IFFTC-Based Procedure for Hidden Tone Detection

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***Abstract*—In this paper a procedure for the detection of hidden tones in the spectrum of a signal is proposed and experimentally evaluated. It is based on the "Corrected Interpolated FFT" algorithm for the estimation of spectral parameters of nonhidden tones, and on the analysis of the disagreement between measured and expected spectra in order to detect hidden tones. The design of the procedure takes advantage of experimental design techniques in the threshold evaluation.**

***Index Terms*—Digital signal processing (DSP), hidden tones, interpolated fast Fourier transform (FFT), spectral resolution.**

I. Introduction

Spectral analysis is one of the most powerful tools in the field of digital signal processing, since the spectral content of a signal is often critical information. The availability of efficient analysis algorithms and the continuous improvement of the performances of microprocessors, and in particular of dedicated digital signal processors (DSPs), have made the applicability of numerical measurement systems for spectral analysis more and more common. In recent years, measurement issues in numerical spectral analysis have been studied, and proposals have been made in order to increase the performances of these measurement systems. A tool of choice in these systems is the fast Fourier transform (FFT) algorithm, allowing estimation of the spectral parameters of a signal (amplitude, frequency and phase of its spectral components) with relatively small computational requirements. For these reasons, researchers are devoting efforts not only to the developments of even more efficient algorithms, but also to the characterization and the improvement of FFT results from a metrological point of view. In particular, the reduction of the residual bias of the spectral parameters estimated with the FFT is a topic of fundamental importance. Thus, different interpolation techniques have been proposed; the widely used being the so-called interpolated-FFT algorithms (IFFT), which allow improving the measurement accuracy by correcting deterministic errors on frequency, amplitude and phase due to the spectral leakage phenomenon [1]–[5].

All the FFT-based techniques adopt a tone detection procedure which finds the relative maxima of the amplitude spectrum of the signal. An essential hypothesis for these techniques is that the frequencies of tones to be analyzed are sufficiently distant. Also the interpolation-based algorithms are limited in the resolution of close tones by the physical resolution of the FFT, which depends exclusively on the number of samples, on the sampling frequency and on the window function.

However, this hypothesis of sufficient separation of frequency components is not adequate in many application fields, such as the vibration analysis of mechanical parts, the radar detection, and the conditioning of acoustic systems. In these cases, because of the harmonic interference, the analyzed signals typically exhibit tones closely located in frequency, and stronger spectral components often obscure weaker ones.

This paper proposes a method able to indicate the occurrence of hidden tones, i.e., tones obscured by near stronger components [5], [6]. It is based on an enhancement of the IFFT, previously developed by the authors in [7], [8], named "Corrected Interpolated FFT" (IFFTc), which allows correcting the estimation bias due to harmonic interference. The proposed technique applies the IFFTc to the estimation of spectral parameters of nonhidden tones, and then analyzes the disagreement between measured and expected spectra in order to detect hidden tones, where the expected spectrum is synthesized on the basis of the previous IFFTc results. Following this approach, a threshold has to be set in order to signal the occurrence of a hidden tone when the disagreement is greater than the threshold value. The high number of parameters to be considered for the choice of the optimal threshold would require too high computational effort. For this purpose, an experimental design (ED) approach [9] has been followed in order to cover the entire possible experimental plan with an adequate number of tests. Such an approach allows the definition of an experimental plan in advance, in order to maximize the amount of "information" that can be obtained for a given amount of experimental effort.

In the paper, at first the IFFTc algorithm is explained in detail; then the proposed technique is introduced and illustrated. A section is devoted to the description of the ED procedure followed in order to find the threshold value; in particular, the Latin Hypercube Sampling is used. Several numerical tests are carried out for the evaluation of the detection procedure performance.

II. Corrected Interpolated FFT (IFFTc)

In this paragraph a brief recall of the so-called IFFTc algorithm, proposed by the authors in [7], [8], will be given. Basically, it applies a two-point IFFT [1]–[3] and corrects the effects of the harmonic interference.

The amplitude spectrum of a signal with P frequency components

$$x(t) = \sum_{i=1}^{P} A_i \mathrm{sen}(2\pi f_i t + \phi_i) \quad (1)$$

is composed of P peaks, if no tone is hidden. The peak corresponding to the ith tone frequency is located at index k_i: $k_i = \mathrm{int}(f_i/\Delta f)$, where Δf is the DFT frequency resolution. The frequency of the ith component is evaluated as

$$f_i = (k_i + \delta_i) \cdot \Delta f \qquad \text{where } -1/2 \leq \delta_i < 1/2. \quad (2)$$

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The 2-point IFFT evaluates δ_i considering the ratio, α_i, between the two largest samples corresponding to the peak

$$\alpha_i = \frac{|X(k_i+\varepsilon_i)|}{|X(k_i)|} \tag{3}$$

where $\varepsilon_i = 1 \cdot \mathrm{sign}(|X(k_i+1)| - |X(k_i-1)|)$.

