New CAD/CAM process: an elaboration of the geometrical matrices of rosette

M. Giordano [(a)], S. Barba [(a)], L. A. Lopresti [(b)], G. H. Defranco [(b)]

[(a)] University of Salerno, Italy
[(b)] National University of La Plata, Argentina

Article Information

Keywords:
Modeling,
3D creation,
Milling software,
Prototyping,
Tracery.

Corresponding author:
Salvatore Barba
Tel.: +39 089 96 4062
Fax.: +39 089 96 4343
e-mail: sbarba@unisa.it
Address:
Università degli Studi di Salerno
Facoltà di Ingegneria
via Ponte don Melillo
84084 Fisciano (SA), Italia

Abstract

The research, developed under the project of International Interuniversity Cooperation "Laboratory DRAWING AND STRUCTURAL ANALYSIS", was applied to the study of decorative composition of uniplanar geometrical matrices, with the aim of identifying a key to understanding the structure of the work leading to a procedure coded for successive reproduction with automated systems.

In the development of possible combinations of basic geometric shapes in the plane, the problem of replica is always present (for example, restoration actions), as the repeatability of a single part - foundation of his own existence - gives rise to the entire composition. It has been sought, therefore, from a careful analysis of the laws of symmetry that dictate the generative logic of the ornamental design for a precise canonization of themes and motifs, to identify the geometric patterns, allowing the rational control of the decoration through the identification of iteration processes in series. The detail is not returned simply as it appears, but the theoretical decomposition of the work helps to find, in the subsequent reconstruction graphics, the nature and location of each item: a critical perspective that leads to a correct and realistic reproduction. The formal models tend to become so in geometric models, symbols of a deeper structure, which can be grasped with the help of topological patterns. And so, knowing the harmonic law that governs the entire composition, as defined by the rigid motion of a default form on a flat reticle, the tessellation can easily be reproduced starting from one unit as an entity that can be, with the use of machines numerical control, repeated mechanically.

The possibilities offered by digital modeling and parametric development applications that allow the decoration to be reproduced as well as to be deformed have been experimented and tested. These applications create new wrapping surfaces of the ornamental theme. The procedure has been applied to case studies appropriate to analyze the principles of repeatability in series and of laboratory operations. This is possible due to the re-design and digital production tools with numerical control technology, to codify the technical reproducibility, immaterial and material, of the architectural decorations.

1 Introduction

«Beauty is closely associated with the symmetry» (Hermann Klaus Hugo Weyl).

Symmetry has been source of inspiration for many artists, including Leonardo da Vinci.

The first conceptions of symmetry in the architecture are identified with proportion and beauty. This search of balance is reflected not only in the design of large building structures but also in its components - such as columns and openings - and in especially in decorations. In modern architecture, this concept is dissociated from the theory of proportion and is bound equal parts of the same figure. This is how a figure can be defined symmetric: if equal parts that compose it are transformed into another one being, the figure unchanged. This makes possible the development of the concept of symmetry in the mathematical sense [1].

The time symmetry can be found in the following architectural decorative elements: friezes, mosaics and rosette. In the friezes, the composition is given by the direction of a single vector and it is an infinite group in which seven different possible combinations of the base element or module.

In the mosaics, also called plane crystallographic groups, there are 17 possible combinations and composition is given by the management of more than one vector not collinear [2].

In the rose windows, there are two groups:

- cyclic, generated by a rotation through $2\pi/n$;
- dihedral, generated by rotation and reflection of the base element or module [3].

Each group has a motif that makes up a module, simple or complex, where develops move onto a grid, generating the entire composition.

The motif is the simple element that composes it (triangle, square, circle, etc.); the module is the element consisting of the motif, where circle, square, triangle, etc. are repeated and developed.

The grid, which moves on the module, is a network of horizontal lines, vertical or diagonal according to the modular structure.

The module on the grid takes any of the following transformations or combinations: identity, translation, rotation around a point, reflection (or simply symmetry) on a line and glide reflection [4].

2 Method and result. Geometry of rosette

The rosette here presented is in Salerno's Cathedral of Matthew the Apostle, Italy.
The church was built between 1080-1085; it has a Latin cross plan and an external colonnade whose columns are topped by round arches decorated with inlaid pilaster strip of volcanic rock. Columns on the docks and superb examples of rosette, yellow and different black tuff, give more vitality to the whole.

