The role of optimal damping allocation to control torsional seismic response in asymmetric-plan systems

L. Petti, M. De Iuliis, B. Palazzo
Department of Civil Engineering – University of Salerno- Italy

ABSTRACT: A comprehensive study on the torsional seismic response controlling capacity of extra-structural dissipation devices in the asymmetric-plan buildings is herein presented. Effects of the plan-wise distribution of supplemental damping on torsionally dynamic behaviour have been investigated by using modal analysis techniques in the state space representation. Parametrical analysis leads to the optimal plan-wise design for different allocation patterns of damping resources on varying the dynamic characteristics of the asymmetric-plan system. The numerical constraints on the mechanical parameters related to the pratical application of the proposed control strategy are explicitally take into account. Results are carried out by applying H_2 and H_∞ norm control methods.

1 INTRODUCTION

Studies on the seismic response of asymmetric-plan systems have always aroused considerable interest in the scientific community, Hejal & Chopra (1987), Goel & Booker (2001). The importance of torsional effects on the seismic behaviour of structures having an irregular plan distribution of mass and stiffness is generally known and it is taken into account in aseismic provisions and guidelines for the design of seismic-resistant systems.

From the beginning of the 1990s, assessment studies started to evaluate the possibilities of utilising extra-structural damping in order to reduce seismic demand in asymmetric-plan systems, Goel (1998). Recent studies have demonstrated the effectiveness of such a control strategy in reducing both the linear, Goel (2000), Lin & Chopra (2001), and non-linear, Goel & Booker (2001), seismic response of asymmetric systems through the use of viscous-fluid devices. These studies have pointed out the importance of the plan-wise distribution of additional damping devices by supplying a sort of design guideline. The result of these studies is to arrange the supplemental dampers such that the damping eccentricity respect to the mass centre takes on the largest value with algebraic sign opposite to the structural eccentricity. This means locate the supplemental damping centre on the flexible edge side. Recently, the design concept of "torsional balance" was presented, De La Llera et al. (2004). It's defined as a property of asymmetric structure that leads to similar deformation demand in structural members equidistant from the geometric center of the building plan. This concept allows for the definition of a new statistical approach to optimally locate extra-structural dissipation devices.

The present study faces the problem locate the "Empirical Center of Balance (ECB)" at equal distance from both edges of the building plan through methodologies of "vibration control theory". In particular, linear seismic response of non-proportional damped systems is investigated through the use of modal analysis techniques and H_2 and H_∞ transfer function norms. Parametrical analyses are carried out for the definition of supplemental damping design criteria by considering both mass and stiffness properties of the asymmetric system and numerical constraints on the mechanical parameters related to the pratical application of the control strategy. Finally, the proposed design criteria have been tested through the dynamic analysis of asymmetric systems subjected both to synthetic and recorded seismic events, having the aim to demostrate its effectiveness in terms of perfomance and robustness.

2 DYNAMIC ANALYSIS OF IRREGULAR SYSTEMS EQUIPPED WITH SUPPLEMENTAL DAMPING

Let us consider the structure shown in figure 1 described in a co-ordinate system where the origin of the axes is located to coincide with the centre of stiffness C_K and the direction of x-axis is described by the line connecting C_K with the centre of masses C_M. System asymmetry is defined by eccentricity

e , which is the distance between the centres of mass and stiffness.

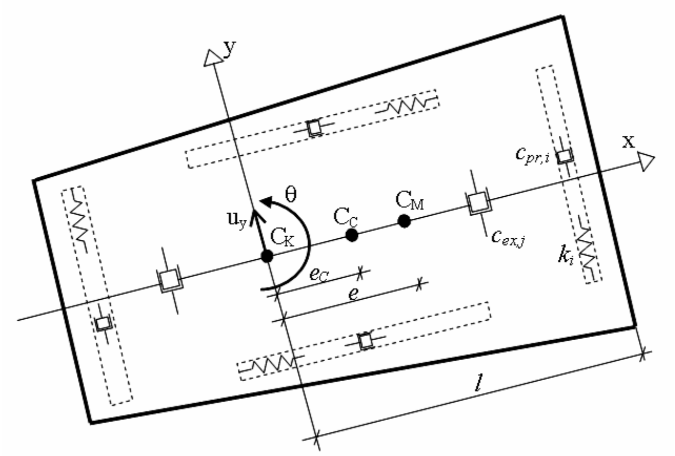

Figure 1. Asymmetric structure equipped with extra-structural dampers

The system is characterized by its natural damping parameters proportional to its mass and stiffness by means of Rayleigh coefficients, α and β, and with extra-structural damping devices described by viscous damping constants $c_{x,i}$ and $c_{y,i}$ respectively in directions x and y. The equations of motion for are derived for the coupled two degrees of freedom as:

