Estimation of Signal Parameters in the Frequency Domain in the Presence of Harmonic Interference: A Comparative Analysis

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***Abstract*—In this paper, a novel method for the estimation of the parameters of the spectral components of a signal, also in the case of harmonic interference, is characterized and compared to other methods proposed in literature. The comparison criteria include the evaluation of residual errors and uncertainties on estimated parameters for different multicomponent signals.**

***Index Terms*—Digital signal processing, fast Fourier transform (FFT), interpolated fast Fourier transform (IFFT), uncertainty.**

I. Introduction

In the field of spectral waveform analysis, digital techniques are very common, especially for the signal parameter estimation. Most of these techniques are based on the windowed discrete Fourier transform (DFT) implemented by the fast Fourier transform (FFT). In particular, many algorithms are presented in literature that allow the estimation of signal parameters, namely the frequency, amplitude, and phase of their spectral components, starting from the DFT samples. The most common approaches suggest processing the DFT samples in the neighborhood of the relative maxima of the amplitude spectrum in order to evaluate the frequency, amplitude, and phase of each spectral component. Consequently, the implementation of these methods include a preliminary analysis of the amplitude spectrum to search for these maxima; then, for each maximum, the component frequency is evaluated, and its amplitude and phase are calculated. Some differences can be observed among the algorithms used in evaluating the frequency, amplitude, and phase of spectral components; they can be grouped as an energy-based approach [1] and an interpolated FFT (IFFT) [2]–[6]. When adjacent spectral components occur, parameter estimations are affected, since tones very close to each other may cause spectral interference. The DFT samples in the neighborhood of the ith peak depend not only on the ith sinusoidal component, but also on the other sinusoids. Neglecting this phenomenon can cause errors in the signal characteristic estimation; in the worst case, a higher harmonic component may hide a lower one. However, some of these methods do not take into account the effects of the harmonic interference. In reference to this problem, Liguori *et al.* [7] proposed a two-step method based on the two-point IFFT, thereinafter called IFFTc, in which the contributions of spectral components altered by the interference are estimated and compensated. The aim of this paper is a comparison between some traditional methods present in literature and the one proposed by the authors in terms of residual bias and uncertainty. With reference to this last point, a "white-box" approach will be followed [7]–[12].

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II. Considered Methods

Since all the methods considered for the comparison are founded on the DFT theory, the terms involved in the DFT calculation will be introduced. Considering a signal $x(t)$ sampled with a sampling period T_S, we have an N-point sequence $\{x(n)\} = \{x(t)|_{nT_s}\}$. The DFT of the signal weighted with a window function $\{w(n)\}$ is defined as follows:

$$X(k) = \frac{1}{S}\sum_{n=0}^{N-1} w(n)\cdot x(n)\mathrm{e}^{-jk\beta_n} \qquad k = 0, \ldots, N-1 \tag{1}$$

where

$$S = \sum_{n=0}^{N-1} w(n) \quad \text{and} \quad \beta_n = \frac{2\pi n}{N}.$$

For all the methods, the performance also depends on the window function employed [5]. In the analysis, only the Hanning window is used since its good capability of filtering the harmonic interference is highlighted in literature [13]. In fact, the rectangular window function would exhibit a heavier bias in the parameter estimation due to the scallop loss while other window functions exhibit a larger main lobe and a worse tone resolvability.

The amplitude spectrum of a signal with P frequency components is

$$x(t) = \sum_{i=1}^{P} A_i \,\mathrm{sen}\,(2\pi f_i t + \phi_i) \tag{2}$$

characterized by P peaks if no tone is hidden. The peak corresponding to the ith tone frequency will be located at index k_i: $k_i = \mathrm{int}(f_i/\Delta f)$ where Δf is the DFT frequency resolution, $\Delta f = f_s/N$. The considered methods [1], [5], [6] evaluate the frequency of ith component as follows:

$$f_i = (k_i + \delta_i)\Delta f, \quad \text{where} -\frac{1}{2} \leq \delta_i \frac{1}{2}. \tag{3}$$

However, these methods differ in the way the δ_i, the amplitude, and the phase are evaluated for each peak. In the following, a brief review of the compared methods is reported.

A. Energy-Based Approach

The energy-based approach provides the parameters δ_i, A_i and ϕ_i by evaluating certain energy parameters related to each spectral component of the signal being analyzed in the frequency domain [10].

In particular, with reference to the ith detected tone on the signal spectrum $X(k)$, the following quantities are evaluated: the energies of the tone E_{x_i} of the tone first derivative E_{xd_i} and of the conjugate symmetric of the tone E_{xc_i}

$$\begin{aligned} E_{x_i} &= \sum_{\ell \in B} M(k_i+\ell)^2 \\ E_{\mathrm{xd}_i} &= \sum_{\ell \in B} \ell \cdot M(k_i+\ell)^2 \\ E_{\mathrm{xc}_i} &= \sum_{\ell \in B} R(k_i+\ell)^2. \end{aligned} \tag{4}$$

The tone characteristics are obtained as follows:

$$\delta_0 = \frac{E_{\mathrm{xd}_i}}{E_{x_i}} \quad A_i = 2 \cdot S\sqrt{\frac{E_{x_i}}{E_w}} \quad \cos^2(\beta_i) = \frac{E_{c_i}}{E_{x_i}} \tag{5}$$

where $E_w = \sum_{k=0}^{N} |W(k)|^2$ is the window energy parameter.

