Dimensional Analysis in Selective Laser Melting Process

F. Cardaropoli[1], F. Caiazzo[1], V. Sergi[1]

[1] Dept. of Mechanical Engineering, University of Salerno, Via Ponte Don Melillo, Fisciano, 84084, Italy

Abstract

This paper aims to use dimensional analysis technique for evaluating the properties of laser sintered components manufactured with Selective Laser Melting (SLM) process from metallic powders. The complexity of SLM does not allow to define an exhaustive mathematical model which involves all governing parameters and, therefore, the dimensional analysis might be a powerful tool for the expression of output parameters as function of dimensionless numbers appropriately defined from the input parameters set. As an example of the developed procedure, the paper explains the construction of a response function for the expression of relative density.

Keywords:

Dimensional Analysis, SLM, Output Parameters

1. Introduction

The estimation of temperature field during the process of Selective Laser Melting (SLM) is vital to define the dynamics of density increase and optimize the window of process parameters in order to improve properties of sintered parts. Unfortunately, the formulation of an exhaustive mathematical model is far to be simple because of thermal conduction in a non-homogeneous material with formation of liquid phase.

The number of input parameters involved in the process is very high. One of the first attempts to classify the factors connected with Selective Laser Sintering – characterized by partial melting – was carried out by Williams [1], but he focused attention only on user defined parameters, leaving out the material properties.

Later many researchers studied the correlation between governing and governed parameters in SLM process [2-5] and sometimes with opposed results, which were apparent by the examination of the different processing windows in Childs and Kruth regarding balling phenomenon. Simchi developed an empirical formula for the expression of the densification in Direct Metal Laser Sintering as function of a calibration constant, connected with the material, and the specific energy input, related to process parameters. The weak point of his formulation is the fact that the influence of every powder characteristic – grain size, dynamic viscosity, surface free energy – on densification factor was not explicit.

The difficulties in the definition of an exhaustive mathematical model lead to finding another approach for the evaluation of output parameters. The dimensional analysis may be a powerful tool for this purpose.

The principle of dimensional analysis is the theorem of Buckingham-Pi, who states that: if *n* variables are connected by an unknown dimensionally homogeneous equation and the units of measurement of these variables can be represented in terms of *k* independent fundamental physical quantities, the original expression is equivalent to an equation involving a set of *n-k* dimensionless variables constructed from the original ones.

The parameters are said to have independent dimensions if none of these quantities has dimensions which can be expressed as products of powers of the remaining quantities dimensions.

Barenblatt [6] suggests that the problem can be written in the form:

$$G = f(G_1,...,G_k,G_{k+1},...,G_n) \tag{1}$$

where G is the governed parameter and G_1, …, G_n the governing parameters. In particular, G_1, …, G_k have independent dimensions, while the dimensions of parameters G_{k+1}, … , G_n can be expressed as products of powers of G_1, … , G_k. From the previous considerations the introduction of new parameters emerges:

$$\Pi_1 = \frac{G_{k+1}}{G_1^{p_{k+1}}...G_k^{r_{k+1}}},\dots,\Pi_i = \frac{G_{k+i}}{G_1^{p_{k+i}}...G_k^{r_{k+i}}},\dots,$$
$$\Pi_{n-k} = \frac{G_n}{G_1^{p_n}...G_k^{r_n}} \tag{2}$$

where the exponents of the governing parameters with independent dimensions are chosen such that all the parameters $\Pi_1,\dots,\Pi_{n-k}$ are dimensionless. The governing equation can be rewritten replacing G_{k+1}, …, G_n as:

$$\Pi = \frac{f(G_1,...,G_k,G_{k+1},...G_n)}{G_1{}^p...G_k{}^r} \rightarrow \Pi = \Phi(\Pi_1,\Pi_2,...,\Pi_{n-k}) \tag{3-4}$$

Whereas the formula (1) can be expressed as:

$$G = G_1^p...G_k^r\Phi\left(\frac{G_{k+1}}{G_1^{p_{k+1}}...G_k^{r_{k+1}}},...,\frac{G_n}{G_1^{p_n}...G_k^{r_n}}\right) \tag{5}$$

This is possible if the response function possesses the property of generalized homogeneity or symmetry. Besides the reduction in the governing parameters, another benefit of dimensional analysis is the construction of dimensionless groups, which could have a specific meaning in SLM process.

