

New Method for Measuring the Detail Preservation of Noise Removal Techniques in Digital Images

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Abstract: It is known that cancelling the noise without blurring the image details is a very difficult task for any image denoising technique. The availability of metrics for accurate evaluation of filtering distortion is thus of paramount importance for the development of new filters. Peak signal-to-blur ratio PSBR is a recently introduced measure of detail preservation that overcomes the limitations of the sole peak signal-to-noise ratio (PSNR) and other metrics in evaluating the performance of image denoising filters. Formally, the PSBR is the PSNR component that deals with the detail blur, so the method that is adopted for blur estimation plays a key role. This paper presents a novel algorithm for PSBR computation that offers significant advantages over the first method: it is simpler, more robust and much more accurate. Furthermore, this paper presents new validation tools for evaluating the accuracy of this kind of metrics when some well known classes of linear and nonlinear filters are considered. Results of many computer simulations dealing with images corrupted by different combinations of Gaussian and impulse noise show that the proposed PSBR algorithm outperforms the most effective metrics in the field.

Key-Words: - Image filtering, image denoising, Gaussian noise, impulse noise, image quality, PSNR.

1 Introduction

Digital images are very often corrupted by noise, hence the development of effective techniques for data denoising is a key issue in many research and application areas such as robotics, medical imaging, remote sensing, video surveillance, consumer electronics etc.. Image denoising, however, is not a trivial task because the noise should be cancelled while preserving image details and textures [1-7]. Thus, metrics that can measure the filtering blur are necessary in order to analyze the performance of any new denoising method. Peak signal-to-blur ratio (PSBR) is a recently introduced full-reference measure that aims at overcoming the limitations of the sole peak signal-to-noise ratio (PSNR) and other methods in evaluating the performance of greyscale image denoising filters [8]. Indeed, it is known that the PSNR performs badly in distinguishing noise cancellation from detail preservation. On the other hand, the same limitation also affects metrics that aims at mimicking the human perception of image quality [9-14], because different combinations of unfiltered noise and detail blur can give the same score [15]. Conversely, the PSBR represents the

PSNR component that deals with the unwanted distortion produced during noise filtering, so it can be used in conjunction with the classical and widespread adopted PSNR in order to fully characterize the key behavior of a denoising system. This approach also reinforces the validity of PSNR-based quality metrics in image processing [16-19]. In the field of image denoising, that is the subject of this work, many different attempts have been performed to measure the amount of detail blur caused by filtering. Some techniques have adopted edge detectors in order to focus on the filtering errors affecting image details [20-21]. However, the extension of image regions degraded by blur depends upon the filtering parameters and the window size. As a result, when large filtering windows are used, the detail blur can cross the boundaries of the edge regions and its amount is typically underestimated. Furthermore, unfiltered noise still present on the image edges can wrongly be classified as detail blur. Other approaches do not resort to any edge map [22]: for each image pixel, the type of noise correction is analyzed and the filtering error is classified as blur when an excess of

smoothing is revealed. Unfortunately, any wrong classification of pixels in the uniform regions (where no details are present) produces an erroneous result and the detail blur is overestimated.

In the PSBR approach we aimed at overcoming all the mentioned drawbacks. The first algorithm for PSBR computation [8] was not affected by apparent inaccuracies as commonly occurs for other metrics [21-22]. This method can also yield the exact value of the detail preservation when edge and uniform regions are located in different areas of a synthetic (i.e., not real) test picture.

This paper presents a novel PSBR algorithm that offers significant advantages over our previous one:

- the new method is simpler (it does not need any offset-correction procedure),
- it is more robust (it does not require the heuristic choice of thresholds),
- it is much more accurate and can yield the true values of detail preservation for real images, whereas all other techniques fail. In this respect, a collection of new validation tools is provided in the paper for evaluating the accuracy of this kind of metrics.

The rest of the paper is organized as follows. Section 2 briefly reviews the PSBR approach, Section 3 presents the novel method, Section 4 provides new tools for metrics validation, Section 5 discusses the results of many computer simulations and, finally, Section 6 reports the conclusions.

2 The PSBR Approach

Let us deal with digitized images having Q gray levels (typically $Q=256$). Let $r_{i,j}$ be the pixel luminance at location (i,j) in the reference (noise-free) image ($i=1, \dots, L$; $j=1, \dots, M$).

Let $x_{i,j}$ and $y_{i,j}$ be the pixel luminances at the same location in the input noisy image and in the filtered one, respectively. It is well known that the PSNR is expressed by the following relationship:

$$\text{PSNR} = 10 \log_{10} \frac{(Q-1)^2}{\frac{1}{LM} \sum_{i=1}^L \sum_{j=1}^M e_{i,j}^2} \quad (1)$$

where $e_{i,j} = y_{i,j} - r_{i,j}$ is the filtering error. Now, let B represent a measure of the detail blur. If $B \neq 0$ (as commonly occurs during noise smoothing), we can split the PSNR into two components, namely *peak signal-to-blur ratio* (PSBR) and *degradation caused by noise* (D), as expressed by the following relationships [8]:

$$\text{PSNR} = \text{PSBR} - D \quad (2)$$

$$\text{PSBR} = 10 \log_{10} \frac{(Q-1)^2}{B} \quad (3)$$

$$D = 10 \log_{10} \frac{\frac{1}{LM} \sum_{i=1}^L \sum_{j=1}^M e_{i,j}^2}{B} \quad (4)$$

The PSBR measures how good a filter is at preserving image details, whereas D defines the loss in image quality caused by unfiltered noise. If no noise is added to the input image, we have $D=0$ and thus $\text{PSNR}=\text{PSBR}$, according to (2).

