

## Planck 2013 results. XXV. Searches for cosmic strings and other topological defects

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### ABSTRACT

*Planck* data have been used to provide stringent new constraints on cosmic strings and other defects. We describe forecasts of the CMB power spectrum induced by cosmic strings, calculating these from network models and simulations using line-of-sight Boltzmann solvers. We have studied Nambu-Goto cosmic strings, as well as field theory strings for which radiative effects are important, thus spanning the range of theoretical uncertainty in the underlying strings models. We have added the angular power spectrum from strings to that for a simple adiabatic model, with the extra fraction defined as  $f_{10}$  at multipole  $\ell = 10$ . This parameter has been added to the standard six parameter fit using COSMOMC with flat priors. For the Nambu-Goto string model, we have obtained a constraint on the string tension of  $G\mu/c^2 < 1.5 \times 10^{-7}$  and  $f_{10} < 0.015$  at 95% confidence that can be improved to  $G\mu/c^2 < 1.3 \times 10^{-7}$  and  $f_{10} < 0.010$  on inclusion of high- $\ell$  CMB data. For the abelian-Higgs field theory model we find,  $G\mu_{\text{AH}}/c^2 < 3.2 \times 10^{-7}$  and  $f_{10} < 0.028$ . The marginalized likelihoods for  $f_{10}$  and in the  $f_{10}$ - $\Omega_b h^2$  plane are also presented. We have additionally obtained comparable constraints on  $f_{10}$  for models with semilocal strings and global textures. In terms of the effective defect energy scale these are somewhat weaker at  $G\mu/c^2 < 1.1 \times 10^{-6}$ . We have made complementarity searches for the specific non-Gaussian signatures of cosmic strings, calibrating with all-sky *Planck* resolution CMB maps generated from networks of post-recombination strings. We have validated our non-Gaussian searches using these simulated maps in a *Planck*-realistic context, estimating sensitivities of up to  $\Delta G\mu/c^2 \approx 4 \times 10^{-7}$ . We have obtained upper limits on the string tension at 95% confidence of  $G\mu/c^2 < 8.8 \times 10^{-7}$  with modal bispectrum estimation and  $G\mu/c^2 < 7.8 \times 10^{-7}$  for real space searches with Minkowski functionals. These are conservative upper bounds because only post-recombination string contributions have been included in the non-Gaussian analysis.

**Key words.** Astroparticle physics – cosmology: cosmic background radiation – cosmology: observations – cosmology: theory – cosmology: early Universe

## 1. Introduction

This paper, one of a set associated with the 2013 release of data from the *Planck*<sup>1</sup> mission (Planck Collaboration I 2013), describes the constraints on cosmic strings, semi-local strings and global textures. Such cosmic defects are a generic outcome of symmetry-breaking phase transitions in the early Universe (Kibble 1976) and further motivation came from a potential role in large-scale structure formation (Zeldovich 1980; Vilenkin 1981a). Cosmic strings appear in a variety of supersymmetric and other grand unified theories, forming at the end of inflation (see, for example, Jeannerot et al. 2003). However, further interest in cosmic (super-)strings has been motivated by their emergence in higher-dimensional theories for the origin of our Universe, such as brane inflation. These superstring variants come in a number of  $D$ - and  $F$ -string forms, creating hybrid networks with more complex dynamics (see, e.g., Polchinski 2005). Cosmic strings can have an enormous energy per unit length  $\mu$  that can give rise to a number of observable effects, including gravitational lensing and a background of gravitational waves. Here, we shall concentrate on the impact of strings on the cosmic microwave background (CMB), which includes the generation of line-like discontinuities in temperature. Comparable effects can also be caused by other types of cosmic defects, notably semi-local strings and global textures. As well as influencing the CMB power spectrum, each type of topological defect should have a counterpart non-Gaussian signature giving us the ability distinguish between different defects, alternative scenarios, or systematic effects. The discovery of any of these objects would profoundly influence our understanding of fundamental physics, identifying GUT-scale symmetry breaking patterns, perhaps even providing direct evidence for extra dimensions. Conversely, the absence of these objects will tightly constrain symmetry breaking schemes, again providing guidance for high energy theory. For a general introduction to cosmic strings and other defects, refer to Vilenkin & Shellard (2000); Hindmarsh & Kibble (1995); Copeland & Kibble (2010).

High resolution numerical simulations of cosmic strings using the Nambu-Goto action indicate that cosmological networks tend towards a scale-invariant solution with typically tens of long strings stretching across each horizon volume. These strings continuously source gravitational perturbations on sub-horizon scales, the magnitude of that are determined by the dimensionless parameter:

$$\frac{G\mu}{c^2} = \left( \frac{\eta}{m_{\text{Pl}}} \right)^2, \quad (1)$$

where  $\eta$  is the energy scale of the string-forming phase transition and  $m_{\text{Pl}} \equiv \sqrt{\hbar c/G}$  is the *Planck* mass. String effects on the CMB power spectrum have been estimated using a phenomenological string model and, with *WMAP* and SDSS data, these estimates yield a  $2\sigma$  upper bound of  $G\mu/c^2 < 2.6 \times 10^{-7}$  (Battye & Moss 2010). A consequence is that strings can be responsible for no more than 4.4% of the CMB anisotropy signal at multipole  $\ell = 10$ .