Considering the window frequency spectrum $(W(k))$, we have [1]–[3]

$$\frac{|W(\varepsilon_i-\delta_i)|}{|W(-\delta_i)|} = \frac{|X(k_i+\varepsilon_i)|}{|X(k_i)|}. \tag{4}$$

The δ_i is obtained from (3) and (4); in particular, simple relationships exist for the Hanning window

$$\begin{aligned} \delta_i &= \varepsilon_i \frac{2\alpha_i - 1}{1+\alpha_i} & (5)\\ A_i &= 2\,|X(k_i)| \cdot \frac{\pi\delta_i \cdot \left(1-\delta_i^2\right)}{\mathrm{sen}(\pi\delta_i)} & \\ \phi_i &= \arg\left(X(k_i)\right) + \frac{\pi}{2} - \pi\delta_i. & (6) \end{aligned}$$

Equation (4) is valid for a single tone signal but it cannot be extended to multifrequency signals when harmonic interference is present. In fact, $X(k_i)$ and $X(k_i+\varepsilon_i)$ depend not only on the main lobe of the windowed ith sinusoidal component but also on those of the other sinusoids, since each discrete Fourier transform (DFT) sample can be obtained as

$$X(k) = \frac{1}{S}\left[\sum_{i=1}^{P} V_i \cdot W\left(\frac{k\Delta f - f_i}{\Delta f}\right) + \sum_{i=1}^{P} V_i^* \cdot W\left(\frac{k\Delta f + f_i}{\Delta f}\right)\right] \tag{7}$$

$$\begin{aligned} X(k_i) &= \frac{1}{S}\left[V_i W(-\delta_i) + \sum_{r\neq i} V_r W\left(\frac{f_{ki}-f_r}{\Delta f}\right) + \sum_{r=1}^{p} V_r^* W\left(\frac{f_{ki}+f_r}{\Delta f}\right)\right] \\ &= \frac{V_i}{S} W(-\delta_i) + F_i \quad \Rightarrow \\ &\qquad W(-\delta_i) = \frac{S}{V_i}\left(X(k_i) - F_i\right) \\ X(k_i+\varepsilon_i) &= \frac{1}{S}\left[V_i W(\varepsilon_i-\delta_i) + \sum_{r\neq i} V_r W\left(\frac{f_{\varepsilon i}-f_r}{\Delta f}\right) + \sum_{r=1}^{p} V_r^* W\left(\frac{f_{\varepsilon i}+f_r}{\Delta f}\right)\right] \\ &= \frac{V_i}{S} W(\varepsilon_i-\delta_i) + B_i \quad \Rightarrow \\ &\qquad W(\varepsilon_i-\delta_i) = \frac{S}{V_i}\left(X(k_i+\varepsilon_i) - B_i\right) \end{aligned} \tag{8}$$

being $V_i = (A_i/2j)e^{j\phi_i}$; $V_i^* = -(A_i/2j)e^{-j\phi_i}$, $S = \sum_{n=0}^{N-1} w(n) = (N/2)$.

Equation (7) can be rewritten in the neighborhood of k_i, highlighting the contribution due to the ith sinusoid; termed $f_{ki} = k_i \cdot \Delta f$ and $f_{\varepsilon i} = (k_i+\varepsilon_i)\cdot\Delta f$, the relationships (8) follow.

Substituting (8) in (4) we obtain

$$\frac{|W(\varepsilon_i-\delta_i)|}{|W(-\delta_i)|} = \frac{|X(k_i+\varepsilon_i) - B_i|}{|X(k_i) - F_i|} = \alpha_i'. \tag{9}$$

Using this corrected α_i', the new δ_i' can be evaluated, and the amplitude and the phase of the ith spectral components can be calculated for the Hanning window

$$\begin{aligned} \delta_i' &= \varepsilon_i \frac{2\alpha_i' - 1}{1+\alpha_i'} & (10)\\ A_i' &= 2 \cdot \frac{\pi\delta_i'\left(1-\delta_i'^2\right)}{\mathrm{sen}\left(\pi\delta_i'\right)} \cdot |X(k_i) - F_i| & \\ \phi_i' &= \arg\left(X(k_i) - F_i\right) + \frac{\pi}{2} - \pi\delta_i'. & (11) \end{aligned}$$