Since most of the window's energy does not spread over a wide range around the center frequency, the energy parameters are evaluated on a few spectral samples, taken within a very narrow frequency band B located around the peak $B \equiv [-K, K]$ (in our tests $K = 5\Delta f$ since it was shown in [1] that this choice is characterized by the best performance).

B. Interpolated DFT

These techniques realize an interpolation of the DFT output based on the window spectrum. In particular, the two-point interpolated FFT determines the spectral component frequency, considering only the two largest samples corresponding to the tone peak [2], [4], [5]. Specifically, this technique evaluates δ_i considering the ratio α_i between the two largest samples corresponding to the peak

$$\alpha_i = \frac{|X(k_i+\varepsilon_i)|}{|X(k_i)|} \tag{6}$$

where $\varepsilon_i = 1 \cdot \mathrm{sign}(|X(k_i+1)| - |X(k_i-1)|)$.

Furthermore, considering the window frequency spectrum $W(k)$, we have [7], [8]

$$\frac{|W(\varepsilon_i - \delta_i)|}{|W(-\delta_i)|} = \frac{|X(k_i+\varepsilon_i)|}{|X(k_i)|} \tag{7}$$

and from (6) and (7), we can obtain δ_i.

For the Hanning window, we have

$$\delta_i = \varepsilon_i \frac{2\alpha_i - 1}{1+\alpha_i} \tag{8}$$

$$\begin{aligned} A_i &= 2\,|X(k_i)| \cdot \frac{\pi\delta_i \cdot \left(1-\delta_i^2\right)}{\mathrm{sen}(\pi\delta_i)} \\ \phi_i &= \arg\left(X(k_i)\right) + \frac{\pi}{2} - \pi\delta_i. \end{aligned} \tag{9}$$

A similar approach is followed for the multipoint IFFT, which evaluates the amplitude and frequency for the ith spectral component, processing the three or five (IFFT3p and IFFT5p, respectively) DFT samples [6], [14] in the neighborhood of the index k_i, shown in (10) and (11) at the bottom of the next page.

C. Proposed Method

The proposed method applies a two-point IFFT, but corrects the effect of the harmonic interference. In particular, the relationships of the IFFT are derived from (9); they are valid for a single tone signal, but cannot be extended to multifrequency signals when harmonic interference is present. In fact, $X(k_i)$ and $X(k_i+\varepsilon_i)$ depends not only on the main lobe of the windowed ith sinusoidal component, but also on those of the other sinusoids. In particular, each DFT sample can be obtained as follows:

$$X(k) = \frac{1}{S}\left[\sum_{i=1}^{P} V_i \cdot W\left(\frac{k\Delta f - f_i}{\Delta f}\right) + \sum_{i=1}^{P} V_i^* \cdot W\left(\frac{k\Delta f + f_i}{\Delta f}\right)\right] \tag{12}$$

with $V_i = (A_i/2j)\mathrm{e}^{j\phi_i}$ $V_i^* = -(A_i/2j)\mathrm{e}^{-j\phi_i}$.

Equation (12) can be rewritten in the neighborhood of k_i, highlighting the contribution due to the ith sinusoid, termed $f_{ki} = k_i \cdot \Delta f$ and $f_{\varepsilon i} = (k_i + \varepsilon_i) \cdot \Delta f$; therefore, we have

$$\begin{aligned} X(k_i) &= \frac{1}{S}\left[V_i W(-\delta_i) + \sum_{r \neq i} V_r W\left(\frac{f_{ki} - f_r}{\Delta f}\right) + \sum_{r=1}^{p} V_r^* W\left(\frac{f_{ki} + f_r}{\Delta f}\right)\right] \\ &= \frac{V_i}{S} W(-\delta_i) + F_i \Rightarrow W(-\delta_i) \\ &= \frac{S}{V_i}\left(X(k_i) - F_i\right) \\ X(k_i+\varepsilon_i) &= \frac{1}{S}\left[V_i W(\varepsilon_i - \delta_i) + \sum_{r \neq i} V_r W\left(\frac{f_{\varepsilon i} - f_r}{\Delta f}\right) + \sum_{r=1}^{p} V_r^* W\left(\frac{f_{\varepsilon i} + f_r}{\Delta f}\right)\right] \\ &= \frac{V_i}{S} W(\varepsilon_i - \delta_i) + B_i \Rightarrow W(\varepsilon_i - \delta_i) \\ &= \frac{S}{V_i}\left(X(k_i+\varepsilon_i) - B_i\right). \end{aligned} \tag{13}$$

Substituting (13) in (7), we obtain

$$\frac{|W(\varepsilon_i - \delta_i)|}{|W(-\delta_i)|} = \frac{|X(k_i + \varepsilon_i) - B_i|}{|X(k_i) - F_i|} = \alpha'_i. \tag{14}$$