The rosette selected belongs to the dihedral group and has twelve lines of symmetry. This determines a grid where it is possible to see two modules (fig. 1 and fig. 2).

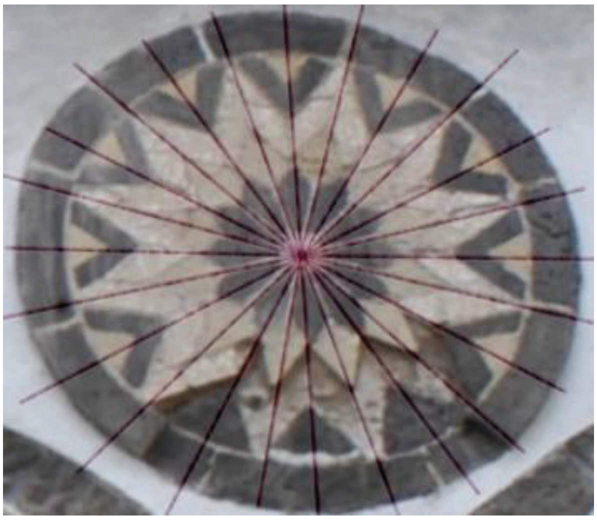


Fig. 1 Rosette of Salerno´s Cathedral with symmetry axis.

Using the tool McNeel Rhinoceros 4.0[1] to rebuild the module, considering only one of them, it has been chosen the one in red. Next, detailed as obtained in fig. 2 and consequently fig. 5.

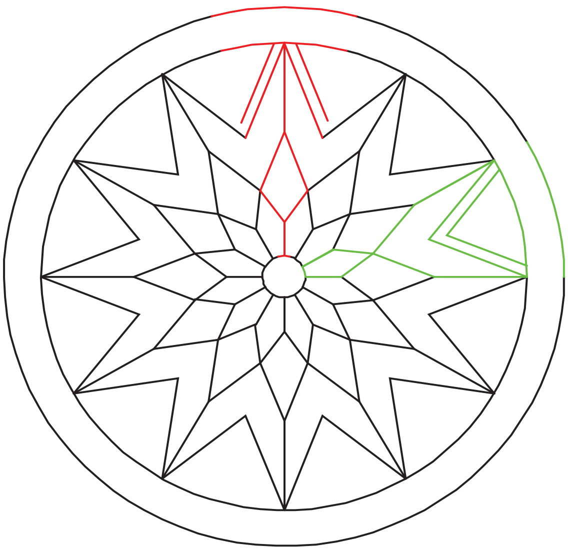


Fig. 2 Schematically, two possible modules of rosette.

In the reconstruction of the module, the gold ratio has been found to use it in the design of rosette. In each segment are the radii of the circles of rosette remains, considering the golden ratio: *Ø ≈ 1.6180339887.*

The proportions as shown in eq. 1:

$$\overline{Ox}/\overline{OE} = \overline{OC}/\overline{Ox} = \overline{OB}/\overline{OC} = \overline{OA}/\overline{OB} \approx 1.618 \quad (1)$$

note that the circle whose radius is Ox, is not used constructively in the rosette.

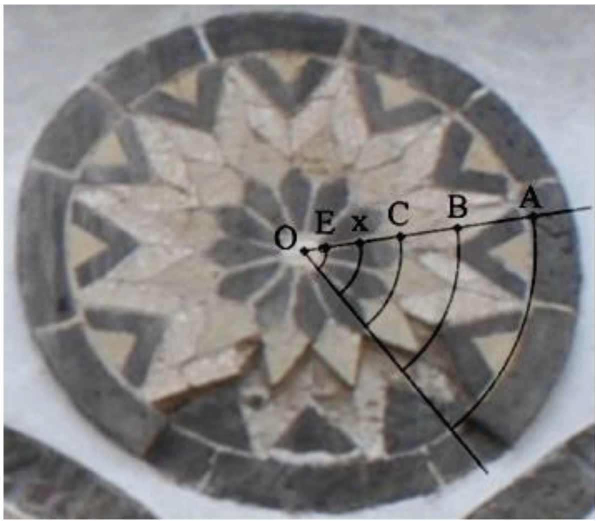


Fig. 3 Golden ratio of rosette.