The correction factors F_i and B_i, depend on the frequency, amplitude and phase of the signal tones. The proposed solution consists in using the value measured with the 2-point IFFT in the evaluation of the correction factors, $\hat{F}_i$ and $\hat{B}_i$. As expected, the obtained correction factors are still corrupted by harmonic interference, but the residual errors obtained using these values, instead of the actual ones, F_i and B_i are negligible, as shown in [7] and [8]. The proposed procedure is, thus, the following:

- application of the 2-point IFFT for each peak in order to estimate frequency, amplitude and phase of the corresponding spectral component, neglecting the harmonic interference effects;
- $\hat{F}_i$, $\hat{B}_i$ evaluation using the 2-points IFFT estimations;
- computation of the correction factors for each peak;
- determination of frequencies $\hat{f}_i = (k_i + \hat{\delta}_i)\cdot\Delta f$, amplitudes, $\hat{A}_i$, and phases $\hat{\phi}_i$, corrected by the harmonic interference effects using (10) and (11).

This procedure could be iterated more than once, but the improvement achievable with more than one step is negligible. As a consequence only one step of correction is run; this solution represent a good compromise between accuracy and computational burden. With respect to the iterative methods presented in [10], [11], the proposed IFFTc allows to estimate all the components at the same time and with lower computational burden, especially in case of signal with several spectral components. Moreover, parametric methods require the *a priori* knowledge of some parameter of the signal, such as the number of its spectral components, and some methods presented in literature (such as the ESPRIT method) do not allow to estimate the amplitudes of spectral components directly, but only their frequencies.

III. Proposed Method for Hidden Tone Detection

As illustrated in [7] and [8], if hidden tones are not present, the IFFTc algorithm corrects all the significant systematic effects which can occur in the signal parameter estimation. Using the so measured parameters, the amplitude spectrum, $\hat{M}(k)$, can be reconstructed

$$\hat{M}(k) = \frac{1}{S}\left|\sum_{n=1}^{N} \hat{x}(n)\cdot w(n)\cdot e^{-j2\pi kn/N}\right| \tag{12}$$

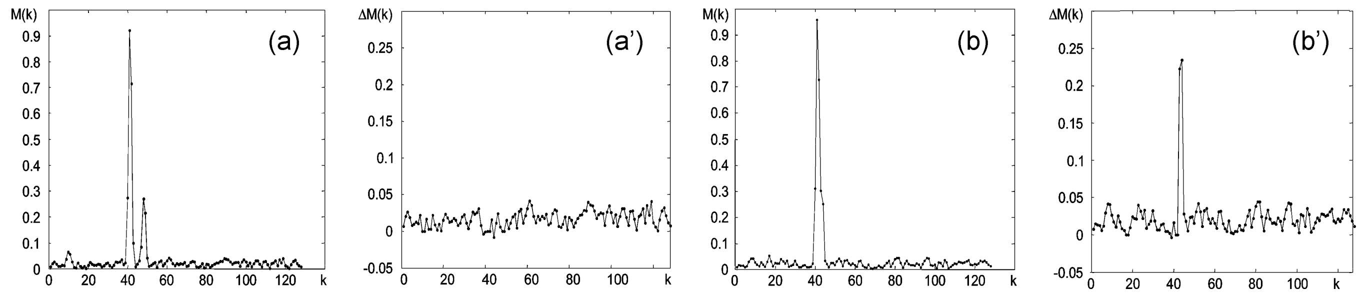

Fig. 1. (a) Windowed amplitude spectrum and (a′) $\Delta M(k)$ with two interfering but resolvable tones. (b) Windowed amplitude spectrum and (b′) $\Delta M(k)$ in presence of a hidden tone.

where

$$\hat{x}(n) = \sum_{i=0}^{P} \hat{A}_i \cdot \sin(2\pi \hat{f}_i n T_s + \hat{\phi}_i) = \sum_{i=0}^{P} \hat{A}_i \cdot \sin\left(2\pi(k_i + \hat{\delta}_i)/N + \hat{\phi}_i\right). \quad (13)$$

In this case, this spectrum is very similar to the measured one $M(k)$

$$M(k) = |X(k)| = \frac{1}{S}\left|\sum_{n=1}^{N} x(n) \cdot w(n) \cdot e^{-j2\pi kn/N}\right|. \quad (14)$$

In particular, $\hat{M}(k)$ differs from $M(k)$ for the residual error [7], [8], which is very low, in the estimation of the characteristics of the signal and for the effects of the uncertainty on the input samples.