2. Parameters identification

In the application of Buckingham-Pi theorem, when *f* is unknown, the selection of governing parameters is connected with phenomenon dynamics and experience. The

SLM process is characterized by several parameters, but the exclusion of less influential factors is likely.

The model developed in this paper starts from Van Elsen one [7] and uses 16 parameters for the evaluation of governed ones (Table 1). They are all independent and connected with laser sintering dynamics, powder properties and process parameters.

Table 1
Selective Laser Melting parameters and corresponding physical dimensions (SI).

Factor		Symbol	Unit of measurement	Dimensions
1	Oxygen level	$O_{2\%}$	-	-
2	Laser beam quality	M^2	-	-
3	Scan length	l	[m]	[L]
4	Powder size	s	[m]	[L]
5	Initial bed density	ρ_0	[kg m^{-3}]	[M L^{-3}]
6	Dynamic viscosity of the molten metal	μ	[kg m^{-1} s^{-1}]	[M L^{-1} T^{-1}]
7	Scan speed	u	[m s^{-1}]	[L T^{-1}]
8	Surface free energy of the molten metal	σ	[J m^{-2}]	[M T^{-2}]
9	Heat capacity at constant pressure	c_m	[J kg^{-1} K^{-1}]	[L^2 T^{-2} Θ^{-1}]
10	Latent heat of fusion	L_F	[J kg^{-1}]	[L^2 T^{-2}]
11	Laser specific energy	E_V	[J m^{-3}]	[M L^{-1} T^{-2}]
12	Scan angle	β	-	-
13	Bulk density	ρ	[kg m^{-3}]	[M L^{-3}]
14	Spot diameter	d_S	[m]	[L]
15	Average thermal diffusivity	α_m	[m^2 s^{-1}]	[L^2 T^{-1}]
16	Temperature difference	ΔT	[K]	[Θ]

The percentage of oxygen $O_{2\%}$ in the process chamber is a measure for the probability that a molten molecule of the metal interacts with an oxygen molecule. The oxidization has a negative effect on sintered parts properties, for this reason it is considered in the set. Conversely, a previous work showed that the influence of the gas type used for the purification of sintering atmosphere was not very pronounced [5].

In order to involve the features of laser system, the beam quality M^2 is included between key factors. According to the ISO Standard 11146 [8] there is a relationship among the beam quality, the wave length, the spot diameter and the beam divergence:

$$\Delta = M^2 \frac{2\lambda}{\pi d_S} \tag{6}$$

An increase in M^2 causes lower laser quality due to a higher divergence.

Building strategies influence many factors, such as the laser specific energy, scan length and speed, spot size and scan angle. In particular, the laser specific energy is function of laser power P, hatch spacing h, scan speed u and layer thickness t:

$$E_V = \frac{P}{h\ u\ t} \tag{7}$$

The scan length is included in the set as, together with the scan speed, it influences the delay period between successive irradiation exposures and, therefore, the sintered part mean temperature [1].

For the same reason the scan angle between scan paths of different layers is considered in the governing parameters.

Other factors are dependent on the material properties. As far as the powder grain size is concerned, in literature contrasting theories are present. Tolochko showed that the absorptance of nickel particles is not connected with such factor [9]. Conversely, Simchi pointed out that the powder grain size affects the densification factor [5] and, therefore, it is included in the input parameters.