Given this scheme of proportions (fig. 3) and the grid (fig. 1), reconstruct the V-shaped motif with 72° angle resting on the circle of radius OA. This angle is determined by the relationship between the golden ratio and the study of the Pentagon [5]. In it the triangle aureus, has internal angles of 36°, 72° and 72°. The reason is then repeated in the circle of radius OB.

In fig. 4 is shown the circles of radius $\overline{OA}, \overline{OB}, \overline{OC}, \overline{OE}$.

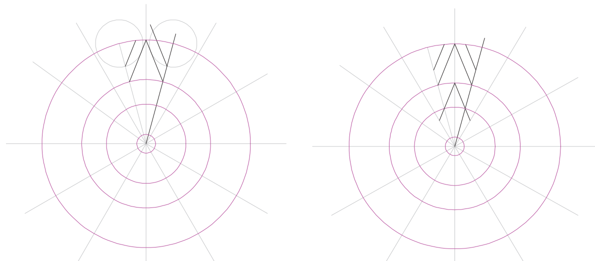


Fig. 4 Motif and module of rosette.

The entire composition is generated by the operation of "Serie polare" which specifies the center of rosette as a center of rotation, it is determined that there are 12 items (includes a base module) and is distributed through 360°. This distributes the module into the grid.

The motive, the module, the grid and transformation by rotation is built geometrical matrix of rosette (fig. 5).

[1] *Please note that we are using the Italian version of the Rhinoceros 4.0SR8, that is why the names of the tools are in Italian.*

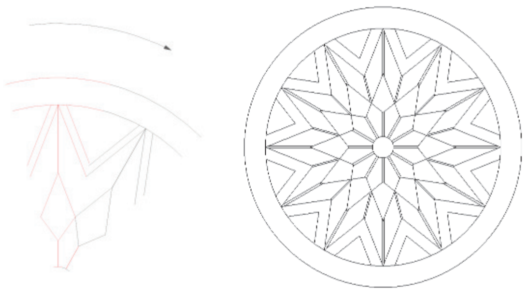
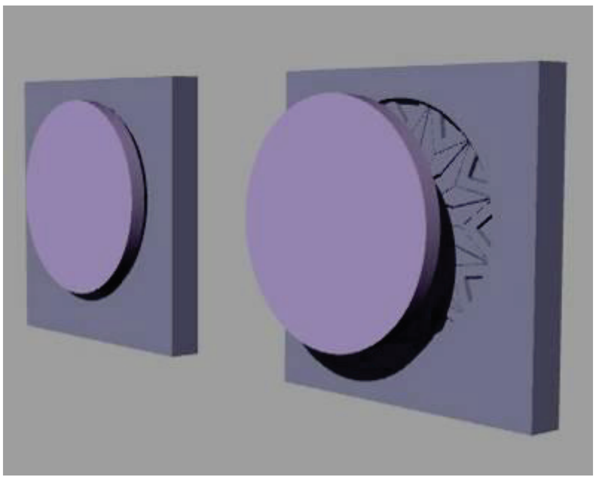


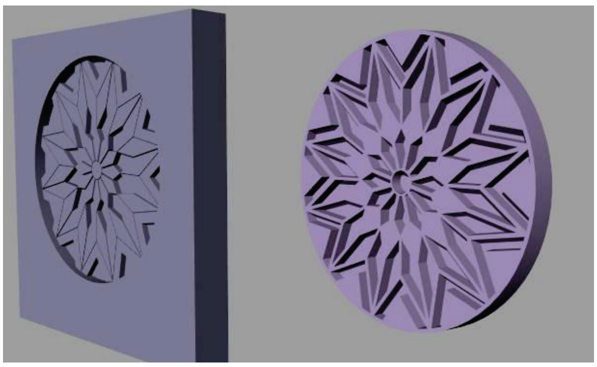
Fig. 5 Modulo, partial and total rotation on the grid.

2.1 Adequacy of rosette for mass production

As an alternative process to manufacture the rosette real, is it possible to adjust the digital model. This procedure is done considering all aspects, including color and form. It is composed of three colors determined by three materials. That is why we consider two alternative productions:

- rosette: reproduce the piece in a milling machine with computer numerical control (CNC) considering the use of low relief to color later, either by casting a colored material or simply the application of paint color;
- rosette mould: building the mould in the CNC milling machine to perform, followed by a casting. This process consists of two stages, the first one is using the mould (fig. 6 and fig. 14) for casting resin or plaster, the following is used the product obtained from the first casting as a mould. This can result in a casting of two or three colors and, if desired, of different materials.





