Multiple factor analysis for time-varying two-mode networks

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Abstract

Most social networks present complex structures. They can be both multi-modal and multi-relational. In addition, each relationship can be observed across time occasions. Relational data observed in such conditions can be organized into multidimensional arrays and statistical methods from the theory of multiway data analysis may be exploited to reveal the underlying data structure. In this paper, we adopt an exploratory data analysis point of view, and we present a procedure based on multiple factor analysis and multiple correspondence analysis to deal with time-varying two-mode networks. This procedure allows us to create static displays in order to explore network evolutions and to visually analyze the degree of similarity of actor/event network profiles over time while preserving the different statuses of the two modes.

Keywords: *multiple factor analysis, multiple correspondence analysis, two-mode networks, time-varying networks*

1 Introduction

Heterogeneous networks (Memon & Wiil, 2013) may arise when we consider two-mode networks where the two modes are linked by several different relationships or by the same type of relationship, and they may also be observed across time occasions. In the first two cases, we have a three-way data structure, while in the last case we observe a four-way data structure (actors-by-events under different types of relationships across time occasions).

This type of network is of interest in a number of fields, such as organizational, economic, sociological, and political studies. For example, in organizational studies, there is considerable interest in interlocking directorates and in the processes of directors' appointment to and departure from boards over time (Seierstad & Opsahl, 2011; Conaldi et al., 2012; Koskinen & Edling, 2012; Opsahl, 2013). In economic studies, the performances of public programs in the R&D sector can be evaluated in terms of outcome trajectories over time (Gordon & Heinrich, 2004) or by comparing actors' collaboration trajectories over time (D'Esposito et al., 2014b). In sociological studies, interest may lie in the evolution of web communities, group memberships and Facebook-like forum participation, employment preferences, scientific collaboration (Holme et al., 2007; Kang et al., 2007; Leydesdorff & Schank, 2008; Memon & Wiil, 2013; Opsahl, 2013; Snijders et al., 2013). In political studies, examples of two-mode longitudinal networks are found in the analysis of the dynamics of people's affiliation to organizations and their participation in political events (Faust et al., 2002; Gagliolo et al., 2014). Examples of multi-modal and multi-relational networks are also found in the marketing research when considering the market basket analysis where customers buy, rent, or rate products (Horvat & Zweig, 2012) or in biological studies where diseases and genes are linked with respect to the different criteria (Davis et al., 2011).

In this paper, we focus on time-varying two-mode networks. Analytical methods for such networks are rare (Horvat & Zweig, 2012) and often rely on the one-mode projection procedure. On the other hand, several visual exploration methods have been proposed, mainly based on the graph drawing techniques, which can furnish either a static or a dynamic network representation (Batagelj & Mrvar, 1998; Freeman, 2000; Baur et al., 2002; Tversky et al., 2002; Moody et al., 2005; Bender-deMoll & McFarland, 2006; Perer & Shneiderman, 2006; Brandes et al., 2007; Kang et al., 2007; Ghani et al., 2012; Memon & Wiil, 2013).

As these observed time-varying two-mode networks can be organized into multidimensional arrays, statistical methods from the theory of multiway data analysis (Kroonenberg, 2008; Coppi, 1994) may be exploited to reveal the underlying data structure. Among all such existing methods, we propose to consider factorial methods, and we focus on the Multiple Factor Analysis (MFA) (Escofier & Pagés, 1988; Escofier & Pagés, 1994). To this end, we present a procedure based on the MFA and Multiple Correspondence Analysis (MCA) (Greenacre, 2006; D'Esposito et al., 2014a). In order to preserve the complexity of the network, we adopt a direct approach (Borgatti & Halgin, 2011) without projecting the two-mode network into two one-mode networks. We obtain a set of statistical visualizations (Correa & Ma, 2011) that incorporate both the temporal dimension and the network analytical description and allow us to create static displays in order to analyze network evolutions over time while preserving the different statuses of the two modes. We exemplify our approach by using a dataset provided by MacFarland (1999). These data concern the extracurricular memberships—entailing various forms of interactions, such as sports, interschool contests, and collective performances—of 36 classes in two schools (River High and Magnet High) over three years from 1996 to 1998. The data are an example of a time-varying affiliation network. The students represent the actors, and the events are the student's extracurricular activities and association to clubs.

The paper is organized as follows. In Section 2, we present and discuss our approach providing analytical details of the MFA applied to two-mode network data, while in Section 3, we highlight the features of the proposed methods through MacFarland data. Section 4 offers some concluding remarks.

2 MFA for two-mode heterogeneous networks

A time-varying two-mode network can be represented by a set of K two-mode networks $\{\mathscr{G}_k\}_{k=1,\dots,K}$. The k index refers to different time points, and in the following we will generally refer to them as occasions. Each two-mode network $\mathscr{G}_k$ consists of two sets of relationally connected units (which we will call actors and events, as in the affiliation network setting, without loss of generality), and can be represented by a triple $\mathscr{G}_k\,(V_{1k}, V_{2k}, \mathscr{R}_k)$ composed of two disjoint sets of nodes—V_{1k} and V_{2k} of cardinality N_k and J_k, respectively—and one set of edges or arcs, $\mathscr{R}_k \subseteq V_{1k} \times V_{2k}$. By definition, $V_{1k} \cap V_{2l} = \emptyset, \forall k$.

For the sake of presentation, in this paper we assume that $V_{1k} = V_1 \;\; \forall k$.[1] The set $V_1 = \{a_1, a_2, \dots, a_N\}$ represents the set of N actors, whereas $V_{2k} = \{e_{1k}, e_{2k}, \dots, e_{J_k}\}$ represents the set of J_k events. We are thus considering networks in which the set of actors is fixed over time while the events can be fixed or fleeting occurrences over time. The edge $r_{ijk} = (a_i, e_{jk})$, $r_{ijk} \in \mathscr{R}_k$ is an ordered couple and indicates whether or not an actor a_i attends an event e_{jk}.