Using this corrected α'_ι, the new δ'_i can be evaluated, and from its value, the amplitude and the phase of the ith spectral components are calculated. In particular

$$\delta'_i = \varepsilon_i \frac{2\alpha'_i - 1}{1 + \alpha'_i} \tag{15}$$

$$A_i = 2 \cdot \frac{\pi\delta'_i \left(1 - \delta'^2_i\right)}{\text{sen}\left(\pi\delta'_i\right)} \cdot |X(k_i) - F_i|$$

$$\phi_i = \arg\left(X(k_i) - F_i\right) + \frac{\pi}{2} - \pi\delta'_i. \tag{16}$$

It has to be noted that in order to estimate the correction factors F_i and B_i, the frequency, amplitude, and phase of the signal tones would need to be known. However, in this application, the values of the signal parameters are the unknown quantities that have to be measured. We propose a two-step procedure in which, firstly, the two-point IFFT is applied, and then the measured values are used in the evaluation of the correction factors $\hat{F}_i$ and $\hat{B}_i$. Obviously, the obtained correction factors are still corrupted by harmonic interference; using these values, instead of the actual ones, F_i and B_i, causes the correction to be incomplete (the residual errors will be evaluated in the next section). Thus, the proposed procedure is described as follows.

1) The two-point IFFT is applied to each peak in order to estimate frequency, amplitude, and phase of the corresponding spectral component, neglecting the harmonic interference effects.
2) By using these estimations, the correction factors for each peak are determined.
3) The relationships (15) and (16) are used in order to calculate the frequencies, amplitudes, and phases of each spectral component corrected by the harmonic interference effects.

III. Residual Error Estimation

In order to evaluate the performance of the proposed method and to make a comparison with traditional solutions, suitable signals for the specific analysis are used. Multifrequency signals $x(t) = \sum_{i=1}^{P} A_i$ sen $(2\pi f_i t + \phi_i)$ are considered and the tests are made for different values of P, N, f_i, A_i, and ϕ_i. In particular, for terms $f_i = (k_i + \delta_i) \cdot \Delta f$, $A_i = \beta_i \cdot A_0$, $d_{ij} = (f_j - f_i/\Delta f)$, the experiments are made by changing the value of k_i, δ_i, d_{ij}, β_i, ϕ_i, and N in order to analyze the dependence of the harmonic interference effects on the signal characteristics and the measurement system's settings. Once the signal and the measurement parameters are fixed, the signal samples are generated and the algorithms are run on these points, and finally, the systematic errors are evaluated. The following analysis is mainly concentrated on the estimation of δ. Since the frequency, amplitude, and phase are calculated from δ, the bias and the uncertainty of δ propagates through the algorithms, affecting the estimations of the other parameters. In the first tests $(A–E)$, $P = 2$ is fixed.

A. Sensitivity to Frequency

Tests are carried out for different values of the first tone frequency $f_1 = (k_1 + \delta_1) \cdot \Delta f$, having a fixed value of d_{12}. As an example, the maximum measured errors for each k_1 versus δ_1 are reported in Fig. 1(a) (where $N = 128$), and the errors versus δ_1 are shown in Fig. 1(b), having fixed $k_1 = N/4$. For all the considered methods, the results show that the errors decrease with k_1, as long as k_1 remains less than a threshold (about nine), and then they become constant. A little dependence on δ_1 has been evidenced, and this dependence is less noticeable for the IFFTc.

Hereinafter, the k_1 and δ_1 values will be kept constant; $k_1 = N/4$, since k_1 does not affect the error provided it is greater than nine, and $\delta_1 = 0.25$, in order to have an intermediate behavior [see Fig. 1(b)] between 0 (no error) and $\delta_1 = 0.5$ (worst case).

B. Sensitivity to the Tone Distance d_{ij}

In Fig. 2 and in Table I, the envelopes of the absolute errors of δ versus the frequency and the percentage of errors on the amplitude of the first tone versus the distance d_{12} between tones are shown. Some considerations are discussed below.

1) The energy-based approach is strongly affected by the interference; the error becomes comparable with the other methods only for d_{12} greater than eight.
2) The three-point IFFT and the five-point IFFT do not give very good results for tones which are very close $(d_{12} < 5)$, but results are similar to those obtained with the two-point IFFT. This is due to the use of DFT samples that are altered heavily by the near tone $X(k_i \pm 2)$. However, the five-point IFFT gives better results among the considered traditional methods for higher distances $(d_{12} > 10)$.
3) The proposed IFFTc is characterized by a residual error that is lower than that of all the other methods for very small distances, and for a higher d_{12}, the errors still remain the smallest.

$$_3\delta_i = \varepsilon_i \frac{|X(k_i+1)| - |X(k_i-1)|}{2 \cdot |X(k_i)| + |X(k_i+1)| + |X(k_i-1)|} \tag{10}$$

$$_5\delta_i = 3\frac{2 \cdot [|X(k_i+1)| - |X(k_i-1)|] + \varepsilon_i [|X(k_i+2)| + |X(k_i-2)|]}{6 \cdot |X(k_i)| + 4 \cdot [|X(k_i+1)| + |X(k_i-1)|] + \varepsilon_i [|X(k_i+2)| - |X(k_i-2)|]}. \tag{11}$$