As we shall discuss, the evolution of Nambu-Goto string networks is computationally challenging and quantitative uncertainties remain, notably in characterizing the string small-scale

structure and loop production. An alternative approach has been to use field theory simulations of cosmological vortex-strings. These yield a significantly lower number of strings per horizon volume (less than half), reflecting the importance of radiative effects on the microphysical scales being probed numerically. The degree of convergence with Nambu-Goto string simulations is difficult to determine computationally at present, but there are also global strings for which radiative effects of comparable magnitude are expected to remain important on cosmological scales. It is prudent in this paper, therefore, to constrain both varieties of strings, labelling the field theory constraints as AH from the abelian-Higgs (local  $U(1)$ ) model used to describe them. Given these quantitative differences, such as the lower density, field theory strings produce a weaker constraint  $G\mu_{\text{AH}}/c^2 < 5.7 \times 10^{-7}$  using *WMAP* data alone (Bevis et al. 2008) and (Urrestilla et al. 2011). The shape of the string-induced power spectrum also has a different shape, which allows up to a 9.5% contribution at  $\ell = 10$ . These *WMAP* constraints can be improved by adding small-scale CMB anisotropy in a joint analysis. The Nambu-Goto strings limit improves to become  $G\mu/c^2 < 1.7 \times 10^{-7}$  (using SPT data, Dvorkin et al. 2011) and field theory strings yield  $G\mu_{\text{AH}}/c^2 < 4.2 \times 10^{-7}$  (Urrestilla et al. 2011). Power-spectrum based constraints on global textures were studied in Bevis et al. (2004) and (Urrestilla et al. 2008), with the latter paper giving a 95% limit of  $G\mu/c^2 < 4.5 \times 10^{-6}$ . Urrestilla et al. (2008) also provide constraints on semi-local strings,  $G\mu/c^2 < 5.3 \times 10^{-6}$ .

Constraints on cosmic strings from non-Gaussianity require high resolution realisations of string-induced CMB maps that are extremely challenging to produce. Low resolution small-angle and full-sky CMB maps calculated with the full recombination physics included, have indicated some evidence for a significant kurtosis from strings (Landriau & Shellard 2011). More progress has been made increasing high resolution maps from string lensing after recombination (see Ringeval & Bouchet (2012) and references therein) and identifying, in principle, the bispectrum and trispectrum, which can be predicted for strings analytically (Hindmarsh et al. 2009, 2010; Regan & Shellard 2010). The first *WMAP* constraint on cosmic strings using the analytic CMB trispectrum yielded  $G\mu/c^2 < 1.1 \times 10^{-6}$  at 95% confidence (Fergusson et al. 2010b). An alternative approach is to fit pixel-space templates to a map, this method was applied to global textures templates in Cruz et al. (2007) and Feeney et al. (2012a,b).

The most stringent constraints that are claimed for the string tension arise from predicted backgrounds of gravitational waves that are created by decaying loops (Vilenkin 1981b). However, these constraints are strongly dependent on uncertain string physics, most notably the network loop production scale and the nature of string radiation from cusps, i.e., points on the strings approaching the speed of light  $c$ . The most optimistic constraint based on the European Pulsar Timing Array is  $G\mu/c^2 < 4.0 \times 10^{-9}$  (van Haasteren et al. 2011), but a much more conservative estimate of  $G\mu/c^2 < 5.3 \times 10^{-7}$  can be found in Sanidas et al. (2012), together with a string parameter constraint survey and an extensive discussion of these uncertainties. Such gravitational wave limits do not apply to global strings or to strings for which other radiative channels are available.

Alternative topological defects scenarios also have strong motivations and we report limits on textures and global monopoles in this paper as well. Of particular recent interest are hybrid networks of cosmic strings where the correlation length is reduced by having several interacting varieties (e.g.,  $F$ - and

<sup>1</sup> *Planck* (<http://www.esa.int/Planck>) is a project of the European Space Agency (ESA) with instruments provided by two scientific consortia funded by ESA member states (in particular the lead countries France and Italy), with contributions from NASA (USA) and telescope reflectors provided by a collaboration between ESA and a scientific consortium led and funded by Denmark.

$D$ -strings) or a small reconnection probability,  $p < 1$ . We expect to investigate these models using the *Planck* full mission data.

The outline of this paper is as follows. In Sect. 2 we briefly describe the different types of topological defects that we consider, and their impact on the CMB anisotropies. We also discuss how the CMB power spectrum is computed and how we obtain CMB maps with a cosmic string contribution. In Sect. 3 we present the defect constraints from the CMB power spectrum (with numbers given in Table 2), while Sect. 4 discusses searches for topological defects with the help of their non-Gaussian signature. We finally present the overall conclusions in Sect. 5.

## 2. Theoretical Modelling and Forecasting

### 2.1. Cosmic strings and their cosmological consequences

#### 2.1.1. String network evolution

A detailed quantitative understanding of the cosmological evolution of string networks is an essential pre-requisite for making accurate predictions about the cosmological consequences of strings. Fortunately, all string network simulations to date have demonstrated convincingly that the large-scale properties of strings approach a self-similar scale-invariant regime soon after formation. If we treat the string as a one-dimensional object, then it sweeps out a two-dimensional worldsheet in spacetime

$$x^\mu = x^\mu(\zeta^a), \quad a = 0, 1, \quad (2)$$

where the worldsheet parameters  $\zeta^0$  and  $\zeta^1$  are time-like and space-like respectively. The Nambu-Goto action that governs string motion then becomes

$$S = -\mu \int \sqrt{-\gamma} d^2\zeta, \quad (3)$$

where  $\gamma_{ab} = g_{\mu\nu} \partial_a x^\mu \partial_b x^\nu$  is the two-dimensional worldsheet metric ( $\gamma = \det(\gamma_{ab})$ ) induced by the spacetime metric  $g_{\mu\nu}$ . The Nambu-Goto action Eq. (3) can be derived systematically from a field theory action, such as that for the abelian-Higgs model describing  $U(1)$  vortex-strings:

$$S = \int d^4x \sqrt{-g} \left[ (D_\mu \phi)^* (D^\mu \phi) - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2 \right], \quad (4)$$

where  $\phi$  is a complex scalar field,  $F^{\mu\nu}$  is the  $U(1)$  field strength and  $D_\mu = \partial_\mu + iA_\mu$  is the gauge-covariant derivative with  $e$  and  $\lambda$  dimensionless coupling constants. The transverse degrees of freedom in  $\phi$  can be integrated out provided the string is not strongly curved, that is, the string width  $\delta \approx \hbar c / \eta \ll L$  where  $L$  is the typical radius of curvature. For a cosmological string network today with  $G\mu/c^2 \sim 10^{-7}$ , these two lengthscales are separated by over 40 orders of magnitude, so this should be a valid approximation.

