Network Data as Contiguity Constraints in Modeling Preference Data

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Abstract In the last decades the use of regression-like preference models has found widespread application in marketing research. The Conjoint Analysis models have even more been used to analyze consumer preferences and simulate product positioning. The typical data structure of this kind of models can be enriched by the presence of supplementary information observed on respondents. We suppose that relational data observed on pairs of consumers are available. In such a case, the existence of a consumer network is introduced in the Conjoint model as a set of contiguous constraints among the respondents. The proposed approach will allow to bring together the theoretical framework of Social Network Analysis with the explicative power of Conjoint Analysis models. The combined use of relational and choice data could be usefully exploited in the framework of relational and tribal marketing strategies.

1 Introduction

Recent studies provided a conceptual framework to model consumer relationships where consumers are both individuals and members of social groups. The consumer, through these relationships, tends to develop its identity/self-concept and to socially identify itself in a group (Brewer and Gardner 1996). In this framework, firms tend to adapt their marketing and communication strategies, finalizing their behavior

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to supporting individuals in their social context. The study of the role played by individuals as actors embedded in social relationships is one of the aims of Social Network Analysis (Wasserman and Faust 1994). The network analysis highlights the position of single individuals in the net, describing their peculiar attitude to improve the sharing of resources (knowledge, friendship, confidence, advise, etc.).

The characterization of individuals as potential consumers and social actors, at the same time, leads to a new perspective when analyzing the elicitation preference process. This process has been traditionally analyzed in the context of Conjoint Analysis (Green and Srinivasan 1990). Nowadays, by recognizing the role of the individual as a consumer embedded in a (not only virtual) community, we highlight the relevance to incorporate such knowledge when modeling individual utility functions.

The aim of this paper is to introduce a new model of Conjoint Analysis that makes it possible to take into account: (a) the overall notes of preference revealed on a set of stimuli profiles; (b) the attribute-levels describing the stimuli profiles and (c) the relational data giving information about the set of pairwise ties among respondents and their knowledge exchange in the community. The main goal is to obtain aggregate part-worths estimates considering the neighboring of each individual consumer (ego-network) and comparing the estimated individual utility function with the aggregate utility function associated to the ego-network. Such a comparison might give a major insight about the presence of an ego-network effect in the elicitation preference process.

2 The Metric Conjoint Analysis Model: Factorial Approximation and External Information

Conjoint Analysis (Green and Srinivasan 1990) is one of the most popular statistical techniques used in Marketing to estimate preference functions at both individual and aggregate level. In this contribution, we refer to the metric approach to Conjoint Analysis by assuming that a fractional factorial design is realized, the full profile concept evaluation is used in the data collection phase, the part-worth utility function is estimated on a rating scale by means of a metric (OLS) regression. Initially, we consider two sets of variables: the dependent variables in the matrix $\mathbf{Y}$ $(N \times J)$ and the explicative ones in the design matrix $\boldsymbol{X}$ $(N \times (K - p))$, where N is the set of Stimuli, J is the number of judges – i.e. the preference responses – and p is the number of experimental factors expanded in K attribute-levels. The Metric Conjoint Analysis model is written as the following multivariate multiple regression model:

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{B} + \boldsymbol{E}_x \tag{1}$$

Let $\mathbf{B}$ be the $(K - p) \times J$ matrix of individual part-worth coefficients and let $\boldsymbol{E}_x$ be the $(N \times J)$ matrix of error terms for the set of J individual regression models. Lauro et al. (1998) proposed a particular factorial decomposition (see (2)) of the estimated coefficient matrix $\hat{\boldsymbol{B}}$ to be read in the within of Principal Component Analysis and related variants such as the Principal component analysis onto a reference subspace (D'Ambra and Lauro 1982) and Redundancy Analysis (Israels 1984). The part-worths decomposition is defined as follows:

$$\hat{\boldsymbol{B}}'\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{B}}\boldsymbol{u}_\alpha = \boldsymbol{\Lambda}\boldsymbol{u}_\alpha \qquad \boldsymbol{\alpha} = 1, \ldots, (K - p) \tag{2}$$

In the psychometric literature, several authors have contributed to the statistical analysis of data matrices with external information on rows and columns by means of suitable factorial decompositions (Takane and Shibayama 1991). Giordano and Scepi (1999) extended the model (1) to the case of external information observed on the respondents to the Conjoint Analysis task.