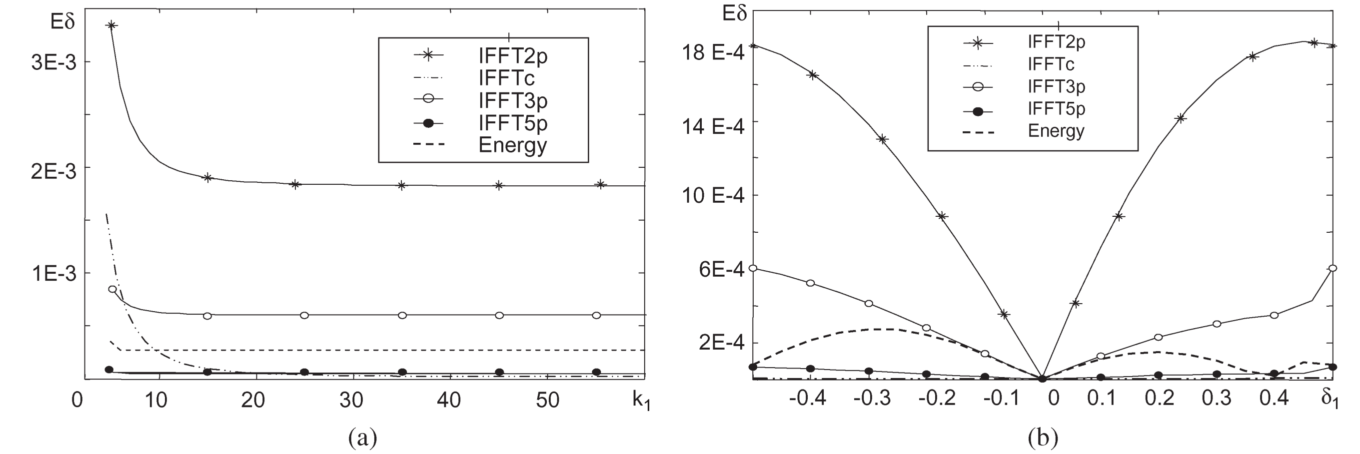

Fig. 1. For a signal with $P = 2$, $d_{12} = 10$, $\beta_1 = \beta_2 = 1$; $f_1 = f_2 = 0$. (a) Maximum measured E_δ for all possible values of δ_1 versus k_1. (b) E_δ versus δ_1 with $k_1 = N/4$.

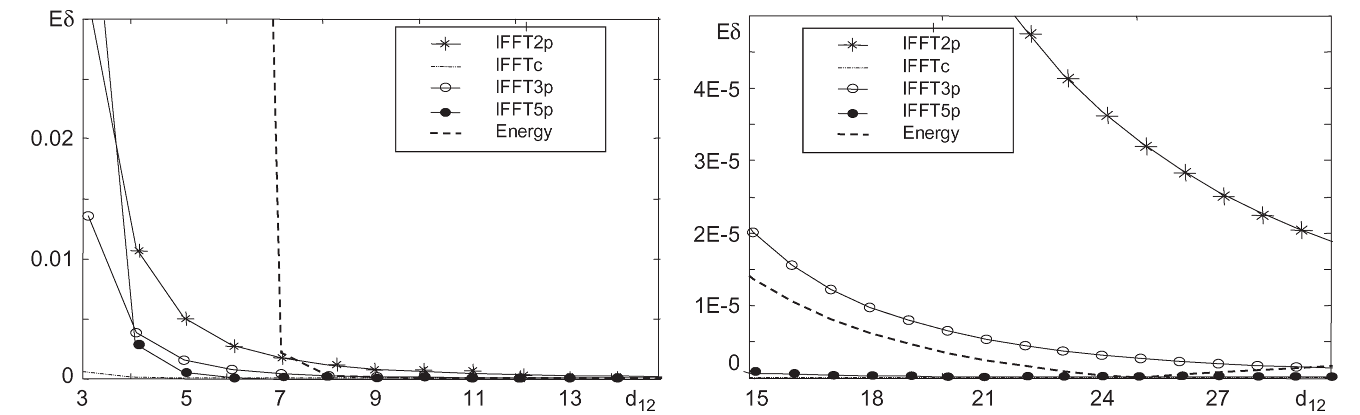

Fig. 2. E_δ versus d_{12} for the different methods using $P = 2$, $k_1 = N/4$; $\delta_1 = 0.25$; $\beta_1 = \beta_2 = 1$; $\phi_1 = \phi_2 = 0$; $N = 256$.

TABLE I
MAXIMUM E_δ AT DIFFERENT d_{12}; $P = 2$, $k_1 = N/4$; $\delta_1 = 0.25$; $\beta_1 = \beta_2 = 1$; $\phi_1 = \phi_2 = 0$; $N = 256$

d_{12}	IFFT-2p	IFFTc	IFFT-3p	IFFT-5p	Energy
3	3.2E-2	5.6E-4	1.4E-2	4.6E-2	1.6E-1
4	1.1E-2	1.0E-4	3.8E-3	3.0E-3	2.0E-1
5	5.0E-3	2.7E-5	1.5E-3	6.7E-4	1.8E-1
10	5.4E-4	4.4E-7	9.9E-5	8.8E-6	5.5E-5
15	1.5E-4	4.0E-8	2.0E-5	7.8E-7	1.4E-5
20	6.4E-5	7.4E-9	6.5E-6	1.4E-7	3.5E-6
25	3.2E-5	2.1E-9	2.7E-6	3.7E-8	1.2E-7
30	1.9E-5	8.5E-10	1.3E-6	1.3E-8	1.6E-6

The subsequent analysis will be focused on the case of small distances between the tones where the harmonic interference effects are significant.