In an expanding universe, the Nambu-Goto action Eq. (3) yields a Hubble-damped wave equation governing the string motion. These equations can be solved numerically, provided ‘‘kinks’’ or velocity discontinuities are treated carefully. However, they can also be averaged analytically to describe the scale-invariant evolution of the whole string network in terms of two quantities, the energy density  $\rho$  and the r.m.s. velocity  $v$ . Any string network divides fairly neatly into two distinct populations of long (or ‘‘infinite’’) strings  $\rho_\infty$  stretching beyond the Hubble radius and the small loops  $\rho_l$  with length  $l \ll H^{-1}$  that the long strings create Kibble (1985). Assuming the long strings form a

**Table 1.** Summary of numerical simulation results for the string density parameter  $\zeta$  defined in Eq. (5). The Nambu-Goto string simulations are respectively labelled as MS (Martins & Shellard 2006), RSB (Ringeval et al. 2007), and BOS (Blanco-Pillado et al. 2011). This is contrasted with the much lower density results from lattice field theory simulations of vortex-strings labelled as MMS (Moore et al. 2002) and BHKU (Bevis et al. 2007b).

Epoch	MS	RSB	BOS	MSM	BHKU
Radiation	11.5	9.5	11.0	5.0	3.8
Matter	3.0	3.2	3.7	1.5	1.3

Brownian random walk characterised by a correlation length  $L$ , we have

$$\rho_\infty = \frac{\mu}{L^2} \equiv \frac{\zeta\mu}{l^2}, \quad (5)$$

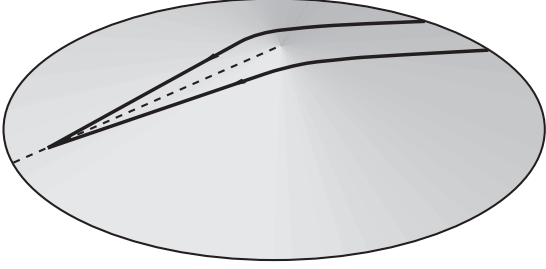
and the averaged equations of motion become simply

$$\begin{aligned} 2 \frac{dL}{dt} &= 2HL(1 + v^2) + \tilde{c}v, \\ \frac{dv_\infty}{dt} &= (1 - v^2) \left[ \frac{k(v)}{L} - 2Hv \right], \end{aligned} \quad (6)$$

where  $\tilde{c}$  measures the network loop production rate and  $k(v)$  is a curvature parameter with  $k \approx 2\sqrt{2}(1 - \sqrt{2}v)$ . This is the velocity-dependent one-scale (VOS) model and, with a single parameter  $\tilde{c}$ , it provides a good fit to both Nambu and field theory simulations, notably through the radiation-matter transition (Martins & Shellard 1996).

A general consensus has emerged from the three main simulation codes describing Nambu-Goto string networks (Martins & Shellard 2006; Ringeval et al. 2007; Blanco-Pillado et al. 2011). These independent codes essentially solve for left- and right-moving modes along the string using special techniques to handle contact discontinuities or kinks, including ‘‘shock fronting’’, artificial compression methods and an exact solver for piecewise linear strings, respectively. The consistency between simulations is shown in Table 1 for the string density parameter  $\zeta$  defined in Eq. (5). Averaging yields the radiation era density  $\zeta = 10.7$  and a matter era value  $\zeta = 3.3$ . Note that these asymptotic values and the intervening matter-radiation transition can be well-described by the VOS model Eq. (6) with  $\tilde{c} = 0.23$ . The matter era VOS value appears somewhat anomalous from the other two simulations, but this is obtained from larger simulations in a regime where convergence is very slow, so it may more closely reflect the true asymptotic value. These simulations have also advanced the study of string small-scale structure and the loop distribution, about which there had been less consensus (see, e.g., Blanco-Pillado et al. 2011). However, note that CMB anisotropy is far less sensitive to this issue compared to constraints from gravitational waves.

Field theory simulations using lattice gauge techniques have also been employed to study the evolution of string networks in an expanding universe. Comparatively, these three-dimensional simulations are constrained to a lower dynamic range and the simulations require the solution of modified field equations to prevent the string core width shrinking below the lattice resolution. On the other hand, field theory simulations include field radiation and therefore provide a more complete account of the string physics. In Table 1 the lower string densities obtained



**Fig. 1.** The spacetime around a cosmic string is conical, as if a narrow wedge were removed from a flat sheet and the edges identified. For this reason cosmic strings can create double images of distant objects. Strings moving across the line of sight will cause line-like discontinuities in the CMB radiation.

from two sets of abelian-Higgs simulations are given (Moore et al. 2002; Bevis et al. 2007b). The evolution can be fitted with a VOS model Eq. (6) with  $\tilde{c} = 0.57$ , which is 150% higher than for Nambu-Goto strings. Field theory simulations have further important applications, particularly for describing delocalised topological defects such as textures, for describing models that do not form stable defects like semilocal strings, and because they include radiative effects naturally. Radiative effects observed in current abelian-Higgs simulations are comparable to the radiative damping anticipated for cosmological global strings and so the AH analysis below should offer some insight into this case.

### 2.1.2. String gravity and the CMB

Despite the enormous energy per unit length  $\mu$ , the spacetime around a straight cosmic string is locally flat. The string has an equation of state  $p_z = -\rho$ ,  $p_x = p_y = 0$  (for one lying along the  $z$ -direction), so there is no source term in the relativistic version of the Poisson equation  $\nabla^2\Phi = 4\pi G(\rho + p_x + p_y + p_z)$ . The straight string exhibits no analogue of the Newtonian pull of gravity on any surrounding matter. But this does not mean the string has no gravitational impact; on the contrary, a moving string has dramatic effects on nearby matter or propagating CMB photons.