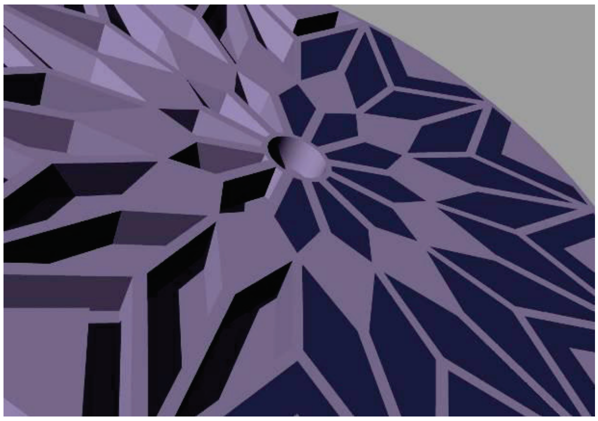


Fig. 6 Mould, part a result of the casting and second casting.

Another consideration is related to the use of digital model of rosette to generate the digital model of the array.

The vertical sides of the digital model, not vertical, have a small angle that makes it easy, and often occurs, the stripping of the part of the mould and prevents breakage.

To modeling rosette and the mould, have been considered the adjustments mentioned above, to use the 2D scheme presented in fig. 5.

2.2 3D Digital models

In the modeling rosette, we will continue to use the CAD software *Rhinoceros 4.0* presented earlier, but now this software is a model of polygons and Non-Uniform Rational B-Splines.

NURBS are mathematical representations of 3-D geometry in a way that accurately described. This software can create, edit, analyze, document, render, animate, and translate NURBS curves, surfaces, and solids with no limits on complexity, degree, or size and can supports polygon meshes and point clouds.

As a modeling, strategy divides the model into NURBS and flat surfaces set up a closed polysurface with 868 surfaces. Then, details the process.

Based on the 2D scheme (fig. 5) an area larger than the outline of rosette is drawn, indicating four-point “Superficie da 3 o 4 vertici”. In addition, is placed behind the scheme to “Sposta” (fig. 7).

This is done to obtain an area bounded by the outline drawn. It is a quick way to obtain, from a 2D drawing, a surface.

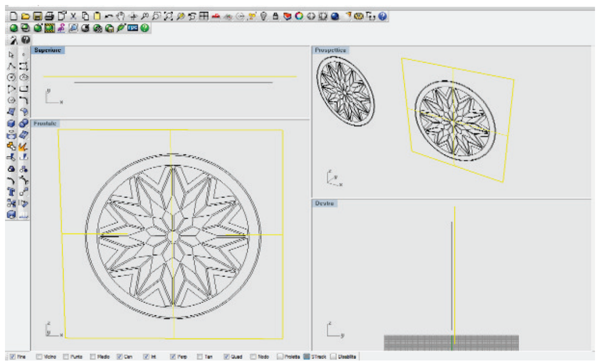


Fig. 7 Base schema and flat square back.

Proceeds to extrude the scheme with “Estrusione lineare” (fig. 8) until they exceed the previously created surface.

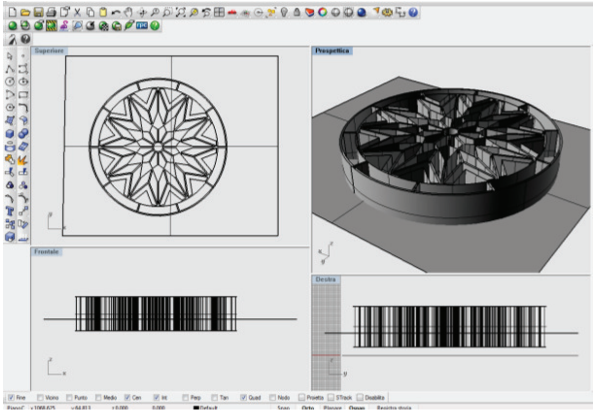


Fig. 8 Extrusion of the base scheme.

