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Chemical Engineering Science 58 (2003) 5269–5278

Chemical Engineering Science

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On the role and the origin of the gas pressure gradient in the discharge of fine solids from hoppers

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Received 7 October 2002; received in revised form 8 August 2003; accepted 19 August 2003

Abstract

The Brown and Richards (Principles of Powder Mechanics, Pergamon Press, Oxford, UK, 1970) correlation for the discharge rate of fine powders from a hopper was modified to account for the gas pressure gradient near the outlet. According to Donsì et al. (Chem. Eng. Sci. 52 (1997) 4291) there is a transition between a granular flow region and a suspended flow region near the hopper outlet. Brown and Richards (1970) stated that the particle discharge rate depends on the flow conditions just above this transition surface. In the modified equation that is developed to account for the gas pressure, a term including the gas pressure gradient at this surface appears. The gas pressure gradient is evaluated from the literature experimental results by considering the Donsì et al. (1997) finding that a significant part of the gas pressure gradient near the hopper outlet is due to the suspended motion. Furthermore, a simplified analysis is carried out to evaluate from the experimental results the voidage variation within the solids phase that is responsible for the onset of the gas pressure gradient.

Keywords: Hopper discharge; Fine granular solids; Gas pressure gradient; Discharge rate; Free fall arch

1. Introduction

The influence of the solid–fluid interactions is generally neglected in the description of the motion of coarse granular solids discharging from a bin. This is the case in the Beverloo et al. (1961) empirical correlation which states

$$W_s = 0.58\rho_b g^{1/2}(D_0 - 1.5d_p)^{5/2}, \qquad (1)$$

where W_s is the solids flow rate, ε_b is the bulk voidage, ρ_s is the particle density of the solids, g is the acceleration due to gravity and D_0 is the outlet diameter. Similarly, Rose and Tanaka (1959) modified Eq. (1) for conical hoppers. According to this equation the flow rate values have to be greater than those predicted by Beverloo et al. (1961).

One of the first theoretical predictions of mass flow rate from discharging mass flow hoppers was proposed by Brown and Richards (1970). Their theory was based on the concept of the "free fall arch" that refers to a surface in a flowing powder below which the solid particles are not constrained and therefore are allowed to fall freely under gravity. The existence of the "free fall arch" was proved experimentally by Brown and Richards (1965) in a two-dimensional bin. The equation obtained for the discharge rate is

$$W_s = \frac{\pi}{6}\rho_b g^{1/2}(D_0 - kd_p)^{5/2}\frac{1-\cos^{3/2}\alpha}{\sin^{5/2}\alpha}. \qquad (2)$$

The mass flow rates predicted by the Brown and Richard's correlation (2) for a conical hopper are lower than the values obtained by the Beverloo et al. (1961) correlation using the corrective factor by Rose and Tanaka (1959), thus, appearing to be closer to the experimental values.

Davidson and Nedderman (1973), Williams (1977) and Brennen and Pearce (1978) approached the modelling of the solids discharge by means of continuum mechanics. Davidson and Nedderman (1973) proposed what they called the hour-glass theory. According to this theory, the mass flow rates are about twice the values determined experimentally. Williams (1977) proposed the limits of the range in which the experimental discharge rates are expected by accounting the wall friction effect. The upper limit of this range produces equations very similar to Eqs. (1) and (2). Brennen and Pearce (1978), who worked with wedge-shaped hoppers, and Nguyen et al. (1979), who worked with conical hoppers, applied a perturbation technique to modify the Williams (1977) model. In spite of the approximate

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doi:10.1016/j.ces.2003.08.022

approach, the mass flow rate predicted was about 50% less than the rate predicted by the smooth walled theory and 10% higher than the experimentally measured values.

The empirical and theoretical correlations proposed above tend to overestimate the mass flow rate of fine solids (< 500 μm) from hoppers by as much as a factor of 10. This effect has been commonly related to the interstitial fluid pressure gradient in the vicinity of the outlet.

Altenkirch and Eichhorn (1981) extended the Brown and Richards (1970) model to the case of fine particles by accounting for gas–solid interactions. To this end, the authors considered the total energy balance of the system by including both the solid- and the gas-phase contributions. The two-phase system was considered as a single-pseudo-homogenous phase. They neglected the solids stress and assumed that the total pseudo-pressure should be assumed in the limit of a voidage equal to one. In order to evaluate the pressure gradient at the arch, Altenkirch and Eichhorn (1981) used the Darcy's equation assuming a radial gas flow. The final equation contains two parameters (the bulk voidage and the ratio between the solids and the gas mass flow rate) which were used to fit the experimental data by Resnick et al. (1966) and Crewdson et al. (1977).

Crewdson et al. (1977) argued that the interstitial fluid pressure gradient acts as an extra body force on solid particles equal to the vertical pressure gradient. Consequently, these authors proposed to add a term to the Beverloo et al. (1961) correlation to take into account the action of the pressure gradient

$$W_s = C\rho_b \left(g + \frac{1}{\rho_b} \left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0} \right)^{1/2} (D_0 - kd_p)^{5/2}. \tag{3}$$

According to the authors, the best value of the C constant in Eq. (3) to fit experimental data is 1.21. Crewdson et al. (1977) completed the analysis by using the Darcy law to express the bed permeability and some available constitutive equation to describe the solids compaction due to compression.