$$\begin{cases} m(\ddot{y}+\frac{e}{l}(l\ddot{\theta}))+\left[\alpha m(\dot{y}+\frac{e}{l}(l\dot{\theta}))+\beta k_y\dot{y}\right] \\ \qquad +c_{y,ext}(\dot{y}+\frac{e_C}{l}(l\dot{\theta}))+k_y y=-m\ddot{u}_{g,y} \\ m(\frac{e}{l}\ddot{y}+\frac{\rho_m^2}{l^2}(l\ddot{\theta}))+\left[\alpha m(e\dot{y}+\frac{\rho_m^2}{l^2}(l\dot{\theta}))+\beta k_y\frac{\rho_k^2}{l^2}(l\dot{\theta})\right] \\ \qquad +c_{y,ext}(e_C\dot{y}+\frac{\rho_C^2}{l^2}(l\dot{\theta}))+k_y\frac{\rho_k^2}{l^2}(l\theta)=-m\frac{e}{l}\ddot{u}_{g,y} \end{cases} \quad (1)$$

In (1), the supplemental damping system is described by eccentricity e_C between the centre of damping $C_C \equiv (x_C, y_C)$ and the centre of stiffness C_K, the radius of gyration ρ_C and the overall damping coefficient $c_{y,ext}$ in direction y:

$$x_C=\sum_{i=1}^{n}c_{y,i}x_i\Big/\sum_{i=1}^{n}c_{y,i} \quad y_C=\sum_{i=1}^{n}c_{x,i}y_i\Big/\sum_{i=1}^{n}c_{x,i}$$
$$\rho_C=\sqrt{\frac{\sum_{i=1}^{n}c_{y,i}x_i^2+\sum_{i=1}^{n}c_{x,i}y_i^2}{\sum_{i=1}^{n}c_{y,i}+\sum_{i=1}^{n}c_{x,i}}} \quad c_{y,ext}=\sum_{i=1}^{n}c_{y,i} \quad (2)$$

In the following, l indicates the projection along the x-axis of the edge of the system, n is the number of dampers, and finally, x_i and y_i the coordinates which identify the position of the i-nth device.

Equations (1) can be rewritten in the form:

$$\begin{bmatrix}1 & \varepsilon\\ \varepsilon & \lambda_M^2\end{bmatrix}\begin{Bmatrix}\ddot{y}\\ l\ddot{\theta}\end{Bmatrix}+\left(\alpha\begin{bmatrix}1 & \varepsilon\\ \varepsilon & \lambda_M^2\end{bmatrix}+\beta\omega_y^2\begin{bmatrix}1 & 0\\ 0 & \lambda_K^2\end{bmatrix}\right)\begin{Bmatrix}\dot{y}\\ l\dot{\theta}\end{Bmatrix}+$$
$$+2\xi_y\omega_y\begin{bmatrix}1 & \varepsilon_C\\ \varepsilon_C & \lambda_C^2\end{bmatrix}\begin{Bmatrix}\dot{y}\\ l\dot{\theta}\end{Bmatrix}+\omega_y^2\begin{bmatrix}1 & 0\\ 0 & \lambda_K^2\end{bmatrix}\begin{Bmatrix}y\\ l\theta\end{Bmatrix}= \quad (3)$$
$$=-\begin{bmatrix}1 & \varepsilon\\ \varepsilon & \lambda_M^2\end{bmatrix}\begin{Bmatrix}1\\ 0\end{Bmatrix}\ddot{u}_{g,y}$$

where the dynamic properties of the system are described by means of the following set of parameters:

- the translation vibration circular frequency of the system, ω_y;
- the supplemental damping, ξ_y;
- the structural eccentricity, $\varepsilon = e/l$;
- the spreads of mass and stiffness about their distribution centroids, respectively $\lambda_M = \rho_M / l$ and $\lambda_K = \rho_K / l$;
- the supplemental damping eccentricity, $\varepsilon_C = e_C / l$;
- the spread of damping about its centroid, $\lambda_C = \rho_C / l$.