Each set $V_1 \times V_{2k}$ can be fully represented by a binary *affiliation matrix* $\mathbf{F}_k = (f_{ijk})$, $i = 1, \dots, N$, $j = 1, \dots, J_k, k = 1, \dots, K$, with $f_{ijk} = 1$ if $(a_i, e_{jk}) \in \mathscr{R}_k$ and 0 otherwise. Given $\mathbf{F}_k$, the row and column marginals $f_{i.k} = \sum_{j=1}^{J_k} f_{ijk}$ and $f_{.jk} = \sum_{i=1}^{N} f_{ijk}$ coincide with the degree d_{ik} of the ith actor at occasion k and the size s_{jk} of the jth event at occasion k, respectively, i.e., $f_{i\cdot k} = d_{ik}$ and $f_{\cdot jk} = s_{jk}$. The set of all affiliation matrices $\{\mathbf{F}_k\}_{k=1,\dots,K}$ can be represented through a data table, usually called the *grand table*, $\boldsymbol{F} = [\mathbf{F}_1|\dots|\mathbf{F}_K] = (f_{ijk})$, with $f_{ijk} \in \{0, 1\}$, $i = 1, \dots, N$, $j = 1, \dots, J_k$, $k = 1, \dots, K$. The grand table $\boldsymbol{F}$ is built up by stacking the subtables $\{\mathbf{F}_k\}_{k=1,\dots,K}$ side by side; its form is depicted in Figure 1. Given $\boldsymbol{F}$, for each i, $i = 1, \dots, N$, the row marginal $f_{i..} = \sum_{j=1}^{J_k} \sum_{k=1}^{K} f_{ijk}$ coincides with the total degree of the ith actor, $d_i = d_{i..}$, and $L = \sum_{i=1}^{N} \sum_{j=1}^{J_k} \sum_{k=1}^{K} f_{ijk}$ is the total number of edges over all the occasions.

We aim to (i) analyze the structure embedded in each affiliation matrix $\mathbf{F}_k$ in terms of relational similarity between actors/events, (ii) cumulatively analyze the K occasions by looking at the general relational structure embedded in $\boldsymbol{F}$, (iii) highlight the differences between the K occasions in terms of global structure, and (iv) analyze the *variations* of actors/events over the occasions. Points (iii) and (iv) allow us to study the longitudinal change and the behavior over time of the relational patterns for both modes.

In order to pursue the previously listed aims, we will use the MFA, which provides a unifying and general framework to deal with multiple-way matrices. The MFA is an extension of factorial techniques (Escofier & Pagés, 1994) tailored to handle multiple data tables (the same observations by different sets of variables *o*r the same variables measured on different sets of observations). This makes it possible to jointly analyze quantitative and qualitative variables, providing displays in which representations of the set of individuals associated with each group of variables are superimposed.

[1] The case of nodes in V_{1k} that partially change over k can be treated by considering V_1 as the overall set of nodes that includes all the V_{1k}'s, i.e., $V_1 = \cup_{k=1}^{K} V_{1k}$.

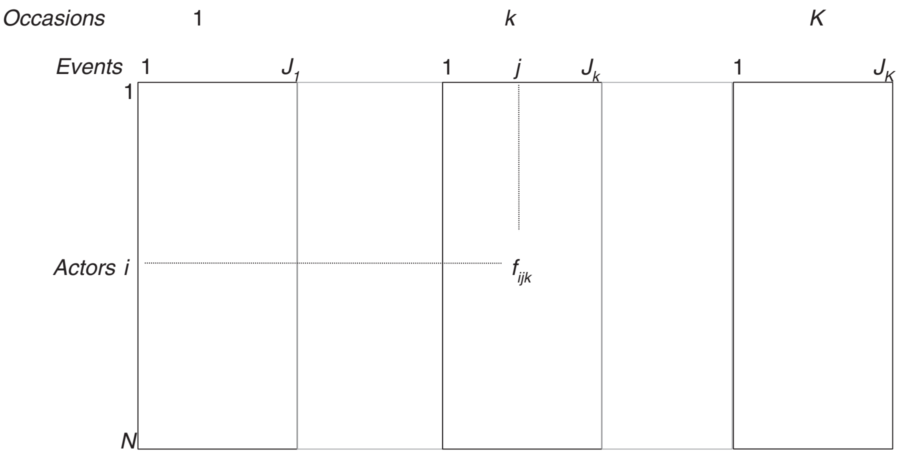

Fig. 1. The grand table $\mathbb{F}$ for a time varying two-mode network. The f_{ijk} element of the affiliation matrix $\mathbf{F}_k$ codes the participation/non-participation of actor i in event j at occasion k.

In the perspective of MFA, we identify the following four steps corresponding to the previously listed aims:

1. Analyze each affiliation matrix $\mathbf{F}_k$, $k = 1,\ldots,K$ through a suitable factorial analysis method (partial analysis);
2. Analyze the grand table $\mathbb{F}$ through a suitable factorial analysis (global analysis);
3. Analyze which (if any) structural changes arise among the occasions (i.e., the change of the relational patterns over time for both modes); and
4. Analyze actor and event variation over the occasion by projecting the weighted affiliation matrices $\mathbf{F}_k$ on the global factorial plane.

2.1 *Partial analyses*

A key issue in MFA is the choice of factorial method (i.e., principal component analysis, correspondence analysis, or MCA) to use in the separate analyses and in the global analysis. Given the nature of two-mode networks, we assume that the corresponding $\mathbf{F}_k$ matrices are *case-by-variable* data (Gower, 2006) matrices in which a different status is assigned to the rows and columns. In such a case, MCA (Richards & Seary, 2000; Greenacre, 2006) may be appropriate as MCA assigns different roles to actors and events, thus allowing distinct features to be highlighted in each mode; moreover, it makes it possible to add covariates to the analysis in order to improve results interpretation and affords visualizations that can be easily interpreted in terms of the similarities among actors/events network relations (D'Esposito et al., 2014a; D'Esposito et al., 2014c). The use of MCA in the MFA procedure makes it possible to interpret changes in the patterns over time as corresponding changes in the relational similarity among actors/events.

In order to perform MCA on each $\mathbf{F}_k$, we consider actors as observational units and their participation in events as dichotomous categorical variables and apply the usual correspondence analysis (CA) algorithm—singular value decomposition

(SVD) of the doubled normalized and centered profile matrix (Greenacre, 2006)—to the *multiple indicator matrix*, or simply *indicator matrix*, $\mathbf{Z}_k$ derived from $\mathbf{F}_k$ through a full disjunctive coding. To this end, we consider each event e_{jk} as a dichotomous variable with categories e^+_{jk} and e^-_{jk}, where e^+_{jk} is a dummy variable coding the participation in the event and e^-_{jk} is a dummy variable coding the non-participation. Each $\mathbf{Z}_k$ matrix is a $N \times 2J_k$ matrix of the form: $\mathbf{Z}_k \equiv \left[\mathbf{F}^+_k, \mathbf{F_k}^-\right]$, where $\mathbf{F}^+_k = (e^+_{jk}) = \mathbf{F}_k$ and $\mathbf{F}^-_k = (e^-_{jk}) = \mathbf{1} - \mathbf{F}^+_k = \mathbf{1} - \mathbf{F}_k$, with $\mathbf{1}$ the $N \times J_k$ all-ones matrix. The indicator matrix $\mathbf{Z}_k$ turns out to be a *doubled matrix*.