Nedderman et al. (1983) further completed the analysis by Crewdson et al. (1977) by including in Eq. (3) the Rose and Tanaka correction factor. Then they also corrected the hour-glass theory to account for the opposed gas pressure gradient. A more complete analysis based on mass and momentum balance on the gas and solids phase is given by Gu et al. (1992b) who managed to correlate a large set of experimental data by the use of a single adjustable parameter. This parameter corresponds to the ratio between the whole bed height and the position of the pressure minimum in the gas pressure profile.

Donsì et al. (1997) found, from the measurements of axial pressure profiles in a conical hopper, the existence of an ideal arch somewhere above the outlet where the fine solids motion switches from a granular type to a suspended one. This result is conceptually close to the "free fall arch", the main difference relying in the type of motion below this arch due to the different interaction between solids and interstitial fluid. For the coarse particles used by Brown and Richards (1965) to validate their model, the gas–solids interaction was negligible and therefore particles fell freely under gravity. Differently, fine particles, such as those used by Donsì et al. (1997), are seriously affected by the interaction with the interstitial gas. According to them, a suspended flow is established below the arch, responsible for a non negligible portion of the pressure gradient near the hopper outlet and, therefore it should not be accounted for in the evaluation of the gas pressure gradient in the granular flow region just above the arch. In this paper a modification of the Brown and Richards (1965) model, similar to that proposed by Altenkirch and Eichhorn (1981), will be developed. Experimental results regarding the discharge of fine powders found in literature will be analysed in the light of the interpretation of the gas pressure gradient proposed by Donsì et al. (1997). The voidage variation responsible for the gas pressure gradient in the granular flow region will also be assessed.

2. Analysis of fine solids discharge

The whole theoretical treatment which follows relies on the assumption that the flow of fine solids in proximity of the hopper outlet can be divided into two different motion regimes: (1) a granular motion regime, which applies far above the hopper outlet and is characterised by the prevalence of frictional forces between particles; (2) a suspended motion regime, which applies closely above the orifice and is characterised by the fact that the solids fall in the interstitial fluid like a suspension and, consequently, frictional interactions are negligible. Furthermore, it is assumed that the granular flow region is the one in which the solids' discharge rate is determined. This means that only the portion of the gas pressure drop in the region of granular flow regime should be considered in the evaluation of the particle discharge rate.

These assumptions were suggested by some observations on the gas pressure gradients as observed by Donsì et al. (1997). In particular, assumption (2) on suspended motion in the vicinity of the orifice relies on the measured pressure gradients at the outlet. In fact, these pressure gradients are large enough to be compared with those that are measured in gas–solid suspensions such as a fluidised bed or a settling suspension. On the other hand, the existence of the granular regime in assumption (1) is evident from any hopper discharge observation.

The idea that the solids flow is determined in the granular flow region and cannot be determined in the suspended flow region derives from the extension of the principle that, in a suspension, the effects of a local variation cannot travel faster than particle concentration waves. This implies that any physical effect on the suspended region cannot overcome the stationary solid concentration wave represented by the transition between the regions in which the two regimes apply.

Brown and Richards (1965) followed an approach which more closely describes the above hypothesis on the flow transition between regimes. Kaza and Jackson (1984) objected this approach due to some conceptual difficulty in admitting powder incompressibility above the free fall arch and a bulk density variation below the arch. By admitting both circumstances, in fact, an inconsistent increase of the bulk density would result in the free fall region if a non zero stress gradient was hypothesised on the arch. However, we believe that this paradox is resolved following the physical picture of our model, according to which it is possible to distinguish between a granular and a suspended flow region. In the first region finite mechanical interparticle interactions occur, which are responsible for the stress within the solids phase. In the second region only fluid dynamic interactions are to be accounted for. The soundness of this view is proved by some direct voidage measurements carried out by Fickie et al. (1989) and by Hosseini-Ashrafi and Tüzün (1993), which seem to prove the existence of a continuous voidage profile near the hopper outlet. Nevertheless, in the application of our model for the calculation of the solids discharge rate, the bulk density of the granular solids is for simplicity considered constant because we suppose that at the arch it does not change much with respect to the bulk. For the above reasons, the Brown and Richards (1965) approach is assumed as a basis to predict the solids discharge rate. In particular, this model is extended to the discharge of fine powders by taking into account the contributions coming from the gas–solid interactions. Furthermore, a proof that small voidage variations that are considered negligible for the determination of the solids bulk density can produce significant gas pressure gradients within the granular flow region will be given by the application of a simplified one-dimensional analysis based on similar assumptions.

Beyond any physical observation on the transition from a granular to a suspended regime, a sharp transition from one regime to the other is the simplest case to be treated in the model. Nevertheless, we expect that this approach can provide at least with the first order effect of the observed discharge phenomena. The correctness of this approximation will be verified by the comparison between the model results and the experiments.