In a matrix form (3) can be represented as:

$$\mathbf{M}\ddot{\boldsymbol{\zeta}}+(\alpha\mathbf{M}+\beta\mathbf{K})\dot{\boldsymbol{\zeta}}+\mathbf{C}_{ext}\dot{\boldsymbol{\zeta}}+\mathbf{K}\boldsymbol{\zeta}=-\mathbf{M}\mathbf{I}_{\mathbf{g}}\ddot{u}_g \quad (4)$$

where $\mathbf{M}, \mathbf{C}_{ext}$ and $\mathbf{K}$ are the mass, supplemental damping and stiffness matrices respectively, $\mathbf{I}_g$ is the influence vector of the ground motion and $\boldsymbol{\zeta}=[y \quad l\theta]^T$ the displacement vector. he supplemental damping may be considered as a control action described by $\mathbf{u}=\mathbf{C}\dot{\boldsymbol{\zeta}}$. Equation (4) can therefore be rewritten in the form:

$$\mathbf{M}\ddot{\boldsymbol{\zeta}}+(\alpha\mathbf{M}+\beta\mathbf{K})\dot{\boldsymbol{\zeta}}+\mathbf{K}\boldsymbol{\zeta}=-\mathbf{M}\mathbf{I}_{\mathbf{g}}\ddot{u}_g-\mathbf{u} \quad (5)$$

which in state space representation leads to:

$$\begin{Bmatrix}\dot{\boldsymbol{\zeta}}\\ \ddot{\boldsymbol{\zeta}}\end{Bmatrix}=\begin{bmatrix}\mathbf{0}_{2x2} & \mathbf{I}_{2x2}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}(\alpha\mathbf{M}+\beta\mathbf{K})\end{bmatrix}\begin{Bmatrix}\boldsymbol{\zeta}\\ \dot{\boldsymbol{\zeta}}\end{Bmatrix}+$$
$$+\begin{bmatrix}\mathbf{0}_{2x1}\\ -\mathbf{I}_{\mathbf{g}}\end{bmatrix}\ddot{u}_g+\begin{bmatrix}\mathbf{0}_{2x2}\\ -\mathbf{M}^{-1}\end{bmatrix}\mathbf{u} \quad (6)$$

or rather, in symbolic form:

$$\begin{cases}\dot{\mathbf{Z}}=\mathbf{A}\mathbf{Z}+\mathbf{B}_{\mathbf{g}}\ddot{u}_g+\mathbf{B}_{\mathbf{u}}\mathbf{u}\\ \mathbf{u}=\mathbf{K}_{\mathbf{1}}\mathbf{Z}\end{cases} \quad (7)$$

where $\mathbf{Z}=[\zeta \quad \dot{\zeta}]^T=[y \quad l\theta \quad \dot{y} \quad l\dot{\theta}]^T$ is the state vector of the system, with $\mathbf{A}$ the state matrix for the uncontrolled structure and with $\mathbf{K}_{\mathbf{1}}=[\mathbf{0}_{2x2} \quad \mathbf{C}_{ext}]^T$ the gain matrix that connects the control action to the system state.

In the complex Laplace space, equation (7) can be written as:

$$\begin{cases} s\mathbf{IZ} = \mathbf{AZ} + \mathbf{B_g} s^2 U_g + \mathbf{B_u u} \\ \mathbf{u} = \mathbf{K_1 Z} \end{cases} \quad (8)$$

By defining $\mathbf{H} = (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B_g} s^2$ the transfer matrix relating the complex response of the uncontrolled system to the input seismic action, the system (8) can be rewritten as:

$$\begin{cases} \mathbf{Z} = \mathbf{H} U_g + \mathbf{H} \dfrac{\mathbf{B_u}}{\mathbf{B}_g s^2} \mathbf{u} \\ \mathbf{u} = \mathbf{K_1 Z} \end{cases} \quad (9)$$

From (9) the control block diagram, shown in fig. 2, which is representative of the behaviour of an asymmetric-plan system equipped with extra-structural damping is obtained, Palazzo & Petti (1997).