Given the affiliation matrices $\mathbf{F}_k$, in order to perform the separate MCAs, the profile matrices $\mathbf{P}_k$ and weight matrices $\mathbf{D}_{ak}$ and $\mathbf{D}_{ek}$, $k = 1, \ldots, K$, which are characteristic of the CA algorithm and involved in the normalization step, are defined as follows:

$$\mathbf{P}_k = \frac{\mathbf{Z}_k}{NJ}$$

with $J = \sum_{k=1}^{K} J_k$, the total number of events over all the occasions,

$$\mathbf{D}_{ak} = diag\left(\frac{1}{N}, \ldots, \frac{1}{N}\right)$$
$$\mathbf{D}_{ek} = diag\left(\frac{s_{1k}}{NJ}, \ldots, \frac{s_{2J_k}}{NJ}\right)$$

Note that each actor a_i has weight $1/N$ at each occasion k. Then $\mathbf{D}_{ak} = \mathbf{D}_a$, $\forall k$. For each occasion k, $k = 1, \ldots, K$, the centered and doubled normalized $\mathbf{Z}_k$ is the matrix $\mathbf{S}_k$

$$\mathbf{S}_k = \mathbf{D}_a^{-1/2}\left(\frac{\mathbf{Z}_k}{NJ} - \mathbf{D}_a\mathbf{1}\mathbf{1}^T\mathbf{D}_{ek}\right)\mathbf{D}_{ek}^{-1/2} = \sqrt{N}\left(\frac{\mathbf{Z}_k}{NJ} - \frac{1}{N}\mathbf{1}\mathbf{1}^T\mathbf{D}_{ek}\right)\mathbf{D}_{ek}^{-1/2}$$

where $(1/N)\mathbf{1}$ is the vector of the actor weights and $\mathbf{1}^T\mathbf{D}_{ek}$ is the vector of the event weights.

The SVD of $\mathbf{S}_k$ gives

$$\mathbf{S}_k = \mathbf{U}_k\boldsymbol{\Lambda}_k\mathbf{V}_k^T$$

where $\boldsymbol{\Lambda}_k$ is the diagonal matrix of singular values, and $\mathbf{U}_k$, $\mathbf{V}_k$ are the matrices of the left and right singular vectors, respectively.

The principal coordinates for the row and column categories, respectively, are defined as:

$$\boldsymbol{\Phi}_k = \mathbf{D}_a^{-1/2}\mathbf{U}_k\boldsymbol{\Lambda}_k = \sqrt{N}\mathbf{U}_k\boldsymbol{\Lambda}_k \tag{3}$$
$$\boldsymbol{\Psi}_k = \mathbf{D}_{ek}^{-1/2}\mathbf{V}_k\boldsymbol{\Lambda}_k \tag{4}$$

and the standard coordinates for the row and column categories, respectively, are defined as:

$$\boldsymbol{\Gamma}_k = \sqrt{N}\mathbf{U}_k \tag{5}$$
$$\boldsymbol{\Delta}_k = \mathbf{D}_{ek}^{-1/2}\mathbf{V}_k \tag{6}$$

The first two columns of $\boldsymbol{\Phi}_k$ and $\boldsymbol{\Psi}_k$, or of $\boldsymbol{\Gamma}_k$ and $\boldsymbol{\Delta}_k$, are used to construct two-dimensional maps in which the data are graphically represented. In the two-dimensional maps thus derived, the distances between actors and between events optimally approximate the ones in the original spaces (Greenacre, 2006).

As usual in factor analysis, the quality of the approximation of the two-dimensional maps can be obtained by evaluating the proportion of inertia explained by the first two factorial axes, i.e., by the sum of the first two singular values over the sum of all singular values $(\lambda_{1k} + \lambda_{2k})/trace(\mathbf{\Lambda}_k)$. In our case, since we are considering MCA, it is known that the total inertia of $\mathbf{Z}_k$ is inflated (Greenacre, 2006), and all the percentages of inertia of the principal axis are artificially low. In order to overcome this problem, for the sth factorial axis, the adjusted values for each $\lambda_{sk} \geqslant 1/J_k$ are calculated as

$$\lambda_{sk}^{adj} = \left(\frac{J_k}{J_k - 1}\right)^2 \left(\lambda_{sk} - \frac{1}{J_k}\right)^2 \tag{7}$$

and used to evaluate the proportion of inertia explained by the first two factorial axes.

The quality of representation of each single data point (actor or event) can also be measured by the squared cosine between the vector from the origin to the data point and its projection on the axis. The squared cosines for the ith actor (or for the jth event) on the sth factorial axis for each k can be easily evaluated using the principal coordinates in Equations 3 and 4 as follows:

$$\cos^2_{i,sk} = \frac{\phi^2_{i,sk}}{\sum_s \phi^2_{i,sk}}$$

and

$$\cos^2_{j,sk} = \frac{\psi^2_{j,sk}}{\sum_s \psi^2_{j,sk}}.$$

If the squared cosines are close to one, the corresponding elements are well projected on the axis, and the distances between them can thus be correctly interpreted.

The principal coordinates are also used to evaluate the contribution of each row (column) in determining the factorial axes as follows:

$$contr_{i,sk} = \frac{\phi^2_{i,sk}}{\sum_i \phi^2_{i,sk}} \tag{8}$$

and

$$contr_{j,sk} = \frac{\psi^2_{j,sk}}{\sum_j \psi^2_{j,sk}}. \tag{9}$$

Within these steps, the relational patterns at each occasion can be visually analyzed by:

i. *Representing events in the actor space*: Events can be represented in a two-dimensional map by using the first two columns of the principal coordinates $\mathbf{\Psi}_k$ in Equation (4). Each event e_{jk} in the actor space is represented by two opposite vectors, corresponding to the two poles e^+_{jk} and e^-_{jk}, lying on the same direction and passing through the origin. The cosine of the angle between two event segments is the "correlation" between participation patterns in the events. Then, if two event segments form a small angle and present the positive pole on the same side, the corresponding events will have similar participation patterns. If the two segments form a small angle, but the positive poles are opposite, the events will have opposite participation patterns.

ii. *Representing actors in the event space*: Actors can be represented in a two-dimensional map by using the first two columns of the principal coordinates $\mathbf{\Phi}_k$ in Equation (3). In this map, each actor corresponds to a point, and two actors corresponding to two close points in the map have similar participation patterns. Actors corresponding to points close to the axes' origin have a common participation habit, while actors with corresponding points far from the center have an unusual participation pattern. Isolated actors and groups of actors can also be detected.

iii. *Jointly representing actors and events*: In order to represent actors and events in a joint two-dimensional map, we can use the asymmetric biplot (Gabriel, 1995; Greenacre, 2010), where the actors are represented in principal coordinates (Equation (3)) and the events in standard coordinates (Equation (6)) (Greenacre, 2010). The direction vector defined by each event is the biplot axis. By projecting the points representing actors onto each biplot axis, we can approximately appreciate their event participation profiles. This allows us to characterize actors' closeness or farness in terms of event participation.