Let $\mathbf{Z}$ $(J \times (H - q))$ be the matrix of external information on the columns of $\mathbf{B}$ holding (H-q) dummy variables related to q categorical variables observed on J judges, then the following multiple multivariate regression model is considered:

$$\hat{\boldsymbol{B}}' = \boldsymbol{Z}\boldsymbol{\Theta} + \boldsymbol{E}_z \tag{3}$$

where the estimated part-worth coefficients of the Conjoint Analysis are described as a function of the external information on the judges/consumers. The coefficients in the matrix $\boldsymbol{\Theta}$ explain the effect of the individual' characteristic on the stimuli feature, (for instance: "How worth is the *red-painted* car for *women*?"). The matrix $\boldsymbol{\Theta}$ is estimated by:

$$\hat{\boldsymbol{\Theta}} = (\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{Y}'\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{4}$$

thus the factorial decomposition proposed by Giordano and Scepi (1999) is obtained by the following Singular Value Decomposition (SVD):

$$\boldsymbol{\Theta} \approx \boldsymbol{W}\boldsymbol{\Delta}\boldsymbol{V}' \qquad with \qquad \boldsymbol{W}'\boldsymbol{W} = \boldsymbol{V}'\boldsymbol{V} = \boldsymbol{I} \tag{5}$$

where $\mathbf{W}$ and $\boldsymbol{V}'$ are the left and right singular vectors with singular values in the diagonal matrix $\boldsymbol{\Delta}$. In the next section the approach will be particularized to the case of relational data as external information on consumers considered as social actors in a network.

3 Network Data Matrix as Relational Constraints on Judges

Social Network Analysis deals with ties (relational data) among interacting units to describe patterns of social relationships. In the Economics field, relationships can emerge from different settings: exchange of information, acquaintanceship; trustee; business activity, spatial or geographic contiguity, and so on. The typical data matrix of Social Network Analysis (SNA) is the *Adjacency Matrix*, a squared matrix of order J, the number of actors in the net. In this paper, we only refer to the case of binary (presence/absence of ties), symmetric relationships (mutual exchange), so that the adjacency matrix is symmetric and binary valued. Let $\boldsymbol{A}$ $(J \times J)$ be an adjacency matrix, its generic element a_{ij} is 1 if the two corresponding actors i and j are tied, otherwise it is set to 0; $a_{ij} = a_{ji}$ (symmetry); it is assumed that $a_{ii} = 0$ $(i = 1, \ldots, J)$. We particularize now the Giordano and Scepi (1999) approach to the (metric) Conjoint Analysis model with adjacency data as external information on consumers. With the notation introduced in Sect. 2, let $\boldsymbol{A}$ be a squared, binary and symmetric adjacency matrix among J consumers sharing mutual information in a network. The ties can be established by considering an ad-hoc survey on the quality and quantity of exchanged information or by investigating them in a web-based brand forum. Let us consider the model (3) and restate it as:

$$\hat{\boldsymbol{B}}' = \boldsymbol{A}\boldsymbol{\Theta} + \boldsymbol{E}_a \tag{6}$$

its solution leads to:

$$\hat{\boldsymbol{\Theta}} = (\boldsymbol{A}\boldsymbol{A})^{-1}\boldsymbol{A}\boldsymbol{Y}'\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1} \equiv (\boldsymbol{A}\boldsymbol{A})^{-1}\boldsymbol{A}\hat{\boldsymbol{B}}' \tag{7}$$

since $\mathbf{A}$ is symmetric.

The solution (7) is formed by the matrix multiplication of two quantities, let them be referred as:

$$\hat{\boldsymbol{\Theta}} = (\boldsymbol{A}\boldsymbol{A})^{-1}\boldsymbol{A}\hat{\boldsymbol{B}}' \equiv \boldsymbol{\Sigma}^{-1}\boldsymbol{\Upsilon} \tag{8}$$

where $\boldsymbol{\Sigma} = (\boldsymbol{A}\boldsymbol{A})$ and $\boldsymbol{\Upsilon} = \boldsymbol{A}\hat{\boldsymbol{B}}'$. A deeper look at these two quantities shows that:

- $\boldsymbol{\Sigma}$ is the matrix holding the number of pathways of length two from each actor to each other actor. They are a measure of the network connectivity. Its diagonal elements are the actor's degree, i.e. the number of actors belonging to the ego-network of each actor.
- The matrix $\boldsymbol{\Upsilon}$ holds the generic element υ_{ik} which represents the sum of the part-worth coefficients for the attribute X_k estimated for all the actors belonging to the ego-network of actor i.
- The generic term of $\hat{\boldsymbol{\Theta}}$, $\hat{\theta}_{i,k}$ is a function of the overall part-worth coefficients of each attribute-level measured on the ego-network of i, inversely weighted by the connection strength between each pair of actors/judges.

