FREQUENCY-SHAPING METHOD TO OPTIMALLY CONTROL BASE ISOLATED SYSTEMS EQUIPPED WITH HYBRID MASS DAMPERS

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A new structural system conceived to control Base Isolated System vibrations under seismic excitations is analyzed. The new system combines the Hybrid Mass Damping strategy with the Isolation concept in order to obtain a new system which principally attenuates the effects of the seismic excitation components with frequencies close to the fundamental natural vibration one. The aim is achieved by using a control algorithm based on the frequency shaping technique. Comparisons of system performance show that under the considered processes the control power required to achieve similar performances using control algorithm in the frequency domain is less than using algorithm in the time domain.

1 Introduction

The mass damping technique has been applied mainly in Japan and the United States for the vibration reduction in tall buildings due to strong wind and moderate seismic activity. The efficiency of TMD (Tuned Mass Damper) for the reduction of response of systems with several degrees of freedom, subject to strong seismic excitations, is considered substantially low[1,4]. A significant improvement of building vibration control capacity using mass damping was obtained with the introduction of active control criteria first proposed at the AIJ Confernce by T. Kobori[5]. In 1994, the authors proposed a new system derived from the combination of Base Isolation System (BIS) and mass damping strategy[6]. Preliminary tests have shown that the efficiency of this system (BIS&TMD) in the amplitude reduction of seismic motion is significant when compared to the case without TMD. The positive behaviour is due to the appropriate combination of three fundamental properties of the combined system : 1) the reduction of the ground motion transmission to the superstructure; 2) the vibration mode modification due to the isolation; 3) the first vibration mode reduction by means of the damping mass tuned to this frequency.

However, the system presents some difficulty in tuning the TMD optimal parameters to a natural vibration frequency, variable with the excitation intensity, caused by non-linearity of the isolation system behaviour. In this study, a new hybrid system, conceived by the authors, deriving from a combination of Base Isolation, Mass Damping and Active Control strategies is investigated. Hybrid Mass Dampers-HMD. For an isolated system the aim of an Active Control should reduce the absolute accelaration and the relative displacements. Since the typical frequency response of Base Isolation presents amplitude peaks in a narrow frequency range, the frequency-shaping method is proposed as a control algorithm for these systems.

A frequency-shaped performance index has been introduced using weighting matrices defined in the frequency domain. The applied technique allows the designer to simultaneously control some state variables with frequency specifications compared to the well known time performance index.

2 The model

Figure 1 shows the model of a base isolated structure equipped with a hybrid mass damper located at the base. The superstructure is considered to be linearly elastic because designed as so. The isolation system is also considered as an equivalent linear system according to some equivalence criteria[2,3]:

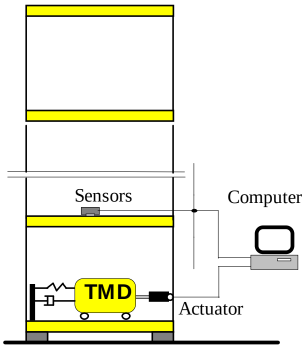

Fig. 1 : The BIS and HMD model

The motion of system model, Assuming the state variable $\mathbf{x} = [\mathbf{y}, \dot{\mathbf{y}}]^T$, is described by the following

$$\begin{bmatrix} \dot{\mathbf{y}} \\ \ddot{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{O} \\ \mathbf{b}_g \end{bmatrix} \ddot{u}_g + \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{b}_u \end{bmatrix} f_u \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ respectively represent mass, damping and stiffness matrices of the model, u_g and f_u are the ground displacement and the control force acting on the TMD.

3 Shaping the Dynamic Response by Frequency Specifications

The design of the feedback controller, assumed to be linearly dependent on the state, is based on the minimization of the following index in the frequency domain[7]

$$J = \frac{1}{4\cdot\pi}\int_{-\infty}^{+\infty}\left[\mathbf{X}^*(j\cdot\omega),\mathbf{U}^*(j\cdot\omega)\right]\cdot\begin{bmatrix}\mathbf{Q}(j\cdot\omega) & \mathbf{O}\\ \mathbf{O} & \mathbf{R}(j\cdot\omega)\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}(j\cdot\omega)\\ \mathbf{U}(j\cdot\omega)\end{bmatrix}d\omega \quad (4)$$

where $\mathbf{X}(\omega)$ and $\mathbf{U}(\omega)$ represent the Fourier transform of the state variable and control force, and where $*$ denotes the complex conjugate transpose of the matrices.