2.1. Extension of the Brown and Richards (1970) model

By assuming the presence of a "free fall arch", the Brown and Richards (1970) theory hypothesises total energy conservation below the arch. The term "total energy" refers to the mechanical energy connected to the solids motion. For fine powders, gas–solids interactions cannot be neglected and a two-phase approach is necessary. In other words, the application of the total energy conservation should take into account the energy contribution associated with the gas phase. The conservation of total energy in the derivation by Brown and Richards (1970) is supported by the argument that the dissipation of the mechanical energy is due to friction forces acting only in the granular flow region and, therefore, the total energy derivative at the "free fall arch" has to be nil. In the application of the two-phase model this condition is maintained despite the fact that the presence of a slip velocity between the phases should be associated to some dissipation also in the region of suspended flow. The underlying hypothesis is that the gas–solid frictional dissipation is much smaller than that due to solid–solid friction. In the analysis of the solids discharge from conical hoppers, a system of spherical coordinates is assumed. The origin is placed at the imaginary apex of the conical surface that is defined by the hopper walls. A scheme of this reference is given in Fig. 1. For the sake of simplicity, downward velocities will be considered positive.

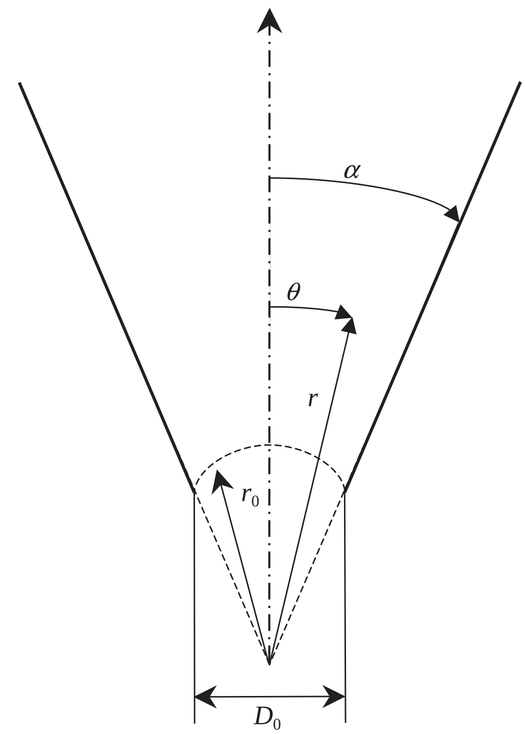

Fig. 1. Scheme of the coordinate system used in the analysis of the solids discharge.

According to Brown and Richards (1970) the following hypotheses are made:

(i) the bulk material dilation in the vicinity of the "free fall arch" is neglected in the continuity equation for the solids;
(ii) below the "free fall arch" the variation of the solids stress σ is assumed to be negligible with respect to the variation of the other contributions to the total energy;
(iii) the solids flow field is assumed to be radial;
(iv) the location of the ideal surface of the "free fall arch" is supposed to be at $r=r_0$ which is the radial coordinate of the orifice edge.

In the present model these further hypotheses regarding the gas phase are made:

(v) the total energy is the sum of the solid- and the gas-phase energy;
(vi) below the "free fall arch", similarly to the solids stress σ in point (ii), the variations along the radial coordinate of the kinetic and gravitational energy in gas phase are assumed to be negligible;
(vii) consistently with the solids flow field, the interstitial gas pressure field is assumed to be radial.

With regard to hypothesis (i), the dilation of powders will be used to explain the onset of gas pressure gradients opposed to the solids motion. In relation to experimental pressure gradients, dilation values will be found small enough to be neglected in the framework of the present analysis in the application of the continuity equation and the "total energy" conservation. With regard to hypothesis (iv), in the one-dimensional analysis carried out by Donsì et al. (1997), the location of the transition surface measured along the hopper axis was approximately at a distance of half the orifice diameter above the orifice itself. This finding was associated to measurements in wide angled and flat bottomed hoppers and it is acknowledged to be almost equivalent to hypothesis (iv) which better agrees with hypotheses (iii) and (vii) of radial flow.

According to the above hypotheses the continuity equation for the solids in spherical coordinates is the following:

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}[\rho_s(1-\varepsilon)v_s(r,\theta)r^2]=0, \tag{4}$$

where v_s is the solids velocity, ρ_s is the solid particle density and ε is the local voidage in the bed of granular material. According to assumption (iii), the solids velocity is given by

$$v_s(r,\theta)=\frac{f(\theta)}{r^2}, \tag{5}$$

where $f(\theta)$ is an unknown function of the angular coordinate θ. The total energy of the solids phase T_s and that of the gas phase T_f are, respectively

$$T_s=\sigma+\rho_s(1-\varepsilon)\frac{v_s^2}{2}+\rho_s(1-\varepsilon)gr\cos\theta, \tag{6}$$

$$T_f=p+\rho_f\varepsilon\frac{v_f^2}{2}+\rho_f\varepsilon gr\cos\theta, \tag{7}$$

where g is acceleration due to gravity, p is the interstitial gas pressure, ρ_f is the gas density and v_f is the gas velocity.