C. Sensitivity to the Tone Amplitude β_i

In Fig. 3, the trends of the errors of δ for both tones are reported versus the amplitude of the second tone (β_2), for the proposed IFFTc and for the five-point IFFT.

As expected, for the other methods, the residual errors increase on the first sinusoid and decrease for the second one when β_2; consequently, A_2 increases. The higher tone gives rise to higher effects and is less susceptible than the smaller one, while if the amplitudes are equal, the harmonic interference effects have comparable intensities. Instead, the IFFTc residual errors are always very low, as well as very slightly influenced by β_2, which proves the capability of the proposed method in correcting the harmonic interference. A small increase is shown, since the estimations of the correction factors get slightly worse because of the high mutual interference.

D. Sensitivity to the Tone Phases

In order to highlight the relationships between the difference of tone's initial phases and the residual errors, tests were made by fixing all the signal parameters and changing ϕ_2. In Fig. 4, the errors of δ_1 for the IFFTc and the five-point IFFT are reported. The interference is stronger for $\phi_2 = \phi_1 + \pi/2$; also, in this case, the IFFTc is characterized by better performance (an order of magnitude of lower errors), and it is less influenced by the signal characteristics.

E. Sensitivity to the Number of Points N

In Fig. 5, the residual errors versus N are reported for a given signal. There is no dependence of the residual errors on

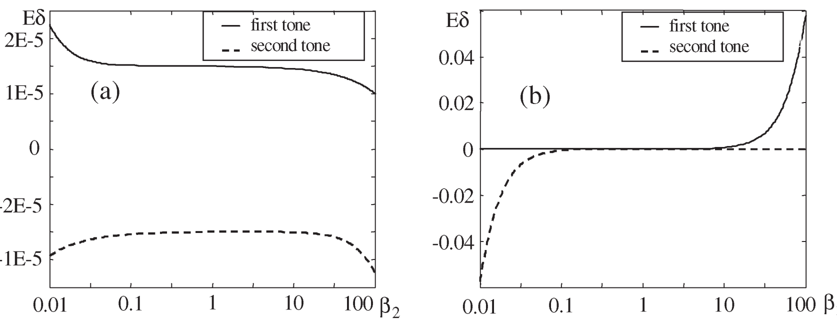

Fig. 3. $E_{\delta 1}$ $E_{\delta 2}$ versus β_2 for (a) IFFTc. (b) IFFT5p. $P = 2$, $k_1 = N/4$; $\delta_1 = 0.25$; $d_{12} = 4.5$; $\phi_1 = \phi_2 = 0$; $N = 256$.

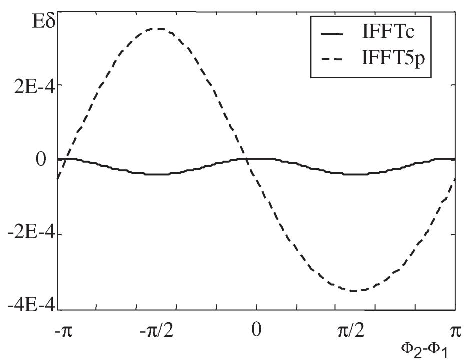

Fig. 4. E_δ versus $\phi_2 - \phi_1$ for IFFTc and IFFT5p $P = 2$, $d_{12} = 4.5$; $\beta_1 = \beta_2 = 1$; $N = 256$.

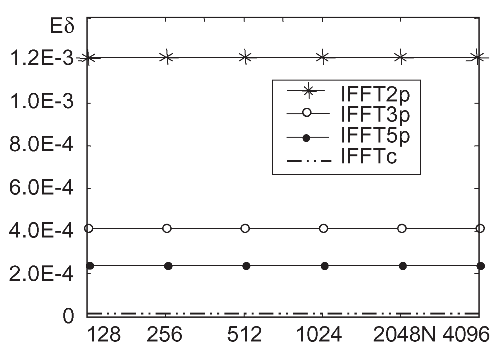

Fig. 5. E_δ versus N. $P = 2$, $d_{12} = 4.5$; $\beta_1 = \beta_2 = 1$; $\phi_1 = \phi_2 = 0$; $N = 256$.

the number of the processed points. This is expected, since the error of δ is considered, not the error on the frequency. With the sampling frequency fixed the greater the number of points, the lesser the spectral resolution and the lesser the error on frequency. The best performance of the IFFTc are confirmed for each N.