Vice versa, if in $M(k)$ there are hidden tones, these alter the signal parameter estimations ($\hat{A}_i$, $\hat{f}_i$, $\hat{\phi}_i$), and consequently the obtained amplitude spectrum $\hat{M}(k)$ differs significantly from the measured one $M(k)$ in frequency neighborhoods centered on the hidden tone frequencies. Then, in order to detect possible hidden tones, we propose to evaluate $\hat{M}(k)$ and analyze the difference

$$\Delta M(k) = \left|M(k) - \hat{M}(k)\right|. \quad (15)$$

If all the tones are detected, $\Delta M(k)$ is very low for each k_i, while it increases significantly near an unresolved tone $\Delta M(k)$ [see Fig. 1(b′)]. Thus, the hidden tones are detected by comparing the difference spectrum $\Delta M(k)$ with a suitable threshold.

The choice of the threshold is a very crucial task since it directly affects the sensitivity and the reliability of the detection algorithm; moreover the threshold value depends on the hardware configuration [mainly on the uncertainty on the input samples due to the analog-to-digital (ADC) converter] and on the operating conditions (number of processed points N).

IV. Threshold Tuning

Aiming to optimally and automatically set the threshold value, the performance of the IFFTc, in terms of $\Delta M(k)$, has to be evaluated for different input signals and operating conditions. In the following, two contributions to $\Delta M(k)$ will be distinguished: the former due to the residual errors in the parameter estimations with the IFFTc algorithm, and the latter due to the uncertainty of the input samples, as it propagates through the measurement algorithm.

A. Contribution to $\Delta M(k)$ due to the Residual Errors

For the evaluation of the maximum residual error on $\hat{M}(k)$, E_M, analytical or experimental approaches can be followed. The analytical approach can not be applied due to the complexity of the formulation and to the high number of parameters which it depends on. As a consequence an experimental approach is preferred. Because of the dependence of the error from the signal characteristics, this characterization would require a unmanageable number of tests; in order to run a reduced but effective number of tests, an ED [9], [12]–[15] approach was followed. The basic idea of ED is to obtain the best possible characterization in terms of coverage of the entire experimental plan. To this aim, the sampling of the whole L-dimensional space to be explored plays a key role. The simple random sampling is inefficient because it typically assigns higher probability to the middle of a distribution rather than to its tails, especially in the case of a higher number of dimensions. A more efficient scheme should sample the tails quickly.

The Latin Hypercube Sampling (LHS) method is one of such schemes [12]–[15]. Stein [14] showed that this scheme can behave substantially better than simple random sampling. The Latin hypercube sampling provides an orthogonal array that randomly samples the entire design space broken down into r^L equal-probability regions (where r is the number of runs, and L is the number of input variables).

Suppose we have a L-dimensional random vector $\mathbf{Y} = (Y_1, \ldots, Y_L)$ and we want to get a sample of size M from the joint distribution of $\mathbf{Y}$. If the components of Y are independent, then the scheme is simple, namely:

- divide the range of each component random variable in M intervals of equal probability;
- rndomly sample one observation for each component random variable in each of the corresponding M intervals;
- randomly combine the components to create the test set.

The first step in the LHS implementation is the definition of the design space, i.e., the definition of $\mathbf{Y}$ and of its variability range. In the following, the LHS method will be adopted in determining the residual error level E_M for a representative number of cases. Since E_M essentially depends on the signal

TABLE I
$C_R(k)$ AND $C_I(k)$ VALUES VERSUS K

	$C_R(k)$	$C_I(k)$
k=0; k=N/2	0.7500	0
k=1; k=N/2-1	0.4375	0.3125
All other k	0.3750	0.3750

spectrum contents and on the frequency resolution, the samples $\{x(n)\}$ of multifrequency signals [see (1)] have to be considered

$$x(n) = \sum_{i=1}^{P} A_i \text{sen}\left(2\pi(k_i + \delta_i)/N + \phi_i\right), \quad \text{for } n = 0..N. \tag{16}$$

An exhaustive analysis should be done for all the possible combinations of the parameters $(P, N, A_i, k_i, \delta_i, \phi_i)$, where the number of A_i, k_i, $\delta_i\phi_i$ depends on P; however, some simplifications can be made using the results obtained in [7] and [8], where the evaluation of the performance of the IFFTc parameter estimation is reported.