According to the total energy conservation below the arch

$$\left.\frac{\mathrm{d}T}{\mathrm{d}r}\right|_{r=r_0}=\left.\frac{\mathrm{d}T_s}{\mathrm{d}r}\right|_{r=r_0}+\left.\frac{\mathrm{d}T_f}{\mathrm{d}r}\right|_{r=r_0}=0, \tag{8}$$

where r_0 can be expressed as a function of the orifice diameter D_0 and of the hopper half-angle α

$$r_0=D_0/2\sin\alpha. \tag{9}$$

By substituting Eqs. (6) and (7) into Eq. (8) and taking into account assumptions (ii) and (vi), we have

$$\rho_s\frac{\mathrm{d}}{\mathrm{d}r}\left[(1-\varepsilon)\frac{v_s^2}{2}+(1-\varepsilon)gr\cos\theta\right]\Bigg|_{r=r_0}+\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0}=0. \tag{10}$$

Substituting Eq. (5) into Eq. (10), we obtain

$$f(\theta)=\sqrt{\frac{r_0^5}{2}\left(g\cos\theta+\frac{1}{\rho_s(1-\varepsilon_0)}\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0}\right)} \tag{11}$$

and, consequently, Eq. (5) for the solids velocity becomes

$$v_s(r,\theta)=\frac{1}{r^2}\sqrt{\frac{r_0^5}{2}\left(g\cos\theta+\frac{1}{\rho_s(1-\varepsilon_0)}\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0}\right)}. \tag{12}$$

The mass flow rate, W_s is obtained by integrating the mass velocity over the spherical cap surface corresponding to the free fall arch

$$W_s=\rho_s(1-\varepsilon_0)2\pi r_0^2\int_0^\alpha v_s(r_0,\theta)\sin\theta\,\mathrm{d}\theta \tag{13}$$

which, including the empirical argument of the "free annulus", finally gives

$$W_s=\frac{\pi}{4}\rho_s(1-\varepsilon_0)(D_0-kd_p)^{5/2}\left[\frac{2}{3}g^{1/2}\frac{1-\cos^{3/2}\alpha}{\sin^{5/2}\alpha}+\left(\frac{1}{\rho_s(1-\varepsilon_0)}\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0}\right)^{0.5}\frac{1-\cos\alpha}{\sin^{5/2}\alpha}\right]. \tag{14}$$

This equation is similar to the equation introduced by Crewdson et al. (1977) that corrected the semi-empirical equation by Beverloo et al. (1961) by using a gas pressure gradient term. In this case the numerical coefficients multiplying the gravity acceleration term and the gas pressure gradient are dissimilar due to the fact that the former is vertical and the latter is radial in direction.

2.2. Evaluation of the solids dilation during the discharge

In order to make Eq. (14) fully predictive of the solids mass flow rate, it is necessary to provide an equation for the interstitial gas pressure gradient corresponding to the "free fall arch". As a consequence, in this section an approximate model of the interstitial gas flow setting up during the steady-state gravity discharge of fine powders from hoppers is proposed. The scope is to evaluate the material dilation occurring during the discharge from the measured interstitial gas pressure profiles available in the literature.

The dilation of the solids approaching the hopper outlet was first observed and measured by Van Zuilichem and Van Egmond (1974) and after by Fickie et al. (1989) and

Hosseini-Ashrafi and Tüzün (1993). All these studies registered reductions of the solids volumetric fraction near the outlet of the order of 0.1. However, all these systems include relatively coarse particles for which the gas–solid interactions arising from the observed dilation are not significant. In some cases, this solids dilation was related to the solid stress reduction towards the conical hopper apex (Crewdson et al., 1977). Another possible explanation might rely in the increase of the shear action that is experienced by the powder particles approaching the hopper outlet. However, the determination of the physical causes of the solids dilation is beyond the purpose of the present work in which this phenomenon is assumed to explain the onset of gas pressure gradients that are opposed to the solid flow near the hopper outlet.

The following assumptions are made:

(a) the wall friction is neglected and the hopper half angle α is sufficiently small so that the particle and gas velocities and the material voidage can be assumed to be independent from the angular coordinate;
(b) the dilation of the granular material during the flow is taken into account: as a result, the voidage is assumed to vary along the radial coordinate in the hopper;
(c) according to experimental measurements (Crewdson et al., 1977; Donsì et al., 1997), the interstitial fluid pressure gradient is assumed to be nearly equal to zero at a certain radial coordinate value in the bulk r_b;
(d) according to the hypothesis concerning the two different flow regimes, the laws relevant to the permeation of the gas through a compacted bed of solids can describe the percolation of air only above the ideal surface; in this study the Carman–Kozeny law is used according to the low gas velocities involved

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{150\mu_f v_{\mathrm{sl}}(1-\varepsilon)^2}{d_p^2\varepsilon^2}, \tag{15}$$

where μ_f is the gas viscosity, v_{sl} is the slip velocity, ε is the local voidage and d_p is the mean particle Sauter diameter;
(e) the interstitial gas is assumed to be incompressible: the sufficiently small extent of pressure variations involved during the hopper discharge supports this approximation.

Assumption (a) is useful to develop a simple model, but also allows the application of the present analysis on pressure profiles measured at different positions with respect to the hopper axis. With concern to assumption (c), the value of r_b is generally equal to few hopper outlet diameters. Therefore, the model will be applied to discharge experiments in which the bed height is much larger than few hopper outlet diameters and the solids discharge rate appears to be constant (independent of the bed height reduction) over a significant fraction of the entire discharge period.

Material balances, together with assumption (e), suggest that in every hopper section the ratio between the interstitial fluid and the solid volumetric flow rate, respectively Q_f and Q_s, is constant and is given by

$$\frac{Q_f}{Q_s} = \frac{v_f\varepsilon}{v_s(1-\varepsilon)}. \tag{16}$$

As a consequence of assumption (c), according to any of the laws relevant to the flow of a gas through a packed bed of particles (Darcy, Carman–Kozeny), at $r=r_b$ the slip velocity is nil

$$v_{\mathrm{sl}}|_{r=r_b} = v_s|_{r=r_b} - v_f|_{r=r_b} = 0 \tag{17}$$

and Eq. (16) simplifies to the following:

$$\frac{Q_f}{Q_s} = \frac{\varepsilon_b}{1-\varepsilon_b}, \tag{18}$$

where ε_b is the bulk voidage at $r = r_b$.