An application example is reported in Fig.1 that shows portions of processed images having the same $\text{PSNR}=27.527$ (dB) but different amounts of filtering blur. To obtain this result, we adopted the 512×512 test picture ‘‘Lena’’ and we produced a noisy version of it by adding Gaussian noise with standard deviation $\sigma=29.3$. We filtered the noisy data by means of mean filters with different window sizes because their behavior is well known. The results yielded by the 7×7 and the 3×3 mean operators are depicted in Fig.1a and Fig.1b, respectively. Clearly, the 7×7 filter gives a stronger smoothing than the 3×3 operator at the price of a worse detail preservation. The sole PSNR cannot distinguish these effects, whereas the PSBR can. Indeed we have $\text{PSBR}=28.389$ (dB) for the 7×7 filter and $\text{PSBR}=34.736$ (dB) for the 3×3 operator.

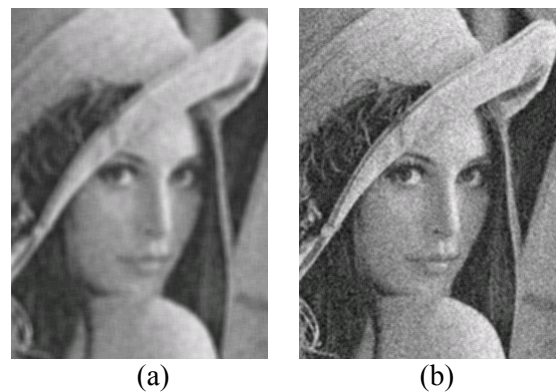


Fig.1 - Portions of pictures having the same PSNR but different amounts of detail blur: (a) result yielded by the 7×7 mean filter ($\text{PSBR}=28.389$ dB); (b) result yielded by the 3×3 mean filter ($\text{PSBR}=34.736$ dB).

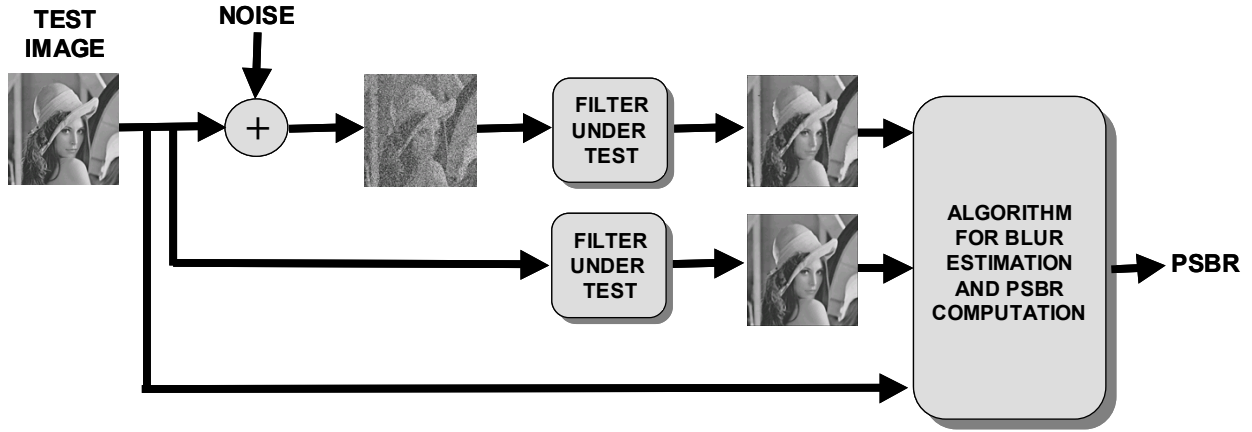


Fig.2 – Block diagram of the overall procedure for performance evaluation of a denoising filter through PSBR estimation.

Notice that the (PSNR, PSBR) pair suffices: the D term can easily be obtained by the difference PSBR–PSNR. In principle, different algorithms can be adopted for measuring the detail blur B.

3 The New Algorithm

The block diagram of the new technique for PSBR computation is shown in Fig.2. The method exploits different kinds of information: the reference image, the picture that is obtained by filtering the noisy data, and the image that is achieved by filtering the (noise-free) reference picture. Formally, let $y_{i,j}^{(r)}$ be the pixel luminance at location (i,j) in the image that is obtained by applying to the reference pixel $r_{i,j}$ exactly the same filtering that is applied to the noisy pixel $x_{i,j}$. Let $e_{i,j}^{(r)} = y_{i,j}^{(r)} - r_{i,j}$ be the corresponding filtering error. In our new approach, the detail blur is evaluated as follows:

$$B = \frac{1}{LM} \sum_{i=1}^L \sum_{j=1}^M b_{i,j}^2 \quad (5)$$

where $b_{i,j}$ (error component representing the detail blur) is given by the following relationship:

$$b_{i,j} = \begin{cases} e_{i,j} & \text{if } r_{i,j} < y_{i,j} \leq y_{i,j}^{(r)} \text{ or } y_{i,j}^{(r)} \leq y_{i,j} < r_{i,j} \\ e_{i,j}^{(r)} & \text{if } r_{i,j} < y_{i,j}^{(r)} < y_{i,j} \text{ or } y_{i,j} < y_{i,j}^{(r)} < r_{i,j} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

According to (6), three cases (or rules) are devised.

1) $b_{i,j} = e_{i,j}$

This case occurs for positive $(r_{i,j} < y_{i,j} \leq y_{i,j}^{(r)})$ or negative $(y_{i,j}^{(r)} \leq y_{i,j} < r_{i,j})$ filtering errors. Let us focus on positive errors. The condition $y_{i,j} \leq y_{i,j}^{(r)}$ means that filtering a noisy pixel would produce a smaller error than filtering the corresponding noise-free pixel. Thus, the actual error $e_{i,j}$ is very likely to include detail blur only. A similar situation occurs for negative errors.