In [7], it was shown that the number of tones P does not influence significantly the IFFTc performance, while the error depends strongly on the nearest components. As a consequence in our tests, P has been considered constant and set equal to 2. Moreover, in [7], it was shown also that, after the values of k_i and δ_i have been set in (16), there is no dependence of the residual errors on the number of the processed points N: in our tests it has been considered constant and set equal to 256 (intermediate value in order to satisfy both the exigency of short elaboration time and number of tones in the spectrum). Thus, termed A_0 the A/D full scale, $A_i = \beta_i \cdot A_0$, $d_{12} = (k_2+\delta_2)-(k_1+\delta_1)$, the $\mathbf{Y}$ vector become

$$\mathbf{Y} = (\beta_1, k_1, \delta_1, \phi_1, \beta_2, d_{12}, \phi_2). \tag{17}$$

The range variability of the $\mathbf{Y}$ components is fixed as follows:

- $\beta_1, \beta_2 \subset [0.005, 1]$: resulting in a tone amplitude variability of $A_0/200 \div A_0$;
- $k_1 \subset [10, N/2-5]$: the lowermost value has been chosen equal to 10 since the IFFTc error increases for very low k[7], while the topmost value depends on N and on the maximum d_{12};
- $\delta_1 \subset [-0.5, 0.5]$: the whole range of possible values;
- $\phi_1, \phi_2 \subset [-\pi, \pi]$: the whole range of possible values;
- $d_{12} \subset [3.0, 3.5]$: since errors decrease significantly with the d_{12} [7], only small distances were investigated.

Then, each one of these ranges is divided in M intervals of equal probability; for each parameter, one observation is sampled for each interval, and finally, the observations are randomly composed in order to create the test set. For each element of the test set:

- the signal samples $\{x(n)\}$ are generated;
- the IFFTc algorithm is run on these samples;
- the amplitude spectrum $\hat{M}(k)$ is calculated using the parameters obtained with the IFFTc;
- if all the signal tones are detected, the difference $\Delta M(k)$ is computed, and, E_M the maximum of $\Delta M(k)$ is evaluated.

The maximum acceptable difference ME_M is deducted from all the elements of the test.

B. Contribution to $\Delta M(k)$ due to the Uncertainty

The uncertainty on the input samples $\{x(n)\}$ determines a variability of the amplitude signal spectrum $M(k)$; as a result, also in absence of any error in the signal parameters estimation ($A_i = \hat{A}_i$, $f_i = \hat{f}_i$, $\phi_i = \hat{\phi}_i$), the difference $\Delta M(k)$ differs from zero. The uncertainty on $\{x(n)\}$ determines also a variability of the signal parameters estimation that causes, on turn, an uncertainty contribution on $\hat{M}(k)$, that can be neglected, and consequently $u_{\Delta M(k)} = u_{M(k)}$. As for the uncertainty on $M(k)$, the authors in [16]–[18], obtained analytical relationships between the uncertainty of the $M(k)$ and the uncertainty of the acquired samples $u_{x(n)}$.

As for the sample uncertainty, $u_{x(n)}$, all the contributions are considered as due to the A/D conversion, since in [16] it was shown as the most relevant cause of uncertainty; the following model was adopted:

$$u_{x(n)} = u_q = \frac{2 \cdot A_0 \cdot 2^{-N\text{bit}}}{\sqrt{12}} \tag{18}$$

where A_0 is the A/D full scale and $N\text{bit}$ is the number of effective bit (including also the input noise) of the A/D converter used. With this model, all uncertainty causes are supposed to be taken into account by the effective number of bits. The uncertainty on $x(n)$ can be propagated through the relationships between $M(k)$ and $x(n)$ using the uncertainty combination law suggested by the ISO GUM [19], obtaining, as shown in [17]

$$u_{M(k)}^2 = \frac{C_R(k)R^2(k) + C_I(k)I^2(k)}{S} u_q^2 \tag{19}$$

where $C_R(k)$ and $C_I(k)$ and are reported in Table I.