In this performance index defined by the (4) two terms contribute to the integrated function control: the quadratic forms $\mathbf{X}^*(j\omega)$ $\mathbf{Q}(j\omega)$ $\mathbf{X}(j\omega)$ and term $\mathbf{U}^*(j\omega)$ $\mathbf{R}$ $(j\omega)$ $\mathbf{U}$ $(j\omega)$ which, respectively, penalize the deviation of the state $\mathbf{X}(j\omega)$ from the origin and the control cost. Weighting matrices $\mathbf{Q}(j\cdot\omega)\mathbf{Q}(j\cdot\omega)^* \geq 0$ e $\mathbf{R}(j\cdot\omega)\mathbf{R}(j\cdot\omega)^* > 0$ are diagonal real functions of complex variables. Minimization of the function (4) may be achieved by transforming the problem in an equivalent form written in the time domain.

Let's consider the matrix factorization of $\mathbf{Q}(j\cdot\omega)$ and $\mathbf{R}(j\cdot\omega)$ matrices as:

$$\mathbf{Q}(j\cdot\omega) = \mathbf{P}_Q^*(j\omega)\cdot\mathbf{P}_Q(j\omega) \qquad \mathbf{R}(j\cdot\omega) = \mathbf{P}_R^*(j\omega)\cdot\mathbf{P}_R(j\omega) \quad (5)$$

where matrix $\mathbf{P}_R(j\omega)$ has $r\times r$ dimensions, while matrix $\mathbf{P}_Q(j\omega)$ has $\sigma\times n$ dimensions, with σ the $\mathbf{Q}(j\cdot\omega)$ matrix rank. Let's introduce the two $\mathbf{x}_p(t)$ and $\mathbf{u}_p(t)$ vectors chosen so that the Fourier transform is given by:

$$\mathbf{X}_P(j\cdot\omega) = \mathbf{P}_Q(j\cdot\omega)\cdot\mathbf{X}(j\cdot\omega) \qquad \mathbf{U}_P(j\cdot\omega) = \mathbf{P}_R(j\cdot\omega)\cdot\mathbf{U}(j\cdot\omega) \quad (6)$$

The performance index (4) can be rewritten as:

$$J = \frac{1}{4\cdot\pi}\int_{-\infty}^{+\infty}\left[\mathbf{X}_P^*(j\cdot\omega),\mathbf{U}_P^*(j\cdot\omega)\right]\cdot\begin{bmatrix}\mathbf{I} & \mathbf{O}\\ \mathbf{O} & \mathbf{I}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}_p(j\cdot\omega)\\ \mathbf{U}_P(j\cdot\omega)\end{bmatrix}d\omega \quad (7)$$

In (7) weighting matrices are constant in the frequency domain. Using Parseval's theorem the performance index can be written as:

$$J = \int_{0}^{+\infty}\left[\mathbf{x}_P^T(t),\mathbf{u}_P^T(t)\right]\cdot\begin{bmatrix}\mathbf{x}_P(t)\\ \mathbf{u}_P(t)\end{bmatrix}dt \quad (8)$$

Therefore the optimum gain problem is expressed in terms of another set of state variables $\mathbf{x}_p(t)$ and $\mathbf{u}_p(t)$, which consider the penalties described above in the frequency domain.

From (6), the new variables $\mathbf{x}_p(t)$ and $\mathbf{u}_p(t)$ can be considered as output of two new auxiliary systems:

$$\mathbf{X}_P(j\cdot\omega) = \mathbf{P}_Q(j\cdot\omega)\cdot\mathbf{X}(j\cdot\omega) \quad <> \quad \begin{cases} \dot{\mathbf{z}}_Q(t) = \mathbf{A}_Q\cdot\mathbf{z}_Q(t) + \mathbf{B}_Q\cdot\mathbf{x}(t) \\ \mathbf{x}_P(t) = \mathbf{C}_Q\cdot\mathbf{z}_Q(t) + \mathbf{D}_Q\cdot\mathbf{x}(t) \end{cases} \tag{9}$$

$$\mathbf{U}_P(j\cdot\omega) = \mathbf{P}_R(j\cdot\omega)\cdot\mathbf{U}(j\cdot\omega) \quad <> \quad \begin{cases} \dot{\mathbf{z}}_R(t) = \mathbf{A}_R\cdot\mathbf{z}_R(t) + \mathbf{B}_R\cdot\mathbf{u}(t) \\ \mathbf{u}_P(t) = \mathbf{C}_R\cdot\mathbf{z}_R(t) + \mathbf{D}_R\cdot\mathbf{u}(t) \end{cases} \tag{10}$$