In the model average thermal properties are considered since there is heat conduction over all the bed powder. Using the symbol k_m for the average thermal conductivity and c_m for the heat capacity, the definition of the average thermal diffusivity is:

$$\alpha_m = \frac{k_m}{\rho_0 c_m} \tag{8}$$

The amount of the specific melt enthalpy during manufacturing process is given by the following expression containing the latent heat of fusion and the difference between the melting temperature T_F and the building platform temperature T_0:

$$\Delta h = c_m (T_F - T_0) + L_F \tag{9}$$

Conductivity, heat capacity and temperature difference involve temperature dimension in their units of measurement and, therefore, they must be included for the application of Buckingham-Pi theorem. Nevertheless, these factors through expressions (8) and (9) are dependent, thereby, only one is considered in the construction of a complete, dimensionally independent subset. The better choice is:

$$\Delta T = T_F - T_0 \tag{10}$$

The building platform temperature T_0 is included as Shiomi proved that this parameter affects the residual stresses and sintered part properties [10].

Before evaluating dimensionless numbers, it is important to underline that average values of input parameters are considered, as these factors change during laser processing. If the variations are noteworthy, it is advisable to use the approach of Langhaar [11] in which a linear transformation law is adopted with the addition of a further parameter to the model. Obviously, such choice implies an increase in the number of experiments for the definition of output parameters.

3. Evaluation of dimensionless numbers and output

The governing parameters have 4 fundamental dimensions and, consequently, the evaluation of 12 dimensionless numbers is possible. Choosing bulk density, spot size, thermal diffusivity and temperature difference, all fundamental dimensions are involved in the subset. The reference group G_0 is:

$$G_0 = \rho^A \cdot d_S^B \cdot \alpha_m^C \cdot \Delta T^D \tag{11}$$

Introducing the dimensions of every parameter, it is obtained that:

$$[G_0] = [M^A L^{-3A+B+2C} T^{-C} \Theta^D] \tag{12}$$

The other dependent governing parameters are multiplied for the fundamental group in order to carry out the nondimensionalization. Oxygen level and beam quality are already dimensionless. As regards the scan length, powder size and initial bed density, the procedure is immediate exploiting subset parameters:

$$\pi_3 = \frac{l}{d_S} \tag{13}$$

$$\pi_4 = \frac{s}{d_S} \tag{14}$$

$$\pi_5 = \frac{\rho_0}{\rho} \tag{15}$$

The dimensionless dynamic viscosity is calculated by:

$$\pi_6 = G_0\mu = \left(\rho^{-1} d_S^0 \alpha_m^{-1} \Delta T^0\right)\mu = \frac{\mu}{\rho\alpha_m} = Pr \tag{16}$$

The Prandtl number (*Pr*) is a function of material properties and is frequently used in fluidodynamics problems for the comparison between viscous effects and thermal diffusivity.

Another important group is obtained through the nondimensionalization of scan speed:

$$\pi_7 = \frac{u\ d_S}{\alpha_m} = Pe \tag{17}$$

The Peclet number (*Pe*) represents the ratio between convective and conductive thermal transfers. Moreover, Simchi used this number for the definition of the temperature field during DMLS process [5]. As far as the other parameters are concerned, the corresponding dimensionless groups are:

$$\pi_8 = \frac{S\ d_S}{\rho\alpha_m^2} \tag{18}$$

$$\pi_9 = \frac{c_m\ d_S^2 \Delta T}{\alpha_m^2} \tag{19}$$

$$\pi_{10} = \frac{L_F\ d_S^2}{\alpha_m^2} \tag{20}$$

$$\pi_{11} = \frac{E_v d_S^2}{\rho\ \alpha_m^2} \tag{21}$$

π_{12} is a function of the scan angle that is already dimensionless.

Table 2 sums up all the exponents calculated through nondimensionalization.