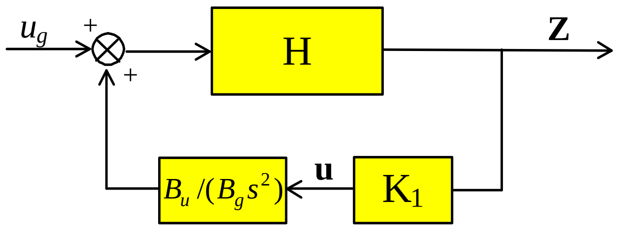

Figure 2. Block diagram of the controlled system

It is interesting to note that the supplemental dissipation is seen by the system as a closed loop controller in which the control action works retroactively on the system. It can be observed that, although the controller input is the complete state of the system, the specific form of the gain matrix, $\mathbf{K_1}$, only allows for direct control of the velocity components. Therefore, displacement control is obtained by indirectly controlling the velocity. By means of supplemental energy dissipation, it is therefore possible to control efficiently the relative displacements coupled to high values of velocity components.
System (7) can also be rewritten as:

$$\dot{\mathbf{Z}} = (\mathbf{A} + \mathbf{B_u K_1})\mathbf{Z} + \mathbf{B_g} \ddot{u}_g \quad (10)$$

In this case, $(\mathbf{A} + \mathbf{B_u K_1})$ represents the state matrix of the controlled system.

3 OPTIMAL PLAN-WISE DISTRIBUTION PROBLEM FOR SUPPLEMENTAL DAMPING

A new approach to solving the optimal plan-wise distribution problem of supplemental damping is herein proposed. In particular, it is based on the study of the transfer function relating the maximum edge displacement of the asymmetric-plan system to the input seismic excitation. The evaluation of the H_∞ and H_2 norms of such a transfer function represent suitable perfomance index for the definition of optimal design criteria for the plan-wise distribution of extra-structural dampers.

3.1 *Physical constraints for dissipation devices location*

Parameters $(\varepsilon_C, \lambda_C)$ characterize the allocation of dissipation devices within the structural systems. However, depending on the stiffness centroid position, that is to the origin of the defined coordinate system, a application domine for couple $(\varepsilon_C, \lambda_C)$ is defined. Such a domine is due to the necessity of placing the devices within the structural system.

In this paper, a single couple of stiff elements placed on system's edges and uniform distribution for mass is considered. Therefore, the structural eccentricity can be defined by mean of a single parameter:

$$\kappa = k_1/(k_1 + k_2) \quad (11)$$

where k_1 and k_2 are the stiffness element values.

It's possible to show that the adimensional values of structural eccentricity and minimum distance between the stiffness centroid and the edges can be write as:

$$\varepsilon = (2\kappa - 1)/(1 + |1 - 2\kappa|) \quad (12)$$

$$\delta_{\lim} = (1 - |1 - 2\kappa|)/(1 + |1 - 2\kappa|) \quad (13)$$

Considering the application of a couple of dissipation devices and using eqs. (12) and (13) the following constraints for $(\varepsilon_C, \lambda_C)$ are carried out:

$$1 > \varepsilon_C > -\delta_{\lim} \quad (14)$$

$$\lambda_C < \delta_{\lim} \quad (15)$$

3.2 *Transfer function of controlled systems. H_2 and H_∞ norms*

Let us consider the Laplace transform of the controlled system. In equation (10) it is possible to evaluate the system's state transform using the following equation:

$$\mathbf{Z} = (s\mathbf{I} - \mathbf{A} - \mathbf{B_u K_1})^{-1} \mathbf{B_g} s^2 U_g \quad (16)$$

where $\mathbf{G}(s) = (s\mathbf{I} - \mathbf{A} - \mathbf{B_u K_1})^{-1} \mathbf{B_g} s^2$ represents the transfer function vector relating the state of the controlled system to the input seismic excitation. Above all, since our interest lies in investigating the seismic response in terms of edge displacements, we define the transfer functions relating the edge displacements $G^{+}(s)$ and $G^{-}(s)$ to the ground motion as follows:

$$G^{+}(s) = [1 \quad 1 \quad 0 \quad 0] \cdot \mathbf{G}(s) \tag{17}$$

$$G^{-}(s) = [1 \quad \delta_{\text{lim}} \quad 0 \quad 0] \cdot \mathbf{G}(s) \tag{18}$$