Therefore it is possible to reformulate the original dynamic problem by using a new state vector describing both the real and auxiliary models (9) and(10), given by:

$$\dot{\mathbf{x}}_A(t) = \mathbf{A}_A\cdot\mathbf{x}(t) + \mathbf{B}_A\cdot\mathbf{u}(t) \tag{11}$$

where the new set of matrices are

$$\mathbf{x}_A = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{z}_Q(t) \\ \mathbf{z}_R(t) \end{bmatrix}; \qquad \mathbf{A}_A = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_Q & \mathbf{A}_Q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_R \end{bmatrix}; \quad \mathbf{B}_A = \begin{bmatrix} \mathbf{B} \\ \mathbf{O} \\ \mathbf{B}_R \end{bmatrix} \tag{12}$$

Considering (11), performance index (8) can be rewritten as:

$$J = \int_0^\infty \left[\mathbf{x}_A{}^T(t), \mathbf{u}^T(t)\right]\cdot\begin{bmatrix} \mathbf{Q}_A & \mathbf{N} \\ \mathbf{N}^T & \mathbf{R}_A \end{bmatrix}\cdot\begin{bmatrix} \mathbf{x}_A(t) \\ \mathbf{u}(t) \end{bmatrix} dt \tag{13}$$

where

$$\mathbf{Q}_A = \begin{bmatrix} \mathbf{D}_Q^T\cdot\mathbf{D}_Q & \mathbf{D}_Q^T\cdot\mathbf{C}_Q & \mathbf{0} \\ \mathbf{C}_Q^T\cdot\mathbf{D}_Q & \mathbf{C}_Q^T\cdot\mathbf{C}_Q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_R^T\cdot\mathbf{C}_R \end{bmatrix} \quad \mathbf{N} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{C}_R^T\cdot\mathbf{D}_R \end{bmatrix} \quad \mathbf{R}_A = \mathbf{D}_R^T\cdot\mathbf{D}_R \tag{14}$$

The new formulation is correct only if $\mathbf{Q}(j\cdot\omega)$ ed $\mathbf{R}(j\cdot\omega)$ matrices can be factorized as in (5) and the terms of $\mathbf{P}_Q(j\omega)$ and $\mathbf{P}_R(j\omega)$ matrices are proper or strictly proper rational functions of $j\omega$. In order for the auxiliary problem to have a solution it needs the condition $\mathbf{R}_A > \mathbf{0}$, and that the new system should be stable and observable. The new system block diagram is represented in figure 2.

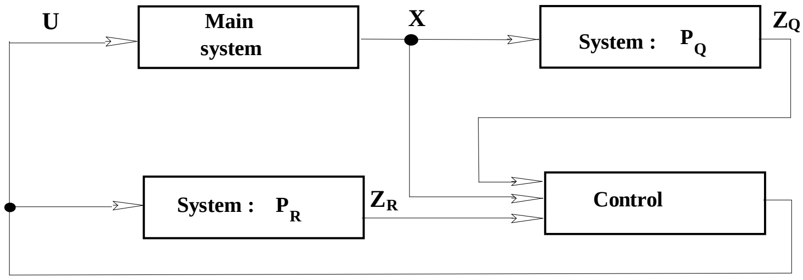

Fig. 2 : Block Diagram of Control Algorithm

Figure 2 shows how this control algorithm may penalize state variables both in time and in the frequency domain.