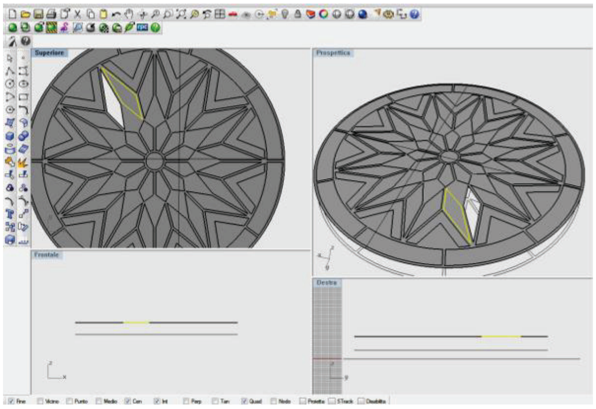


Fig. 9 Subdivision and partial removal of excess surface.

In addition, it has divided the square surface with the surface created by extrusion. Thereafter, surpluses are eliminated (fig. 9).

Work continues, only with the surface to set the main body of rosette (fig. 10).

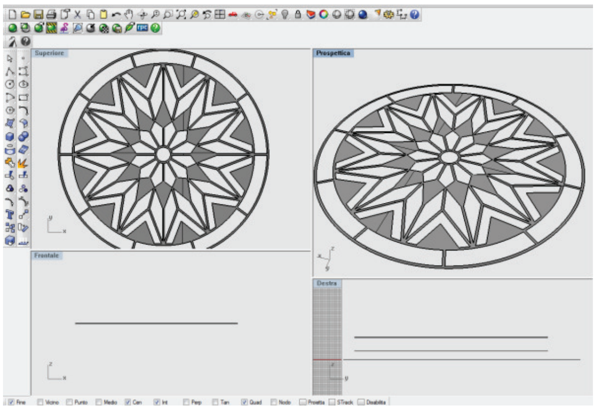


Fig. 10 Base surface for the body of rosette.

This surface gives volume to "Estrudi superficie rastremata" considering an angle of 3 degrees (fig. 11).

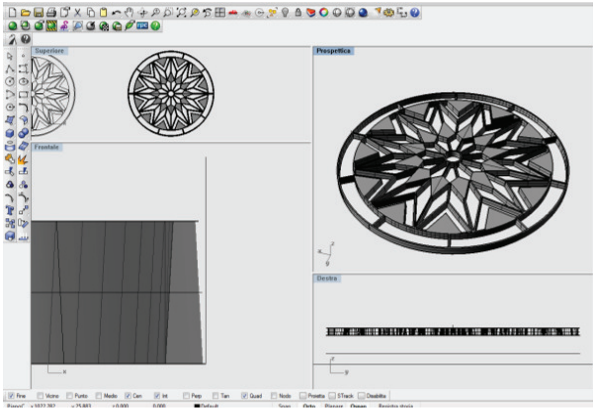


Fig. 11 Extruding volume with angle.

Thickness is increased to give more body to the model, without considering the original scheme rosette but simply taking a circular area at the base of the model and applying once more, "Estrudi superficie rastremata" respecting the same angle (fig.12).

With "Unione booleana" of lower and higher volumes is obtained the rosette body (fig. 13); the finished model of rosette proceed to create the digital model of the mould.

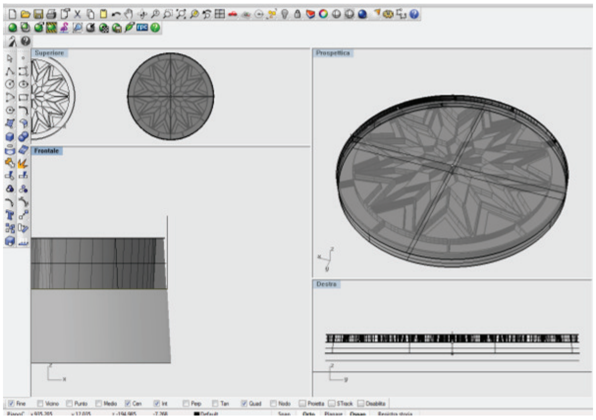


Fig. 12 Second volume extruded angle not straight.

It creates a volume and is removed to "Differenza booleana" model rosette and get a block with the cavity. In fig. 14 can be seen in the foreground, a mould cut and the entire cast.