The plan-wise distribution optimisation has been investigated through the analysis of performance indices defined by the H_∞ and H_2 norms of the transfer functions (20) and (21).

The H_2 norm of a transfer function $\mathbf{G}(s)$ is defined as, Boyd & Barrat (1991):

$$H_2 = \sqrt{tr\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{G}(i\omega)\overline{\mathbf{G}}(i\omega)d\omega\right)} \tag{19}$$

where the symbols *tr* and $\overline{*}$ respectively represent the trace and the transpose complex conjugate operators. The H_2 norm represents a measure of the root mean square (RMS) of the system response to white noise.

The H_∞ norm of a stable system transfer matrix is defined as, Boyd & Barrat (1991):

$$H_\infty = \sup_{re(s)>0} \sigma[\mathbf{G}(s)] \tag{20}$$

where $\sigma(.)$ is the maximum singular value operator defined as:

$$\sigma(\mathbf{G}(s)) = \max_{\mathbf{d}\neq 0} \frac{\mathbf{d}^{*}\mathbf{G}^{*}(s)\mathbf{G}(s)\mathbf{d}}{\mathbf{d}^{*}\mathbf{d}} \tag{21}$$

Such an expression shows that the H_∞ norm represents a measure of the upper extreme of the rms output-input ratio. Therefore, in this sense we are talking about design control in the "worst case".

3.3 *Optimal plan-wise allocation of extrastructural damping resource*

Having fixed the amount of extrastructural damping, $\xi_{ext} = 0.20$, in figures 3,5,7 values of the H_∞ and H_2 norms applied to the transfer function $G^{+}(s)$ and $G^{-}(s)$ on varying the value of structural eccentricity ε and the parameters describing the plan-wise distribution of the extra-structural damping, $(\varepsilon_C, \lambda_C)$, are plotted. In particular, according to the aim of limiting the maximum of both edge displacements, figure 4,6,8 show the maximum value of the H_∞ (figs. 4,6) and H_2 (fig. 8) norm between the two transfer function under consideration.

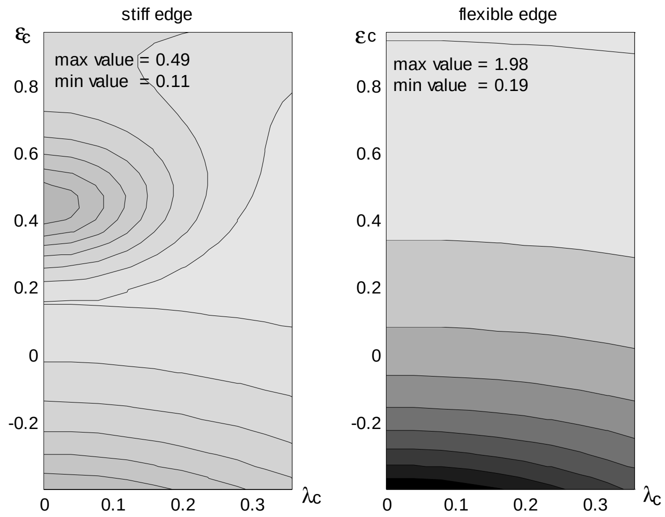

Figure 3. H_∞ norm of $G^{-}(s)$ and $G^{+}(s)$ functions ($\varepsilon = 0.30$, $\xi_{ext} = 0.20$)