Let's consider that $\mu < n$ state variables have to be controlled in the frequency domain with the aim of suppressing the response in a narrow frequency band around ω_i. In this case the correspondent weighting matrix **Q** should be expressed by

$$\mathbf{Q}_{n\times n}(j\omega) = \mathrm{diag}\left\{\mathbf{Q}'_{\mu\times\mu}(j\omega), \mathbf{0}_{n-\mu\times n-\mu}\right\}$$
$$\mathbf{Q}'\left(j\cdot\omega\right)_{\mu\times\mu} = \mathrm{diag}\left\{\frac{a_i^2}{\left(\omega_i^2 - \omega^2\right)^2};\ i = 1,\dots,\mu\right\} \tag{15}$$

which can be factorized using

$$\mathbf{P}_Q(j\omega) = \begin{bmatrix} \mathbf{P}'_Q(j\omega) & \mathbf{0}_{\mu\times n-\mu} \\ \mathbf{0}_{\sigma-\mu\times\mu} & \mathbf{0}_{\sigma-\mu\times n-\mu} \end{bmatrix}$$
$$\mathbf{P}'_Q\left(j\cdot\omega\right) = \mathrm{diag}\left\{\frac{a_i}{\omega_i^2 - \left(j\cdot\omega\right)^2};\ i = 1,\dots,\mu\right\} \tag{16}$$

Note that the generic diagonal term $\frac{a_i}{\omega_i^2 - \left(j\cdot\omega\right)^2}$ represents the response of a simple degree dumpless oscillator. Matrix $\mathbf{P}'_Q$ can be considered as the transfer function of a set of μ simple oscillators. This particular system can be expressed in the state-space by

$$\mathbf{A}_{Q(\eta\times\eta)} = \begin{bmatrix} \mathbf{0}_{(\mu\times\mu)} & \mathbf{I}_{(\mu\times\mu)} \\ \mathrm{diag}\left\{-\omega_i^2, \mathrm{i}=1,..,\mu\right\} & \mathbf{0}_{(\mu\times\mu)} \end{bmatrix} \qquad \mathbf{B}_{Q(\eta\times n)} = \begin{bmatrix} \mathbf{0}_{(\mu\times\mu)} & \mathbf{0}_{(\mu\times n\text{-}\mu)} \\ \mathbf{I}_{(\mu\times\mu)} & \mathbf{0}_{(\mu\times n\text{-}\mu)} \end{bmatrix}$$

$$\mathbf{C}_{Q(\sigma\times\eta)} = \begin{bmatrix} \mathrm{diag}\left\{a_i, \mathrm{i}=1,..,\mu\right\} & \mathbf{0}_{(\mu\times\mu)} \\ \mathbf{0}_{(\sigma\text{-}\mu\times\mu)} & \mathbf{0}_{(\sigma\text{-}\mu\times\mu)} \end{bmatrix} \quad \mathbf{D}_{Q(\sigma\times n)} = \begin{bmatrix} \mathbf{0}_{(\mu\times\mu)} & \mathbf{0}_{(\mu\times n\text{-}\mu)} \\ \mathbf{0}_{(\sigma\text{-}\mu\times\mu)} & \mathbf{0}_{(\sigma-\mu\times n\text{-}\mu)} \end{bmatrix} \qquad (17)$$

In (10), an algebraic problem for these particular cases, the only non-null matrix $\mathbf{D}_R$ has to be set by considering that $\mathbf{R}_A = \mathbf{D}_R^T \cdot \mathbf{D}_R = \mathbf{R}$.

Therefore, problem (11) is characterized by a null dimension vector $\mathbf{z}_R$, a weighting matrix $\mathbf{Q}_{A(n+\eta\times n+\eta)} = \left\{\mathbf{0}_\mu, \mathbf{0}_{n-\mu}, \mathrm{diag}(a_i^2, \mathrm{i}=1..\mu), \mathbf{0}_\mu\right\}$, $\mathbf{N}_{n+\eta\times r} = \mathbf{0}$ and $\mathbf{R}_{A(r\times r)} = \mathbf{R}$. The first μ values in $\mathbf{Q}_A$ represent null penalty in correspondence of frequency penalized main state variables, the next $n-\mu$ values are penalties for main state variables not penalized. The other 2μ values in $\mathbf{Q}_A$ are the weighting in the frequency domain for main state variables.

4 Results

Seismic responses of models controlled by the frequency-shaping algorithm have been obtained using numerical simulations. The seismic response of the same models controlled by using the LQR algorithm in the time domain has also been tested in order to make comparisons. Control forces, energies and powers are also evaluated for both control algorithms in the time and frequency domain.

Figures 3, 4 and 5 show response comparisons of systems controlled by time and frequency-shaping algorithm under the El Centro N-S (1940) earthquake.