At that point it is possible to suppose the existence of an unknown function *f* which expresses an output of Selective Laser Melting process starting from the input parameters. For instance, considering the part final density the governing equation is in the form:

$$\rho_F = f_1\left(O_{2\%}, M^2, l, s, \rho_0, \mu, u, S, c_m, L_F, E_v, g(\beta), \rho, d_S, \alpha_m, \Delta T\right) \tag{22}$$

The problem can be rewritten through the application of Buckingham-Pi theorem reducing the governing parameters and obtaining:

$$\frac{\rho_F}{\rho} = f_{1A}\left(\pi_1, \pi_2, \ldots, \pi_{12}\right) \tag{23}$$

Table 2
Exponents for nondimensionalization of governing parameters and output.

Group	Factor	A	B	C	D
π_1	Oxygen level	0	0	0	0
π_2	Laser beam quality	0	0	0	0
π_3	Scan length	0	-1	0	0
π_4	Powder size	0	-1	0	0
π_5	Initial bed density	-1	0	0	0
π_6	Dynamic viscosity of the molten metal	-1	0	-1	0
π_7	Scan speed	0	1	-1	0
π_8	Surface free energy of the molten metal	-1	1	-2	0
π_9	Heat capacity at constant pressure	0	2	-2	1
π_{10}	Latent heat of fusion	0	2	-2	0
π_{11}	Laser specific energy	-1	2	-2	0
π_{12}	Scan angle	0	0	0	0
ρ_F/ρ	Final density	-1	0	0	0

4. Sub-processes dimensionless parameters

Many authors [12-14] have developed different nondimensionalization models without the construction of a complete set in order to explain some phenomena in sub-processes of SLM. It is possible to point out that these parameters are connected with the defined π_i and furthermore can be introduced in the model developed without compromising its validity.

Certain models have used the Christensen number (*Ch*) [12] defined as:

$$Ch = \frac{u^2 A_{rif}}{\alpha_m^2} \tag{24}$$

where A_{rif} is the melt pool cross section. If A_{rif} is equal to the spot area, it follows that:

$$Ch = \frac{\pi}{4} Pe^2 \tag{25}$$

The Peclet number has already been included in the set.

Other schemes have adopted the Fourier number (*Fo*), which is the ratio of the heat conduction rate to the one of thermal energy storage, as dimensionless time in the diffusion equation [13]:

$$Fo = \frac{\alpha_m t_{rif}}{L_{rif}^2} \tag{26}$$

Using the interaction time between the laser beam and powder as t_{rif} and the spot size as L_{rif}, it implies that *Fo* is also related to *Pe*:

$$Fo = \frac{\alpha_m}{d_S^2} t_i = \frac{\alpha_m}{d_S^2} \frac{d_S}{u} = \frac{1}{Pe} \tag{27}$$

To consider the surface tension in melting and solidification process, the Ohnesorge number (*Oh*) is introduced and relates the viscous forces to inertial and surface tension ones [14]:

$$Oh = \frac{\mu}{\sqrt{\rho\ S\ d_S}} \tag{28}$$

Oh as well as the previous parameters can be obtained through the combination of π_i. In particular, replacing π_8 with another number is possible:

$$\pi_8' = \frac{\pi_6}{\pi_8} = \frac{\mu}{\rho\ \alpha_m}\frac{\rho\alpha_m^2}{S\ d_S} = \frac{\mu\ \alpha_m}{S\ d_S} = \frac{Oh^2}{Pr} \rightarrow \frac{\pi_6}{\sqrt{\pi_8}} = Oh \qquad (29\text{-}30)$$

Another important dimensionless parameter is the Stefan number (*St*) defined as the ratio of the sensible heat to the latent heat of fusion:

$$St = \frac{c_m(T_F - T_0)}{L_F} \rightarrow \frac{\pi_9}{\pi_{10}} = St = \pi_{10}' \qquad (31\text{-}32)$$

In this way a new parameter replacing π_{10} is evaluated involving the Stefan number.

In the study of a melting process other dimensionless factors involving the gravity *g* ought to be introduced, but in this problem the effect of gravity is negligible compared to the surface tension one owing to the small dimensions.