F. Sensitivity to Number of Tones P

Several tests were carried out by changing the number of sinusoidal components P of the test signal and the distance between them. From these, we can say that the proposed IFFTc is influenced by the number of tones in a negligible manner, and on the other hand, the residual errors increase only slightly due to the nonexact estimation of the correction factors for a high number of interfering tones. Conversely, for the other methods, there is a superimposition of effects, and consequently, the estimation gets significantly worse, as shown in Fig. 6, where the errors on the frequency estimation versus d are reported for the cases $P = 2$ and $P = 4$.

IV. Uncertainty

To evaluate the combined standard uncertainty on the tone parameters provided by the different methods, a white-box theoretical approach is followed. The uncertainty propagation law [8] is applied to their relationships. As mentioned in the previous section, in presence of considerable harmonic interference, the residual error on the δ_i estimation is significant especially for traditional methods, and consequently, it has to be taken into account. To this aim, a new quantity is defined ($\breve{\delta}_i$) from which the uncertainties of the signal parameters (frequency, amplitude, and phase for each spectral component) are evaluated. Basically, in presence of interference, the uncertainty propagation law has to be applied to the implemented relationships (16), using $u_{\breve{\delta}_i}$ instead of u_{δ_i}.

For all the considered methods, we define $\breve{\delta}_i$ as follows:

$$\breve{\delta}_i = \delta_i + \delta_E \tag{17}$$

where δ_i is returned by the specific algorithm, and δ_E takes into account the residual error. In particular, δ_E is a random variable with a mean equal to zero; consequently, it does not alter the value estimated by the algorithm and with a standard deviation related to the residual error.

The uncertainty of $\breve{\delta}_i$, u_{δ_i}, has to be evaluated as follows:

$$u^2_{\breve{\delta}_i} = u^2_{\delta_i} + u^2_{\delta_E} \tag{18}$$

where u_{δ_i} is the uncertainty on the estimated δ, while u_{δ_E} is the uncertainty due to the residual error and is equal to the standard deviation of the random variable δ_E.

A. Uncertainty on the Estimated δ

In literature, some studies based on a white-box approach for the uncertainty evaluation of δ for the considered traditional methods are present. In [11], the analytical evaluation of the uncertainty on the signal parameter estimation is obtained by means of the energy-based algorithm and the two-point IFFT. The combined uncertainties on the final results (tone frequency, amplitude, and phase) due to the propagation of the uncertainty on the input samples are analytically evaluated. For both

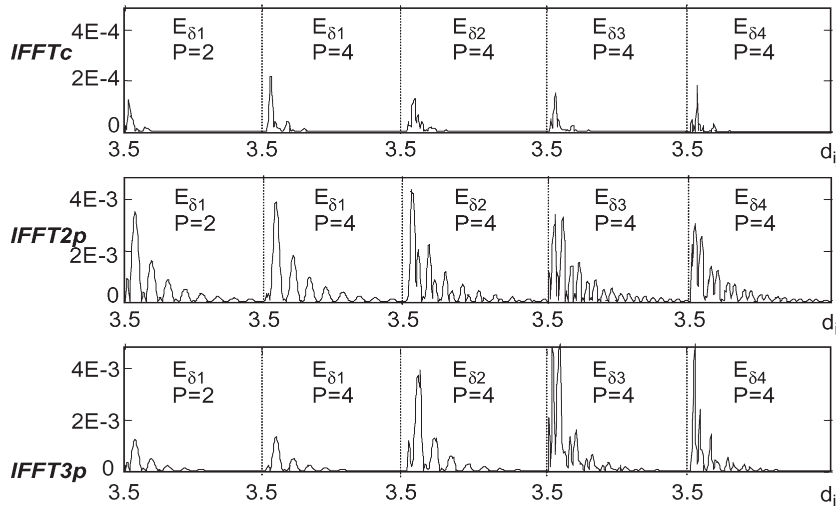

Fig. 6. E_δ versus d_{ij} for two signals with different P. $\beta_i = 1$; $\phi_i = \phi$; $N = 256$.

algorithms, the uncertainty of the final results will be evaluated combining the uncertainty of the DFT samples obtained as in [10]. The results reported in [11] show that the uncertainties obtainable with the two algorithms are very similar, even if the uncertainties obtained by using the energy-based approach is slightly greater than the ones obtained by using the IDFT. Moreover, the uncertainty decreases with N for both methods and particularly for an effective number of bits N_{bit} of the ADC greater than 12.

A very similar approach was followed in [14] in order to characterize the multipoint IFFTs in terms of bias and uncertainty. The author demonstrates that the uncertainty of the three-point IFFT is lower than the five-point IFFT and greater than two-point IFFT; moreover, all the IFFT uncertainties decreases as N increases. Following the same approach, the proposed IFFTc algorithm also has to be characterized. Applying the uncertainty propagation law to (15), we have

$$u_{\delta_i'}^2 = \left(\frac{\partial \delta_i}{\partial \alpha_i}\right)^2 u_{\alpha_i'}^2 = \left(\frac{3}{(1+\alpha_i)^2}\right)^2 \cdot u_{\alpha_i'}^2. \tag{19}$$

The uncertainty of α_i' depends on the uncertainty of the two used DFT samples $X(k_i)$, $X(k_i + \varepsilon_i)$ and on the uncertainty of the correction factors B_i and F_i. In this case, some approximations can be made; for instance, in the term $|X(k_i) - F_i|$, F_i e u_{F_i} are much lesser than $X(k)$ and $u_{X(k)}$, respectively, since the Hanning window has a fast decay and $W(k\Delta f \pm f_i/\Delta f)$ is very small.