Writing Eq. (16) for $r = r_0$ where $v_f = v_s - v_{\mathrm{sl}}$ and substituting Eq. (18) into Eq. (16), we obtain

$$v_{\mathrm{sl}}|_{r=r_0} = v_s|_{r=r_0}\left(\frac{1-\varepsilon_0}{\varepsilon_0}\,\frac{\varepsilon_b}{1-\varepsilon_b} - 1\right). \tag{19}$$

According to the material balance on the solids, the solid velocity at $r = r_0$ is given by

$$v_s|_{r=r_0} = \frac{4W_s}{\rho_s(1-\varepsilon_0)\pi D_0^2}. \tag{20}$$

The surface of the spherical cap, in this case, has been approximated to a circular surface according to the small value of α hypothesised under assumption (a).

Substituting Eqs. (19) and (20) in Carman–Kozeny's equation, the interstitial fluid pressure gradient at the ideal surface is given by the following:

$$\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r=r_0} = \frac{150\mu_f(1-\varepsilon_0)^2}{\mathrm{d}_p^2\varepsilon_0^2}\,\frac{4W_s}{\rho_s(1-\varepsilon_0)\pi D_0^2} \times\left(\frac{1-\varepsilon_0}{\varepsilon_0}\,\frac{\varepsilon_b}{1-\varepsilon_b} - 1\right). \tag{21}$$

Eq. (18) can be used to evaluate voidage values at the hopper outlet ε_0, corresponding to assigned values of local gas pressure gradient and of solids bulk voidage ε_b.

3. Comparison with experiments and discussion

To verify the correctness of Eq. (14) and its hypotheses and to evaluate the voidage value at the hopper outlet ε_0 by Eq. (21), it is necessary to use experimental results on the discharge of fine powders providing both solids discharge rates and gas pressure profiles near the hopper outlet. For this reason results reported by Crewdson et al. (1977), Head (1979) and Donsì et al. (1997) were considered. The most significant properties of the granular materials used by these authors are reported in Table 1.

Table 1
Results of voidage calculation based on experimental data and symbol legend for Figs. 5–7

Symbol	Material	d_p (μm)	ρ_s (kg m^{-3})	ϕ_e	ϕ_w [a]	α_{max} [b]	D_0 (mm)	ε_b	ε_0	Reference
●	FCC	84	1960	30	18	31	15.9	0.50	0.511	Donsì et al. (1997)
							19.1		0.516	
							22.2		0.515	
○	Polymer	116	1320	28	4	55	9.5	0.43	0.433	
							12.7		0.433	
							15.9		0.432	
							19.1		0.432	
							22.2		0.433	
▼	Glass	55	2600	24	11	46	12.7	0.49	0.491	
							15.9		0.491	
							19.1		0.492	
							22.2		0.491	
▽	Sand	90–106	2640				6.9	0.502	0.509	Crewdson et al. (1977)
		212–300	2640					0.455	0.463	
		355–422	2190					0.414	0.418	
■	Sand	86–130	2600				28.0	0.556	0.558	Head (1979)
		86–130					30.4	0.556	0.560	
		186–268					28.0	0.505	0.507	
		186–268					30.4	0.505	0.506	

[a] Data measured on aluminium that was the hopper wall material.
[b] Evaluated according to the Jenike (1961) procedure on the basis of ϕ_e and ϕ_w.

Experimental data taken by Donsì et al. (1997) are referred to the discharge of three different powders from a conical hopper with a 29° half-angle through different outlet diameters. Experimental observation apparently indicated the occurrence of mass flow during discharge. Also the comparison between this hopper half-angle and the maximum hopper half-angle for mass flow according to Jenike (1961) (see Table 1) indicated that mass flow discharge took place in all the considered cases. The pressure was measured at different positions on the hopper axis and the voidage values reported were those corresponding to the bulk. In the majority of the experiments performed in the conical hopper, the pressure profiles measured above the outlet can be approximated to two linear sections with different slopes. Donsì et al. (1997) interpreted this trend by the presence of a granular flow region over a region of suspended soils. Consequently, the pressure gradient corresponding to the granular flow region has been adopted to evaluate W_s in Eq. (14).

In the Crewdson et al. (1977) work on pressure profiles, authors report data corresponding to solids mass flow rates and voidage values measured during the discharge of three different sands from a conical hopper with a 5.9° half-angle through a 6.895 mm orifice. The pressure was measured by using pressure taps located on the wall. The voidage was measured by permeability tests that were carried out in an extension tube fitted to the hopper during the discharge. According to the authors, this voidage value should be that established at the orifice. Differently, in our opinion, such a measurement method should provide the bulk voidage. Consequently, this voidage value has been assumed to correspond to the bulk value in the present calculations. With concern to pressure profiles, since the exact location of the ideal surface is not known, the measured pressure gradient has been taken at r_0 according to assumption (iv).

Head (1979) performed his experiments in a conical hopper with a 15° half-angle through two different orifices (2.80 and 3.04 mm) using two sands with different granulometric distribution. Also in this case, the pressure measurements have been carried out at the wall and the pressure gradients have been evaluated at r_0 for the present analysis.

