Optimal Control Algorithms based on Energy Criteria for Semiactive Isolation

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ABSTRACT

In the present study, methodologies of optimal control of base isolated systems through the use of additional semi-active devices are investigated. Particularly, by analyzing the single contributions to the energy balance of a linear equivalent two-degree of freedom base isolated system, driven law of semi-active devices are carried out. From a comparative analysis of the seismic response of controlled and uncontrolled isolated systems, the effectiveness of the proposed methodologies are investigated.

1. INTRODUCTION

As is known, the efficiency of base isolation depends on the capacity of filtering, through the use of flexible elements, the horizontal components of the excitations with frequencies that are close to the fundamental one of the structure being protected. However, in the case of excitations with high energy content over long periods the isolation elements could be subjected to a high degree of deformation. The need to contain these displacements can be partially resolved by introducing supplementary damping devices; a solution that translates into an increase in the dynamic impedance of the isolating interface which, for high value, reduces the benefits that derive from the isolation since it contaminates the filter effect at higher frequencies [Palazzo,Petti 1995]. In the present study we explore the possibility of controlling the isolator displacements (Fig. 1) through the use of semi-active devices [Kobori T et al. 1994, Inaudi 1996, Kobori et al. 1990, Kawashima et al. 1993, Spencer, Dyke 1996] which are capable of varying the mechanical properties of stiffness and damping at such a level [Inaudi, Kelly 1993, Palazzo,Petti 1995] according to optimal energetic criteria.

2. ANALYTICAL MODEL

Let us consider a base isolated, two-degree of freedom, linear equivalent model which is equipped with semi-active devices capable of varying stiffness and damping (Fig. 2). The dynamic equations, when the system is subjected to a generic base seismic excitation $\ddot{u}_g(t)$, representing the characteristics of the semi-active devices with variable parameters of damping Δc and of stiffness Δk, can be written:

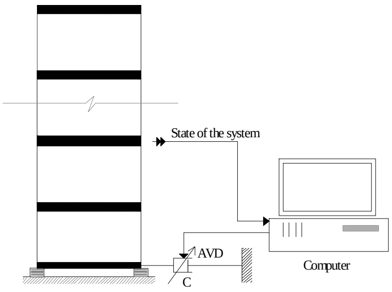

Figure 1. Semi-active Control Scheme

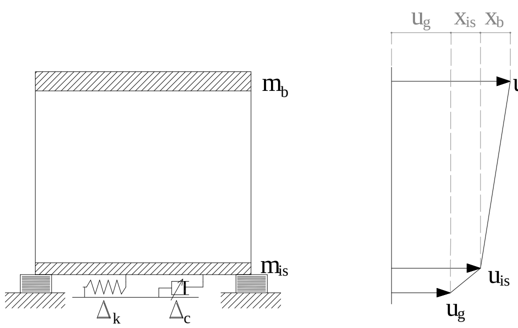

Figure 2. Semi-active BIS system model

$$m_b(\ddot{x}_b+\ddot{x}_{is}+\ddot{u}_g)+c_b\dot{x}_b+k_b x_b=0 \tag{1}$$

$$m_{is}(\ddot{x}_{is}+\ddot{u}_g)+(c_{is}+\Delta c_{is})\dot{x}_{is}+(k_{is}+\Delta k_{is})x_{is}-c_b\dot{x}_b-k_b x_b=0 \tag{2}$$

where, m_b and m_{is} represent respectively the masses of the superstructure and isolated level, c_b and k_b the damping and the stiffness of the superstructure, c_{is} and k_{is} the nominal values of the damping and the stiffness of the isolation, x and u the relative and absolute motion. Equations (1) and (2) can be rewritten thus :

$$\ddot{u}_b+2\xi_b\omega_b\dot{x}_b+\omega_b^2 x_b=0 \tag{3}$$

$$\chi\ddot{u}_b+(1-\chi)\ddot{u}_{is}+2\xi_{is}\omega_{is}\dot{x}_{is}+\omega_{is}^2 x_{is}=-(2\xi_{is}\omega_{is}\alpha_c\dot{x}_{is}+\omega_{is}^2\alpha_k x_{is})=f_c \tag{4}$$