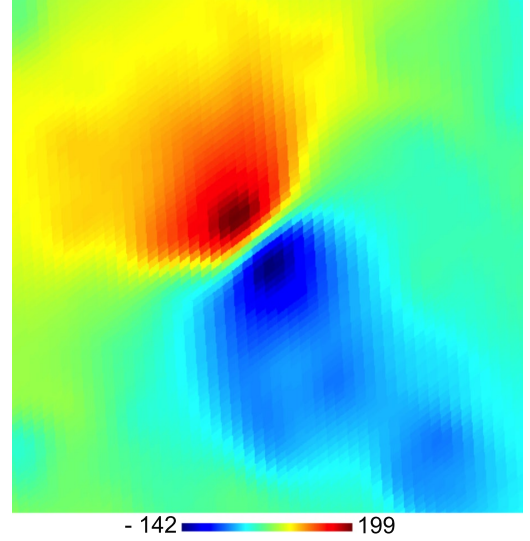
The spacetime metric about a straight static string takes the simple form,

$$ds^2 = dt^2 - dz^2 - dr^2 - r^2 d\theta^2, \quad (7)$$

that looks like Minkowski space in cylindrical coordinates, but for the fact that the azimuthal coordinate  $\theta$  has a restricted range  $0 \leq \theta \leq 2\pi(1 - 4G\mu)$ . The spacetime is actually *conical* with a global deficit angle  $\Delta = 8\pi G\mu$ , that is, an angular wedge of width  $\Delta$  is removed from the space and the remaining edges identified (see Fig. 1). This means that distant galaxies on the opposite side of a cosmic string can be gravitationally lensed to produce characteristic double images.

Cosmic strings create line-like discontinuities in the CMB signal. As the string moves across the line of sight, the CMB photons are boosted towards the observer, causing a relative CMB temperature shift across the string, given by (Gott III 1985; Kaiser & Stebbins 1984)

$$\frac{\delta T}{T} = 8\pi G\mu v_s \gamma_s, \quad (8)$$



**Fig. 2.** Characteristic CMB temperature discontinuity created by a cosmic string. Here, the simulated Nambu-Goto string has produced a cusp, a small region on the string that approaches the speed of light, which has generated a localised CMB signal.

where  $v_s$  is the transverse velocity of the string and  $\gamma_s = (1 - v_s^2)^{-1/2}$ . This rather simple picture, however, is complicated in an expanding universe with a wiggly string network and relativistic matter and radiation components. The energy-momentum tensor  $T_{\mu\nu}(\mathbf{x}, t)$  essentially acts as a source term for the metric fluctuations that perturb the CMB photons and create temperature anisotropies. Essentially, the problem can be recast using Green's (or transfer) functions  $G^{\mu\nu}$  that project forward the contributions of strings from early times to today:

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}, \mathbf{x}_{\text{obs}}, t_0) = \int d^4x G^{\mu\nu}(\hat{\mathbf{n}}, \mathbf{x}, \mathbf{x}_{\text{obs}}, t, t_0) T_{\mu\nu}(\mathbf{x}, t), \quad (9)$$

where  $\hat{\mathbf{n}}$  is the line-of-sight direction for photon propagation and  $\mathbf{x}_{\text{obs}}$  is the observer position. The actual quantitative solution of this problem entails a sophisticated formalism to solve the Boltzmann equation and then to follow photon propagation along the observer's line-of-sight. An example of the line-like discontinuity signal created by a cosmic string in the CMB is shown in Fig. 2. In this case, a string cusp has formed on the string, causing a strongly localised signal and reflecting the Lorentz boost factor in Eq. (8).

### 2.2. Semi-local strings

The tight constraints on the presence of cosmic strings that we will discuss later in this paper start to put pressure on the wide class of inflation models that generate such defects (Hindmarsh 2011). The power of these constraints would be reduced if the strings could be made unstable. This is the basic motivation behind semilocal strings: a duplication of the complex scalar field  $\phi$  in the abelian-Higgs action (4), occurring naturally in a range of inflation models (Dasgupta et al. 2004; Achucarro et al. 2006; Dasgupta et al. 2007), transforms the stable cosmic strings into non-topological semilocal strings (Vachaspati & Achucarro 1991) as the vacuum manifold becomes  $S^3$ , which is simply-connected. The existence and stability of the semilocal strings is thus a question of dynamics rather than due to the topology of the vacuum manifold. In general we do not expect to form

long strings, but rather shorter string segments, as the semilocal strings can have ends. The evolution of these segments is very complicated and arises directly from the field evolution, so that it is only practicable to simulate these defects with the help of field theory (Urrestilla et al. 2008).

### 2.3. Global defects

A large alternative class of defects is due to the breaking of a global  $O(N)$  symmetry (rather than a gauge symmetry as in the case of cosmic strings) of a  $N$ -component scalar field  $\phi$ . The energy density of global defects is significantly less localised than those that result from gauge symmetry breaking due to the absence of the screening by a gauge field, and there are thus long-range forces between the defects. The field self-ordering is therefore very efficient for all types of defects with  $N \geq 2$ , leading to a generic scaling of the defect energy density with the background energy density (see e.g., Durrer et al. 2002). For this reason global monopoles ( $N = 3$ ) do not overclose the Universe as their local counterparts would. In this paper we study specifically the case  $N = 4$  called “texture”, which can arise naturally in many multi-field inflation models that involve a non-zero vacuum expectation value and symmetry breaking. In this case there are no stable topological defects present, but contrary to local texture, global texture can have a non-negligible impact on the perturbations in the cosmos, with the field self-ordering leading to “unwinding events”. In spite of their non-topological nature, the field evolution is closely related to the one of lower-dimensional stable global defects due to the long-range nature of the forces. This is similar to the case of the non-topological semilocal strings of the previous section, and indeed the semilocal example can be seen as an intermediate case between cosmic strings and global texture: Starting from the semilocal action, we can on the one hand revert to the cosmic string action by removing one of the complex scalar fields, and on the other hand we find the texture action if we remove the gauge field.