3.1. Solids discharge rates: comparison between model and experiments

Figs. 2–7 show the comparison between the experimental results and model results calculated according to the original

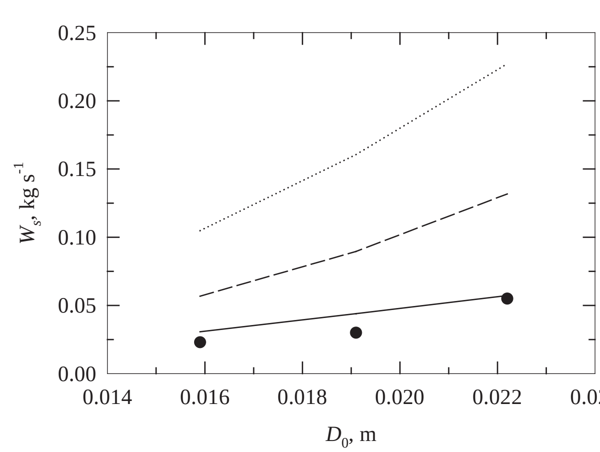

Fig. 2. Solids discharge rate vs. the orifice diameter for a 84 μm FCC powder: ●, experimental results (Donsì et al., 1997); —, Eq. (14); - - -, Eq. (2); ••••••, Eq. (3).

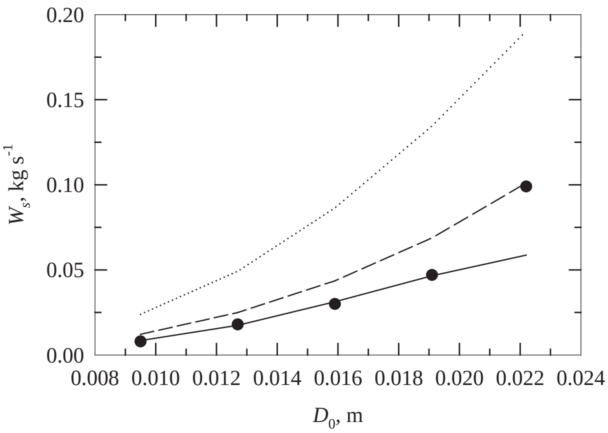

Fig. 3. Solids discharge rate vs. the orifice diameter for a 116 μm polymer powder: •, experimental results (Donsì et al., 1997); —, Eq. (14); - - -, Eq. (2); ••••••, Eq. (3).

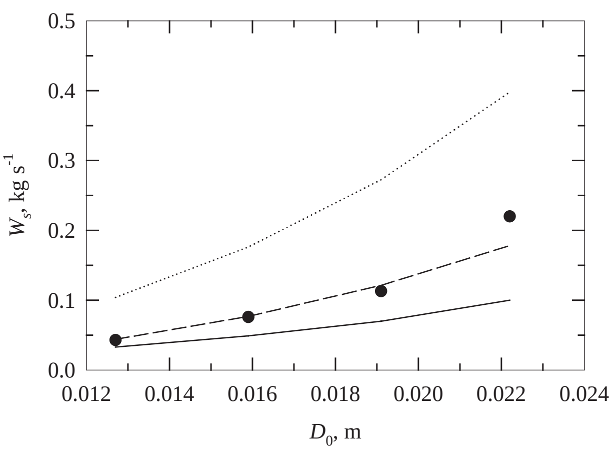

Fig. 4. Solids discharge rate vs. the orifice diameter for 55 μm glass beads: •, experimental results by (Donsì et al., 1997); —, Eq. (14); - - -, Eq. (2); ••••••, Eq. (3).

Brown and Richards (1970) Eq. (2), to its modification Eq. (14) and to the Crewdson et al. (1977) Eq. (3).

Experimental results in Figs. 2–4 refer to the comparison with experimental data by Donsì et al. (1997) for FCC, polymer powder and glass beads, respectively. In these figures, experimental discharge rates are given as a function of the hopper outlet diameter. Inspection of these figures reveals that the results of the present model are in good agreement with the experimental mass flow rate for the FCC and the polymer powders. In particular, according to Fig. 2, the mass flow rates obtained for FCC using Eq. (14) are still greater than the experimental values, but they are much closer than those evaluated according to Eqs. (2) and (3). In the case of the polymer powder (Fig. 3) the results are better than for FCC. In fact, the model previsions fit well with the experimental flow rates for all the orifice diameter values except that for the 2.22×10^{-2} m orifice, for which the model

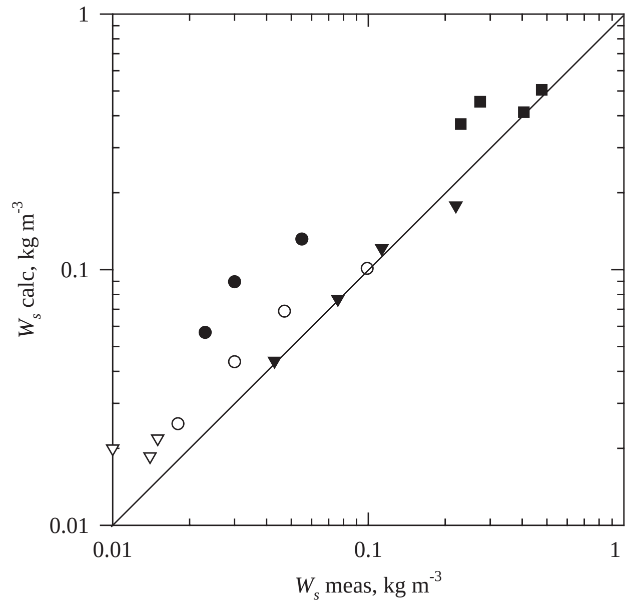

Fig. 5. Experimental and calculated solids discharge rate in a parity plot for Eq. (2) (Brown and Richards, 1970). See Table 1 for the symbol legend.