If we consider only the diagonal elements of $\boldsymbol{\Sigma}$, we define the diagonal matrix $\boldsymbol{\Delta}_{\Sigma}$, and compute

$$\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Delta}_{\Sigma}^{-1}\boldsymbol{\Upsilon} \tag{9}$$

The matrix coefficient in (9) has a more straightforward interpretation, since its generic element $\tilde{\theta}_{ik}$ represents the average part-worth coefficient of attribute X_k estimated for all the actors belonging to the ego-network of actor i. Let us notice that in the i-th row of $\tilde{\boldsymbol{\Theta}}$, the individual utility function (the coefficients in $\hat{\boldsymbol{B}}$) is substituted by the average utility function of judge i's neighbors. The singular value decomposition of $\tilde{\boldsymbol{\Theta}}$ makes it possible to analyze, on a reduced factorial subspace, the proximity between judges sharing similar ego-network preference structures.

Table 1 Estimated part-worth coefficients for 24 artificial response variables

J	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}	J	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}
J1	−2.67	−0.33	−1.00	0.00	0.00	0.67	J13	0.00	3.00	−0.67	0.67	0.33	0.00
J2	−3.00	0.00	−0.67	0.00	0.00	−0.67	J14	0.00	3.00	−0.67	0.00	0.00	0.33
J3	−3.00	0.00	−0.33	0.33	−0.33	0.00	J15	0.00	3.00	0.67	−1.00	0.33	0.00
J4	−3.00	0.00	−0.33	0.33	1.00	−0.67	J16	0.00	3.00	0.00	0.33	0.33	0.00
J5	−3.00	0.00	0.00	−0.33	0.00	−0.67	J17	0.00	3.00	0.33	−1.00	−0.33	0.00
J6	−3.00	0.00	0.67	0.00	0.33	0.00	J18	0.00	3.00	0.33	−0.33	0.00	−0.33
J7	−2.33	−0.67	0.00	0.33	0.33	0.33	J19	3.00	0.00	0.00	−0.33	0.00	0.33
J8	−1.00	−2.00	1.00	−0.33	−0.33	−1.00	J20	3.00	0.00	−0.33	1.00	0.00	−0.33
J9	0.00	−3.00	0.00	−1.00	0.00	0.00	J21	3.00	0.00	0.67	0.00	0.33	0.00
J10	0.00	−3.00	0.00	0.33	1.00	−0.33	J22	3.00	0.00	0.00	0.00	0.00	0.00
J11	−0.11	−2.78	−0.78	0.89	−0.11	−0.11	J23	3.00	0.00	−0.67	0.33	0.33	−0.67
J12	0.11	−3.22	0.11	−0.22	0.11	−0.22	J24	3.00	0.00	−0.33	0.67	−0.33	0.67

4 Example and Validation Study

The proposed strategy will be illustrated by using an artificial dataset where it is supposed that a set of $J = 24$ judges have been involved in a Conjoint experiment about a generic product described by three factors at three levels each. The estimated part-worths obtained by the metric conjoint model are reported in Table 1. Due to constraints imposed on the coefficients, only six out of nine coefficients have been determined. The 24 judges are arranged according to four different groups of six homogeneous units. Judges 1-6 (first group); Judges 7-13 (second group); Judges 14-18 (third group) and Judges 19-24 (fourth group). So we suppose that each group is a potential market segment and we want to relate these groups with their corresponding relational data.

Let us suppose that relational data were collected on the respondents (for instance, a link has been set if two respondents shared information about the product in a web-forum), the adjacency matrix is in Table 2. Notice that the second and the third group share several ties, whereas the first and the fourth group share fewer ties with the other groups.