2.2 Global analysis

In order to extract the information embedded in all the occasions and search for factors which are common to them, it is necessary to look at the grand matrix $\mathbb{F} = [\mathbf{F}_1|\ldots|\mathbf{F}_K]$ by performing a new global MCA. In the MFA approach, the global MCA is performed as follows:

i. Derive the grand indicator matrix $\mathbb{Z} = [\boldsymbol{Z}_1|\ldots|\boldsymbol{Z}_K]$.

ii. Derive the double normalized matrix:

$$\mathbb{S} = \mathbb{D}_a^{-1/2}\left(\frac{\mathbb{Z}}{NJ} - \mathbb{D}_a\mathbf{1}\mathbf{1}^T\mathbb{D}_e\right)\mathbb{D}_e^{-1/2} = \sqrt{N}\left(\frac{\mathbb{Z}}{NJ} - \frac{1}{N}\mathbf{1}\mathbf{1}^T\mathbb{D}_e\right)\mathbb{D}_e^{-1/2}$$

with

$$\mathbb{D}_a = \mathbf{D}_a = diag\left(\frac{1}{N},\ldots,\frac{1}{N}\right), \mathbb{D}_e = \begin{bmatrix} \lambda_{(1)1}^{-1}\mathbf{D}_{e1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda_{(1)2}^{-1}\mathbf{D}_{e2} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \cdots & \lambda_{(1)K}^{-1}\mathbf{D}_{eK} \end{bmatrix}$$

where $\lambda_{(1)k}$ is the largest eigenvalue obtained by the SVD of $\mathbf{S}_k$ in Step 1 for each occasion k. Note that the use of the weights $\lambda_{(1)k}^{-1}$ balance the influence of each occasion k in each global factor. Indeed, the maximum inertia of the partial clouds defined at the different occasions is 1 in any direction (Abascal et al., 2006).

iii. Perform the SVD of $\mathbb{S} = \mathbb{U}\boldsymbol{\Lambda}\mathbb{V}^T$, where $\boldsymbol{\Lambda}$ is the diagonal matrix of singular values and $\mathbb{U}$, $\mathbb{V}$ are the matrices of the left and right singular vectors, respectively, and obtain the principal coordinates

$$\boldsymbol{\Phi} = \mathbb{D}_a^{-1/2}\mathbb{U}\boldsymbol{\Lambda} = \sqrt{N}\mathbb{U}\boldsymbol{\Lambda} \tag{10}$$

$$\boldsymbol{\Psi} = \mathbb{D}_e^{-1/2}\mathbb{V}\boldsymbol{\Lambda} \tag{11}$$

and the standard coordinates for the row and column categories, respectively, defined as

$$\boldsymbol{\Gamma} = \sqrt{N}\mathbb{U} \tag{12}$$

$$\boldsymbol{\Delta} = \mathbb{D}_e^{-1/2}\mathbb{V}. \tag{13}$$

Note that each event e_{jk} is weighted by the quantity $\frac{s_{jk}}{NJ\lambda_{(1)k}}$. The weights of each event involved in the global analysis make it possible to balance variability between the different occasions, and thus each specific occasion cannot exert an important influence on the global solution. By using the first columns of the principal coordinates of $\mathbf{\Phi}$ in Equation (10), it is possible to represent actors in the global event space. Actors with similar participation patterns in all the occasions will be located close together on the factorial planes. By using the first columns of the principal coordinates of $\mathbf{\Psi}$ in Equation (11), it is possible to represent the events cumulatively for all the occasions on the same factorial planes by looking at correlations (i.e., similar attendance patterns) both with respect to the events of the same occasion and with respect to the events of the other occasions. The quality of the approximation and the quality of representation of each data point can be evaluated analogously to the case of partial analysis by using the singular values in $\mathbf{\Lambda}$ and the principal coordinates in $\mathbf{\Phi}$ and $\mathbf{\Psi}$, respectively.

2.3 Longitudinal analysis

In the MFA, approach it is possible to analyze the differences and variations in the occasions (i) of each actor/event relational pattern or (ii) of the whole network structure.

With respect to point (i), in the case of longitudinal analysis, on the factorial planes we can analyze the trajectories along the timeline of points corresponding to actors or to events, if the latter are fixed over time. Trajectories highlight variations in the relational patterns actor-wise and event-wise. For each actor, it is also possible to compare his/her location on factorial planes both among the occasions and with respect to their overall combination. With respect to point (ii), the analysis of variations between the occasions aims at discovering similarities and differences in the whole relational structure of the two-mode network in each occasion.

i. With respect to the actor-wise analysis, we note that the coordinates contained in the $\mathbf{\Phi}$ matrix can be considered a sort of center of gravity or compromise of the partial coordinates $\mathbf{\Phi}_k$. In order to analyze the variations in terms of the participation patterns of each actor at the different occasions, it is possible to project the single occasion table as supplementary points onto the global factorial space (Nenadic & Greenacre, 2007; Abdi et al., 2013). From the transition formula:

$$\mathbf{\Phi} = \mathbf{D}_a^{-1/2} \frac{\mathbf{Z}}{NJ} \mathbf{\Psi} \mathbf{\Lambda}^{-1} = \mathbf{D}_a^{-1/2} \frac{\mathbf{Z}}{NJ} \mathbf{\Delta}$$

which expresses the actor's principal coordinates as the weighted average of the event standard coordinates with weights equal to the actor profiles, the principal coordinates of actors at the k occasions with respect to the global solution are expressed as

$$\mathbf{\Phi}_k = \mathbf{D}_a^{-1/2} \frac{\mathbf{Z}_k}{NJ} \mathbf{\Delta}_k,$$

where $\mathbf{\Delta}_k$ is a sub-matrix of the standard coordinates referring to the occasion k in $\mathbf{\Delta}$ These coordinates make it possible to visualize the actor-wise evolution and variability among the occasions on the factorial planes given by the global solution. We can draw actors' trajectories by jointly representing points with partial coordinates in $\mathbf{\Phi}_k$ and connecting them. Short paths stand for small differences, i.e., actors present a quite stable participation pattern. In addition, for each actor we can use a star-type representation by connecting the points with partial coordinates in $\mathbf{\Phi}_k$ to their compromise point with coordinates in $\mathbf{\Phi}$. Small stars correspond to small variability, i.e., actors that display a similar behavior over all the occasions.