2) $b_{i,j} = e_{i,j}^{(r)}$

Focusing again on positive errors, the condition $y_{i,j}^{(r)} \leq y_{i,j}$ means that filtering a noisy pixel would produce a larger error than filtering the corresponding noise-free pixel. In this case, only a part of the filtering error is represented by detail blur. A reasonable choice is to adopt $e_{i,j}^{(r)}$ for estimating this quantity.

3) $b_{i,j} = 0$

In all remaining situations, residual noise is very likely to constitute the only cause of filtering error, so $b_{i,j} = 0$.

4 New Validation Tools

The validation of the method would clearly benefit from applications where the *true value* of PSBR, namely $PSBR_T$, is known. We shall consider two



Fig.3 – Test pictures: (a) “House” (b) “Peppers”, (c) “Boat”, (d) “Lighthouse”. (e) “Lena”, (f) “Airplane”.

important classes of linear and nonlinear filters, where the $PSBR_T$ can be theoretically evaluated and used for a comparison: the *finite impulse response* (FIR) filters and the *median* operators.

4.1 Computing the $PSBR_T$ for FIR Filters

In case of FIR filters, the $PSBR_T$ can be theoretically evaluated as follows. Let $f_{i,j}$ be the output of a $(2N+1) \times (2N+1)$ FIR filter:

$$f_{i,j} = \sum_{p=-N}^N \sum_{q=-N}^N h_{p,q} x_{i-p,j-q} \quad (7)$$

where $h_{p,q}$ are the filter coefficients. Now, let $n_{i,j}$ be the noise amplitude affecting the pixel at location (i,j) :

$$x_{i,j} = r_{i,j} + n_{i,j} \quad (8)$$

Thus, the filtering error $e_{i,j} = f_{i,j} - r_{i,j}$ can be expressed by the following relationship:

$$e_{i,j} = \left[\sum_{p=-N}^N \sum_{q=-N}^N h_{p,q} r_{i-p,j-q} \right] - r_{i,j} + \sum_{p=-N}^N \sum_{q=-N}^N h_{p,q} n_{i-p,j-q} \quad (9)$$

Let $d_{i,j}$ be the error component dealing with the detail blur:

$$d_{i,j} = \left[\sum_{p=-N}^N \sum_{q=-N}^N h_{p,q} r_{i-p,j-q} \right] - r_{i,j} \quad (10)$$

and let $g_{i,j} = e_{i,j} - d_{i,j}$ denote the error component dealing with the unfiltered noise.

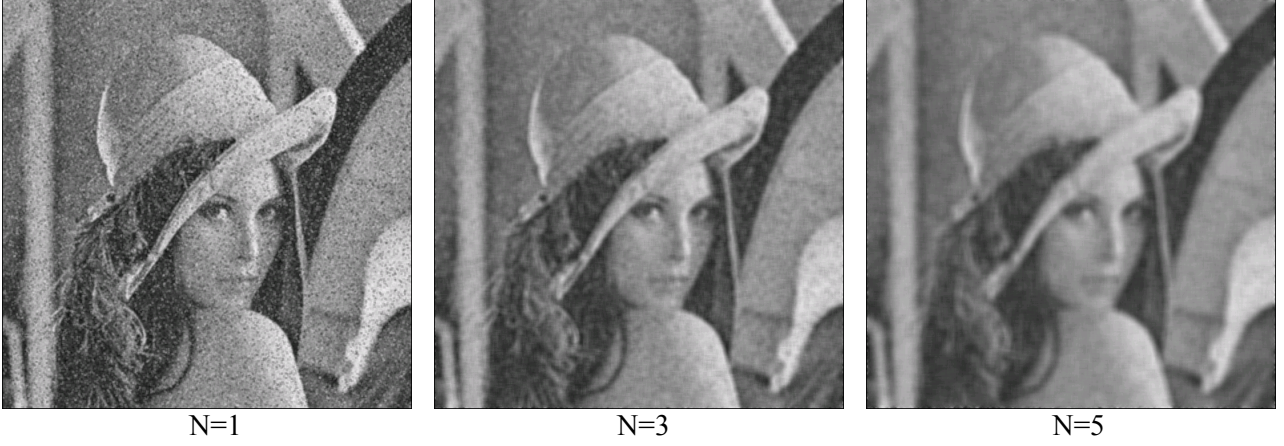


Fig.4 – “Lena” image corrupted by Gaussian ($\sigma=20$) and impulse noise (prob=10%): results given by $(2N+1) \times (2N+1)$ mean filters.

Depending on the signs and amounts of $d_{i,j}$ and $g_{i,j}$, the resulting detail blur $t_{i,j}$ is evaluated as follows.

Case 1: $d_{i,j}$ and $g_{i,j}$ have the same signs. In this case we have $t_{i,j} = d_{i,j}$.

Case 2: $d_{i,j}$ and $g_{i,j}$ have different signs and $|d_{i,j}| \geq |g_{i,j}|$. In this case, blur prevails: $t_{i,j} = d_{i,j} + g_{i,j}$.

Case 3: $d_{i,j}$ and $g_{i,j}$ have different signs and $|d_{i,j}| < |g_{i,j}|$. In this case $t_{i,j} = 0$.