Consequently, we can pose $u_{\delta i} = u_{\delta i}'$.

Such an approximation has been verified numerically, comparing the variability of the estimations of δ_i obtained by the two-point IFFT with the variability of estimations of δ_i' for different kinds of signals and in different operation conditions. The variability of δ_i' has always turned out as significantly lesser than the variability of δ_i $(\sigma_{\delta i'} < 1.1\ \sigma_{\delta i})$; thus, the approximation can be considered to be valid. The description of numerical simulations carried out for the assessment of this approximation has not been reported for sake of brevity.

B. Uncertainty Due to the Residual Error

The uncertainty due to the residual error is evaluated for each method measuring the standard deviation on a set of similar signals. In particular, since the analysis carried out in Section III shows that the residual error depends mainly on d_{ij}, the test sets have been composed for each N and β_{ij}, d_{ij}, changing δ_1 to $[-0.5,\ 0.5]$, ϕ_2 to $[-\pi/2, \pi/2]$, and also d_{ij} is changed to $[d_0 - 0.5, d_0 + 0.5]$. In Table II, the measured standard deviations versus d_0 for the considered methods are reported for a signal with two tones $(P = 2)$. As we can see, the proposed IFFTc is characterized by a $\sigma_{\delta E}$ much lower than the other methods (up to two to three orders of magnitude); also, the variability versus d and N is much lower than that of the other methods. Moreover, the considerable worsening of the traditional methods when the two tones have different amplitudes does not happen for the IFFTc.

C. Examples of Combined Uncertainty

In order to better quantify the contribution of the δ_E on the total uncertainty, it is necessary to introduce some parameters concerning the hardware configuration (number of effective bits N_{bit}, full scale fs of the A/D converter), the operative condition (number of processing points N), as well as the characteristics of the input signal. As an example, Fig. 7 reports $u_{\breve{\delta}_i}$, u_{δ_i} and u_{δ_E} for IFFTc, two-point IFFT, and five-point IFFT in the function of the distance between the two tones d_{12}, the number of processing points, and the number of effective bits of the ADC. Considering the IFFTc algorithm, the contribution of the error is always practically negligible, except for small values of d $(d < 4)$ or for a high number of points (since in this case, the uncertainty is small). For the two-point IFFT, the error is always bigger than the uncertainty, except for low numbers of the ADC effective bits. The five-point IFFT has a similar behavior, but shows a significantly smaller error particularly for high values of d_{12}.

The residual error of the traditional method is maximum especially for very close tones; consequently, in these cases,

TABLE II
MEASURED STANDARD DEVIATIONS $\sigma_{\delta E}$, OF THE RESIDUAL ERROR ON δ VERSUS d_{12}

d_0	N=256;β_{12}=1; f_1=(N/4+δ)·Δf					N=256;β_{12}=0.2; f_1=(N/4+δ)·Δf					N=256;β_{12}=0.1; f_1=(N/4+δ)·Δf					N=256;β_{12}=0.05; f_1=(N/4+δ)·Δf				
	IFFT 2p	IFFTc	IFFT 3p	IFFT 5p	Ener.	IFFT 2p	IFFTc	IFFT 3p	IFFT 5p	Ener.	IFFT 2p	IFFTc	IFFT 3p	IFFT 5p	Ener	IFFT 2p	IFFTc	IFFT 3p	IFFT 5p	Ener
4	5.0E-3	8.8E-5	4.0E-3	1.6E-2	1.3E-1	2.5E-2	8.8E-5	1.9E-2	9.8E-2	2.4E-1	5.0E-2	8.8E-5	3.6E-2	2.0E-1	2.5E-1	1.0E-1	9.5E-5	8.3E-2	5.0E-1	2.6E-1
5	2.4E-3	2.0E-5	1.3E-3	1.2E-3	5.7E-1	1.2E-2	2.0E-5	6.3E-3	1.2E-2	4.5E-1	2.4E-2	2.0E-5	1.2E-2	2.7E-2	3.0E-1	4.8E-2	2.0E-5	2.4E-2	5.7E-2	2.5E-1
6	1.3E-3	6.3E-6	5.5E-4	4.3E-4	5.4E-1	6.6E-3	6.2E-6	2.7E-3	2.8E-3	1.7	1.3E-2	6.2E-6	5.4E-3	8.0E-3	1.8	2.6E-2	6.3E-6	1.1E-2	2.0E-2	1.7
7	8.3E-4	2.4E-6	2.8E-4	2.2E-4	3.2E-2	4.1E-3	2.4E-6	1.4E-3	1.2E-3	4.8E-1	8.2E-3	2.4E-6	2.8E-3	2.8E-3	9.0E-1	1.6E-2	2.6E-6	5.5E-3	7.8E-3	1.3
8	5.5E-4	1.1E-6	1.6E-4	1.3E-4	5.4E-4	2.7E-3	1.1E-6	7.8E-4	6.6E-4	1.4E-2	5.4E-3	1.2E-6	1.6E-3	1.4E-3	5.3E-2	1.1E-2	1.5E-6	3.1E-3	3.4E-3	1.9E-1
9	3.8E-4	5.3E-7	9.8E-5	8.5E-5	9.6E-5	1.9E-3	6.1E-7	4.8E-4	4.3E-4	1.5E-3	3.8E-3	8.2E-7	9.6E-4	8.7E-4	6.2E-3	7.5E-3	1.4E-6	1.9E-3	1.9E-3	2.5E-2
10	2.8E-4	2.9E-7	6.3E-5	5.9E-5	4.7E-5	1.4E-3	4.5E-7	3.1E-4	2.9E-4	3.8E-4	2.8E-3	7.6E-7	6.3E-4	5.9E-4	1.4E-3	5.5E-3	1.4E-6	1.3E-3	1.2E-3	5.6E-3
15	8.3E-5	1.1E-7	1.2E-5	1.5E-5	1.2E-5	4.0E-4	5.3E-7	5.9E-5	7.5E-5	4.1E-5	6.7E-4	1.2E-6	9.2E-5	1.2E-4	6.5E-5	1.6E-3	2.1E-6	2.4E-4	3.0E-4	1.8E-4
20	4.1E-5	1.4E-7	4.7E-6	7.0E-6	9.1E-6	2.0E-4	7.0E-7	2.3E-5	3.5E-5	2.0E-5	4.0E-4	1.4E-6	4.7E-5	6.9E-5	3.6E-5	8.0E-4	2.8E-6	9.3E-5	1.4E-4	7.0E-5