The previous analysis can also be performed event-wise when the events are not fleeting occurrences.

ii. With respect to the analysis of the whole network structure, the aim is to discover similarities and differences among the two-mode networks in the different occasions. In order to pursue this task, a K by K matrix that contains all the Escoufier's $RV_{(k,k')}$ similarity coefficients (Robert & Escoufier, 1976) is computed as

$$RV_{(k,k')} = \frac{Trace[(\mathbf{Z}_k\mathbf{Z}_k^T)(\mathbf{Z}_{k'}\mathbf{Z}_{k'}^T)]}{\sqrt{Trace[(\mathbf{Z}_k\mathbf{Z}_k^T)(\mathbf{Z}_k\mathbf{Z}_k^T)], Trace[(\mathbf{Z}_{k'}\mathbf{Z}_{k'}^T)(\mathbf{Z}_{k'}\mathbf{Z}_{k'}^T])}}. \quad (14)$$

The RV coefficient varies between 0 and 1 in relation to the shared amount of variance between the two matrices, and it is a global measure of the similarities in the co-attendance patterns of actors to events in the different occasions. Note that the RV index is based on the evaluation of the $\mathbf{Z}_k\mathbf{Z}_k^T$ matrix, that is, in the conversion approach, the actor-by-actor projection matrix. However, such matrix contains for each couple of actors not only the number of co-attended events but also the number of not co-attended events because we are adopting the doubling approach.

Moreover, in the case of fixed events among all the occasions, we note that it makes sense to compare the relational structure among the occasions by looking at the event-by-event projection matrices $\mathbf{Z}_k^T\mathbf{Z}_k$. Since $\mathbf{Z}_k^T$ is a doubled matrix, the entries in $\mathbf{Z}_k^T\mathbf{Z}_k$ are the number of actors that attended each pair of events and the number of actors that did not attend each pair of events at the same time.

For each pair of occasions (k,k') a new similarity index $RV_{(k,k')}$ can be evaluated as:

$$RV'_{(k,k')} = \frac{Trace[(\mathbf{Z}_k^T\mathbf{Z}_k)(\mathbf{Z}_{k'}^T\mathbf{Z}_{k'})]}{\sqrt{Trace[(\mathbf{Z}_k^T\mathbf{Z}_k)(\mathbf{Z}_k^T\mathbf{Z}_k)], Trace[(\mathbf{Z}_{k'}^T\mathbf{Z}_{k'})(\mathbf{Z}_{k'}^T\mathbf{Z}_{k'}])}}.$$

In this case, the $RV'_{(k,k')}$ coefficient measures the global similarity in the participation and non-participation patterns among the events in the different occasions.

Factorial maps may be obtained by using the factor scores of eigendecomposition of the K by K between-table of the RV coefficients (and of the RV' coefficients when they can be evaluated). On these maps, each two-mode network at a single occasion (i.e., each sub-table) is represented by a single point. If the global relational structure in two occasions k and k' is similar, the points which represent the corresponding sub-tables will be close to each other (Abdi et al., 2012). Note that this kind of between-table analysis can be applied to both fixed and fleeting events among the occasions.

3 Features of the MFA representations

In the following, we exemplify the use of MFA by exploring the multi-mode time-varying structure of the McFarland dataset. Our aim is not a thorough analysis of the McFarland dataset. We only report the main results useful to illustrate the steps of the methods presented in the previous section. The McFarland data concern extracurricular memberships—entailing various forms of interactions, such as sports, interschool contests, and collective performances—of 36 classes in two schools (River High and Magnet High) over three years, 1996–1998. These data are clearly an example of a time-varying affiliation network. Students represent actors, and events are the students' extracurricular activities and association to clubs. In the following, we deal only with Magnet High school and students attending the 10th grade in the starting year (1996).

As illustrated in the previous discussion, we use MCA for both the partial and the global analyses—and interpret patterns in the event space, in the actor space, and in the joint space.[2].

3.1 Main features of the dataset

First of all, in order to avoid negligible patterns, we select only those associations counting at least 10 members in a given year, i.e., we select only events having $s_{jk} \geqslant 10$. The resulting dataset is composed of $N = 181$ students, and $J_1 = 16$ events for 1996, $J_2 = 17$ events for 1997 and $J_3 = 18$ events for 1998 for a total of 51 events and $K = 3$. Gender (male and female) and racial group (Hispanic, Asian, African American, White, and Native American) are also available as actor attributes. The rate of attendance in the events ranges globally from 5% to 34% with the majority of values below 10%. The actor participation rate values vary from 2% (which corresponds to an actor that attends only one event in the three year period) to 32%, with the majority of values below 10%. Altogether, these results imply that the three affiliation matrices are quite sparse.

In order to have an idea of the overall quality of 2-D approximation of the the MCA and MFA solution, we look (Table 1) at the unadjusted and adjusted values (according to Equation (7)) of the proportion of inertia explained by the first two factorial axes. We note that the adjusted proportions dramatically increase, and they range, for the partial analysis, from 75% to 87%.

Given that the value for the global solution is around 56% and the correlations between the factorial axes of the partial solutions and the global one are all greater than 0.80, highlighting a good match among them, for the sake of presentation, in the following we discuss only the global analysis results.

3.2 Events representation in the global actor space

The space spanned by all actors over all the occasions is represented in Figure 2. In this space, each event is represented by a pair of points connected by a segment.

[2] The analyses for all the steps have been performed by using the fully comprehensive procedure AFMUL implemented in SPAD.TM (SPAD.TM Version 5.5, 2002). The network input data have been preprocessed to obtain the grand indicator matrix $\mathbb{Z}$.