The normalization of global defects is usually given in terms of the parameter  $\varepsilon = 8\pi G\eta^2/c^2$  when using an action like Eq. (4) (with a second complex scalar field but without the gauge fields). However, for a simpler comparison with the cosmic string results we can recast this in terms of  $G\mu/c^2 \equiv \varepsilon/4$  and quote limits on  $G\mu/c^2$  also for the texture model, as in Urrestilla et al. (2008).

### 2.4. CMB power spectra from cosmic defects

The CMB power spectrum from topological defects, like strings, is more difficult to compute than the equivalent for inflationary scenarios that predict a spectrum dominated by an adiabatic component with a possible, but highly constrained, isocurvature component. In defect-based scenarios the perturbations are sourced continuously throughout the history of the Universe, as opposed to adiabatic and isocurvature modes that are the result of initial conditions. In principle this requires knowledge of the source, quantified by the unequal-time correlator (UETC) of the defect stress-energy tensor, from the time of defect formation near the GUT scale to the present day—a dynamic range of about  $10^{52}$ —something that will never be possible to simulate. Fortunately, we can use the scaling assumption to extrapolate the results of simulations with substantially smaller dynamic range. This has allowed a qualitative picture to emerge of the characteristics of the power spectra from defects, though quantitative predictions differ. Here, we will focus on spectra calculated in

two different ways for cosmic strings, as well as spectra from semilocal strings and texture models.

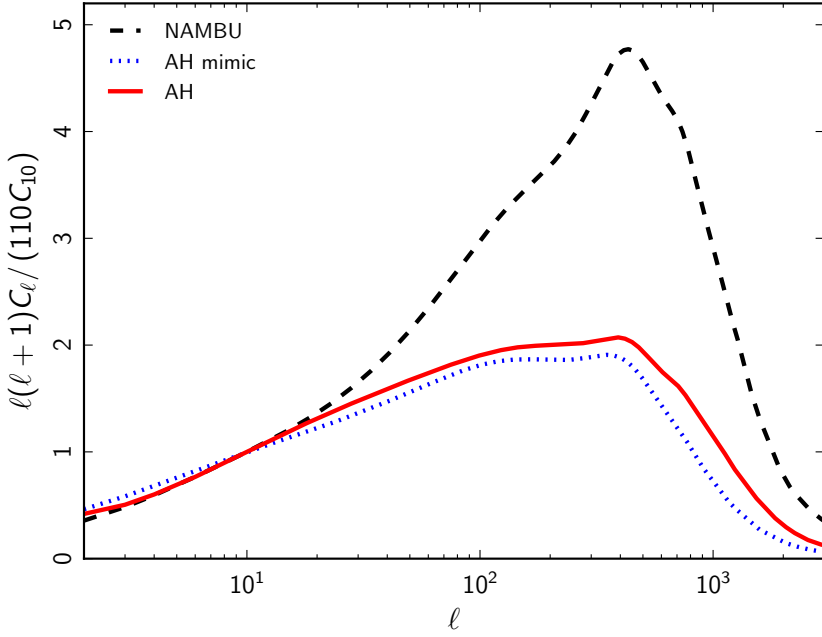
Defect-based power spectra are dominated by different physical effects across the range of angular scales. (i) On large angular scales the spectra are dominated by an integrated Sachs-Wolfe (ISW) component due to the strings along the line-of-sight between the time of last scattering and the present day. The scaling assumption implies that this component will be close to scale invariant, although in practice it typically has a mildly blue spectrum. (ii) At intermediate scale the dominant contribution comes from anisotropies created at the time of last scattering. In contrast to the strong series of acoustic peaks created in adiabatic and isocurvature models, defects produce only a broad peak because their contributions are not coherent. (iii) At very small angular scales, the spectra are again dominated by the ISW effect because, rather than decaying exponentially due to the effects of Silk damping, there is only power-law decay with the exponent being a characteristic of the specific type of defect.

The standard lore is to treat the defect stress energy tensor,  $\theta_{\mu\nu}$ , as being covariantly conserved at first order, which is known as the “stiff approximation”. In principle, this means that it is necessary to measure two independent quantities from the simulations, or model them. The other two component are then computed from the conservation equations. In practice things are a little more complicated since it is necessary to provide the UETC

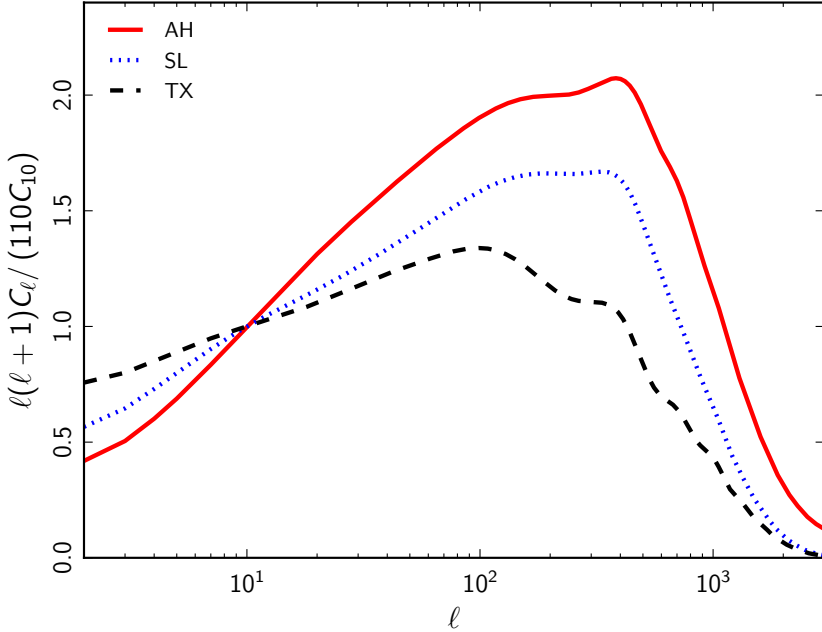
$$U_{\mu\nu\alpha\beta}(k, \tau, \tau') = \langle \theta_{\mu\nu}(k, \tau) \theta_{\alpha\beta}(k, \tau') \rangle, \quad (10)$$