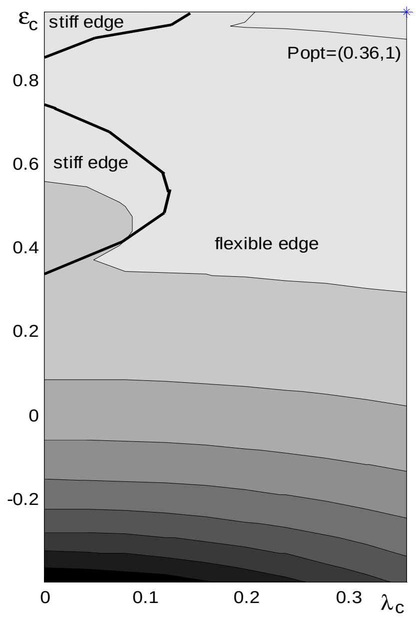

Figure 4. Maximum H_∞ norm ($\varepsilon = 0.30$, $\xi_{ext} = 0.20$)

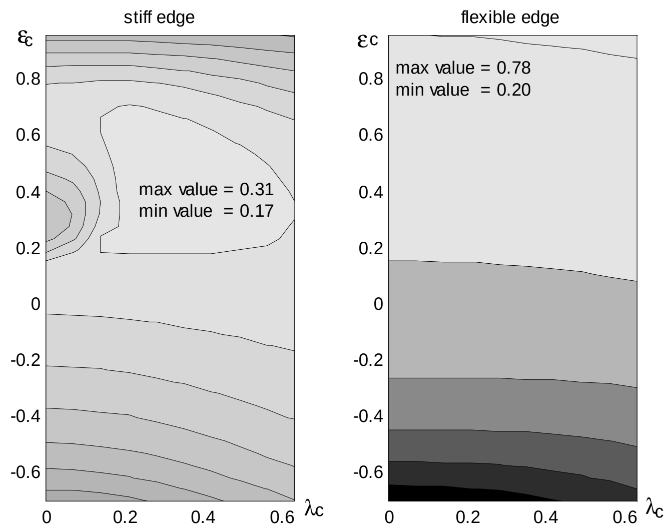

Figure 5. H_∞ norm of $G^{-}(s)$ and $G^{+}(s)$ functions ($\varepsilon = 0.15$, $\xi_{ext} = 0.20$)

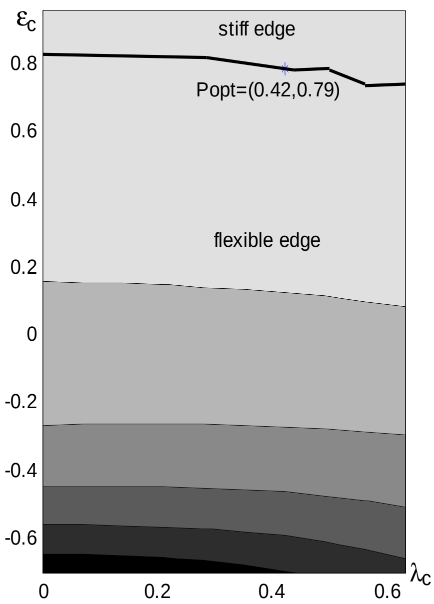

Figure 6. Maximum H_∞ norm ($\varepsilon = 0.15$, $\xi_{ext} = 0.20$)

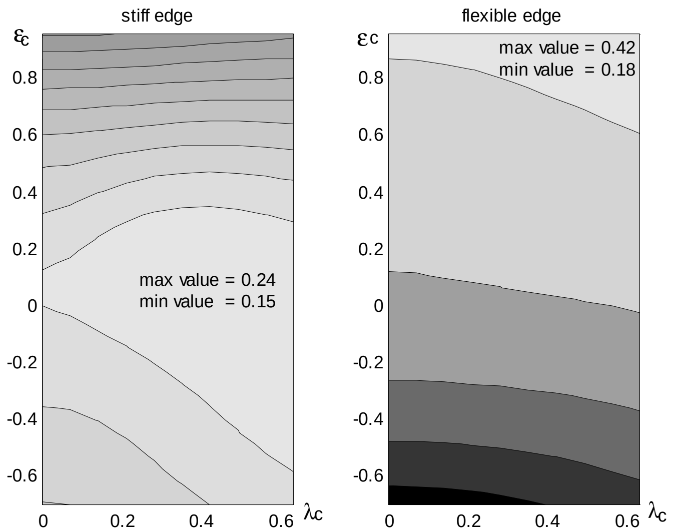

Figure 7. H_2 norm of $G^{-}(s)$ and $G^{+}(s)$ functions ($\varepsilon = 0.15$, $\xi_{ext} = 0.20$)