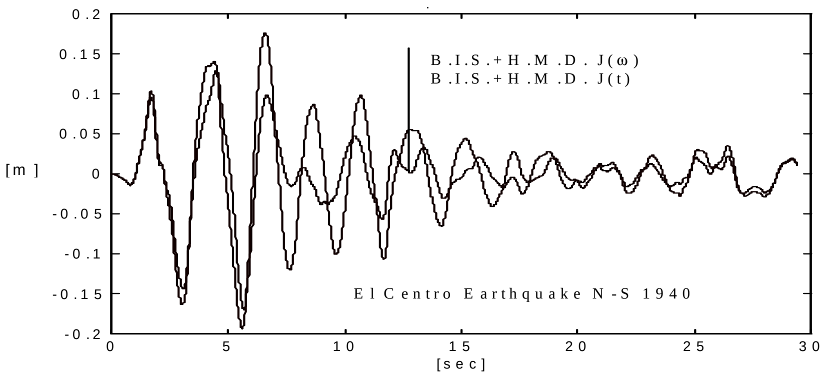

Fig. 3 : Base Relative Displacement Time Histories

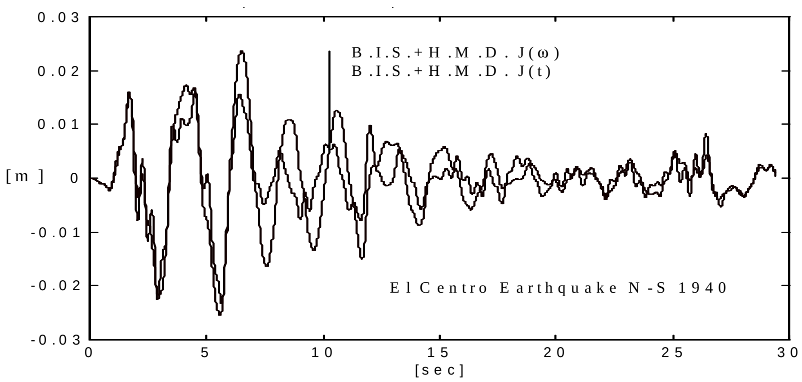

Fig. 4 : Top Relative Displacement Time Histories

In particular figg. 3-4 show base and top relative displacements, while fig. 5 shows the tuned mass damper response.

With the frequency shaping technique the performance index penalizes the response around the fundamental natural damped frequency. Two methods have been compared designing control systems to have similar performance in the structure to investigate the energy, power and force required by the control system.

Figures 6-8 show control force, power and energy time histories for the two control methodologies. Results show that control forces, energy and power required by the frequency shaping method are strongly lower than the ones required by the optimum control in the time domain.