Table 1. *Unadjusted and adjusted proportion of inertia (%) explained by the first two factorial axes.*

	Unadj. Prop. In.	Adj. Prop. In.
1996	24.67	75.31
1997	28.53	87.90
1998	28.67	87.11
AFM	19.67	55.81

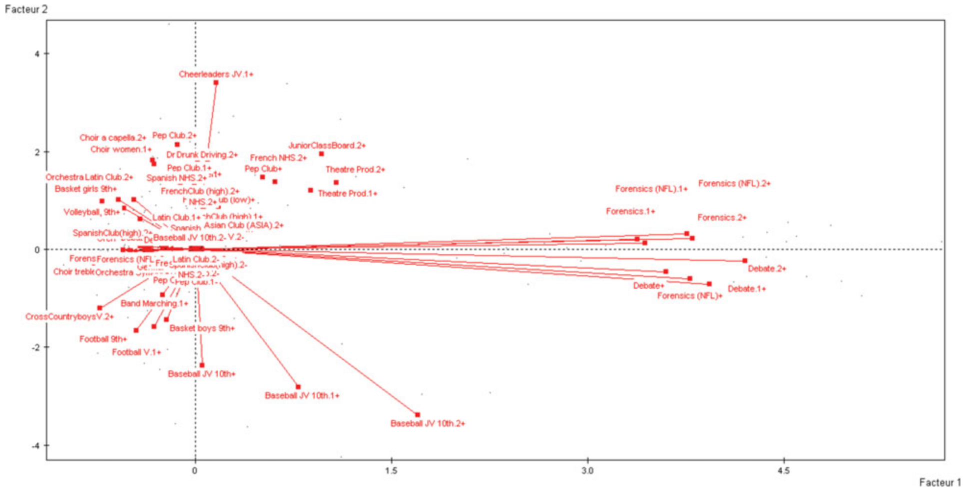

Fig. 2. Global analysis: events' representation in the actor space for all the occasions through principal coordinates. Each event is represented by a couple of points connected by a segment. Proportion of inertia: first factor = 33.37%, second factor 22.43%. (Color online)

Similarities, associations, and changes among attendance patterns of different events and over time occasions can be visually analyzed. In the map, events related to the year 1996 are labeled without suffixes, whereas events in 1997 and 1998 are suffixed by 0.1 and 0.2, respectively. Two elements are of interest in the actor space: (i) the angle between two event vectors, which is proportional to the correlation between event attendance patterns; and (ii) the length of event vectors, which is connected to the variability in the attendance pattern. By looking at the angles between event attendance profiles over the occasions, it is possible to graphically appreciate the presence of some groups of events characterized by a high relational similarity, since the angle formed by their vectors is quite small.

For instance, we note three groups formed by: (1) clubs based on speech and debate—"Forensic," "Forensic (NFL)," and "Debate"—on the right-hand side; (2) mainly male sport clubs—"Football 9th," "Basketball Boys 9th," and "Football V.1'—on the bottom left; (3) mainly female sport clubs—"Volleyball 9th" and "Basketball girls 9th" – on the top left, along with some choir associations, cheerleading and Pep clubs. For the events in each group, the attendance patterns are similar, and they are attended mainly by the same actors. The contribution values (Equation (9)) will furnish greater insights for the interpretation. For example the events of the first group give the larger contribution to the first axis, while all the sports give high contributions to the second and, partially, to the third axes. Hence from this discussion and from a visual inspection of Figure 2, we note that the x-axis can be seen as the axis of the speech and debate clubs (on the right-hand side) versus the other clubs (on the left-hand side), whereas the y-axis can be named the gender axis because female-oriented activities are characterized by large coordinates and male-oriented activities by low coordinates. Around the origin, we find both the most common event profiles, characterized by very low rates of attendance, and events, such the foreign language clubs (French, Spanish, and German), that do not contribute to the first two axes and are not well represented on the corresponding factorial plane. In order to analyze these, we should look at the third and fourth axes, to which these events make larger contributions. This implies that the foreign language group can be interpreted as a fourth group.

In this global analysis, the visualization can also be interpreted event-wise, in the sense that it is possible to study the time evolution of the attendance pattern of the individual events. For instance, the attendance pattern of the event "Baseball JV 10th" changes from 1996 to 1998, since its attendance gradually differs from the other sports clubs along the time points (it might perhaps have been attended at the beginning by students who did not participate in the other sport clubs in later years). It is also important to note that, when events are positioned at opposite poles, their participation patterns are negatively correlated, that is to say that actors belonging to one club do not belong to the other. Figure 2 points out that, for instance, "Baseball JV 10th.2" and "Latin Club.2" are negatively correlated, which means that in 1998, students attending one of these events did not attend the other. This negative correlation is also noticeable between sport clubs (see, for instance, "Cheerleader JV" and "Baseball" in all three occasions), confirming that many sports are segregated by gender (McFarland, 1999).

In this map, the variability of the attendance patterns can also be analyzed by looking at the length of the event vectors. This length can be interpreted as a measure of the event's "peculiarity" or "elitism." For instance, "Debate", "Forensic", and "Forensic (NFL)" on the one hand and "Cheerleader JV" on the other hand are the elite clubs, in which the students taking part in them typically do not attend the others.[3]

3.3 Actors, and attributes, representation in the global event space

The space spanned by all the events over all the occasions is represented in Figure 3. In this space, each point represents one actor, and its coordinates are the weighted average of the coordinates over the three-year span. Students' positions indicate the tendency to be members of some specific associations over time. The proximity of actor points can be interpreted in terms of relational similarities, specifically in terms of the similarity of their participation patterns (i.e., actors who are members of the

[3] In his doctoral dissertation, McFarland (1999) stated that in Magnet High, "Debate" and "Forensic clubs"—clubs set up for inter-school tournaments on academic topics—are attended by strongly motivated students, whereas access to the cheerleader club is highly formalized and competitive, and the peculiarity is that, in order to take part in this club, the students need "to write an essay about why they wanted to be a cheerleader" (McFarland, 1999, p.416).

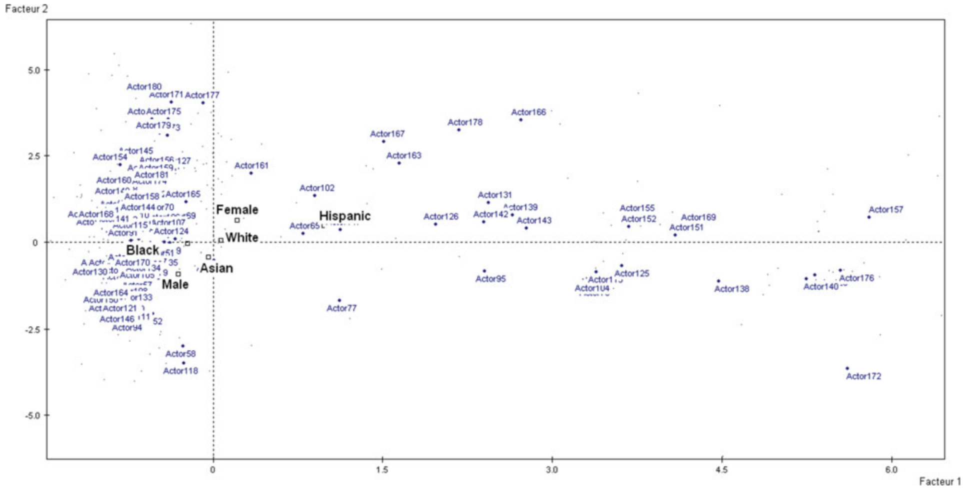

Fig. 3. Global analysis: actors' representation in the event space through principal coordinates. Each point represents one actor and its coordinates are the weighted average of the coordinates over the three-years span. Actors' attributes are added as supplementary points. Proportion of inertia: first factor = 33.37%, second factor 22.43%. (Color online)