When $C_I(k)$ is equal to $C_R(k)$, (19) becomes

$$u_{M(k)}^2 = \frac{C_R(k)}{S} u_q^2 = \frac{2 \cdot C_R(k)}{N} u_q^2. \tag{20}$$

Thus, uncertainty on $M(k)$ and consequently on $\Delta M(k)$ depends on the input uncertainty (it depends on turn on $N\text{bit}$ and A_0) and on the number of processed points N, while it is independent on the signal spectrum contents. Moreover $u_{M(k)}$, depends on k but if $k \neq 0$ it is always less or equal to

$$u_{M(k)} \leq \sqrt{\frac{2 \cdot C_R(1)}{N}} u_q. \tag{21}$$

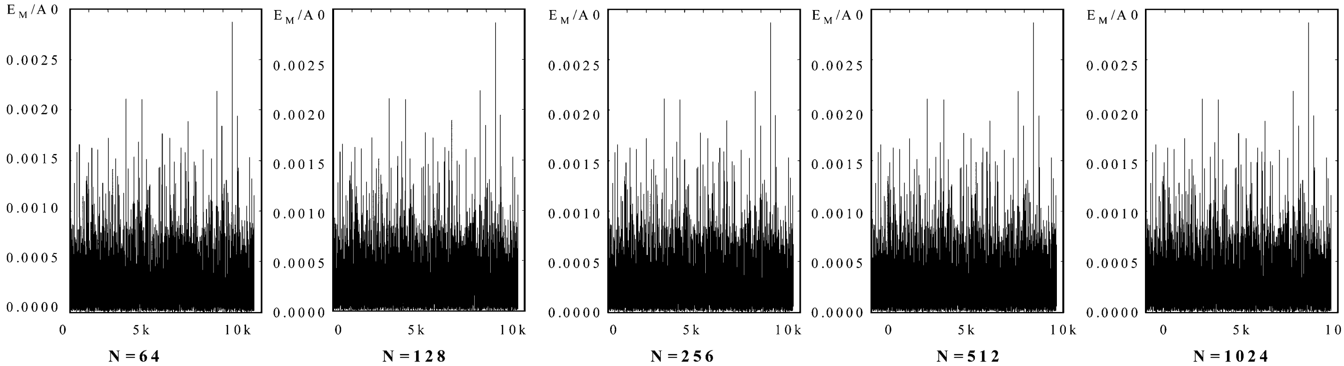

Fig. 2. Observed E_M in a test set of $M = 10000$ elements, for different number of processed points N.

TABLE II
MAXIMUM OF $\Delta M(k)$, E_M, MEASURED FOR DIFFERENT NUMBER OF TONES P

P	2	3	4	5
E_M/A_0	0.0027	0.0031	0.0027	0.0029

C. $\Delta M(k)$: Composition of Contributes

In the evaluation of the suitable threshold th, both the systematic and the random contributions have been taken into account. A simple relationship is assumed

$$th = ME_M + \delta E. \tag{22}$$

ME_M is the maximum $\Delta X(k)$ observed in all the tests and represents the residual errors contribution; δE expresses the contribution of the uncertainty and was posed equal to

$$\delta E = 4 \cdot \sqrt{\frac{2 \cdot C_R(1)}{N}} u_q \tag{23}$$

namely four times the uncertainty on $|X(k)|$. For a Gaussian distribution of the uncertainty, the probability that $\Delta M(k)$ is less than $4 \cdot \delta E$ is greater than 0.999999.

V. NUMERICAL RESULTS: THRESHOLD TUNING

Five different test sets with $M = 50000$ were generated as detailed in Section IV-A, and the maximum value of the $\Delta M(k)$ for each set is measured; the following results were obtained: $E_M/A_0 = (0.0029, 0.0020, 0.0028, 0.0027, 0.0022)$.

In order to verify that the error does not depend on N, the same test sets were made by changing N from 64 to 1024, proving that the maximum of $\Delta X(k)$ does not change significantly. For instance, Fig. 2 shows the measured E_M normalized by A_0 in each one of the signals present in a test set for different values of N. A similar check was made for changing values of P. In particular, tones have been added progressively to a first pair of tones; the results, reported in Table II, prove that the number of tones does not affect the value measured for E_M.

From the abovementioned results, ME_M can be chosen equal to 0.003 A_0, i.e., equal to the maximum E_M measured in all the tests. This choice privileges the reliability of the detection rather than the sensitivity. However, the results of these tests could suggest different values based on a different compromise between a low probability of missed tones (sensibility) and a low probability of false detection (reliability).