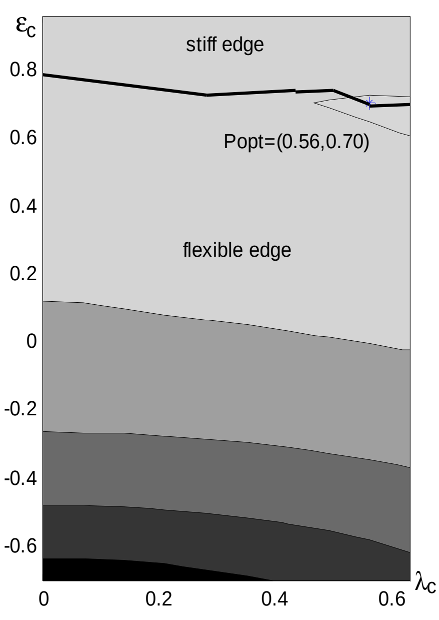

Figure 8. Maximum H_2 norm ($\varepsilon = 0.15$, $\xi_{ext} = 0.20$)

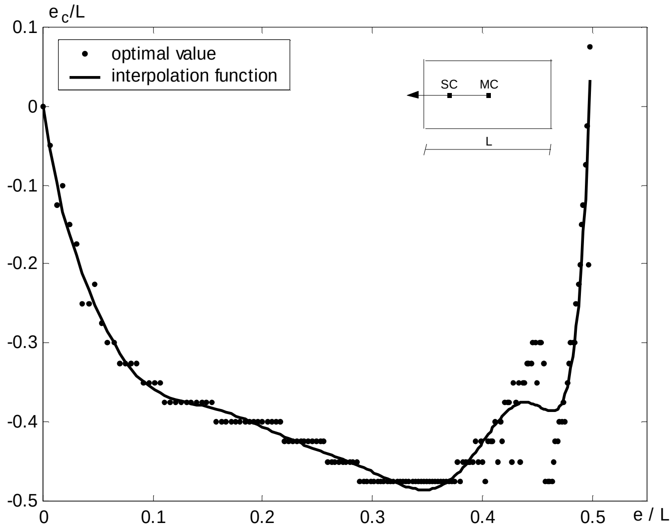

Figure 9. Damping center optimal position - H_∞ norm ($\xi_{ext} = 0.20$)

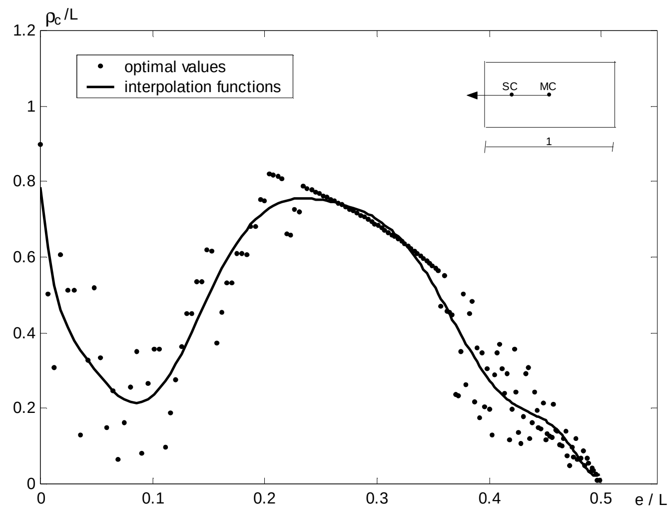

Figure 10. Damping spread optimal values - H_∞ norm ($\xi_{ext} = 0.20$)

