# Optimal Control Algorithms Based on Energy Criteria for Semi-Active Control of Bridges

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### ABSTRACT

The present work analyses the dynamic response of bridges equipped with semi-active isolation devices based on optimal energy control methodologies.

In the case of systems equipped with semi-active controls, optimization response criteria and driving algorithms, derived by energy balance analysis, have been described. The proposed control methodologies are tested by analysing the numeric response of isolated bridge structures to recorded excitations on equivalent linear two degrees of freedom models. By comparing the responses with and without control, it is possible to identify optimal design parameters as well as the effectiveness of the proposed semi-active control methodologies.

### **1. INTRODUCTION**

Through this study we wish to make a contribution to the development of design methodologies for isolated bridges controlled by semi-active devices that allow for the modification of the mechanical characteristics of the isolation level and, therefore, of the dynamic coupling by pile and deck motion. Such modifications of structural parameters (fig.1) may concern the stiffness characteristics (AVS, Active Variable Stiffness) [Kobori et al. 1990], the damping characteristics (AVD, Active Variable Damping) [Kobori et al. 1994, Spencer, Dyke, Sain 1996] or even both [Spencer, Dyke, Sain 1996].

From the study of a linear equivalent 2DOF isolated system equipped with semi-active devices, which allow for an instantaneous variation of stiffness and damping, different criteria of response optimization are investigated. Through an analysis carried out in terms of energy balance, the corresponding driven algorithms for semi-active devices are carried out [Palazzo, Petti, Mauriello 2001] and tested by analysing the numerical response to recorded excitations. By comparative seismic analysis of bridge models with and without semi-active control, it is possible to identify the optimal design control parameters as well as the effectiveness of the proposed methodologies.

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## 2. ANALYTICAL MODEL

Let's consider a linear equivalent 2DOF model of a bridge equipped with semi-active isolation which is capable of varying of both stiffness and damping (fig. 1).



Figure 1: Model of an isolated bridge with semi-active control

The dynamic equations of the system, subjected to seismic excitation  $u_g(t)$ , may be written:

$$\begin{aligned} \ddot{x}_{i} + \ddot{x}_{p} + 2\xi_{i}\omega_{i} \cdot \dot{x}_{i} + \omega_{i}^{2} \cdot x_{i} &= -\ddot{u}_{g} + f_{c} \\ \ddot{\chi} \cdot \ddot{x}_{i} + (1 + \chi) \cdot \ddot{x}_{p} + 2\xi_{p}\omega_{p} \cdot \dot{x}_{p} + \omega_{p}^{2} \cdot x_{p} &= -(1 + \chi) \cdot \ddot{u}_{g} \end{aligned}$$
(1)

where x and u represent respectively the relative and absolute displacement of deck (i) and the pile (p);  $\xi_i$  and  $\omega_i$  the damping factor and the natural frequency of the deck in the case of an infinitely stiff pile and  $\Delta c_i$  and  $\Delta k_i$  set to zero;  $\xi_p$  and  $\omega_p$  the same quantities for a pile disconnected from the bridge deck,  $\chi$  the mass ratio  $m_i/m_p$ . The force  $f_c$  represents the action of the feedback system control, given by:

$$f_c = -(2\xi_i \omega_i \alpha_c \cdot x_i + \omega_i^2 \alpha_k \cdot x_i)$$
<sup>(2)</sup>

where  $\alpha_c$  and  $\alpha_k$  represent respectively the variations of the control parameters  $\alpha_c = \Delta c_i / c_i$ ,  $\alpha_k = \Delta k_i / k_i$ . The semi-active devices, placed between pile and bridge deck, allow for variation of stiffness and damping according to the following two states of regulation:

$$\alpha_{k} = \begin{cases} 0 & semi-active \ devices \ OFF \\ \alpha_{cmax} & semi-active \ devices \ ON \end{cases}$$

By integrating the dynamic equations (1), as regards the relative displacements, it is possible to derive the energy description as follows:

$$E_{k,i} + E_{\xi,i} + E_{e,i} = E_f + E_{i-p}$$
  
 $\chi \cdot E_{i-p} + E_{k,p} + E_{\xi,p} + E_{e,p} = E_i$ 
(3)

In (3)  $E_{k,i}$  and  $E_{k,p}$  represent respectively the kinetic energy of the deck and of the pile,  $E_{\xi,i}$  and  $E_{\xi,p}$  the viscous energy,  $E_{e,i}$  and  $E_{e,p}$  the elastic energy,  $E_i$  the input energy to the overall system,  $E_{i-p}$  the exchanged energy between the deck and the pile and, finally,  $E_f$  the control energy.