where  $\tau$  is the conformal time and  $k$  is the wavenumber. Once one has the UETC, then there two ways to proceed. The first involves creating realisations of the defect stress-energy whose power spectra are computed then averaged to give the total power spectrum. The other approach involves diagonalization of the UETC. During pure matter or radiation domination, the scaling property of defect evolution means that quantities are measured relative to the horizon scale, so that the UETC is only a function of  $x = k\tau$  and  $x' = k\tau'$ . These functions  $U(x, x')$  can be discretized and then are symmetric matrices that we can diagonalize. The resulting eigenvectors can be inserted as sources into a Boltzmann code, and the resulting  $C_\ell$  are then summed up, weighted by the eigenvalues (Pen et al. 1997; Durrer et al. 2002). Even though the power spectrum resulting from each “eigen-source” exhibits a series of acoustic peaks, the summation over many such spectra smears them out, as they are not coherent (unlike inflationary perturbations). This smearing-out explains why defect power spectra generically are smooth, as mentioned above.

There are also several methods to obtain predictions for the UETCs of cosmic strings and other topological defects. The first approach we will consider for cosmic strings is to use what has become known as the Unconnected Segment Model (USM; Albrecht et al. 1997, 1999; Pogosian & Vachaspati 1999). In its simplest form this models the cosmic string energy momentum tensor as that of an ensemble of line segments of correlation length  $\xi d_H(t)$ , moving with an r.m.s. velocity  $\langle v^2 \rangle^{1/2}$ , where  $d_H(t)$  is the horizon distance. In addition one can take into account the effects of string “wiggles” due to small-scale structure via a coefficient,  $\beta = \mu_{\text{eff}}/\mu$  quantifying the ratio of the renormalized mass per unit length to the true value. The model parameters  $\xi$ ,  $\langle v^2 \rangle^{1/2}$  and  $\beta$  are computed from simulations. In our calculations we link the USM sources to the line-of-sight Boltzmann solver CMBACT (Pogosian & Vachaspati 1999) to create an ensemble of realisations from which we find an averaged angular power spectrum.



**Fig. 3.** Cosmic string power spectra used in this analysis: Nambu (black dashed), AH-mimic (blue dotted) and AH (red solid). The spectra have been normalized to equal power at  $\ell = 10$ . The spectra are normalized the observed *WMAP7* value at  $\ell = 10$  and have  $G\mu/c^2 = 1.17 \times 10^{-6}$ ,  $1.89 \times 10^{-6}$  and  $2.04 \times 10^{-6}$  respectively. Note that the limits discussed in this paper mean that the CMB spectra presented here are less than 3% of the overall power spectrum amplitude and hence the differences observed at high  $\ell$  do not have a much effect.



**Fig. 4.** Comparison between global texture (black dashed) and semilocal (blue dotted) string power spectra and the AH field theory strings (red solid), normalized to unity at  $\ell = 10$ . As expected, the SL spectrum lies in between the TX and the AH spectra. The AH spectrum was recomputed for the *Planck* cosmological model with sources from Bevis et al. (2010), and the SL and TX spectra were taken from Urrestilla et al. (2008).

There are two USM-based models that we will use in this analysis which we believe span the realistic possibilities—we note a more general approach marginalizing over three string parameters is proposed in Foreman et al. (2011) (see also recent work in Avgoustidis et al. 2012). The first USM model, which we will refer to as Nambu, is designed to model the observational consequences of simulations of cosmic string simulations performed in the Nambu-Goto approximation. In these simulations the scaling regime is different in the radiation and matter eras, with  $(\xi, \langle v^2 \rangle^{1/2}/c, \beta)_{\text{rad}} = (0.13, 0.65, 1.9)$  and  $(\xi, \langle v^2 \rangle^{1/2}/c, \beta)_{\text{mat}} = (0.21, 0.60, 1.5)$  and the extrapolation between the two is modelled by using the velocity dependent one-scale model (Martins & Shellard 1996). In the second, which we will refer to as AH-mimic, we attempt to model the field theory simulations using the Abelian-Higgs model described below, with  $(\xi, \langle v^2 \rangle^{1/2}/c, \beta) = (0.3, 0.5, 1)$  independent of time.

The other approach that we will consider is to measure the UETC directly from a simulation of cosmic strings in the Abelian-Higgs model, which we will refer to as AH. The Abelian Higgs model involves a complex scalar field  $\phi$  and a gauge field  $A_\mu$  described earlier Eq. (4), for which the dimensionless coupling constants  $e$  and  $\lambda$  are chosen with  $\lambda = 2e^2$ , so that the characteristic scales of the magnetic and scalar energies are equal, (see Bevis et al. (2007b,a) for further details about the model). We then simulate the evolution of the fields on a grid, starting from random initial conditions designed to mimic a phase transition, followed by a brief period of diffusive evolution, to rapidly reach a scaling solution expected to be typical of the configuration found long after the phase transition. As the simulation is performed in comoving coordinates, the string width is effectively decreasing as time passes. To enlarge the dynamical range available, we partially compensate this shrinking with an artificial string fattening. We perform runs for various

values of the fattening parameter to ensure that the results are not affected by it.