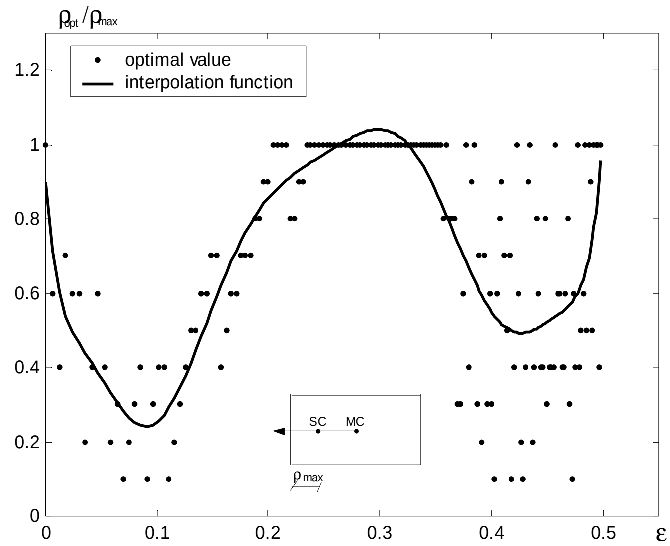

Figure 11. Damping spread optimal values - H_∞ norm ($\xi_{ext} = 0.20$)

These results allow for the evaluation of an optimal plan-wise arrangement of the dissipation resource described by the parameters ($\varepsilon_{C,opt}$, $\lambda_{C,opt}$), considering also the physical limit for a fixed allocation pattern. Specifically, in figures 9-11 the optimal value of the damping centre eccentricity and spread, on varying the structural eccentricity, are plotted. For a better physical understanding, a new coordinate system having the origin in the mass center (MC) and x-axes directed toward the stiffness centroid (SC) has been introduced. An interpolation function to describe the optimal design parameter analytical trend (dotted line) is also represented. The optimal condition, when flexible edge control doesn't completly describe the problem, is characterized by the equality of the H_∞ or H_2 norms for both the transfer functions $G^+(s)$ and $G^-(s)$ (fig. 6,8). From the obtained results the following general considerations can be derived:

- the increase of the supplemental damping centre eccentricity toward the flexible edge reduces the flexible edge displacement, but it worsens the response of the stiff one.
- optimal control of stiff edge displacement is obtained when the supplemental damping centre is located between the mass and stiffness centroids (figs. 3,5,7);
- the increase in the supplemental damping spread has generally a beneficial effect on the control of both flexible and stiff edge displacements (figs. 3-8);
- the optimal position of damping center is generally located in opposite region respect to the stiffness centroid. For structural eccentricity values in the range $[0.1L, 0.4L]$, the damping center optimal position is quite stable between $[-0.4L, -0.5L]$ (fig. 9), while for stiffness centroid position near to the edges or mass centroid high value for optimal position gradient has been observed.
- optimal damping spread presents a not stable trend for high structural eccentricity. In the middle region $\varepsilon = [0.2L, 0.35L]$, the optimal spread takes its maximum potential value (fig. 11), while for little eccentricity cases, a limitated spread of the damping resource improve the structural response (figs. 10-11).
- using a value of the parameter ρ_C that is higher than the optimal value does not significantly affect the H_∞ and H_2 norms.

Results allow for the definition of a "design region" concerning the allocation of extrastructural dampers. In fact, for civil buildings typical eccentricity value, $\varepsilon = [0.1L, 0.35L]$, a dampers allocation having its centre in the range $[-0.4L, -0.5L]$ and the maximum potential spread, is consistent with a notable reduction in seismic response of the irregular system.

4 CONCLUSION

A new approach for defining plan-wise optimal arrangement of supplemental damping in asymmetric-plan systems has been carried out. The dynamic problem has been investigated in the state space representation showing that the supplemental dissipation resources work as a closed-loop feedback control action. This allowed for a better physical understanding of the problem and for the formulation of optimal plan-wise design criteria for additional damping devices showing that moving the damping centre through the flexible edge lead to swap the stiff role between the edges.

Optimal plan-wise arrangement of supplemental damping, obtained by using H_∞ and H_2 norms, take place, for civil buildings typical eccentricity value $\varepsilon = [0.1L, 0.35L]$, when dampers center is located in the range $[-0.4L, -0.5L]$ and the spread has the maximum potential value.

ACKNOWLEDGMENTS

This work is Part of the project of the Campania Regional Center of Competence "Analysis and Monitoring of the Environmental Risk" supported by the European Community on Provision 3.16.

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