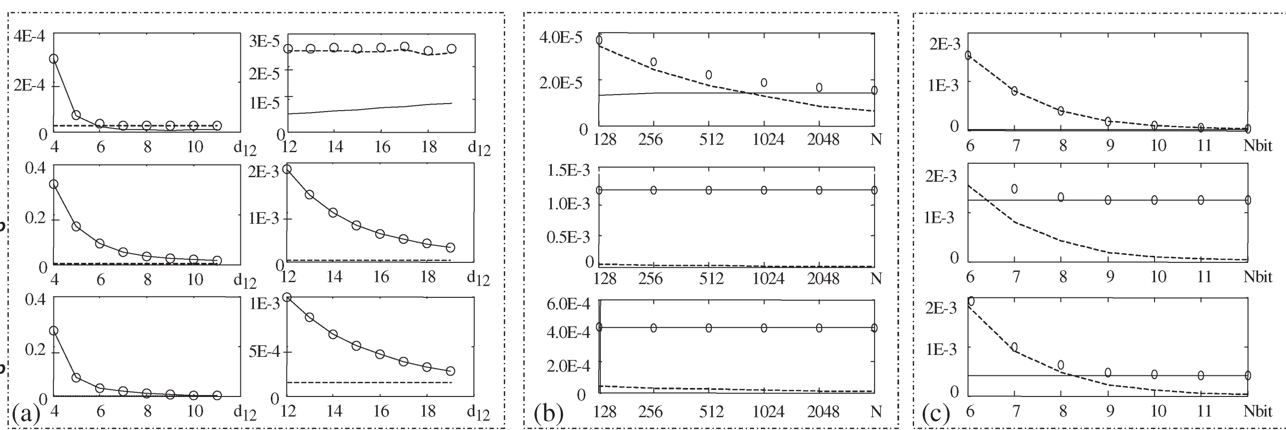

Fig. 7. Contributions to the uncertainty (dotted: $u_{\breve{\delta}_i}$; dashed: u_{δ_i}; solid: u_{δ_E}) for a signal with $P = 2$, $k_1 = N/4$, $\delta_1 = 0.25$, $\beta_1 = \beta_2 = 1$, and an ADC with $\mathrm{fs} = 2A_0$. (a) Versus d_{12} with $N = 256$, $N_{\mathrm{bit}} = 12$. (b) Versus N with $d_{12} = 4.5$, $N_{\mathrm{bit}} = 12$. (c) Versus N_{bit} with $d_{12} = 4.5$, $N = 256$.

the contribution to the combined uncertainty of the provided δ_i of the residual error is very significant.

V. CONCLUSION

In this paper, a novel method (IFFTc) for the evaluation of the parameters of spectral components, which are able to correct the effects of harmonic interference have been characterized in comparison to other methods proposed in literature. The first sequences of numerical tests are run in order to evaluate the sensitivity of the methods to different operating conditions and to different values of typical influence quantities. This analysis has shown that the IFFTc residual errors are always lower than the errors of the other methods, especially when the harmonic interference is significant. Moreover, while traditional methods are sensitive to signal characteristics and to the frequency distance between the spectral components, the IFFTc is much less sensitive to the specific characteristics of the signal.

A second series of tests has allowed a comparison of the levels of uncertainty on the parameter estimations of the methods, obtained through a combination of a study of the propagation of the uncertainty through the algorithms and a statistical treatment of the residual error. These tests have shown the accuracy of the proposed method that has given the lowest uncertainty values for each kind of signal.

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