A COMPARATIVE ANALYSIS OF ACTIVE CONTROL METHODOLOGIES TO REDUCE THE SEISMIC RESPONSE OF STRUCTURES

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ABSTRACT: A comparison between minimal resources required to control structures, fixed or base isolated, by applying different methodologies is presented in this paper. LQR, H_2, H_∞ and IOC control techniques have been considered. Numerical tests have been carried out in order to evaluate the control energy and power necessary to reach similar seismic performances under several recorded excitations. Results are presented by using spectral representations of response and control resources

KEY-WORDS: Active Control, Energy, Resource

I . INTRODUCTION

The primary objectives of a control system is to address a dynamic process according to fixed design objectives and behavior criteria, under excited conditions caused by the outside environment. The seismic response of a system may be controlled through the utilization of appropriate artificial actions according to optimum control algorithms, which require the transfer of energy and power from the outside. The possibility that, at the moment of the earthquake's occurrence, the energy and power necessary for the control of the system's seismic response requires an energy reserve must be taken into account. Therefore, it is necessary to have the instruments available for the prediction of the energies and powers necessary for a control system to adjust the seismic response to predefined levels.

This study investigates the energy resources necessary for the active control of structures subject to seismic excitations comparing several control methods. In particular, with respect to assigned dynamic performances the energy and power necessary for their production are evaluated as a function of different control algorithms: Linear Quadratic Riccati - LQR, "Control Law H_2", "Control Law H_∞" and "Instantaneous Optimal Control - IOC". The analyses take into consideration simple controlled linear systems, fixed or base isolated, subject to recorded excitations.

The results of the analysis of systems subject to different seismic excitations, are represented in spectral form for a direct comparison of the potential performances with variation in active control methodologies considered, as well as in the requirements for the necessary control resources.

II. ANALYTICAL MODEL OF ANALYZED SYSTEMS

The model analyzed is schematically described as a Single Degree Of Freedom or 2 Degrees Of Freedom, subject to base seismic motion $\ddot{\mathbf{u}}_\mathbf{g}$ and controlled by active force $\mathbf{f_c}(\mathbf{t})$ (Figure 1). The motion of the systems are respectively represented by the following equations:

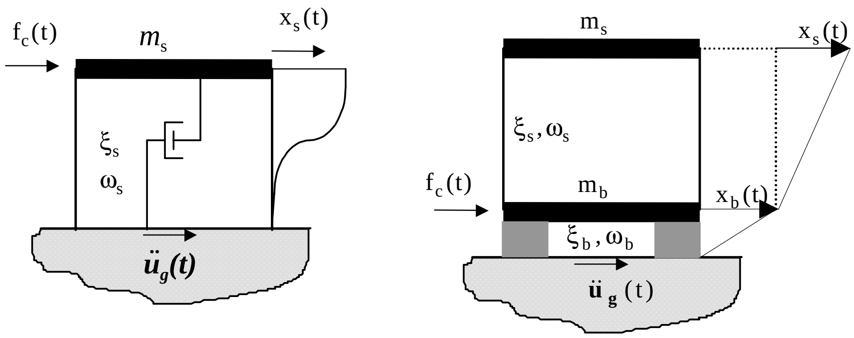

Figure 1: Simple Models for fixed and isolated base structure

$$\ddot{\mathbf{x}}(\mathbf{t}) + 2\xi\omega\,\dot{\mathbf{x}}(\mathbf{t}) + \omega^2\mathbf{x}(\mathbf{t}) = \frac{\mathbf{f_c}(\mathbf{t})}{\mathbf{m}} - \ddot{\mathbf{u}}_\mathbf{g}(\mathbf{t}) \tag{1}$$

$$\begin{cases} \ddot{\mathbf{x}}_\mathbf{s} + \ddot{\mathbf{x}}_\mathbf{b} + 2\xi_\mathbf{s}\omega_\mathbf{s}\dot{\mathbf{x}}_\mathbf{s} + \omega_\mathbf{s}^2\mathbf{x}_\mathbf{s} = -\ddot{\mathbf{u}}_\mathbf{g} \\ \ddot{\mathbf{x}}_\mathbf{b} + \chi\ddot{\mathbf{x}}_\mathbf{s} + 2\xi_\mathbf{b}\omega_\mathbf{b}\dot{\mathbf{x}}_\mathbf{b} + \omega_\mathbf{b}^2\mathbf{x}_\mathbf{b} = -\ddot{\mathbf{u}}_\mathbf{g} \pm \dfrac{\mathbf{f_c}(\mathbf{t})}{\mathbf{M}} \end{cases}, \tag{2}$$