## **3. SEMI-ACTIVE CONTROL METHODOLOGIES**

From the energy description of the dynamic behavior of the system (eq.3) it is possible to identify the following main control criteria:

a) Maximization of the energy dissipated by the control devices;

b) Minimization of the elastic energy of the pile;

c) Minimization of the kinetic energy of the pile;

*d*) *Minimization of the kinetic energy of the bridge deck;* 

e) Minimization of total input energy.

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The state of the semi-active devices is selected governing the parameters  $\alpha_k$  and  $\alpha_c$  according to the given control criterion as following.

In the case of the maximization of energy dissipated by the control devices  $(E_f)$ , managing the power  $P_f = f_c \cdot x_i = -\left(2\xi_i\omega_i\alpha_c \cdot x_i^2 + \omega_i^2\alpha_k \cdot x_i \cdot x_i\right)$  the following control algorithm is carried out:

$\alpha_c = \alpha_{cmax}$	$\alpha_k = \begin{cases} \\ \\ \\ \\ \end{cases}$	$\alpha_{k max}$	se	$x_i \cdot x_i > 0$	maximization of the energy
		$\alpha_{k_{min}}$	se	$x_i \cdot x_i < 0$	dissipated by the control devices

Analogously, having taken account of (1) and according to case a) from the energy balance the methodologies adopted lead to:

$\alpha_c = \langle$	$\alpha_{c_{max}}$ $\alpha_{c_{min}}$	se se	$ \begin{array}{l} x_i \cdot x_p < 0 \\ x_i \cdot x_p > 0 \\ x_i \cdot x_p > 0 \end{array} $	$\boldsymbol{\alpha}_{k} = \begin{cases} \boldsymbol{\alpha}_{k max} \\ \boldsymbol{\alpha}_{k min} \end{cases}$	se $x_i \cdot x_p < 0$ se $x_i \cdot x_p > 0$	minimization of the elastic energy of the pile
$\alpha_{c} = 0$	$\begin{cases} \alpha_{c \max} \\ \alpha_{c \min} \end{cases}$	se se	$x_i \cdot u_p > 0$ $x_i \cdot u_p < 0$	$\boldsymbol{\alpha}_{k} = \begin{cases} \boldsymbol{\alpha}_{k \max} \\ \boldsymbol{\alpha}_{k \min} \end{cases}$	se $x_i \cdot u_p > 0$ se $x_i \cdot u_p < 0$	minimization of the kinetic energy of the pile
$\alpha_c = 0$	$\begin{cases} \alpha_{c_{max}} \\ \alpha_{c_{min}} \end{cases}$	se se	$ \begin{array}{l} \begin{array}{c} \vdots \\ x_i \cdot u_i > 0\\ \vdots \\ x_i \cdot u_i < 0 \end{array} $	$\boldsymbol{\alpha}_{k} = \begin{cases} \boldsymbol{\alpha}_{k_{max}} \\ \boldsymbol{\alpha}_{k_{min}} \end{cases}$	$se  x_i \cdot u_i > 0$ $se  x_i \cdot u_i < 0$	minimization of the kinetic energy of the deck
$\alpha_c = \delta$	$\left\{ egin{aligned} & \alpha_{cmax} \ & \alpha_{cmin} \end{aligned}  ight.$	se se	$x_i \cdot u_g > 0$ $x_i \cdot u_g < 0$	$\boldsymbol{\alpha}_{k} = \begin{cases} \boldsymbol{\alpha}_{kmax} \\ \boldsymbol{\alpha}_{kmin} \end{cases}$	se $x_i \cdot u_g > 0$ se $x_i \cdot u_g < 0$	minimization of total input energy

## 4. EFFECTIVENESS ANALYSIS

In figures 3-20 the maximum seismic responses of the system are represented as a function of the control parameters  $\alpha_c$  and  $\alpha_k$  for different seismic events (fig. 2 and table 1) and proposed control methodologies. The highlighted regions represent the control parameter ranges where the semi-actively controlled response is better than the passive one for the same system.

Seismic even	t [sec]	PGA [cm/s <sup>2</sup> ]	$\begin{bmatrix} S_a \\ A \end{bmatrix}$	
1) Imperial Valley	(1940)	53.8	341.8	$\frac{A_{\text{max}}}{2}$
2) Kern County	(1952)	54.4	175.9	$\frac{1}{4}$
3) Loma Prieta	(1989)	40.0	270.4	$\begin{array}{c c} & - & - & - & - & - & - & - & - & - & $
4) Mexico City	(1995)	180.1	167.91	3 8 1
5) Santa Monica	(1994)	60.0	866.0	2
6) Pacoima	(1971)	41.9	1148.1	
7) Parkfield	(1966)	26.2	269.6	
8) San Fernando	(1971)	59.5	250.0	Figure 2: Response spectra in terms of absolute acceleration