The estimated coefficients $\tilde{\boldsymbol{\Theta}}$ stated in (9) are in Table 3.

In order to analyze the network effect on the estimated part-worth coefficients, we start from the idea that if a group of consumers exchanges information to the extent of influencing their own opinions about the strong and weak features characterizing the product, then they should give coherent ratings and thus show similar utility profiles. This produces the consequence that groups of highly connected people in the net should have homogeneous utility profiles. The comparison of the individual part-worth coefficients with the ego-profile (average of the coefficients for the subset of consumers connected with ego) can be realized by looking at the corresponding elements in the matrices $\hat{\boldsymbol{B}}'$ (Table 2) and $\tilde{\boldsymbol{\Theta}}$ (Table 3).

Since the effect can be different for each individual actor in the net, we are interested to compare the coefficients at the individual level. To this aim, we carry out a permutation test based on the following procedure:

Table 2 The Adjacency matrix **A** for the 24 Judges: higher density of the diagonal blocks simulate the network effect on the four groups of judges

J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}	J_{17}	J_{18}	J_{19}	J_{20}	J_{21}	J_{22}	J_{23}	J_{24}
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
0	0	0	0	1	1	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	1	1	1	0	1	1	0	1	0	1	0	1	1	0	0	0
0	0	0	0	0	1	1	1	0	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	1	1	0	1	1	1	1	0	1	0	1	1	1	1	0	0	0
0	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	0	1	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	0	0	1	0	1	0
0	0	0	0	1	0	1	1	0	1	1	1	0	1	1	1	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	1	0	1	1	1	0	0	0	0
0	0	0	0	0	1	0	1	1	1	0	1	1	1	1	0	1	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1	1	1	0	0	0	0
0	0	0	0	0	1	0	1	0	1	1	1	0	0	1	0	1	0	1	1	0	1	1	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	1	1	1	0	0
0	0	0	0	0	0	0	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0	1	0
0	0	0	0	0	0	0	1	0	1	0	1	1	1	0	1	0	0	1	1	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table 3 The matrix $\tilde{\mathbf{\Theta}}$: coefficients are the average part-worths of the judge's neighbors

J	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}	J	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}
J1	−3,00	0,00	−0,44	0,22	0,22	−0,44	J13	0,42	−0,42	0,06	0,00	0,11	−0,17
J2	−2,89	−0,11	−0,44	0,00	−0,11	0,00	J14	0,36	−0,36	0,12	−0,07	0,12	−0,12
J3	−2,89	−0,11	−0,56	−0,11	0,00	−0,22	J15	0,76	0,72	−0,15	0,01	0,10	−0,03
J4	−1,42	−1,58	−0,42	0,08	0,00	−0,08	J16	0,43	0,06	0,15	−0,16	0,15	−0,10
J5	−1,44	−0,89	−0,11	0,11	0,07	−0,26	J17	0,86	0,43	−0,19	0,19	0,05	−0,05
J6	−0,36	−0,40	−0,03	−0,24	0,14	−0,11	J18	0,73	−0,45	0,09	−0,03	0,12	−0,21
J7	−0,88	−0,63	−0,04	−0,04	0,04	−0,21	J19	1,13	1,13	0,21	−0,08	0,21	−0,13
J8	0,26	0,02	−0,13	0,08	0,14	−0,10	J20	0,66	0,85	0,02	−0,04	0,16	−0,15
J9	−0,44	−0,56	0,11	−0,07	0,15	−0,11	J21	1,11	0,08	−0,12	0,18	0,18	−0,19
J10	0,42	0,08	0,03	0,06	0,08	−0,11	J22	1,50	1,50	−0,17	0,06	0,17	−0,06
J11	−0,32	−0,19	−0,02	−0,13	0,12	−0,24	J23	2,02	−0,04	0,07	0,19	0,02	−0,04
J12	−0,21	0,41	−0,03	−0,10	0,21	−0,14	J24	3,00	0,00	−0,67	0,33	0,33	−0,67

1. For each judge $i = 1, \ldots, J$ compute the mean absolute deviation among the k=K-p coefficients (in this example J =24; K = 9; p = 3; k = 6):

$$D_i = k^{-1} \sum_{j=1,k} |\hat{b}_{ij} - \tilde{\boldsymbol{\theta}}_{ij}| \tag{10}$$