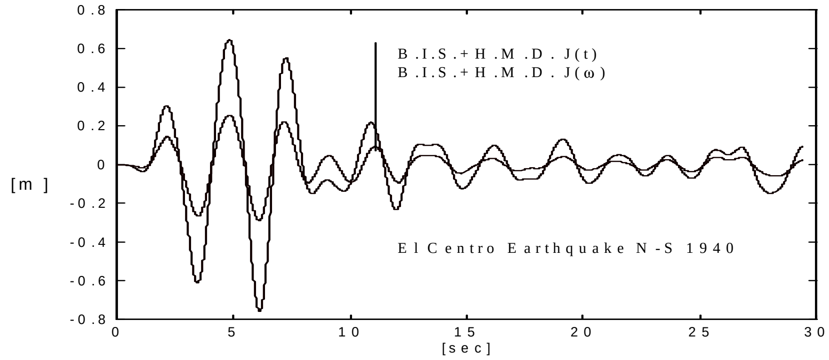

Fig. 5 : TMD Relative Displacement Time Histories

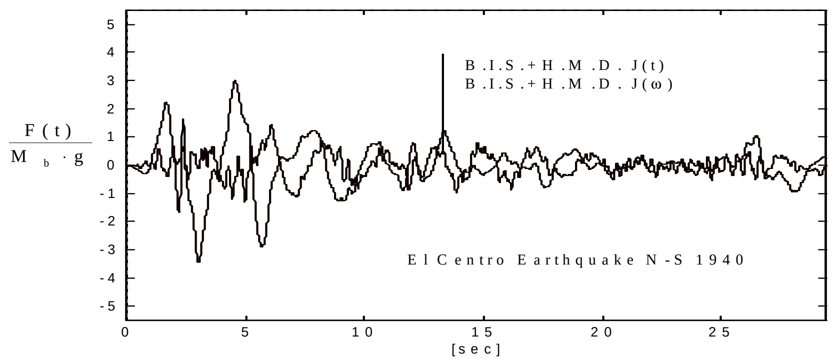

Fig. 6 : Control Force Time Histories

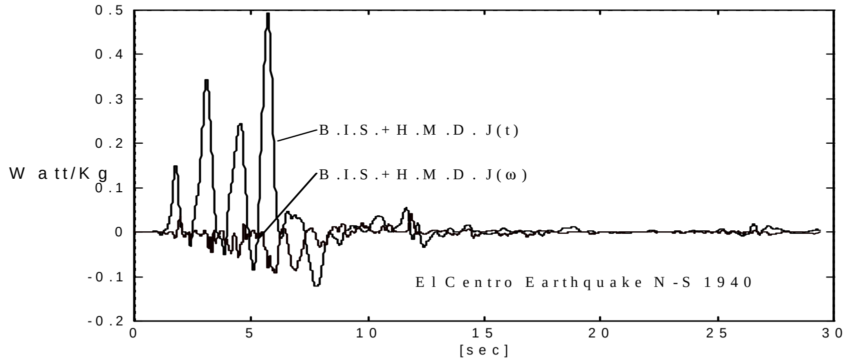

Fig. 7 : Control Power time histories

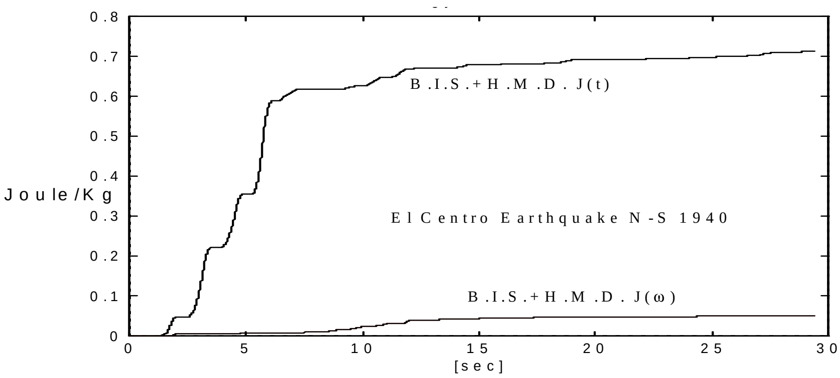

Fig. 8 : Control Energy Time Histories

Figure 9 shows the comparison in the frequency domain of control force for the two considered algorithms. The control power spectrum shows that the power is maximally used to control the assigned frequency band in the frequency-shaping control algorithm.

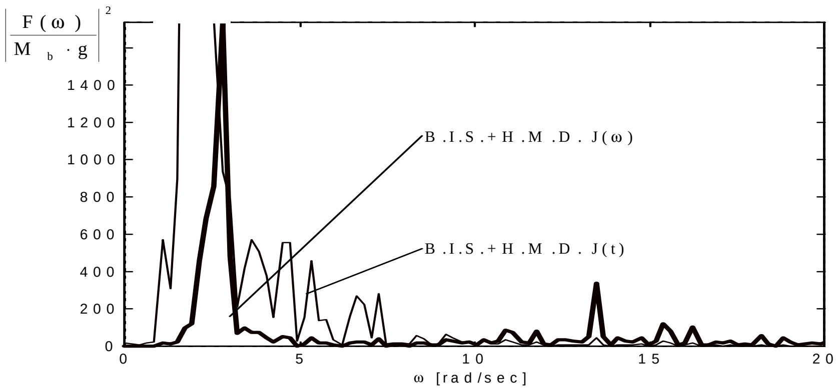

Fig. 9 : Control Force Time Histories

5. Conclusion

This paper describes an active control algorithm based on the frequency shaping control techniques applied to a new hybrid system based on the combination of the isolation strategy with active mass damping.

The performance of the new control method has been verified by numerical simulation in comparison with the response of the same system optimally controlled in the time domain. Responses of the hybrid systems, controlled by the two different methods, have shown that better performance may be obtained using the frequency-shaping techniques which require less control resources.

6 References

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