where ξ_b and ω_b represent the damping factor and the natural frequency of the superstructure, ξ_{is} and ω_{is} the same quantities for the isolation level in the uncontrolled case, χ the mass ratio m_b/m_b+m_{is} , α_c and α_k the ratios $\alpha_c=\Delta c_{is}/c_{is}$, $\alpha_k=\Delta k_{is}/k_{is}$. Such a relationships show that the action of the semi-active devices can be regarded as a feedback control force f_c on the system . By integrating equations (3) and (4) in respect to the relative displacements of the system we obtain:

$$\int_0^t \ddot{u}_b\dot{u}_b\,d\tau+\int_0^t 2\xi_b\omega_b\dot{x}_b^2\,d\tau+\int_0^t \omega_b^2 x_b\dot{x}_b\,d\tau=\int_0^t \ddot{u}_b\dot{u}_{is}\,d\tau \tag{5}$$

$$\chi\int_0^t \ddot{u}_b\,\dot{u}_{is}\,d\tau+(1-\chi)\int_0^t \ddot{u}_{is}\,\dot{u}_{is}\,d\tau+\int_0^t 2\xi_{is}\omega_{is}\,\dot{x}_{is}^{\,2}\,d\tau+\int_0^t \omega_{is}^2 x_{is}\,\dot{x}_{is}\,d\tau= \\ =\chi\int_0^t \ddot{u}_b\,\dot{u}_g\,d\tau+(1-\chi)\int_0^t \ddot{u}_{is}\,\dot{u}_g\,d\tau+\int_0^t f(t)\,\dot{x}_{is}\,d\tau \quad (6)$$

Equations (5-6) in symbolic form, are written into the following energy balance:

$$E_{k_b}+E_{\xi_b}+E_{e_b}=E_{b,is};\quad E_{b,is}+\frac{(1-\chi)}{\chi}E_{k_{is}}+\frac{1}{\chi}(E_{\xi_{is}}+E_{eis})=E_{i_b}+\frac{(1-\chi)}{\chi}E_{i_{is}}+\frac{1}{\chi}E_f \quad (7)$$

where E_{k_b}, $E_{k_{is}}$ represent the kinetic energies of the superstructure and isolation level, E_{ξ_b}, $E_{\xi_{is}}$ the amount of dissipated viscous energy, E_{e_b} and E_{eis} the amount of elastic energy, E_{i_b} and $E_{i_{is}}$ the amount of input energy, $E_{b,is}$ the energy exchange between the superstructure and the isolation, and finally E_f the control energy.

3. PROPOSED OPTIMAL SEMI-ACTIVE CONTROL METHODOLOGIES

The proposed control methodologies are based on the possibility of managing the seismic response in complex systems by regulating individual amounts of the energy balance that describe its dynamic behavior. From the energy description (eq. 7), it is possible to recognize the following control criteria:

a) *the maximization of the energy dissipated by the elements of control*; b) *the minimization of the elastic energy of the isolation level*; c) *the minimization of the kinetic energy of the isolation level; d) the minimization of the kinetic energy of the superstructure*; e) *the minimization of the input energy to the system*.

It is assumed that the semi-active devices allow for a variation of the stiffness and damping properties of isolation according to the following two regulating states:

$$\Delta k=\begin{cases}0\\ \Delta k_{max}\end{cases}\qquad \Delta c=\begin{cases}0 & \text{Semi-active device OFF}\\ \Delta c_{max} & \text{Semi-active device ON}\end{cases}$$

In case “*a)*” (*maximization of the energy dissipated by the elements of control*), by considering that positive values of E_f imply an energy transfer from the system of control to the main structure, in order to reduce the overall system energy, it is necessary to minimize E_f. To achieve this target, it is possible to manage the power P_f :

$$P_f(t)=f_c\cdot\dot{x}_{is}=-\left(2\xi_{is}\omega_{is}\alpha_c\,\dot{x}_{is}^{\,2}+\omega_{is}^2\alpha_k\,x_{is}\,\dot{x}_{is}\right) \quad (8)$$

