processes, then $2n$ observations are usually collected, half are observations of $X_1$ and the other half are observations of $X_2$; alternatively, we might work with samples of size $2n$ if only observations of $X_1$ (or $X_2$) are collected; in both cases the cardinality $c$ of the sample data set is $c = 2n$. If only one of the two quality characteristics, $X_1$ or $X_2$, is measured at each sampling time, then only

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Variable charting statistic; alternate charting statistic; $X$-bar control chart; $T^2$ Hotelling; multivariate processes

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# Design of experiments in the service industry: a critical literature review and future research directions



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**Abstract**

**Purpose** – The purpose of the article is to present the results of a critical literature review (CLR) on Design of experiments (DoE) in the service industry.

**Design/methodology/approach** – A critical review of existing literature review across various databases including Scopus, Web of Science, Google Scholar and Emerald Insight were searched for the identification of relevant papers. The authors searched relevant journal articles for a time period of 25 years (1994–2019).

**Findings** – A total of 29 industry case studies of DoE applications were identified spanning healthcare, retail, logistics, education, marketing, after sales and catering business. The industrial experimentation strategies

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# Design of experiments in the service industry: results from a global survey and directions for further research



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## Abstract

**Purpose** – The purpose of this study is to critically evaluate the current status of Design of experiments (DoE) in the service industry or non-manufacturing processes in the manufacturing sector. This involves the identification of critical success factors and challenges in the use of DoE in the context of the service industry as well as the key skills needed for its successful applications.

**Design/methodology/approach** – In order to identify the critical success factors and challenges, an online

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# Learning lean with lego: developing and evaluating the efficacy of a serious game

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## Abstract

This study presents the use of a serious game developed to teach Lean philosophy. The structure of this game was built from theoretical elements and predefined learning events. Learning outcomes and student motivation were considered in the evaluation of the efficacy of the game. This serious game was applied to four groups of students with different profiles. The evaluation results were compared among these groups of students. It can be concluded that the serious game developed showed positive results in learning and motivation demonstrated by the students, regardless of the group analyzed. The main contributions to the literature presented in this article were the serious game (named 3L) that was developed and the efficacy evaluation method, considering the learning and motivation demonstrated in different profiles of students.

## Keywords

Active learning. ARCS. Business simulation game.

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