where $\mathbf{x}(\mathbf{t})$ and $\mathbf{u_g}(\mathbf{t})$ are the relative movement and the base seismic motion respectively, ξ and ω the damping factor and natural circular frequency and, m is the mass and finally χ the mass ratio $\mathbf{m_s}/(\mathbf{m_s} + \mathbf{m_b})$. The characters “s” and “b” individuate respectively the main structure and the isolation layer properties. In the state space formulation the dynamic behavior of the controlled systems is described by:

$$\begin{aligned} \dot{\mathbf{v}}(\mathbf{t}) &= \mathbf{A}\mathbf{v}(\mathbf{t}) + \mathbf{B}_1\mathbf{w}(\mathbf{t}) + \mathbf{B}_2\mathbf{u}(\mathbf{t}) \\ \mathbf{y}(\mathbf{t}) &= \mathbf{C}_2\mathbf{v}(\mathbf{t}) + \mathbf{D}_{21}\mathbf{w}(\mathbf{t}) \\ \mathbf{z}(\mathbf{t}) &= \mathbf{C}_1\mathbf{v}(\mathbf{t}) + \mathbf{D}_{12}\mathbf{u}(\mathbf{t}), \end{aligned} \tag{3}$$

where $\mathbf{A}$, $\mathbf{B}_1$ and $\mathbf{B}_2$ represent respectively the dynamic matrix of the system, the position matrix of the seismic action and that of the control action, $\mathbf{z}(\mathbf{t})$ the vector of the controlled variables, $\mathbf{C}_1$ represents the matrix of the weight on the components of the state vector and $\mathbf{D}_{12}$ that of the control vector $\mathbf{u}(\mathbf{t})$. Finally, $\mathbf{C}_2$ and $\mathbf{D}_{21}$ the matrices which correlate the vector of the measured variables $\mathbf{y}(\mathbf{t})$ to the state and the seismic excitation.

The dynamic performance of the described systems is evaluated through the following performance indices:

$$\mathbf{I_s} = \int_0^\mathbf{t} [\mathbf{x_s} \quad \dot{\mathbf{x}}_\mathbf{s}] \begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_\mathbf{s} \end{bmatrix} \begin{bmatrix} \mathbf{x_s} \\ \dot{\mathbf{x}}_\mathbf{s} \end{bmatrix} \mathbf{dt} \quad \text{and} \quad \mathbf{I_b} = \int_0^\mathbf{t} [\mathbf{x_b} \quad \dot{\mathbf{x}}_\mathbf{b}] \begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_\mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{x_b} \\ \dot{\mathbf{x}}_\mathbf{b} \end{bmatrix} \mathbf{dt} \tag{4}$$

respectively for the fixed and the isolated base system.

III. CONTROL ALGORITHMS

The control methodologies examined in this study are the following:

LQR Control Law: The LQR methodology assumes as a control force a function that minimizes the performance index $\mathbf{J}$ defined as:

$$\mathbf{J} = \int_{t_0}^{t_f} \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T\mathbf{R}\mathbf{u}(t)\right]\mathbf{dt}\ .$$

IOC Control Law (Instantaneous Optimal Control): This control methodology, useful also for the regulation of the dynamic response of non-linear systems, is based on the minimization of the following instant performance index $\mathbf{J}$:

$$\mathbf{J}(t) = \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T\mathbf{R}\mathbf{u}(t)\ .$$

Control Law H_2: This control methodology has as its objective the minimization of H_2 norm of transfer function $\mathbf{G}(s)$ of the controlled system[1] which ties the transformation in the frequency domain of the vector $\mathbf{z}(t)$ to that of seismic excitation $\mathbf{u}_g(t)$:

$$\mathbf{H}_2 = \sqrt{\mathbf{trace}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{G}(j\omega)\mathbf{G}^*(j\omega)d\omega\right)}\ ,$$

where "*" represents the transposed conjugated matrix function. It is to be noted that H_2 norm of a transfer function represents the mean square deviation of the structural response when the excitation is made up of white noise. Minimizing H_2 norm, is equivalent therefore, in reducing to a minimum the mean square deviation of the response $z(t)$ of the system to the white noise [3].