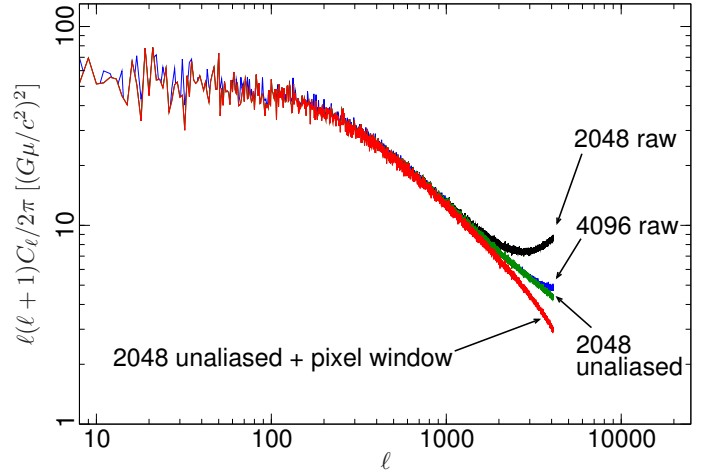
During the simulations, we compute the energy-momentum tensor at regular intervals and decompose it into scalar, vector and tensor parts. We store these components once scaling is reached, and compute the UETCs by correlating them with later values of the energy-momentum tensor. UETCs from several runs are averaged, diagonalized and then fed into a modified version of the CMBEASY Boltzmann code (Doran 2005) to compute the CMB power spectra (both temperature and polarization). The spectra used in this paper were derived from field-theory simulations on a  $1024^3$  grid and used the extrapolation to sub-string scales described in Bevis et al. (2010), which are expected to be accurate at the 10% level to  $\ell_{\max} \approx 4000$ .

In Figs. 3 and 4 we present the spectra we will use in subsequent analysis. The higher dashed black curve is the spectrum computed using the USM for the NAMBU model, and the smaller dashed blue and solid red curves the AH-mimic model and the AH model, respectively. We should note that when normalized to the amplitude of the observed CMB anisotropies on large-scales at  $\ell = 10$ , the three models give  $G\mu/c^2 = 1.17 \times 10^{-6}$ ,  $1.89 \times 10^{-6}$  and  $1.9 \times 10^{-6}$  for the NAMBU, AH-mimic and AH models, respectively. The reasons for differences between the spectra for these two approaches are discussed in Battye & Moss (2010). Briefly, the main reasons for the differences are twofold: First, the overall normalization, which is due to the NAMBU models having smaller values of  $\xi$ , more strings per horizon volume, and larger values of  $\beta$ , with each of the string segments being heavier, than the two AH models. Both these effects mean that a lower value of  $G\mu/c^2$  is required to achieve the same amplitude for the anisotropies. Secondly, the enhanced peak at small angular scales, which is caused by the value of  $\xi$  being smaller in the radiation era than in the matter era, meaning that there are more strings per horizon volume in the radiation era when the small-scale anisotropy is imprinted, and hence more anisotropy on those scales for a given  $G\mu/c^2$ .

The method used for the semilocal strings (denoted SL) and  $O(4)$  global texture (denoted TX) is fundamentally the same as for the AH model: we simulate the field theory on a discretized grid and compute the energy-momentum tensor at regular intervals. From these snapshots we derive the UETCs by correlating the scalar, vector and tensor parts at different times. The only difference is the field-theory action being used in the simulations. In Fig. 4 we present the spectra we used for the semilocal strings and global textures, taken from Urrestilla et al. (2008). These models are also shown with the AH cosmic string model for comparison.

### 2.5. Maps of CMB anisotropies from cosmic strings

In order to go further than the two-point correlation function, we have used numerical simulations of Nambu-Goto cosmic string evolution in an FLRW spacetime to generate various CMB synthetic maps. The use of simulations is crucial to produce realistic string configurations on our past light cone and have been the subject of various code development in the last twenty years (see Albrecht & Turok 1989; Bennett & Bouchet 1989, 1990; Allen & Shellard 1990; Vincent et al. 1998; Moore et al. 2001; Ringeval et al. 2007; Blanco-Pillado et al. 2011). Until recently, the underlying numerical challenges have limited the resolution of the full sky maps to an angular resolution of  $14'$  (corresponding to a HEALPix resolution of  $N_{\text{side}} = 256$ ) in Landriau & Shellard (2003, 2011) (see also early work in Allen et al. 1996). In order to extend the applicability of these maps to the



**Fig. 5.** Integrated Sachs-Wolfe angular power spectra extracted from the full sky cosmic string maps at different resolutions (labelled by  $N_{\text{side}}$ ), with or without applying the anti-aliasing procedure (see text). The anti-aliasing filtering gives back the correct power up to  $\ell_{\max} \lesssim 2N_{\text{side}}$ .