same clubs are likely to lie closer in the factorial plane). Then, it is easy to identify groups of actors and isolated ones. Thanks to the axis interpretation, it is possible to appreciate actor participation profiles through their positions. For instance, the actors characterized by large x coordinates tend to participate exclusively in the debate and speech clubs over the three years, whereas actors positioned close to the origin are mainly characterized by non-participation. A fundamental feature of MFA—and of MCA—is the possibility to enrich the results interpretation by projecting the available actor attributes onto the factorial plane. In Figure 3, we use both gender and racial groups. Attribute positions in the event space confirm that the second factorial axis can be seen as the "gender" axis. Racial groups are instead spanned along the first factorial axis, which discriminates between African American (negative x coordinates) and the other racial groups (positive x coordinates). The average position of the African American group—close to the female attribute and hence to the female-based clubs—is explained by the higher ratio of females over males in this racial group (42 females and only 18 males in the 10th grade) with respect to the others.

3.4 Event and actor joint representation through the biplot

In order to represent actors and events in a joint two-dimensional map, we can use the biplot (Gabriel, 1995). In order to keep the natural asymmetry of actors and events in the representation, we use the so-called asymmetric biplot where the actors are represented in principal coordinates and the events in standard coordinates (Greenacre, 2010). Here the interpretation of actor positions with respect to events is achieved by using the direction vector defined by each event, which represents the biplot axis. By projecting the points representing actors onto each biplot axis, we can get an approximate value of the event participation profile and explain why actors are close together or far apart in the event space. Looking at the map in Figure 4,

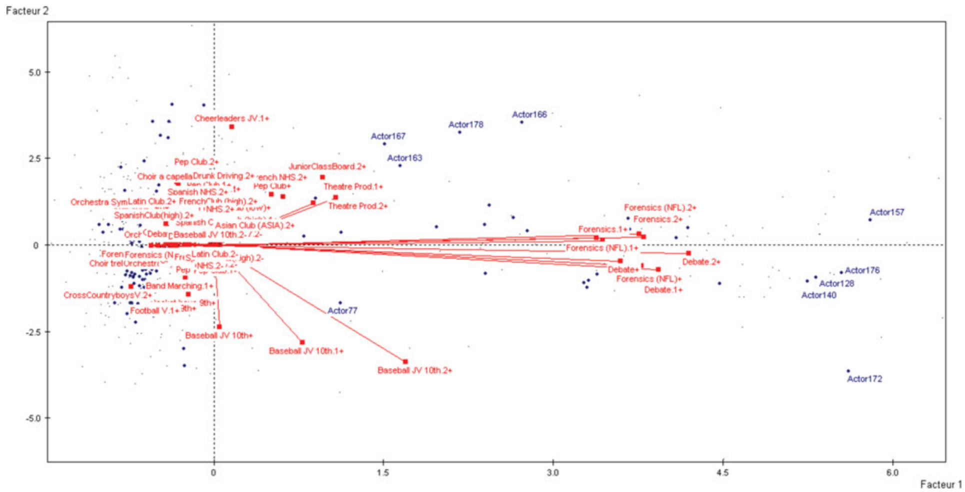

Fig. 4. Global analysis: events' and actors' joint representation. Asymmetric biplot of actors (dots) and events (squares) with some events and actors highlighted. Actors are in principal coordinates, and events are in standard coordinates. Proportion of inertia: first factor = 33.37%, second factor 22.43%. (Color online)

we identify a group of actors (namely, Actors 128,140, 157, and 176) who have the largest coordinates on the biplot axes determined by the speech clubs. These actors are characterized by high participation in these events along the time span. Another group is given by Actors 163, 166, 167, and 178, who have the largest coordinates on the biplot axes determined by their participation in the theater production club. Note that these directions are somehow correlated to the speech clubs' directions. This implies that these clubs share some actors. Furthermore, we can spot some isolated points. For instance, we have Actor 77 who is strongly characterized by his participation in the "Baseball JV 10th" in 1998, and Actor 172 who is located in an extreme position in the lower right corner. This is due to his/her strong participation in the speech club along with the Baseball and Basketball clubs.

3.5 *Variations over time and actor trajectories*

In order to analyze the variations over time of the global relational structure, Table 2 reports the values of the RV coefficients (Equation (14)). The values show that the relation structure over the years changes quite considerably as all the values are lower than 0.5—remember that RV values close to 1 correspond to very similar tables. These results are in line with what can be expected, since these are fleeting events over time. Interestingly, the participation patterns of two subsequent years are closer (1996–1997 and 1997–1998) pointing to gradual changes in the students' affiliation patterns over the years.

In the global event space, we can see the actors' trajectories and variations, which provide a combined visualization of the results of the partial analyses in the global event space and clearly point out the temporal dimension in terms of evolution and variability. Segments connecting points related to the same actor over the partial analyses depict the actor trajectory over time occasions and provide a graphical display of individual attendance pattern evolution. Figure 5 plots the time

