The steady oblique path of buoyancy-driven rotating spheres

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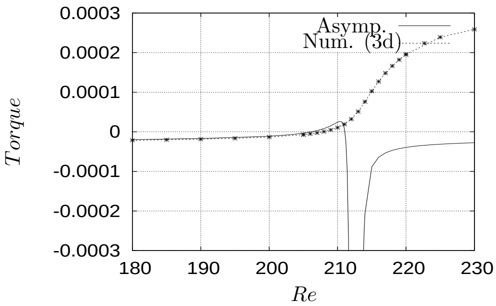
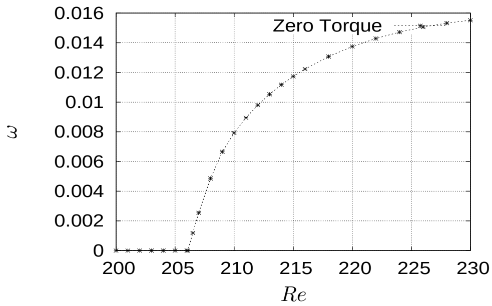
The dynamics of freely falling or rising bodies under the effect of boyancy is an active field of research[1]. A large variety of paths has been reported in experiments and numerical simulations, including fluttering, tumbling, spiral and chaotic motions. For light spheres, in particular[2], a number of regimes characterized by weak deviations with respect to the vertical have been noticed, the simplest one being a steady oblique (SO) trajectory, with the sphere rotating around an axis perpendicular to the path.

In this work, we investigate this situation using two different numerical approaches. The first method[3] is a weakly nonlinear asymptotic expansion in terms of the rotation rate ω. The approach makes use of a modal expansion in the azimuthal direction to reduce the three-dimensional problem into a set of two-dimensional equations, which are solved at each order by the finite-element FreeFem++ software, in the line of ref.[4]. An expression for the lift and torque coefficients as functions of Reynolds number Re and rotation rate ω is derived (see fig (a) for an example). By imposing the torque to vanish, the approach allows to describe the steady oblique (SO) path, and predicts that the latter emerges from a supercritical bifurcation at a critical Reynolds number $Re_{SO} = 206.075$, regardless of the body-to-fluid density ratio (fig (b)).

The second numerical approach is a fully three-dimensional numerical simulation obtained with a combined immersed-boundary multigrid code. Steady solutions for both the flow around the sphere with fixed ω and the SO path with zero torque are obtained using Newton iteration. Results compare well with the weakly nonlinear prediction in the vicinity of the critical Reynolds number and allow to investigate the SO path in the fully nonlinear range. The approach also allows to investigate the stability of the zero-torque SO solutions.



(a) Torque Coefficient for imposed rotation rate ($\omega = 10^{-3}$): asymptotic and 3d numerical data



(b) Bifurcation diagram $\omega - Re$ for zero-torque solutions (3d numerical simulations)

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