Control Law H_∞: The objective of this methodology is to minimize $\mathbf{H}_\infty$ norm of the transfer function $G(s)$ of the controlled system [1]:

$$\mathbf{H}_\infty = \sup_{\mathbf{re(s)>0}} \sigma[\mathbf{G(s)}]\ ,$$

with the single operator $\sigma(\cdot)$ defined by:

$$\sigma(\mathbf{G(s)}) = \max_{\mathbf{d}\neq 0}\frac{\mathbf{d}^*\mathbf{G}^*(s)\mathbf{G}(s)\mathbf{d}}{\mathbf{d}^*\mathbf{d}}\ ,$$

where **d** is a non-zero stationary stochastic process. This expression shows how $\mathbf{H}_\infty$ law permits the adjustment of the system response in the case of worst input; therefore, the $\mathbf{H}_\infty$ methodology produces a solid control with respect to the excitation event.

All control algorithms are designed to divide by 2 the performance indices (4) of the not controlled cases.

IV. DESCRIPTION OF THE SEISMIC EXCITATIONS

The numerical analyses were carried out by considering the accelerometer data recordings of particularly significant events for which the spectral responses in terms of acceleration are shown in Figure 2, and general data is found in the following:

Earthquake	Component	Duration[sec]	PGA [cm/s^2]
Taft 21/07/1952	N69W	30.00	153.83
El Centro 19/05/1940	S00E	53.80	341.82
Mexico City 19/09/1985	N90W	180.08	167.918
Tolmezzo 06/05/1976	NS	36.58	360.03
Petrovac 1979	NS	19.62	429.29
Northridge 17/01/1994	34N 118W	60.00	483.33
Pacoima 1971	S16E	41.90	1148.496

The seismic events under consideration are classifiable in four categories: the first is represented by the Taft earthquake, of low energy content and regular progression in the field of the periods of interest; a second includes the El Centro, Tolmezzo and Northridge earthquakes, characterized by a strong energy content in the 0.1 - 0.8 sec. interval; a third, identified from the Mexico City earthquake, is characterized by elevated peaks of acceleration in the period interval 1.7 - 3 seconds; the fourth and last, is made up of the Petrovac and Pacoima earthquakes, having an elevated energy in the period interval 0.2 - 1.3 seconds.

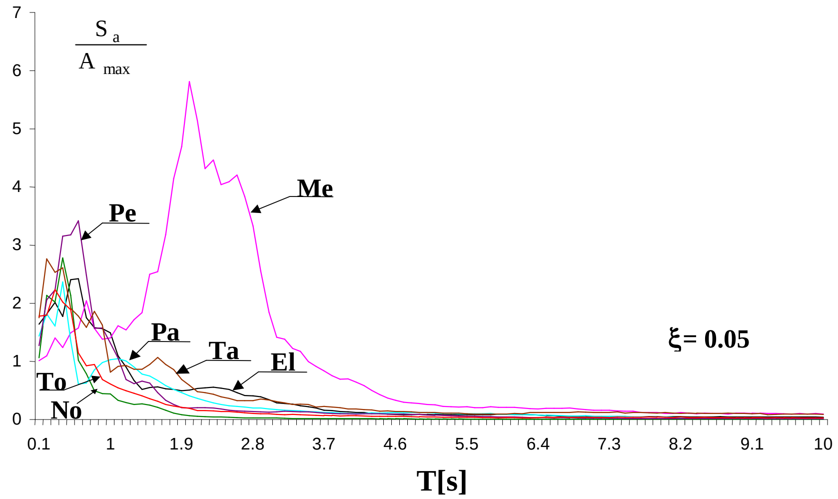

Figure 2: Response spectrum in terms of absolute acceleration

V. NUMERICAL RESULTS

The described systems were subjected to numerical tests for the purpose of determining the energy resources required by the control mechanisms for an assigned performance index which is half that of the uncontrolled cases. In figures 3-6 the spectral representation of the performance index are shown for uncontrolled and controlled cases respectively, with a damping factor $\xi = 0.05$.

For each of the considered control algorithms, the maximum spectral response of the systems are illustrated in figures 7-20. Figs 7-8 shown the worst damping and the elastic energy ratios for the isolation level. In particular, figures plot the ratio between the energies of the controlled and the ones of the uncontrolled systems valued for considered seismic excitations. Figures 9-14 show the worst kinetic energy, dissipated energy and elastic energy ratios for the fixed and isolated base systems. Control force, control energy and control power are finally plotted in the figures 15-20.