Fig. 13 Mostly low relief, partly colored surface.

Is extracted from the CAD, STL (Stereolithography format or interface Standard Triangulation Language)[2] format, two digital models: one of rosette, and other mould to work in the CAM. The rosette this scale 1:2 (300 mm) and the mould in 1:5 (120 mm). The tolerance used to export the models is about 0.05 mm. This means that is the maximum distance between the mesh generated and object modeling *Rhinoceros*.

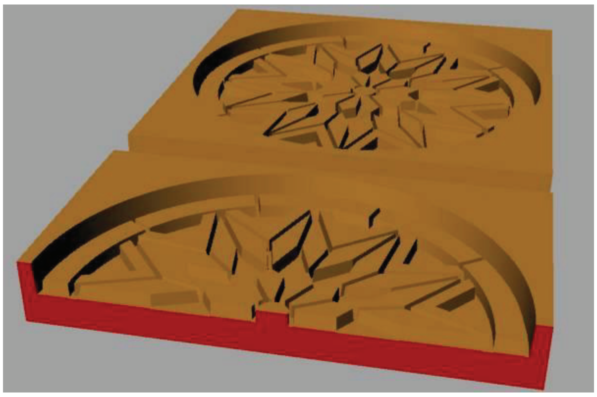


Fig. 14 Final model mould rosette.

2.3 Rosette CAM and mould CAM

The Computer Aided Manufacturing (CAM) is the complement of CAD works best on a project. The CAM allows understanding what happens with the geometric model and translating the language of a CNC machine. The CAM software used is the Abacus company, called *Mayka Expert 7.0*.

This software offers two methods of milling: working in 2D contours extracted from the digital model, or drawn and working in 3D on the STL model.

Contours work involves creating a linear path (straight or curved) where the Z is unchanged. This course may be repeated by varying the Z coordinate manually but always the X and Y will follow the contour established.

Working in 3D is to reproduce the same digital model in the milling machine; the entire surface is transported to the block used as a starting point. This is the method adopted for the experiment and the next step is to determine the best routing strategy used. This requires a set of guidelines to help develop a better-machined surface finish. The guidelines are:

- **□or□ □□pe**: set the strategy used in the tool path may be: Zigzag, horizontal Face, Face not horizontal, constant Z, Outline, Drilling, Work surface, etc.
- **□odel**: defines the model on which it is to perform machining. A scene may contain several models working STL.
- **Area**: is the law that governs the movement of the tool. This can be based on one site in a radial (Polar) or along the x - y (Cartesian).
- **□□pe o□tool**: define the typology of tool use, both its dimensions (diameter and length) and shape (spherical, cylindrical, conical, etc.).
- **□□pe o□ operation**: the machining can be done roughing over-thickness considering a digital model (0.5 mm), or finishing, which considers the final dimensions of the digital model.

Next tab. 1 outlined the stages of milling machine used for the rosette.

Work Type	Model	Area	Type of tool	Type of operation
Zigzag	rosone.stl	Cartesian	Cyl. ø6mm	Rough down
Zigzag	rosone.stl	Polar	Sph ø2mm	Finish
contour	Contour		Cyl. ø3mm	Cut

Tab. 1 Definition of milling parameters. Rosette.

The following table (tab. 2) outlined the stages of milling machine used for rosette mould.

Work Type	Model	Area	Type of tool	Type of operation
Zigzag	mrosone.stl	Cartesian	Sph. ø3mm	Rough down
Zigzag	mrosone.stl	Cartesian	Sph. ø1mm	Finish

Tab. 2 Definition of milling parameters. Mould.

It is notable that for the realization of rosette, it has been placed a support (fig. 15) that can be generated in the software itself *Mayka Expert* to prevent total detachment of the part of the block when the machining process is complete.

This software is used to visualize the whole process simulation (fig. 16 and fig. 17). Here you can check if all the pre-configured parameters are commensurate with the expected result.