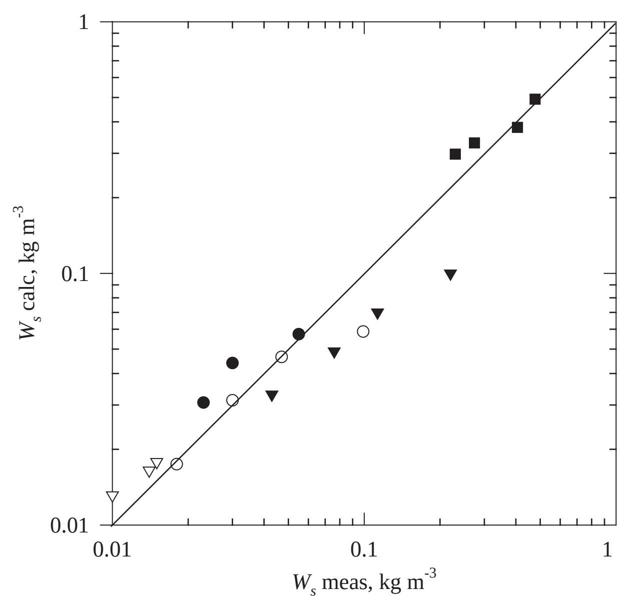

Fig. 6. Experimental and calculated solids discharge rate in a parity plot for Eq. (14). See Table 1 for the symbol legend.

underestimates the solids flow rate. Direct inspection of original data, however, indicates for this discharge condition a certain intrusive effect of the gas pressure measurement procedure which might have affected the flow and therefore the evaluation of the gas pressure profile. This could be attributed to the fact that in this case the approximation of the pressure profile to two linear sections leads to an overestimation of the gas pressure gradient in the zone of granular

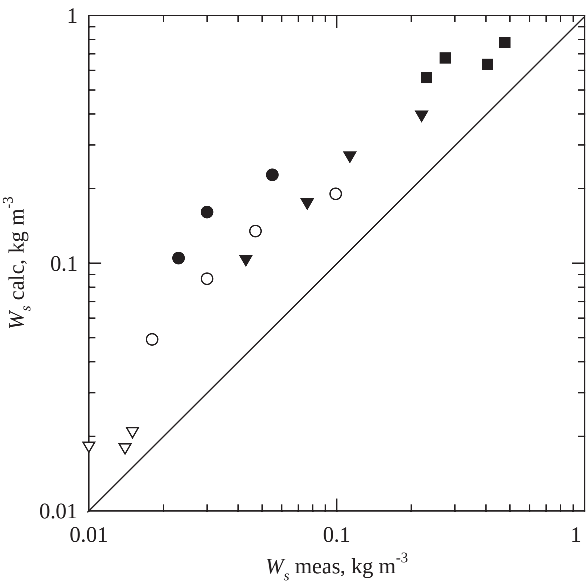

Fig. 7. Experimental and calculated solids discharge rate in a parity plot for Eq. (3) (Crewdson et al., 1977). See Table 1 for the symbol legend.

motion. Differently, the mass flow rates referred to the glass beads (Fig. 4) are well predicted by Eq. (2). As a result, the correction term accounting for the pressure gradient present in Eq. (14) causes an underestimate with respect to the experimental results. Eq. (3) overestimates the solids discharge rates. The discrepancy between Eq. (14) and experimental data can be explained by excluding the occurrence of dilation in the granular flow region during the discharge of this material. Therefore, the whole pressure gradient is to be attributed to a suspended motion regime. This can be justified by the extreme regularity and stiffness of the material which would produce the loss of the mechanical interparticle interaction proper of the granular flow regime with very small solids expansion. Another possibility is that there is a perfectly elastic interaction between particles and, therefore Eq. (8) might apply indifferently to a large range of coordinates far above the transition arch.

Model evaluation was compared also with experimental results by Crewdson et al. (1977) and by Head (1979). The parity plots of the experimental discharge rates found by those authors and Donsì et al. (1997) and those predicted with the use of theoretical models are given in Figs. 5–7. In particular, Fig. 5 shows results obtained by using the original Brown and Richards (1970) model Eq. (2), Fig. 6 shows those obtained by using the modification of the Brown and Richards (1970) model Eq. (14), and Fig. 7 shows those obtained by using the Crewdson et al. (1977) model Eq. (3).

Limiting the analysis to the experimental results by Crewdson et al. (1977) Figs. 5 and 6 show some improvement in the use of Eq. (14) with respect to that of Eq. (2). The presently modified model, in fact, underestimates the experimental findings by not more than 20–30%. Eq. (3) instead overestimates experimental results more than Eq. (2).