With equal performance indices, the control energy is generally minimal for the H_2 and H_∞ control algorithm. The IOC law requires higher values of control power and forces for almost all of the excitations considered. The algorithms H_2 and H_∞, when faced with a greater energy requirement, have lower control force values. Results show that isolated systems required less energy and power resources than the ones necessary to control the fixed base systems. In particular, in the range of interest periods, the control of fixed base system requires energy resources 1.5 times the input ones, while the control of base isolated systems requires less than 0.5.

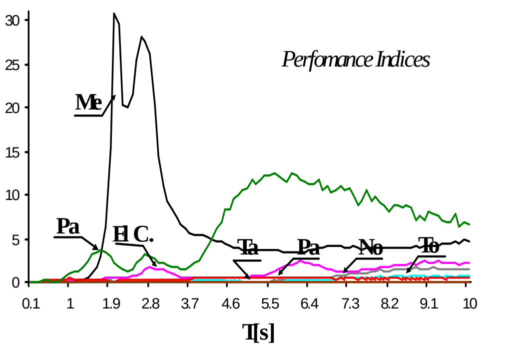

Fig 3: Performance index spectra for non controlled fixed base structures

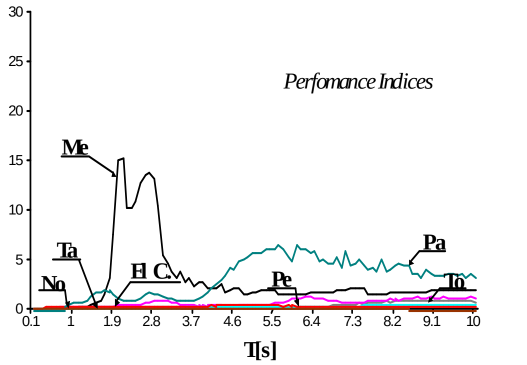

Fig 4: Performance index spectra for controlled fixed base structures

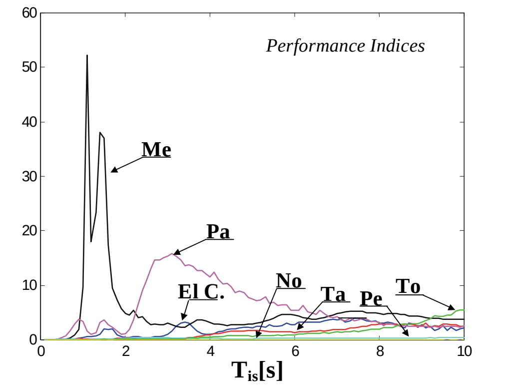

Fig 5: Performance index spectra for non controlled base isolated structures, isolation layer.

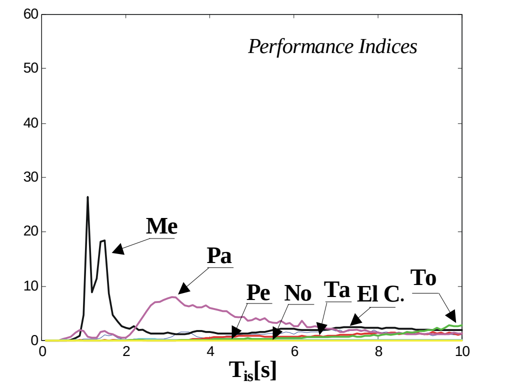

Fig 6: Performance index spectra for controlled base isolated structures, isolation layer.

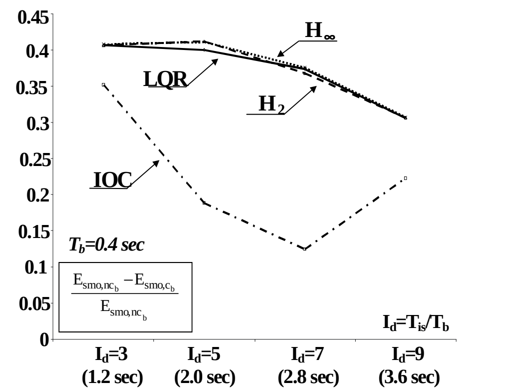

Fig 7: Isolator layer response spectra
(worst case between considered excitations)