From eq. 8, P_f is minimum for α_c and α_k set to the maximum value with the sign according the one of the product $x_{is}\cdot\dot{x}_{is}$. Therefore, the adopted control criterion leads to :

$$\alpha_c = \alpha_{c\,max} \qquad \alpha_k = \begin{cases} \alpha_{k\,max} & se \quad x_{is} \cdot \dot{x}_{is} > 0 \\ \alpha_{k\,min} & se \quad x_{is} \cdot \dot{x}_{is} < 0 \end{cases} \tag{9}$$

Analogously, from the energy balance, having taken account of (1) and according to case a), the criteria adopted lead to:

$$\alpha_c = \alpha_{c_{max}} \qquad \alpha_k = \begin{cases} \alpha_{k\,max} & se \quad x_{is} \cdot \dot{x}_{is} > 0 \\ \alpha_{k\,min} & se \quad x_{is} \cdot \dot{x}_{is} < 0 \end{cases}$$

minimization of the elastic energy of the isolation level

$$\alpha_c = \begin{cases} \alpha_{c\,max} & se \quad \dot{x}_{is} \cdot \dot{u}_{is} > 0 \\ \alpha_{c\,min} & se \quad \dot{x}_{is} \cdot \dot{u}_{is} < 0 \end{cases} \qquad \alpha_k = \begin{cases} \alpha_{k\,max} & se \quad x_{is} \cdot \dot{u}_{is} > 0 \\ \alpha_{k\,min} & se \quad x_{is} \cdot \dot{u}_{is} < 0 \end{cases}$$

minimization of the kinetic energy of the isolation level

$$\alpha_c = \begin{cases} \alpha_{c\,max} & se \quad \dot{x}_{is} \cdot \dot{u}_{b} > 0 \\ \alpha_{c\,min} & se \quad \dot{x}_{is} \cdot \dot{u}_{b} < 0 \end{cases} \qquad \alpha_k = \begin{cases} \alpha_{k\,max} & se \quad x_{is} \cdot \dot{u}_{b} > 0 \\ \alpha_{k\,min} & se \quad x_{is} \cdot \dot{u}_{b} < 0 \end{cases}$$

minimization of the kinetic energy of the superstructure

$$\alpha_c = \begin{cases} \alpha_{c\,max} & se \quad \dot{x}_{is} \cdot \dot{u}_{g} > 0 \\ \alpha_{c\,min} & se \quad \dot{x}_{is} \cdot \dot{u}_{g} < 0 \end{cases} \qquad \alpha_k = \begin{cases} \alpha_{k\,max} & se \quad x_{is} \cdot \dot{u}_{g} > 0 \\ \alpha_{k\,min} & se \quad x_{is} \cdot \dot{u}_{g} < 0 \end{cases}$$

minimization of input energy into the system

4. EFFECTIVENESS ANALYSIS – NUMERICAL TESTS

The system described in fig. 2 has been tested by considering the registered excitations shown in the following table.

Sisma	T[sec]	PGA[cm/s^2]
1) Imperial Valley(1940)	53.8	341.82
2) Kern County (1952)	54.42	175.90
3) Loma Prieta (1989)	40.00	270.36
4) Mexico City (1995)	180.1	167.91
5) Santa Monica (1994)	60.0	865.97
6) Pacoima (1971)	41.90	1148.10
7) Parkfiled (1966)	26.18	269.60
8) San Fernando (1971)	59.0	250.00

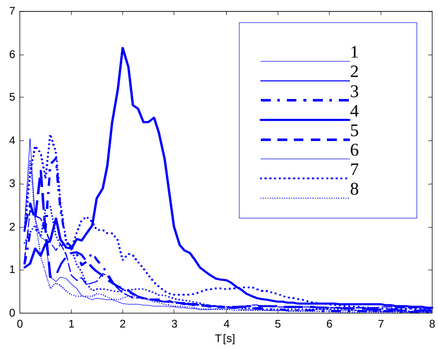

Figure 3. Response spectra in terms of absolute acceleration

Figures 4-19 show the comparison between the maximum values of the response of the system equipped with and without semi–active controls, according to the pre-established criteria on varying the control parameters α_k and α_c.