With regard to experimental results provided by Head (1979) Figs. 5 and 6 show that Eq. (14) overestimates the mass flow rate by a maximum of 20–30%. However, the discharge rates predicted by Eq. (14) are closer to the experimental values than those predicted by Eqs. (2) and (3).

It should be pointed out that in both the Crewdson et al. (1977) and the Head (1979) experiments the pressure was measured at the wall, while the value corresponding to the bulk could be different. Moreover, the very steep pressure profiles in the vicinity of the outlet do not allow to distinguish between the two sections of the pressure profiles corresponding to the granular flow zone and the suspended solids zone. Consequently, these features might explain the partial disagreement between the present model results and the experimental ones.

3.2. Evaluation of the solids dilation during the discharge

In order to complete the application of the model proposed, Table 1 reports the results of the analysis leading to the evaluation of the solids dilation taking place in the vicinity of the outlet during the discharge. Results are given in terms of voidage at the outlet ε_0 which can be compared with the bulk voidage ε_b. In particular, the results derived from the data by Donsì et al. (1997) indicate that the maximum dilation is less than 1% for glass beads and polymer powder, and it is less than 4% for FCC powder. With concern to the data taken from Crewdson et al. (1977), the maximum relative variation of the voidage does not exceed 2%. In the case of the results derived by Head (1979) the voidage at the arch does not differ from that in the bulk for more than 1%.

These results on the voidage variation suggest some considerations. On the one hand, the limited extent of the dilation of the granular material occurring in the vicinity of the outlet can be neglected in the continuity equation for the solids without seriously affecting the mass flow rate prediction. On the other hand, the material expansion is responsible for the gas pressure gradient that affects the fine powders flow during the discharge. Consequently, any predictive model relevant to the discharge of fine solids does need to account for the dilation and to estimate it. However, according to our knowledge, all the constitutive equations proposed in the literature correlating the material voidage with the mean of the normal solids stresses (see for example Schofield and Wroth, 1968; Gu et al., 1992a) seem not to be sufficiently accurate to describe voidage variations of the order of a few points per cent. Thus, even a model that is correctly taking into account the relative importance of the gravity force on particles and of the gas–particles interactions has very small chances to correctly evaluate the material dilation and, therefore, to correctly predict the solids flow rate.

4. Conclusions

The extension of the "free fall arch" theory to the case of fine powders represents a good learning model of the problem. In particular, this extension is able to highlight that the pressure drop corresponding to the granular region in the hopper is responsible for the interaction between the particles and the gas. Indeed, the use in Eq. (14) of the pressure gradient corresponding to the solids suspension would lead to an underestimate of the solids mass flow rate. This pressure gradient is, in fact, larger than the pressure gradient in the granular flow region which is used in the application of the model shown above.

However, some discrepancies with the experimental results still remain. Some of these might be attributed to the approximations made in the model for the sake of simplicity while others can be explained by some uncertainties derived from the experimental data used for the comparison. There are also some that can be explained within the framework of the model hypothesis by the onset of particular experimental conditions.

The evaluation of the solids dilation during the discharge indicates that very limited solids expansions are sufficient to produce the experimentally measured gas pressure gradients, which are able to significantly affect the solids discharge rate. The magnitude of the solids dilation is so small that, to the best of our knowledge, none of the presently available models relating the bulk voidage with the solids consolidation is able to provide it with sufficient accuracy.

Notation

C	discharge coefficient, dimensionless
d_p	powder particle mean diameter, m
D_0	hopper outlet diameter, m
f	function introduced in Eq. (5), $m^3\,s^{-1}$
g	acceleration due to gravity, $m\,s^{-2}$
k	particle shape constant, dimensionless
p	pressure, Pa
Q_f	gas flow rate, $m^3\,s^{-1}$
Q_s	solids flow rate, $m^3\,s^{-1}$
r	radial coordinate taken in the hopper, m
r_b	radial coordinate where the pressure gradient is nil, m
r_0	radial coordinate of the hemispherical surface at the orifice, m
T	total mechanical energy, $kg\,m^{-1}\,s^{-2}$
T_f	mechanical energy of the gas phase, $kg\,m^{-1}\,s^{-2}$
T_s	mechanical energy of the solid phase, $kg\,m^{-1}\,s^{-2}$
v_f	interstitial fluid velocity, $m\,s^{-1}$
v_s	solid velocity, $m\,s^{-1}$
v_{sl}	slip velocity, $m\,s^{-1}$
W_s	solids discharge mass flow rate, $kg\,s^{-1}$

Greek letters

α	hopper half-angle measured from the vertical, dimensionless
α_{max}	maximum hopper half-angle for mass flow discharge, dimensionless
ε	local solids voidage, dimensionless
ε_0	solids voidage at r_0, dimensionless
ε_b	solids voidage at r_b, dimensionless
θ	zenithal angular coordinate, dimensionless
μ_f	interstitial fluid viscosity, Pa s
ρ_b	solids bulk density, $kg\,m^{-3}$
ρ_f	interstitial fluid density, $kg\,m^{-3}$
ρ_s	particle density, $kg\,m^{-3}$
σ	normal stress within the solids phase in the radial direction, Pa
ϕ_e	effective angle of internal friction, dimensionless
ϕ_w	angle of wall friction, dimensionless

Acknowledgements

This work was partially funded by the Italian Ministry of Education, University and Research (MIUR) by a grant for Research Projects of National Relevance (PRIN).

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