2. Produce a random simultaneous permutation of the rows and columns of the observed adjacency matrix, let it be denoted with $\mathbf{A}_{rnd}$.
3. Use $\mathbf{A}_{rnd}$ in computing the coefficients: $\tilde{\boldsymbol{\Theta}}_{rnd}$ according to (9).
4. Get the random individual mean absolute deviations:

$$D_{i,rnd} = k^{-1} \sum_{j=1,k} |\hat{b}_{ij} - \tilde{\boldsymbol{\theta}}_{ij,rnd}| \tag{11}$$

5. Repeat steps from 2 to 4 a very large number of times (we produced 10,000 permutations).
6. Obtain J cumulative distribution functions (*cdf*) of the random individual deviations $D_{i,rnd}$ with $i = 1, \ldots, J$.
7. For each judge, compare the *i-th* observed D_i with the random *cdf* percentiles of the correspondent $D_{i,rnd}$.

In case of a significant effect, the observed D_i should lie in the left tail of the random distribution with a value smaller than a given percentile (say 1%).

The experiment is now conducted on the artificial data set. Having fixed the matrix $\hat{\boldsymbol{B}}$ of the estimated part-worths, we generate 10,000 adjacency matrices by using the `rmperm()` function of the software package *sna* (ver. 2.1-0; Butts 2010) implemented in the *R* environment (R Development Core Team 2010).

Table 4 CDF percentiles of the random MAD compared with the observed one (*Obs. MAD*) for 24 judges. The values smaller than the fifth percentile are in bold

J	1%	2%	5%	10%	20%	*Obs.MAD*	*J*	1%	2%	5%	10%	20%	*Obs.MAD*
J1	0,56	0,63	0,69	0,74	0,79	**0.46**	J13	0,53	0,59	0,67	0,72	0,78	0.94
J2	0,39	0,54	0,62	0,68	0,73	**0.20**	J14	0,46	0,54	0,61	0,67	0,71	0.86
J3	0,44	0,52	0,60	0,65	0,70	**0.24**	J15	0,59	0,67	0,76	0,81	0,86	0.86
J4	0,58	0,65	0,74	0,79	0,85	0.85	J16	0,39	0,46	0,54	0,59	0,64	0.71
J5	0,44	0,50	0,57	0,63	0,68	0.58	J17	0,55	0,63	0,72	0,78	0,84	0.93
J6	0,49	0,56	0,62	0,67	0,71	0.71	J18	0,43	0,48	0,57	0,63	0,68	0.83
J7	0,56	0,62	0,70	0,75	0,80	0.81	J19	0,43	0,50	0,57	0,62	0,68	0.68
J8	0,65	0,69	0,76	0,81	0,88	1.03	J20	0,48	0,58	0,68	0,73	0,79	0.82
J9	0,54	0,59	0,65	0,70	0,75	0.70	J21	0,50	0,54	0,62	0,67	0,72	**0.55**
J10	0,54	0,61	0,68	0,74	0,79	0.82	J22	0,33	0,41	0,48	0,53	0,58	0.57
J11	0,58	0,65	0,70	0,75	0,80	0.83	J23	0,50	0,59	0,69	0,75	0,81	**0.48**
J12	0,46	0,50	0,56	0,61	0,66	0.73	J24	0,53	0,66	0,75	0,81	0,88	**0.45**

In Table 4 we have reported the results of the experiment. For each of the 24 judges, we compare the observed mean absolute deviation (MAD) between the two kinds of part-worth coefficients. Small values of the observed MAD indicate the presence of the network effect. In order to assess the significance of the MAD values, we compare them with the lower percentiles of the simulated distributions. The MAD values smaller than the fifth percentile are in bold. Let us notice that three out of six members of group 1 (J1, J2, J3) and three members of group 4 (J21, J23, J24) show significant presence of the network effect. The groups 2 and 3 which share numerous ties (see Table 2) pertain to two different utility profiles (see Table 1) so that for judges labeled from J7 to J18 the network effect is not relevant.

5 Concluding Remarks

An extension of the metric approach to the full profile Conjoint Analysis method has been provided. Adjacency data – derived by links in a social network of a brand community – have been used as external information in a multiple regression model with multivariate responses. A validation procedure has been proposed in the framework of random permutation tests. It needs further work to evaluate how to perform the proposed method applied to networks different in size, density and topology.

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