The analysis of the system without controls is defined by the following parameters :

$$\chi = 0.8 \qquad \xi_b = 0.02 \qquad \xi_{is} = 0.05 \qquad T_b = 0.6\ sec \qquad I = T_{is}/T_b \in [2,8]$$

where T_b is the natural period of the superstructure and T_{is} is the one for the isolated system. Figures 4-11 show the response of the system subject to the Imperial Valley earthquake (El Centro 1940).

Figures 4-7 show the response comparison in terms of absolute acceleration $\ddot{u}_b / \ddot{u}_{b\,nc}$ and relative displacements of the superstructure $x_b / x_{b\,nc}$ respectively for cases of passive control (semi-active control devices set to maximum values) and proposed optimum control criteria. Figures 8 and 9 show the comparison between the responses of the system in terms of relative base displacements with passive and semi-active control following the maximization of extra-structural dissipation, or rather, the minimization of elastic energy on the plane of isolation. Results shows that semi-active control methodologies lead to better seismic performances for the overall system in a wide ranges of control parameters α_k and α_c.

Figures 10-11 show the comparison between the responses of the system for different control criteria, respectively in terms of absolute acceleration, or rather, relative displacements of the superstructure and relative displacements of the isolation layer (ON= passive control system ; a = maximization of the energy dissipated by the extra-structural system ; b = minimization of base level elastic energy; c = minimization of the kinetic energy of the level of isolation; d = minimization of the kinetic energy of the superstructure ; e = minimization of system input energy). For each criterion of control, the figures represent the ranges which delineate the regions where there is better behavior from those where the response worsens when compared to uncontrolled cases.

Figures 12-19 show the minimum gain obtained in the response of the system to the different seismic excitations. Particularly, figures 12-15 show the comparisons between the responses of the system in terms of absolute acceleration and relative displacements of the superstructure respectively for the cases of passive control and semi-active control following the criteria: maximization of energy dissipated extra-structurally or rather the minimization of base elastic energy; minimization of the kinetic energy of the superstructure; minimization of the kinetic energy of the base. Figures 16 and 17 show the comparison between the responses of the system in terms of relative displacements of the base for passive control and semi-active control following the maximization of the dissipation of energy extra-structurally or rather the minimization of the elastic energy of the level of isolation. And finally, figures 18-19 show the input energies comparisons. An analysis of the results shows that in the case of the semi-active controlled system, there exists a wide range of control parameters α_k and α_c which lead to better seismic performance for the overall system. The figures show that the proposed control methodologies of isolated systems allow us to obtain assigned minimum performance levels independently of the input signal’s spectrum features. It is therefore possible to recognize that hybrid control obtained through a combination of semi-active control of the base isolation is a “robust control strategy” with respect to the uncertainty of the input signal and the mechanical parameters of the overall system.

5. CONCLUSION

The present study has discussed new methodologies of semi-active control. From an energy analysis of the dynamic behavior of equivalent linear two-degree of freedom base isolated systems, optimum energy regulating criteria have been identified. The proposed control algorithms have been numerically tested, and the response of passive and semi-actively controlled systems subject to recorded accelerograms comparatively analyzed.

With semi-active control the system response is always better when compared to uncontrolled and passive controlled cases. The control algorithms we have discussed are more efficient in achieving their pre-assigned targets. The superstructure shows greater overall performance with the control criterion of its own kinetic energy, while the isolation level shows the best behavior with the control criterion of the minimization of the elastic energy of the same level.

Semi-active regulation of the mechanical parameters of the isolation level allow us to obtain the assigned minimum performance levels independently of the input signal's spectrum features. It is therefore possible to recognize that hybrid control obtained through a combination of semi-active control of the base isolation is a "robust control strategy" with respect to the uncertainty of the input signal and the mechanical parameters of the overall system.