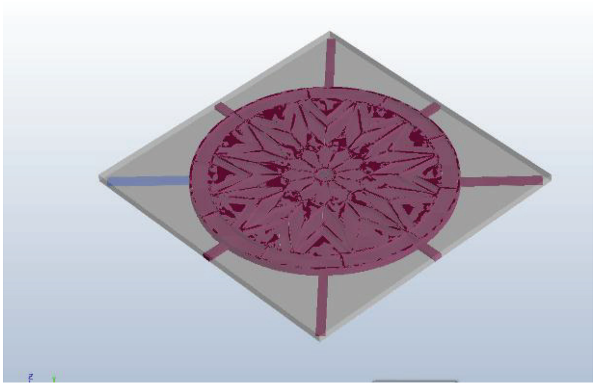


Fig. 15 CAM, support rosette.

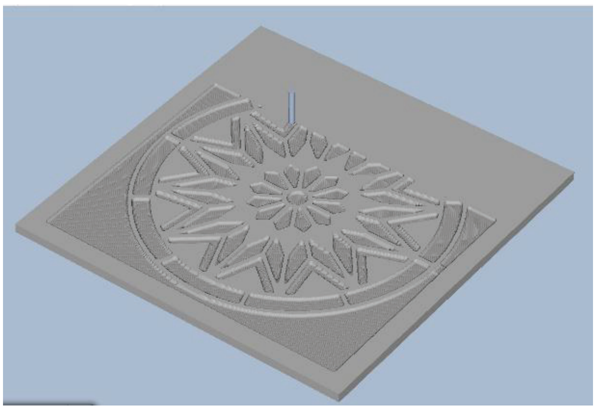


Fig. 16 Simulation of machining in Mayka (rosette).

[2] *This file format is supported by many other software packages; it is widely used for rapid prototyping and computer-aided manufacturing. STL files describe only the surface geometry of a three dimensional object without any representation of color, texture or other common CAD model attributes. The STL format specifies both ASCII and binary representations.*

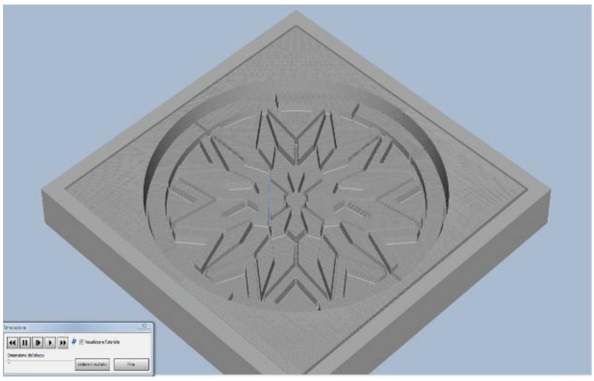


Fig. 17 Simulation of machining in Mayka (mould).

The simulation can be performed in each operation separately or with all the operations in a row to the other in each of the models. The result can be seen as a surface that can be measured and observed to even export to CAD software for better analysis.

CAM software is extracted, using the *NCP Euromond 3 axis post processor*, an NCP file that can be read by software that contains the control CNC milling machine.

2.4 Rosette CNC and mould CNC

The milling machine, model series modular AbaMill and Flatcom EUROMOD, is the company Abacus and is four axes (X, Y, Z and R), where the fourth strand is the turn of a head. The software uses this milling machine is *RemoteWin*.

The procedure is common to any milling machine. The block of material is positioned on the worktable and then be adjusted manually.

Placing the tool on the head, the machine prepares the following details:

- **□ome**: absolute zero in the milling machine, in all three axes.
- **□ero** work: is the parameter that must match the chosen zero in the CAM. In this type of milling machine, this should be in the upper face of the block of material. Preferably, this should be placed in a vertex of the block and that the approach must be done manually.
- **□ig□□**: is the same position as zero work, but the Z axis should be 0. This level is considered a safety and milling machine used before going to the Home.

NCP load the file and run the program. When you go from roughing to finishing the tool change is specified in the software that has been done.

This will automatically measures the length of the new tool and adjust the points made above (high-z and Zero work).

Rosette post-machining finishing operations are performed, such as cutting the supports that connect to the block of material, with the mould is necessary to casting tests.