The $PSBR_T$ is thus evaluated by means of the following relationships:

$$PSBR_T = 10 \log_{10} \frac{(Q-1)^2}{B_T} \quad (11)$$

$$B_T = \frac{1}{LM} \sum_{i=1}^L \sum_{j=1}^M t_{i,j}^2 \quad (12)$$

4.2 Computing the $PSBR_T$ for Median Filters

The method for computing the true PSBR for median filters is similar. Let $f_{i,j}^{(med)}$ be the output of a $(2N+1) \times (2N+1)$ median filter:

$$\begin{aligned} f_{i,j}^{(med)} &= x_{i-u,j-v} \\ &= \text{MEDIAN}_{p,q} \{ x_{i-p,j-q} \} \end{aligned} \quad (13)$$

where $-N \leq p \leq N$, $-N \leq q \leq N$. According to (8), the filtering error $e_{i,j}^{(med)} = f_{i,j}^{(med)} - r_{i,j}$ can be expressed by the following relationship:

$$\begin{aligned} e_{i,j}^{(med)} &= x_{i-u,j-v} - r_{i,j} \\ &= r_{i-u,j-v} - r_{i,j} + n_{i-u,j-v} \end{aligned} \quad (14)$$

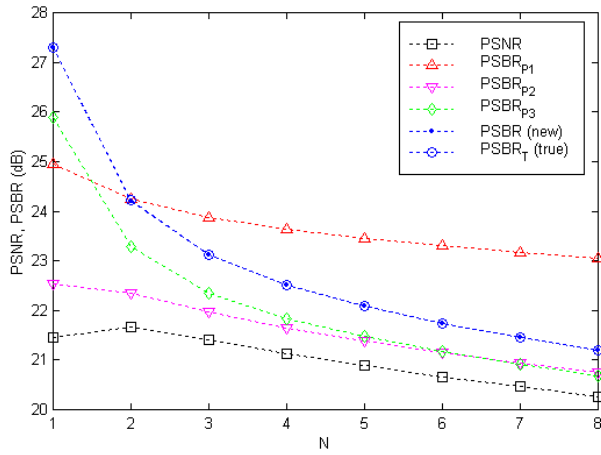
Let $d_{i,j}^{(med)}$ be the error component dealing with the detail blur:

$$d_{i,j}^{(med)} = r_{i-u,j-v} - r_{i,j} \quad (15)$$

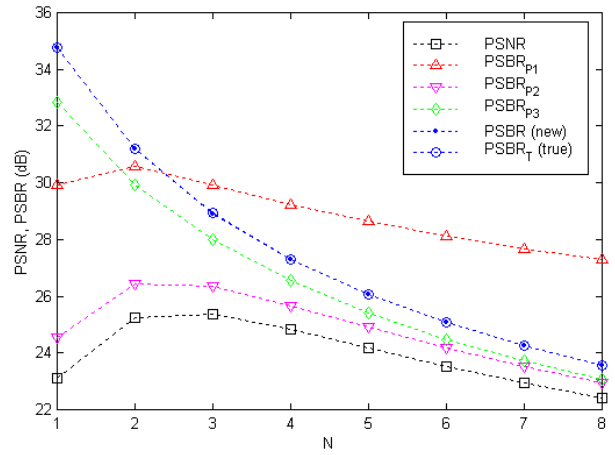
and let $g_{i,j}^{(med)} = e_{i,j}^{(med)} - d_{i,j}^{(med)} = n_{i-u,j-v}$ denote the error component dealing with the unfiltered noise. Depending on the signs and amounts of $d_{i,j}^{(med)}$ and $g_{i,j}^{(med)}$, the resulting detail blur $t_{i,j}$ can be evaluated as in the previous case.

5 Results of Computer Simulations

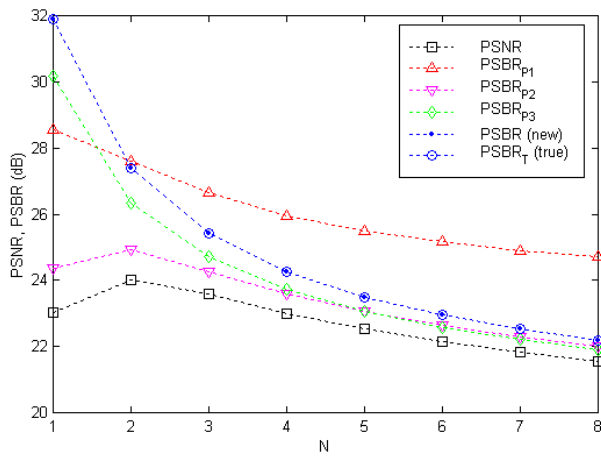
In order to evaluate the performance of the proposed method we considered for a comparison our previous PSBR technique [8] and two measures of detail preservation based on edge distortion ($RMSE_{\tilde{B}}$) [21] and collateral distortion ($RMSE_{CD}$) [22]. Since all comparisons should be performed in terms of PSBR, let $PSBR_{P1}$ and $PSBR_{P2}$ denote the PSBR evaluations that are achieved when $(RMSE_{\tilde{B}})^2$ and $(RMSE_{CD})^2$ are respectively adopted in (3) to estimate B. Let $PSBR_{P3}$ represent our previous technique [8] and let $PSBR$ briefly denote our new method described by relations (5)-(6). We performed several tests dealing with the following



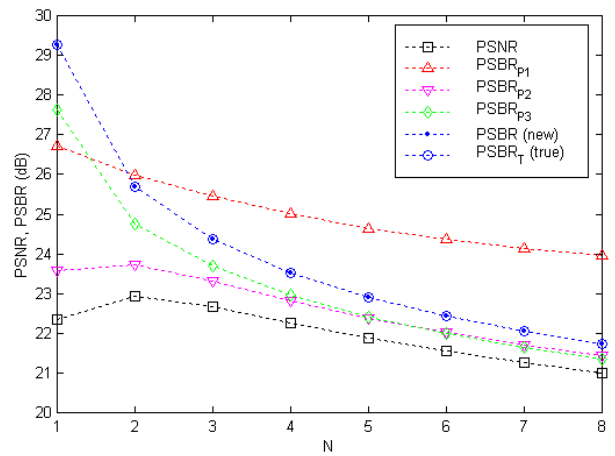
(a)