As the uncertainty is concerned, (22) is used in order to evaluate δE for different values of processed points N and effective

TABLE III
CONTRIBUTION DUE TO THE UNCERTAINTY δE ON THE ADC FULL SCALE A_0 ($\delta E/A_0$) VERSUS THE NUMBER OF PROCESSED POINTS N AND THE NUMBER OF THE A/D EFFECTIVE BITS $N\mathrm{bit}$

Nbit \ N	64	128	256	512	1024
6	$4.2\cdot10^{-3}$	$3.0\cdot10^{-3}$	$2.1\cdot10^{-3}$	$1.5\cdot10^{-3}$	$1.1\cdot10^{-3}$
7	$2.1\cdot10^{-3}$	$1.5\cdot10^{-3}$	$1.1\cdot10^{-3}$	$7.5\cdot10^{-4}$	$5.3\cdot10^{-4}$
8	$1.1\cdot10^{-3}$	$7.5\cdot10^{-3}$	$5.3\cdot10^{-4}$	$3.7\cdot10^{-4}$	$2.6\cdot10^{-4}$
9	$5.3\cdot10^{-4}$	$3.7\cdot10^{-4}$	$2.6\cdot10^{-4}$	$1.9\cdot10^{-4}$	$1.3\cdot10^{-4}$
10	$2.6\cdot10^{-4}$	$1.9\cdot10^{-4}$	$1.3\cdot10^{-4}$	$9.3\cdot10^{-5}$	$6.6\cdot10^{-5}$
11	$1.3\cdot10^{-4}$	$9.3\cdot10^{-5}$	$6.6\cdot10^{-5}$	$4.7\cdot10^{-5}$	$3.3\cdot10^{-5}$
12	$6.6\cdot10^{-5}$	$4.7\cdot10^{-5}$	$3.3\cdot10^{-5}$	$2.3\cdot10^{-5}$	$1.6\cdot10^{-5}$

TABLE IV
PERFORMANCE OF THE PROPOSED PROCEDURE IN TERMS OF FALSE ALARM (FA) AND MISSED DETECTION (MD) WHEN $ME_M = 0.003A_0$

	N=64		N=256		N=1024	
Nbit	MD	FA	MD	FA	MD	FA
9	1.21%	6.71%	1.00%	7.10%	0.99%	7.16%
12	1.11%	6.96%	0.98%	7.14%	0.99%	7.17%
16	1.08%	7.01%	0.98%	7.15%	0.99%	7.12%

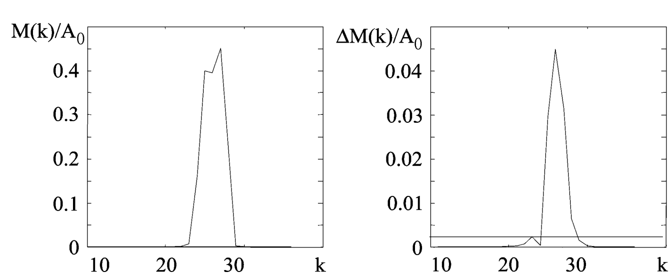

Fig. 3. Example of signal for which the procedure gives an FA.