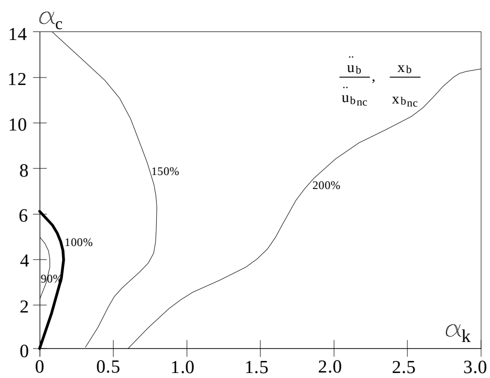

Figure 4. FIXED ON, I=6 – Relative displacements and absolute acceleration of the superstructure. (El Centro 1940)

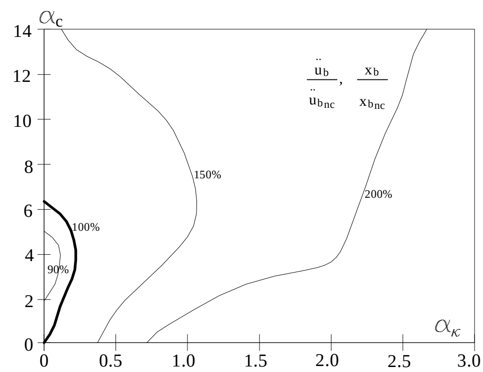

Figure 5. SEMI-ACTIVE ON-OFF, I=6 – Relative displacements and absolute acceleration of the superstructure, maximization of the dissipation of energy extra-structurally . (El Centro 1940)

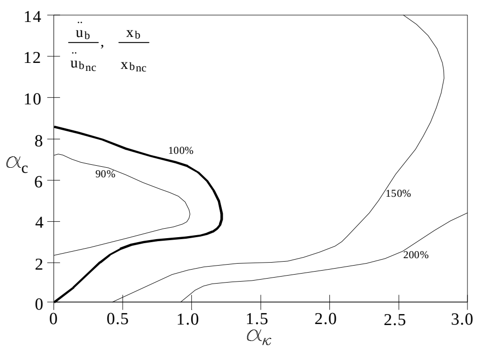

Figure 6. SEMI-ACTIVE ON-OFF, I=6 Relative displacements and absolute acceleration of the superstructure, minimization of the kinetic energy of the superstructure. (El Centro 1940)

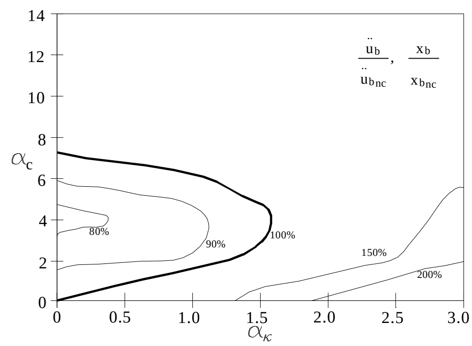

Figure 7. SEMI-ACTIVE ON-OFF, I=6 – Relative displacements and absolute acceleration of the superstructure, minimization of the kinetic energy of the level of isolation. (El Centro 1940)

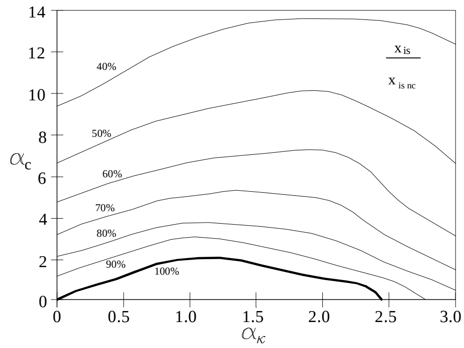

Figure 8. FIXED ON, I=6 – Relative displacements of level of isolation. (El Centro 1940)

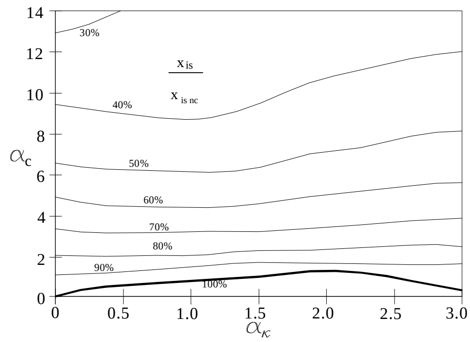

Figure 9. SEMI-ACTIVE ON-OFF, I=6 – Relative displacements of the level of isolation, maximization of the energy dissipated, or rather, of minimization of the elastic energy of the level of isolation. (El Centro 1940)