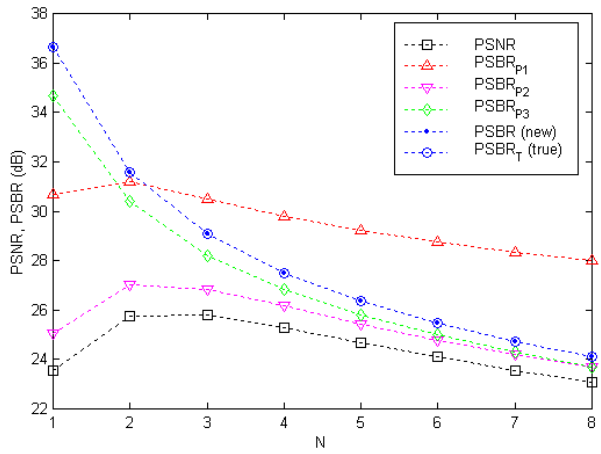
(b)



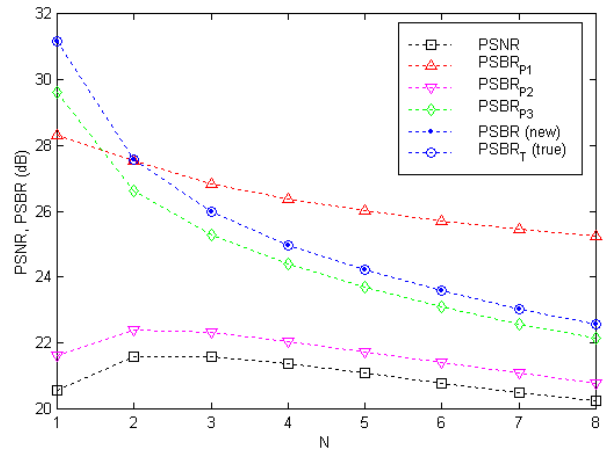
(c)



(d)



(e)

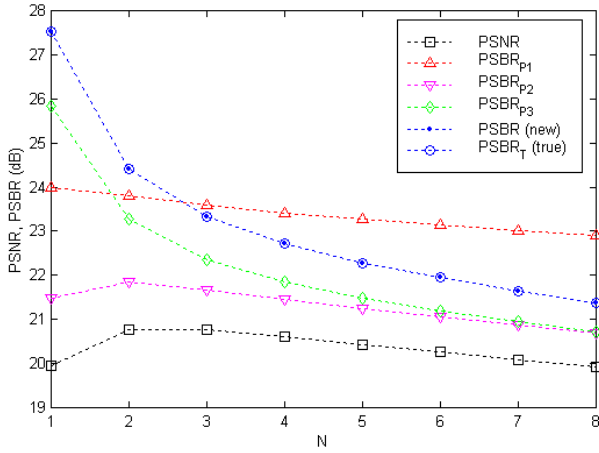


(f)

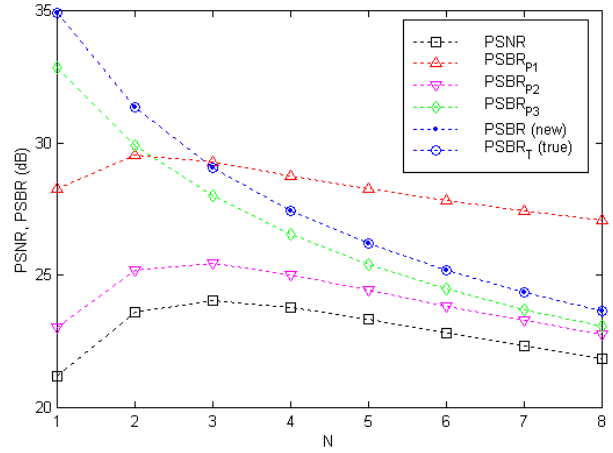
Fig.5 – PSBR values for images corrupted by Gaussian ($\sigma=20$) and impulse noise (prob=10%) and filtered by $(2N+1) \times (2N+1)$ means: (a) “House” (b) “Peppers”, (c) “Boat”, (d) “Lighthouse”, (e) “Lena”, (f) “Airplane”.

512×512 pictures: “House”, “Peppers”, “Boat”, “Lighthouse”, “Lena” and “Airplane” (Fig.3). We corrupted these images by adopting different amounts of Gaussian and impulse noise. We

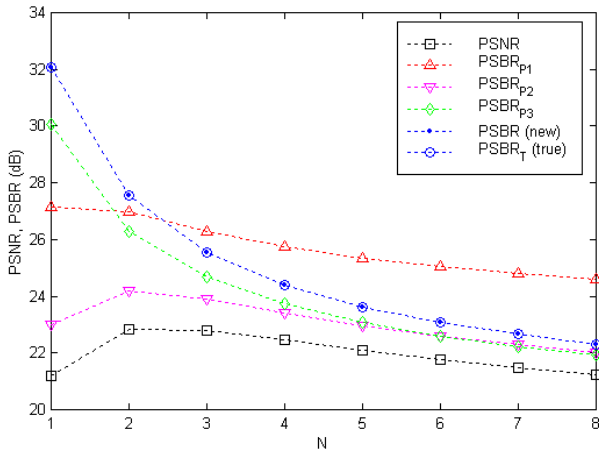
processed the noisy data by means of $(2N+1) \times (2N+1)$ arithmetic mean filters with increasing window size because the theoretical values of the PSBR (namely $PSBR_T$) are known (see