5. Conclusions and future works

In this work a procedure based on principles of dimensional analysis is developed in order to express relationships of output parameters of Selective Laser Melting process with metallic powders.

After the definition of more influential factors, an adequate parameters subset is evaluated with the following properties:

- Independent dimensions.
- Complete representation of all fundamental dimensions involved in the process.

Through the nondimensionalization a set of dimensionless groups connected with SLM is obtained and laser sintered part properties are expressed as functions of these numbers.

The first advantage is the reduction in the governing parameters, which, as a consequence, allows to plan a lower number of experiments for the property characterization. For instance, using Prandtl number is possible to include changes in the viscosity, density or diffusivity.

The dimensionless groups could have a specific meaning in SLM process, and, therefore, they may explain what aspects are more considerable in the process. Moreover, comparisons among the results of separate research groups adopting dimensional analysis can be carried out. Indeed, this approach allows to overcome the differences in process or material parameters and avoid misleading evaluations of some tests undertaken with dissimilar inputs [2,3].

Another important consideration that emerges from the works mentioned before is that the definition of a good processing window with only two factors is unlikely and it is necessary to consider the overall effect of process parameters.

This paper is a grounding for the successive elaboration of an experimental phase. In particular, the attention will be zeroed in on Design of Experiments, developed from the ANOVA analysis for the evaluation of more influential factors on output parameters.

The test optimization may allow to define the processing parameters domain in which sintered parts should be well manufactured.

A further consideration is that the procedure and systematic approach could be adopted for materials different from the metallic powders, choosing adequate factors connected with material characteristics.

References

[1] Williams J.D., Deckard C.R., 1998, Advances in Modelling the Effects of Selected Parameters on the SLS Process, Rapid Prototyping Journal, 4/2: 90-100

[2] Childs T.H.C., Hauser C., Badrossamay M., 2004, Mapping and Modelling Single Scan Track Formation in Direct Metal Selective Laser Melting, CIRP Annals – Manufacturing Technology, 53/1: 191-194

[3] Kruth J.P., Mercelis P., Van Vaerenbergh J., 2005, Binding Mechanisms in Selective Laser Sintering and Selective Laser Melting, Rapid Prototyping Journal, 11/1: 26-36

[4] Rombouts M., Kruth J.P., Froyen L., 2006, Fundamentals of Selective Laser Melting of Alloyed Steel Powders, CIRP Annals – Manufacturing Technology, 55/1: 187-192

[5] Simchi A., 2006, Direct Laser Sintering of Metal Powders: Mechanism, Kinetics and Microstructural Features, Materials Science and Engineering, A/428: 148-158

[6] Barenblatt G.I, 1987, Dimensional Analysis, Gordon and Breach Science Publishers

[7] Van Elsen M., Al-Bender F., Kruth J.P., 2008, Application of Dimensional Analysis to Selective Laser Melting, Rapid Prototyping Journal, 14/1: 15-22

[8] ISO Standard 11146, 2005, Lasers and Laser-related Equipment. Test Methods for Laser Beam Widths, Divergence Angles and Beam Propagation Ratios

[9] Tolochko N.K., Laoui T., Khlopkov Y.V., 2000, Absorptance of Powder Materials Suitable for Laser Sintering, Rapid Prototyping Journal, 6/3: 155-160

[10] Shiomi M., Osakada K., Nakamura K., 2004, Residual Stress within Metallic Model Made by Selective Laser Melting Process, Annals of the CIRP, 53/1: 195-198

[11] Langhaar H.L., 1951, Dimensional Analysis and Theory of Models, Krieger Publishing Company

[12] Christensen N., Davies V., Gjermundsen K., 1965, The Distribution of Temperature in Arc Welding, British Welding Journal, 12/2: 54-75

[13] Steen W.M., 2003, Laser Material Processing, Springer-Verlag

[14] Schiaffino S., Sonin A., 1997, Molten Droplet Deposition and Solidification at Low Weber Numbers, Physics of Fluids, 9/11: 3172-3187