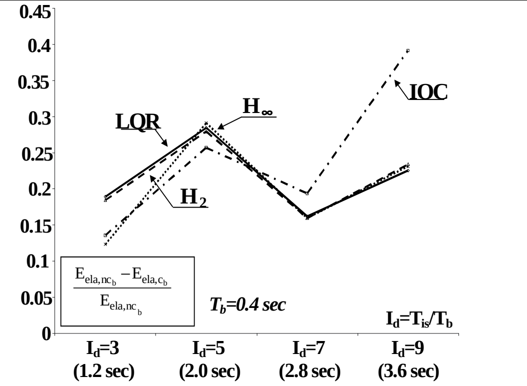

Fig 8: Isolator layer response spectra
(worst case between considered excitations)

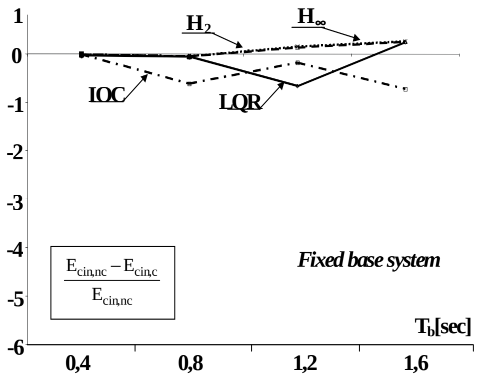

Fig 9: Main structure kinetic energy gain spectra (worst case)

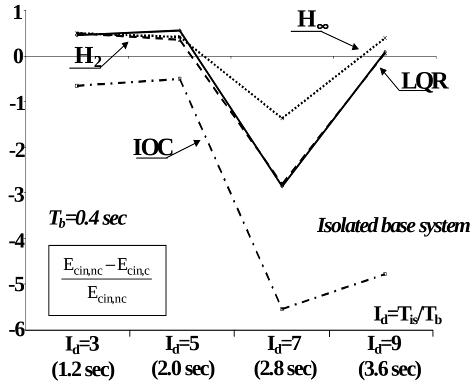

Fig 10: Main structure kinetic energy gain spectra (worst case)

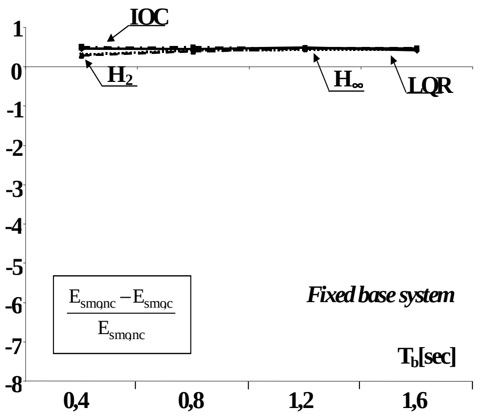

Fig 11: Main structure damping energy gain spectra (worst case)

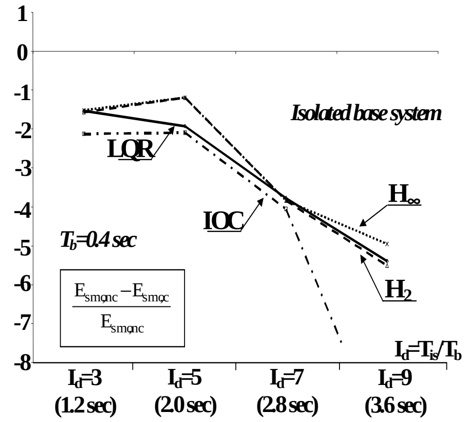

Fig 12: Main structure damping energy gain spectra (worst case)

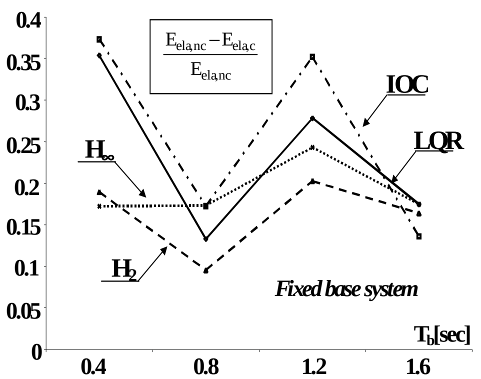

Fig 13: Main structure elastic energy gain spectra (worst case)

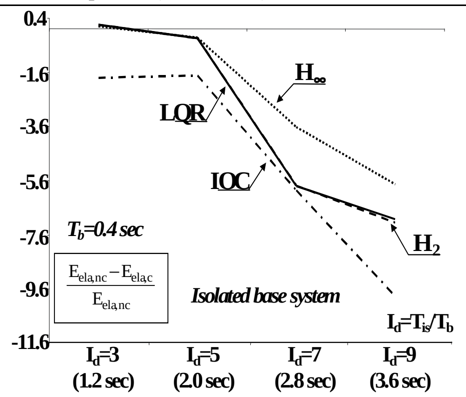

Fig 14: Main structure elastic energy gain spectra (worst case)