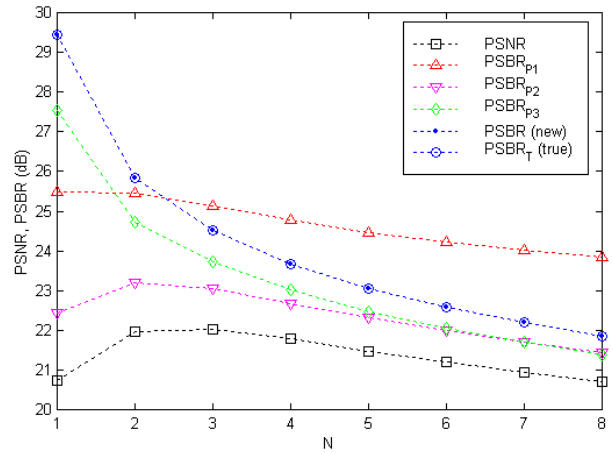
(a)



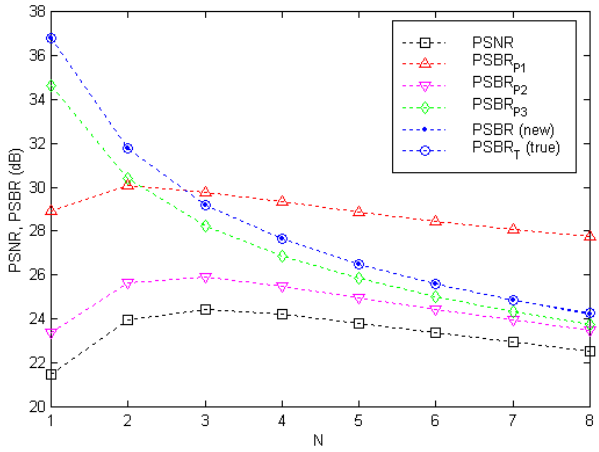
(b)



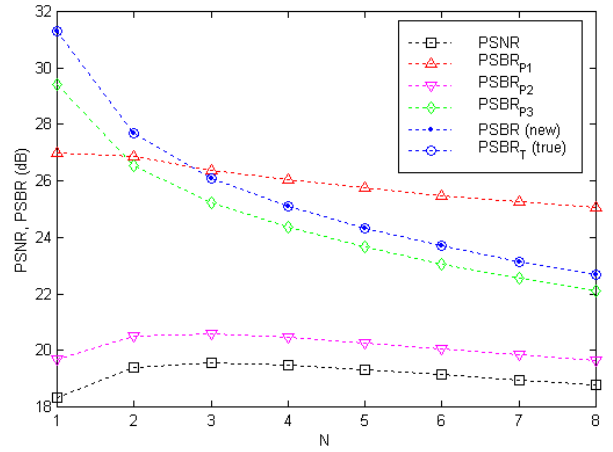
(c)



(d)



(e)

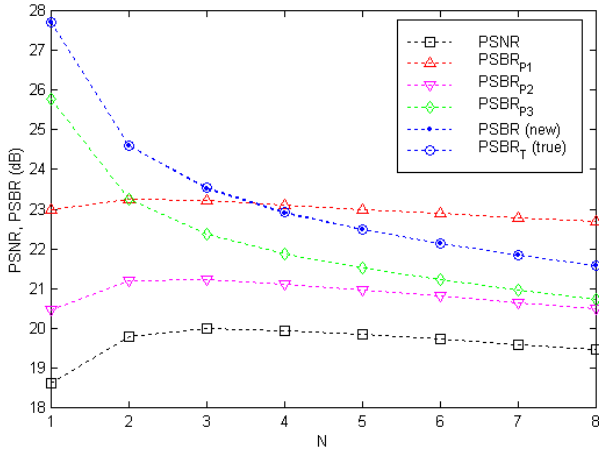


(f)

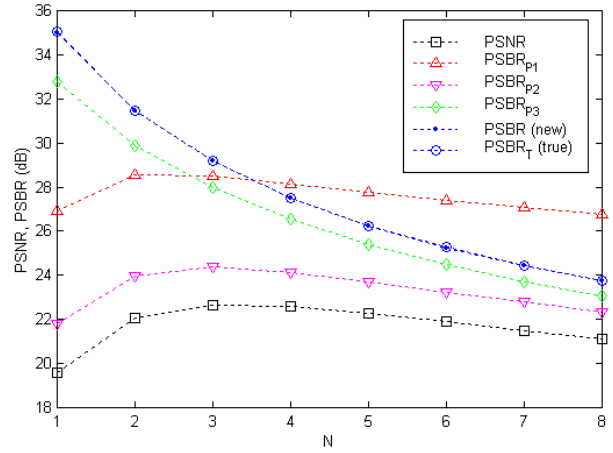
Fig.6 – PSBR values for images corrupted by Gaussian ($\sigma=30$) and impulse noise (prob=15%) and filtered by $(2N+1) \times (2N+1)$ means: (a) “House” (b) “Peppers”, (c) “Boat”, (d) “Lighthouse”, (e) “Lena”, (f) “Airplane”.

Section 4). In the first experiment, we generated six noisy pictures by adding zero-mean Gaussian noise (with standard deviation $\sigma=20$) and by superimposing salt and pepper impulse noise with

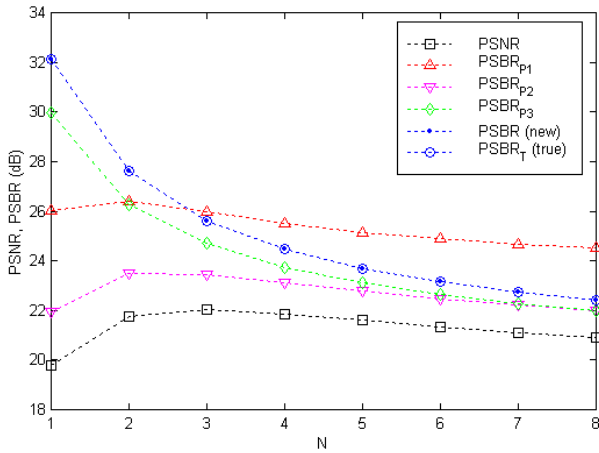
probability 10%. A sample of the processed images is shown in Fig.4. The corresponding PSNR and PSBR values are reported in Fig.5. The superior performance of the novel algorithm is apparent: the