Table 2. *RV coefficients over the years.*

	1996	1997	1998
1996	1	–	–
1997	0.375	1	–
1998	0.198	0.358	1

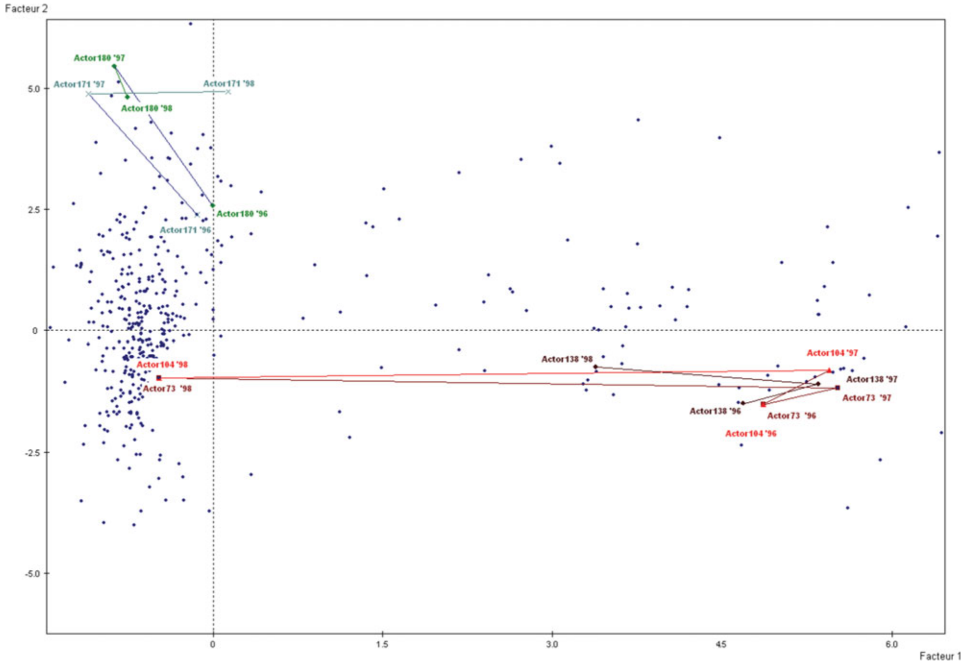

Fig. 5. Global analysis: actors' representation in the event space through principal coordinates. Each point represents one actor at a given time. Some actors and relative trajectories are highlighted. Proportion of inertia: first factor = 33.37%, second factor 22.43%. (Color online)

trajectories of Actors 73, 104, 138, 171, and 180. Actors 73, 104, and 138 have similar trajectories given that they share participation in the Debate Club and Forensic Club in 1996 and 1997. We note that in 1998, Actor 138 remains in the same part of the map as he/she continues to attend the same kind of clubs, while Actors 73 and 104 move to the other side of the map close to the non-participation area (they stop attending the clubs). From the trajectories of Actors 171 and 180, who participate in the Pep Club, Cheerleaders JV, and Female Choirs, we can say that these two actors preserve a similar attendance pattern for all three years with negligible variations.

Together with the evolution of actor behaviors highlighted by the trajectories, we can also evaluate variability in the actors' participation patterns by looking at the points corresponding to the single occasions with respect to their barycenter. In Figure 6, Actors 73, 171, and 172 are highlighted (dots correspond to actor position in each year, diamonds to the barycentric points). The first (Actor 73) presents a high variability with a great change in his/her participation pattern in the third year. Actor 171 presents a smaller variability around the global point, indicating a fairly stable participation pattern. Actor 172 presents an intermediate situation along with

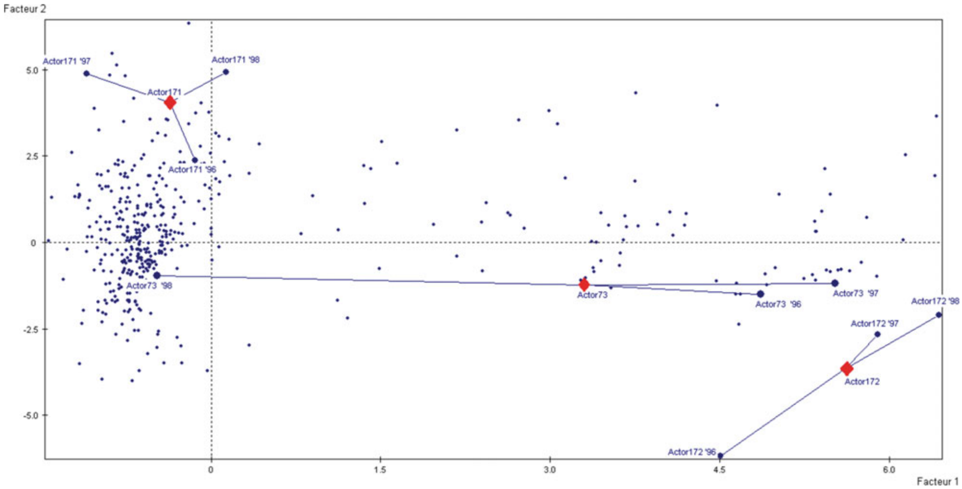

Fig. 6. Small dots represent the actors a_i over all the time of the partial analyses. The positions of Actor 73, Actor 171, and Actor 172 are highlighted by larger dots and diamonds (coordinates of the global analysis). Proportion of inertia: first factor = 33.37%, second factor 22.43%. (Color online)

a regular change in his/her participation pattern passing from being quite isolated to being more exclusively involved in the "Debate" and "Forensic" Clubs.

4 Concluding remarks

In this paper, we have discussed how MFA based on MCA can be used for the visual analysis of time-varying two-mode networks. The proposed method provides a variety of different statistical visualizations that allow different points of view and insights into the data, as well as an interplay between analytical and visual tools. It fits different data structures in which both actors and events may partially change over time. When the same actors are linked to the same set of events in all the occasions, additional analysis, such as event trajectories and comparison of the event-by-event projection matrices, could be undertaken. In any case, the proposed method provides a unique and coherent framework for both static and longitudinal analyses.

Given the analytical properties of MCA, the proposed method allows the user to visually analyze the degree of similarity of actor/event network profiles over time and to characterize actors and events on the basis of their attendance and non-attendance patterns. Distances on the factorial maps allow us to evaluate the structural similarities among actors and events and how they change over time. Furthermore it is also possible to visualize actor attributes or actor groups and their evolution along the occasions.

In addition:

i. The procedure, with few modifications, could be applied to more complex and heterogeneous structures, such as multi-modal, multi-relational networks where different kinds of relationships (possibly observed over time occasions) are considered. In this case, we would consider K different types of relation, and each $\mathbf{F}_k$ would refer to a specific relation;

ii. Due to the geometrical and analytical characteristics of the MFA, the results could be easily included in a dynamic and interactive perspective. Indeed, actors and events for all occasions have coordinates in the unique global space—akin to that of Procrustes analysis. This makes it possible, on the one hand, to draw trajectories in a static layout and, on the other, with few calculations such as linear interpolations, to generate movie-like visualizations of trajectories or of the star-type plot; and

iii. Factorial maps will benefit considerably from the use of interactive tools, e.g., point selection tools based on contributions or measurements of the quality of representations or brushing. In order to achieve this, ad hoc routines should be implemented.

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