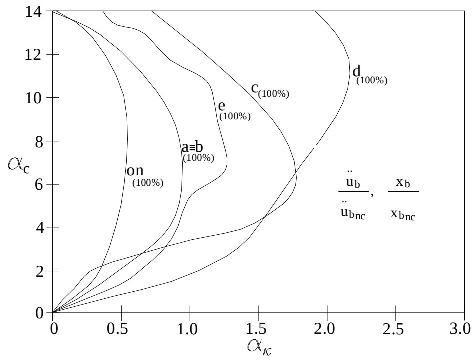

Figure 10. I=5 – Comparison between the delimitation frontiers of the zones of 100% reduction of the response of different types of control in terms of relative displacements and absolute acceleration of the superstructure. (El Centro 1940)

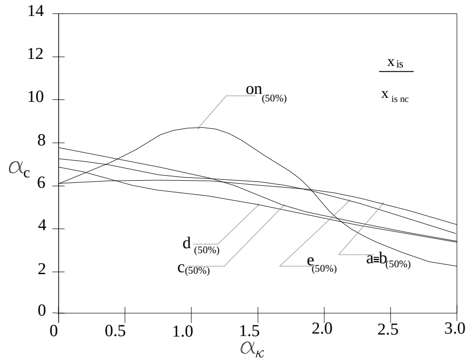

Figure 11. I=5 – Comparison between the delimitation frontiers of the zones of 50% reduction of the response of different types of control in terms of relative displacements of the isolation level. (El Centro 1940)

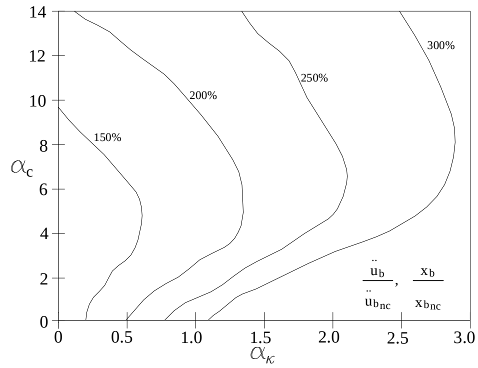

Figure 12. FIXED ON, I=5 – Structural envelope of responses to seismic events considered in terms of relative displacements and absolute acceleration of the superstructure obtained through "Fixed-On" control.

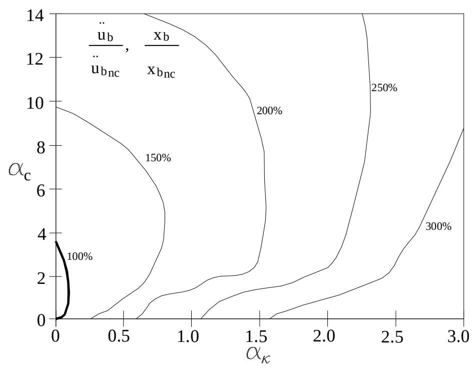

Figure 13. SEMI-ACTIVE ON-OFF, I=5 – Structural envelope of responses to seismic events considered in terms of relative displacements and absolute acceleration of the superstructure obtained through control on the elastic energy of the building base, or rather on the energy dissipated by the extra-structural system.

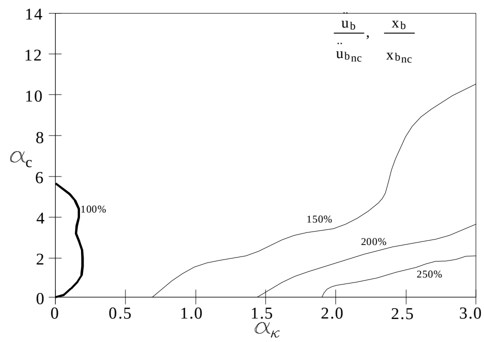

Figure 14. SEMI-ACTIVE ON-OFF, I=5 – Structural envelope of responses to seismic events considered in terms of relative displacements and absolute acceleration of the superstructure obtained through control on the kinetic energy of the superstructure.