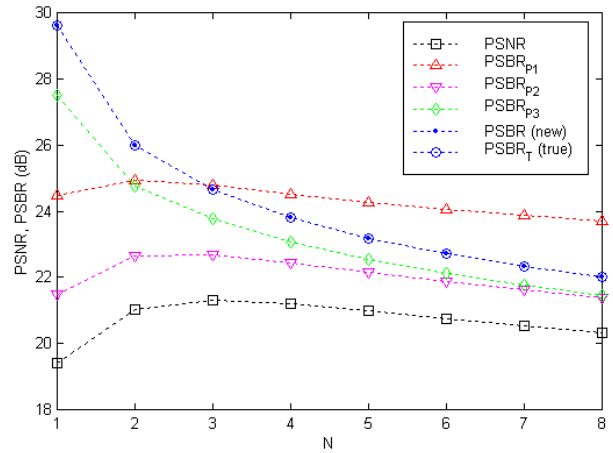
(a)



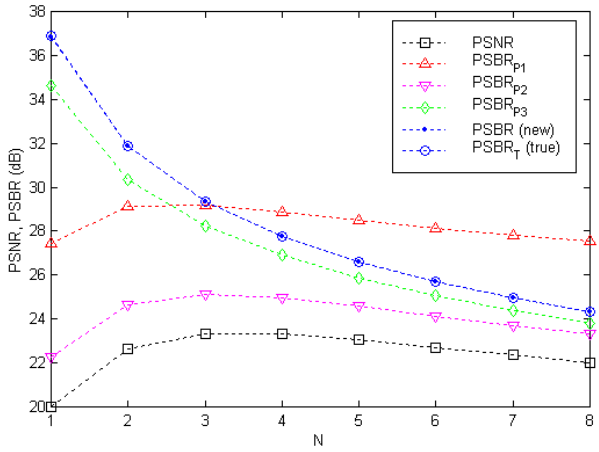
(b)



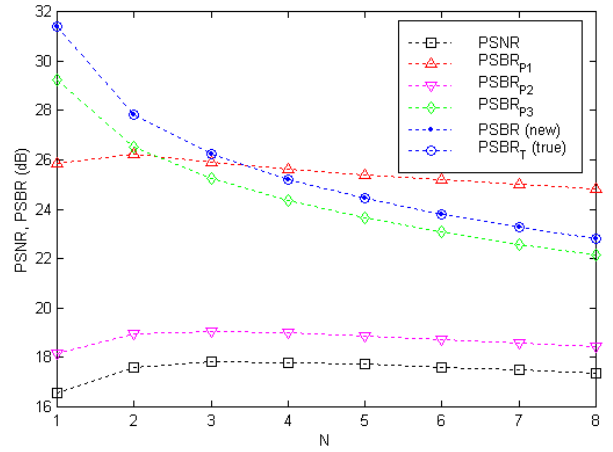
(c)



(d)



(e)

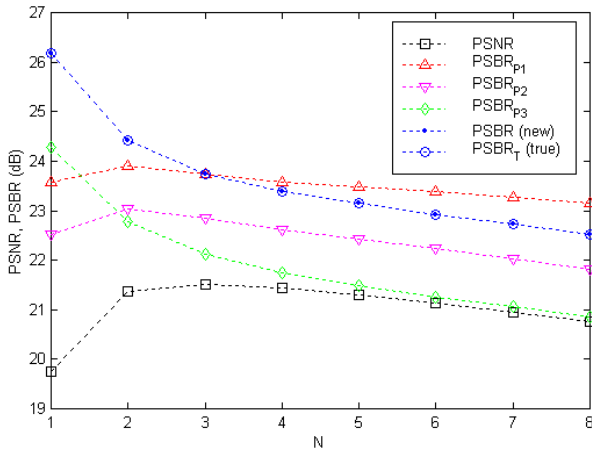


(f)

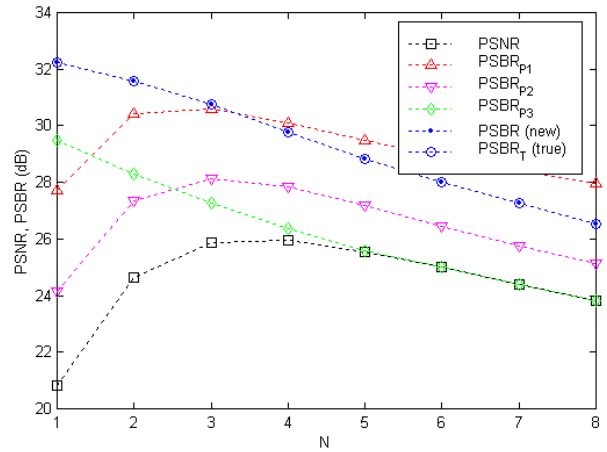
Fig.7 – PSBR values for images corrupted by Gaussian ($\sigma=40$) and impulse noise (prob=20%) and filtered by $(2N+1) \times (2N+1)$ means: (a) “House” (b) “Peppers”, (c) “Boat”, (d) “Lighthouse”, (e) “Lena”, (f) “Airplane”.