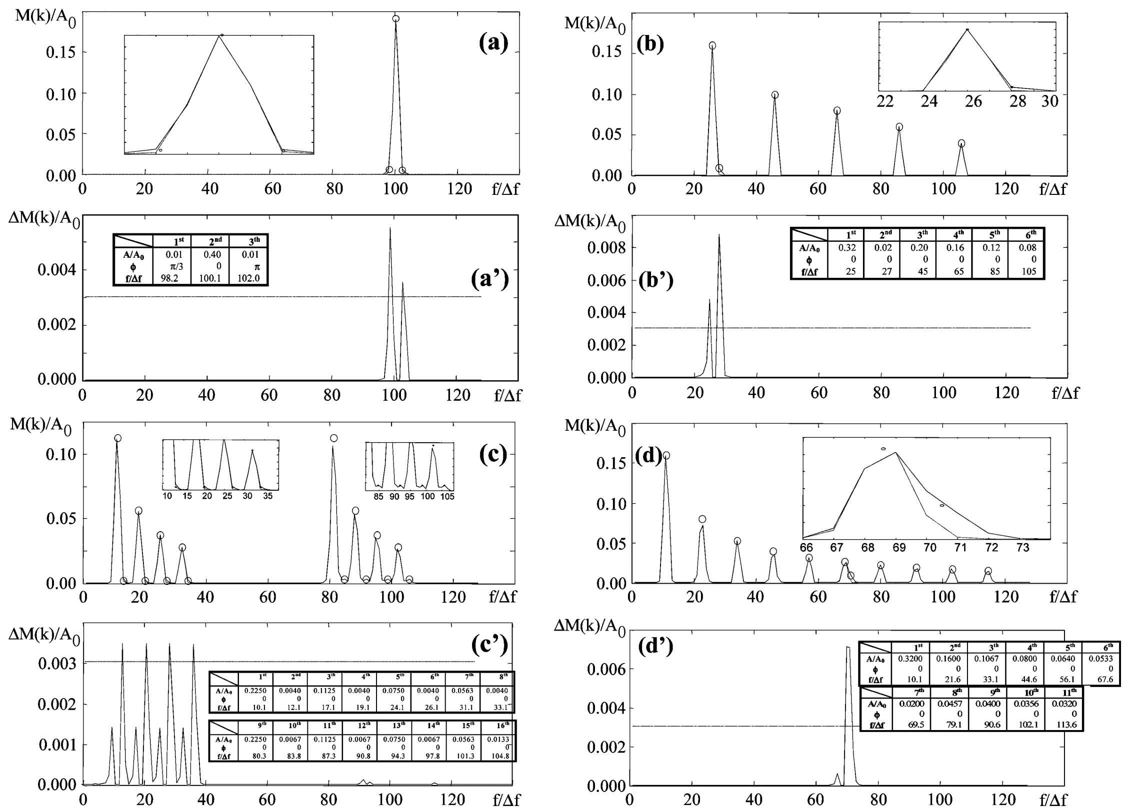

Fig. 4. For four different kinds of signal: the measured amplitude spectrum, $M(k)$. Solid line: the reconstructed one, $\hat{M}(k)$. Dotted line: the position of nominal tones, circles; the calculated $\Delta M(k)$; and the nominal characteristics of the signals.

number of bits of the A/D converter $N\mathrm{bit}$. The results are summarized in Table III. As can be seen, δE is comparable with E_M only for low numbers of bits (very high uncertainty on the input samples).

VI. Numerical Results: Performance Evaluation

After having obtained the thresholds, tests were carried out in order to evaluate the reliability and the sensitivity of the proposed method. Also for the validation phase the LHS approach was followed: tests have been done with signals with 2 tones and a low d_{12}: $d_{12} \subset [1.0, 3.0]$, in order to evaluate the capability of detection of hidden tones. Table IV shows the percentages of missed detection of a hidden tone and of false alarms (i.e., the detection of a nonexistent hidden tone), for different input uncertainties reported as effective numbers of bits, and for different N. The results are fully suitable, and show that the choices of the two contributions are correct since the percentages do not change significantly when operating conditions change. Only for low numbers of points and of bits, a little increase appears due to the increase of the contribution due to the uncertainty. Moreover, false alarms occur in cases of very close tones and when the resolvability is very low (as can be seen in Fig. 3). On the other hand, a missed detection of a tone occurs if it is very close $(d12 < 1.5)$ and its amplitude is significantly lower than the amplitude of the tone that hides it $(\beta_i/\beta_j < 80)$.

Finally, tests have been carried out in the case of one hidden tone among a number of other tones. The results are summarized in Fig. 4, which shows the measured and the reconstructed amplitude spectra, and the difference for signals having a different spectral content. As can be seen, the detection is good both in case of a few tones [Fig. 4(a) and (d)] and for a greater number of tones [Fig. 4(b) and (c)] (it does not depend on P) and regardless of the amplitude of the hiding tones. In Fig. 4(a), the results concerning a signal with two tones hidden by a greater one are reported; as you can see, because of the missed detection of the two low tones, the $\Delta M(k)$ increases in correspondence of both frequencies. In the case of Fig. 4(c), it is interesting to note that the tones in the first group are so close and so low that they are not detected by the IFFTc and then cause an increase of $\Delta M(k)$, causing the signaling of hidden tones. The tones of the second group are distant and strong enough to be detected by the IFFTc; then $\Delta M(k)$ is low and hidden tones are not signaled, as expected.

VII. Conclusions

A method for the detection of spectral components hidden by the spectral leakage of other components in an FFT analysis of a signal has been described in its analytical formulation. The proposed method thresholds the difference between the expected amplitude spectrum based on the results of a first detection of spectral components obtained with the IFFTc and the real amplitude spectrum. The choice of the threshold value has been discussed, and a procedure which follows an ED approach has been adopted.

The performances have been evaluated with several tests in terms of percentages of missed and of false alarms, and have been discussed versus the parameters of the analysis and appear more than interesting.

Moreover, one of the aims of the design and of the implementation of the algorithm was to maintain the computational burden at reasonable levels in order to employ it in applications of digital signal processing where the response time is a fundamental issue, together with the enhanced performance of resolvability. Early results are encouraging under this point of view, and future work will be devoted to the implementation of this improvement in real instrumentation like a digital oscilloscope of new generation.

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