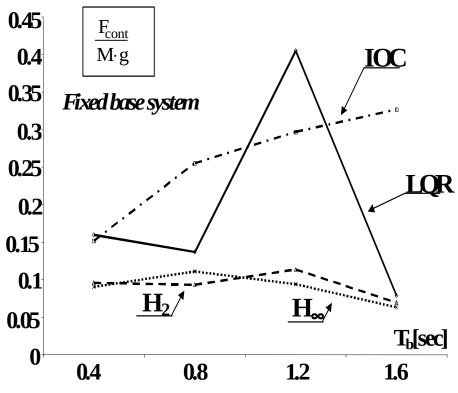

Fig 15: Control force spectra (worst case)

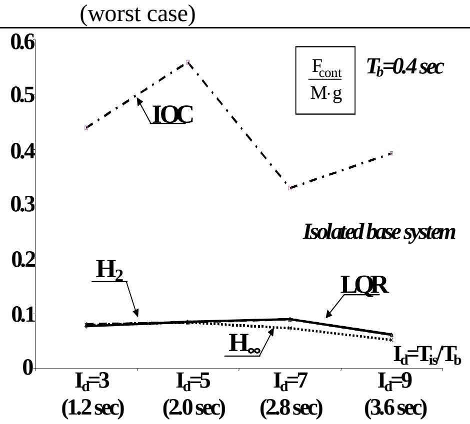

Fig 16: Control force spectra (worst case)

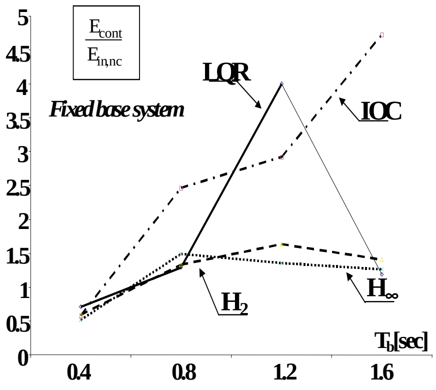

Fig 17: Control energy spectra
(worst case)

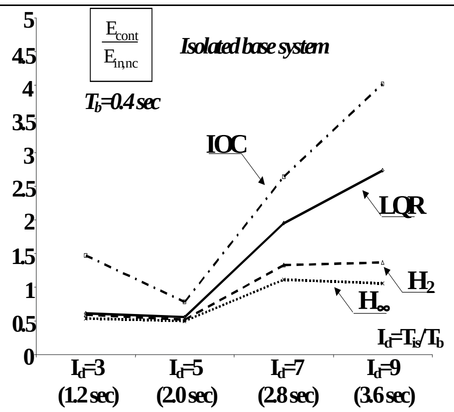

Fig 18: Control energy spectra
(worst case)

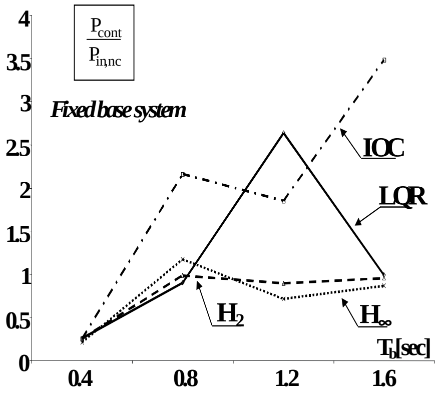

Fig 19: Control power spectra
(worst case)

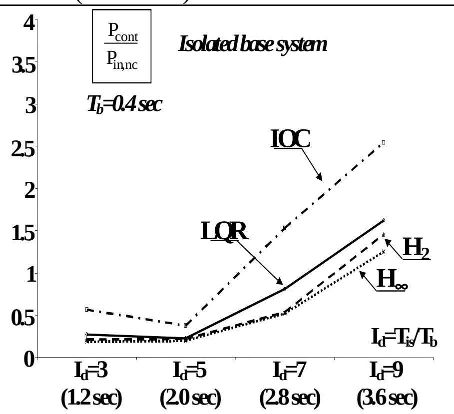

Fig 20: Control power spectra
(worst case)

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