PSBR evaluations (blue points) perfectly estimate the true values (blue circles), whereas the previous method cannot. The proposed PSBR is better than the PSBR_{p3} (green points) and largely outperform

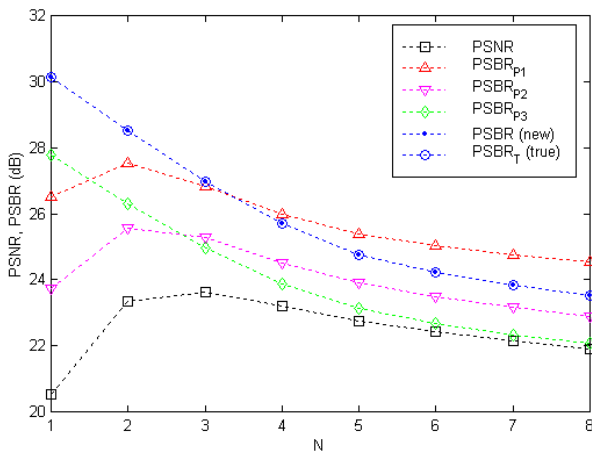
both the PSBR_{p1} (red points) and the PSBR_{p2} (magenta points). The incorrect behavior of PSBR_{p1} and PSBR_{p2} is anyway apparent. This kind of metrics should never increase when the filtering blur



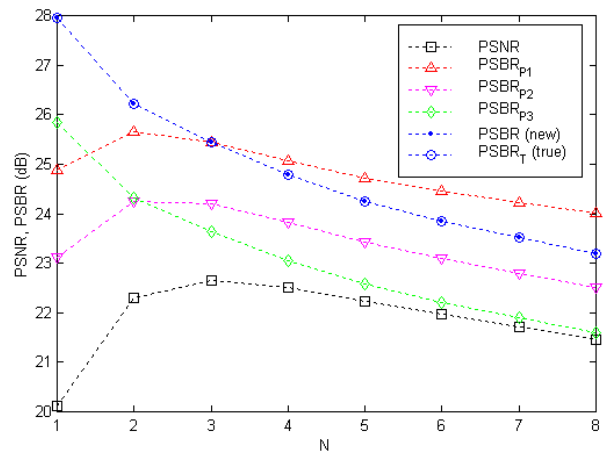
(a)



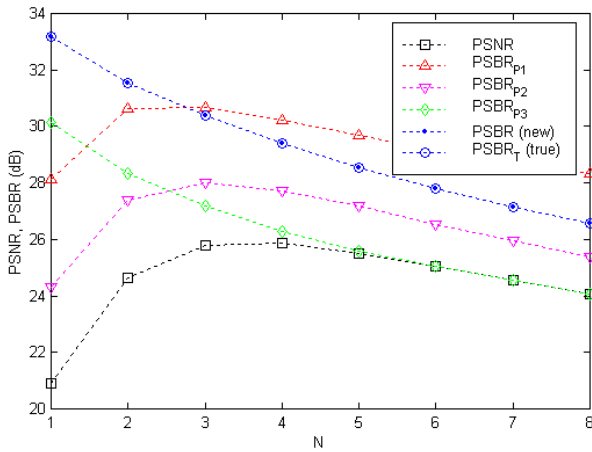
(b)



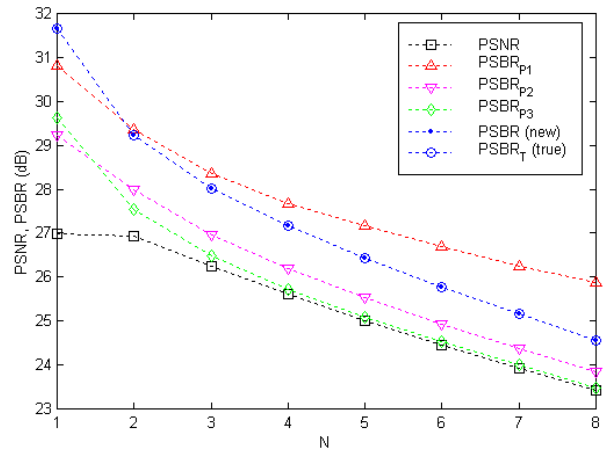
(c)



(d)



(e)



(f)

Fig.8 – PSBR values for images corrupted by Gaussian ($\sigma=40$) and impulse noise (prob=20%) and filtered by $(2N+1) \times (2N+1)$ medians: (a) “House” (b) “Peppers”, (c) “Boat”, (d) “Lighthouse”, (e) “Lena”, (f) “Airplane”.

grows, because higher PSBR (and PSNR) values denote better quality. On the contrary, the proposed (and so the previous) PSBR decreases as N increases, as it should be. The largest value of the

proposed PSBR is correctly obtained for the smallest value of N that produces the largest detail preservation. The results of experiments dealing with increasing amounts of Gaussian and impulse

noise are reported in Figs.6-7. In all cases, the novel PSBR is in perfect agreement with the theoretical values, whereas all other metrics fail.

Finally, we processed the noisy data by means of $(2N+1) \times (2N+1)$ median filters with increasing window size. We computed the theoretical values of the $PSBR_T$ in Section 4, so we can investigate the accuracy of the proposed and other metrics when this important class of nonlinear filters is adopted. The PSBR evaluations for test images corrupted by Gaussian ($\sigma=40$) and impulse noise (prob=20%) are reported in Fig.8. Again, the superior performance of the proposed method is apparent. In all cases, the new algorithm can yield results in very good agreement with the theoretical values, whereas the competing metrics cannot.

6 Conclusions

Achieving accurate measurements of detail preservation is of paramount importance for analyzing the performance of a denoising filter. In this paper we have presented a novel peak signal-to-blur ratio (PSBR) algorithm that significantly improves our previous technique. The new approach is simpler and more effective. It does not require the choice of thresholds and does not need any offset-correction procedure. Results of many computer simulations dealing with pictures corrupted by different amounts of Gaussian and impulse noise have shown that the novel PSBR is much more accurate than previous metrics in the literature. In particular, the new PSBR can correctly estimate the true values of detail preservation, whereas all other metrics fail.

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