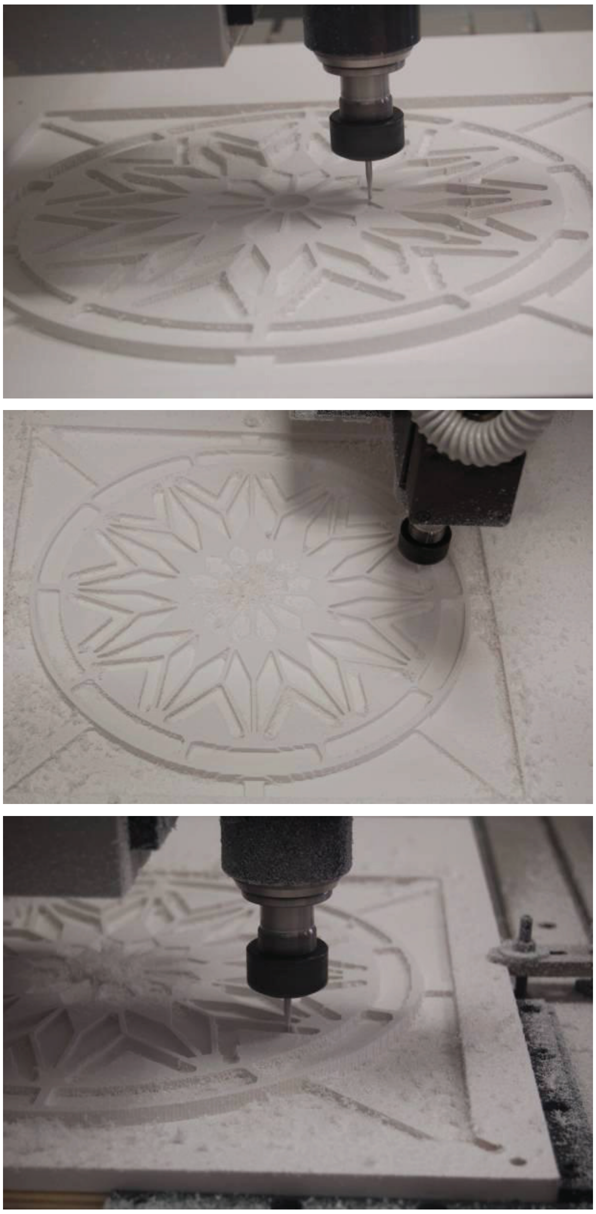


Fig. 18 Roughing mill the rosette, CNC.

3 Conclusion

What can be drawn, can be constructed [6]. The procedure helps analyze and compare the shapes materialize, allowing you to find best solutions for problems associated with the restoration, preservation and documentation, and leads to a new and more complete instructions for use. The creation of the 3D digital model is a good starting point to study the problem of restoring the individual parts, to analyze the geometry, to try and assess the reliability of the position, ultimately, a possible reconstruction.

The reconstruction and the process of prototyping are intrinsically linked by their mathematical model of the object, while that for the first technique is the objective, the second becomes the starting point. Is important, before creating the model, whether physical or digital, will study and organize the entire process in order to avoid delays and design errors.

The procedures and instrumentations described can be a powerful method for Control, verification and monitoring of architectural structures, allowing making metric interactive database can provide information about the object detected at any time.

The reproducibility of mass production, which allows the passage of the geometric models and prototypes, are unquestionable benefits and features that will open up new possibilities for a rigorous development of integrated survey and analysis of artifacts.

Acknowledgement

All our thanks to this instance of project that allowed the International Interuniversity work. A project partners who selflessly supported at all times: special thanks goes to Fausta Fiorillo - Laboratorio Modelli UNISA.

References

[1] H. Weil. Simetría. McGraw-Hill, Madrid, 1990.

[2] University of Roma website: http://www.mat.uniroma3.it/users/magrone/2010_11/magistrale/ accessed 1 Feb 2011.

[3] N. Sala, G. Cappellato. *Viaggio Matematico nell´arte e nell'architettura*. Franco Angeli, Milano, 2003.

[4] M. Giordano. *Matrici geometriche del decoro architettonico. Photo scanning, reverse engineering e processi CAD/CAM.* Dottorato di Ricerca in Ingegneria delle Strutture e del Recupero Edilizio ed Urbano, IX Ciclo Nuova Serie, Università degli Studi di Salerno, 2011.

[5] A. Montù. *Appunti ed annotazione su sezione aurea e forme pentagonali*, E.R.A.T., Milano, 1980.

[6] A. Estévez. *Arquitectura biodigital*. *Gráfica Digital e Informática Aplicada*. SIGRADI/CUJAE, 2008, La Habana, Cuba.