 Table 1. Considered Seismic Events

The numerical analyses were carried out by using the following design parameters:

$$\chi = m_i/m_p = 1$$
,  $\xi_p = 0.02$ ,  $\xi_i = 0.30$ ,  $T_p = 0.5 \, sec$ ,  $I = T_i/T_p = 2$ 

For the Imperial Valley (1940) ground motion, figs. 3-8 show the energy response of controlled systems (ON) in comparison to the no-controlled one (OFF) (Control on the Kinetic Energy of the pile Figs. 3-4, Control on the Kinetic Energy of the deck Figs. 5-6, Control on the Input Energy Figs. 7-8). The analysis of the results show that semi-active control leads to good results in energy terms, independently of the selected optimization criterion. With semi-active control the system response is always better when compared to uncontrolled and passive controlled cases.

Figs. 9-14 show the relative displacements response of the pile. Finally, Figs. 15-20 represent the worst responses of the system for the considered seismic excitations. The figures analysis show that the proposed control algorithms are more efficient in reaching the targets for which they have been designed. The pile shows the best performances in terms of relative displacements for the criteria of control of the minimization of elastic and kinetic energy while, on the other hand, the response of the deck is disadvantaged. The control criterion based on the minimization of input energy allows for averagely good performance of the overall system.

It is possible to recognize that hybrid control obtained through a combination of semiactive control of the bridge isolation is a "robust control strategy" with respect to the uncertainty of the input signal and the mechanical parameters of the overall system. For assigned performance targets is possible to design the optimum control parameters of the semi-active devices.

## **5. CONCLUSIONS**

This paper has discussed new methodologies of semi-active control for isolated bridges. The proposed control strategies have been numerically tested through a comparative analysis of the seismic response of systems, with and without semi-active devices, to recorded accelerograms.

The control algorithms derived here are more efficient in reaching the objectives for which they have been designed. The pile shows the best performances in terms of relative displacements for the criteria of control of the minimization of elastic and kinetic energy while, on the other hand, the response of the deck is disadvantaged. The control criterion based on the minimization of input energy allows for averagely good performance of the overall system.

Semi-active regulation of the mechanical parameters allows the structure to reach assigned minimum performance levels, independently of the spectral characteristics of the input signal. Therefore, it is possible to recognise that the proposed control methodologies obtained through the combination of semi-active control and bridge isolation are robust control strategies with regard to the uncertainty concerning the input signal and the mechanical parameters of the overall system.

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**Figure 3.** Response comparison in terms of kinetic energy of the pile – FIXED ON – Imperial Valley earthquake



**Figure 5.** Response comparison in terms of kinetic energy of the deck – FIXED ON – Imperial Valley earthquake



**Figure 7.** Response comparison in terms of of input energy to the overall system – FIXED ON – Imperial Valley earthquake



**Figure 4.** Response comparison in terms of kinetic energy of the pile – Minimization of the pile kinetic energy algorithm– Imperial Valley earthquake



**Figure 6.** Response comparison in terms of kinetic energy of the deck – Minimization of the deck kinetic energy algorithm– Imperial Valley earthquake



**Figure 8.** Response comparison in terms of input energy to the overall system – Minimization of the input energy algorithm – Imperial Valley earthquake



**Figure 9.** Response comparison in terms of relative displacements of the pile – FIXED ON – Imperial Valley earthquake



**Figure 11.** Response comparison in terms of relative displacements of the pile – Minimization of the elastic energy of the pile algorithm–Imperial Valley earthquake



Figure 13. Response comparison in terms of relative displacements of the pile – Minimization of the kinetic energy of the deck algorithm–Imperial Valley earthquake



**Figure 10.** Response comparison in terms of relative displacements of the pile – Maximization of the dissipated energy algorithm– Imperial Valley earthquake



**Figure 12.** Response comparison in terms of relative displacements of the pile – Minimization of the kinetic energy of the pile algorithm–Imperial Valley earthquake



**Figure 14.** Response comparison in terms of relative displacements of the pile – Minimization of the input energy to the overall system algorithm– Imperial Valley earthquake



**Figure 15.** The worst response in terms of kinetic energy of the pile – FIXED ON



**Figure 17.** The worst response in terms of kinetic energy of the deck – FIXED ON



Figure 19. The worst response in terms of input energy to the overall system – FIXED ON



**Figure 16.** The worst response in terms of kinetic energy of the pile – Minimization of the kinetic energy of the pile algorithm



**Figure 18.** The worst response in terms of kinetic energy of the deck – Minimization of the kinetic energy of the deck algorithm



**Figure 20**. The worst response in terms of input energy to the overall system – Minimization of the input energy algorithm