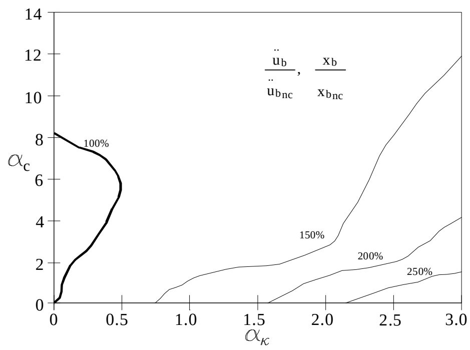

Figure 15. SEMI-ACTIVE ON-OFF, I=5 –Structural envelope of responses to seismic events considered in terms of relative displacements and absolute acceleration of the superstructure obtained through control on the kinetic energy of the level of isolation.

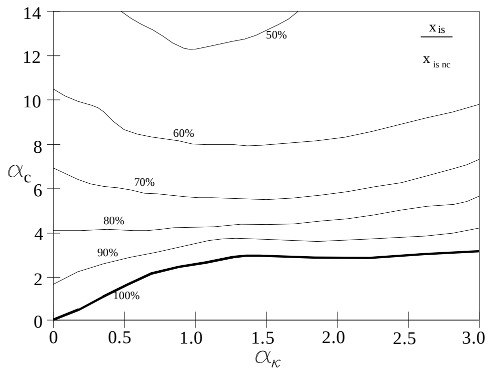

Figure 16. FIXED ON, I=5 – Structural envelope of responses to seismic events considered in terms of relative displacements of the isolation level and obtained through "Fixed-On" control.

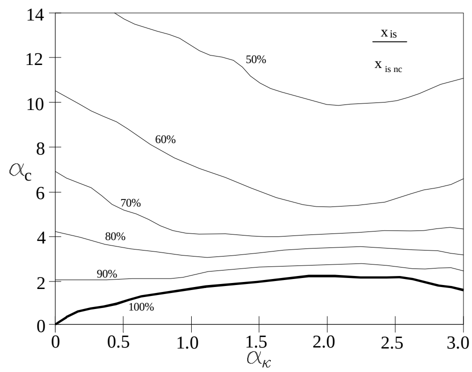

Figure 17. SEMI-ACTIVE ON-OFF, I=5 – Structural envelope of the responses in terms of relative displacement of the level of isolation obtained through control on the elastic energy of the building base.

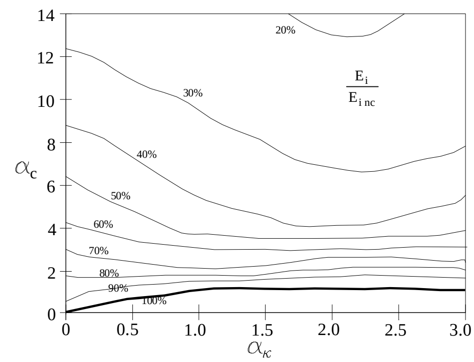

Figure 18. SEMI-ACTIVE ON-OFF, I=5 – Structural envelope of the responses to seismic events considered in terms of input energy to the system obtained through controls on this energy.

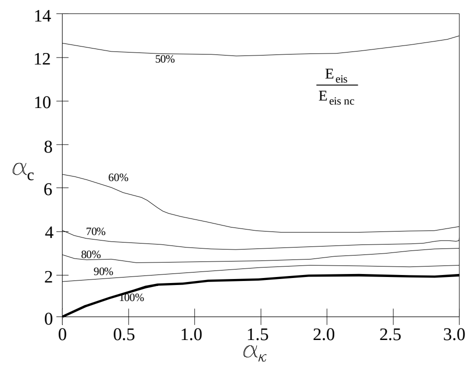

Figure 19. SEMI-ACTIVE ON-OFF, I=5 – Structural envelope of the responses to seismic events considered in terms of elastic energy at the building base obtained through controls on this energy.

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