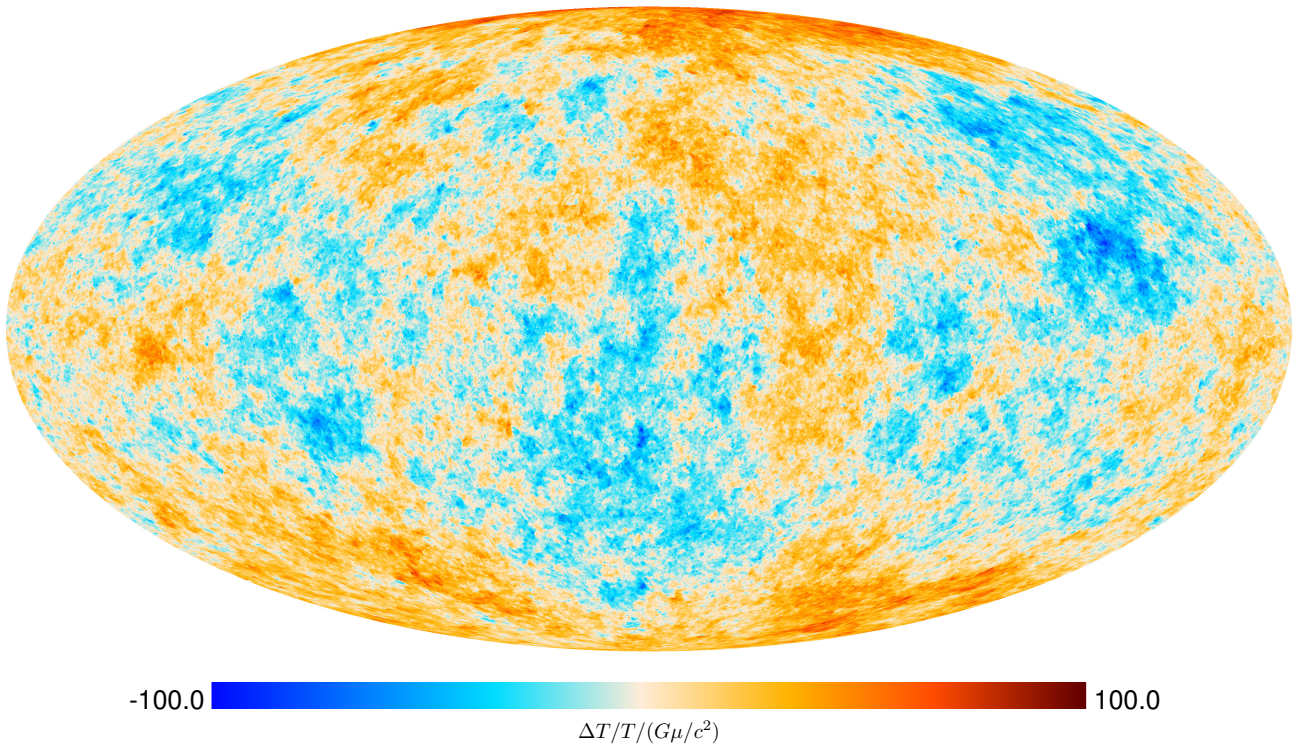
small scales probed by Planck, we have used the maps described in Ringeval & Bouchet (2012) that have an angular resolution of  $0.85'$  ( $N_{\text{side}} = 4096$ ). This map is obtained by considering the ISW contribution from (9), sourced by the Nambu-Goto stress tensor, and which can be recast into the form (Stebbins & Veeraraghavan 1995)

$$\frac{\Delta T}{T}(\hat{n}) = -\frac{4G\mu}{c^2} \int_{X \cap x_y} \left[ \dot{X} - \frac{(\hat{n} \cdot X') \cdot X'}{1 + \hat{n} \cdot \dot{X}} \right] \cdot \frac{X\hat{n} - X}{(X\hat{n} - X)^2} dl. \quad (11)$$

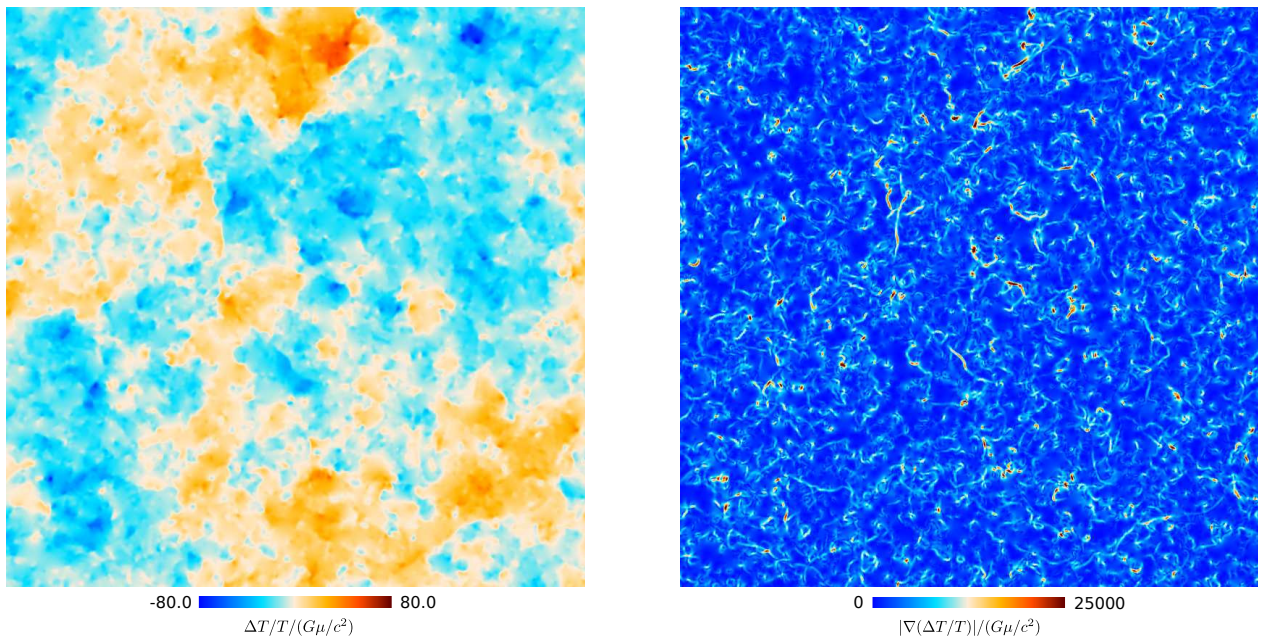
The integral is performed over all string position vectors  $X = \{X^i\}$  intercepting our past line cone (in the transverse temporal gauge). Primes and dots denote differentiation with respect to the spatial and time-like worldsheet coordinates  $\zeta^1$  and  $\zeta^0$  respectively, while  $dl$  is the invariant string length element. Taking the limit  $X\hat{n} \rightarrow X$  gives back the small angle and flat sky approximation used in Hindmarsh (1994); Bouchet et al. (1988); Fraisse et al. (2008). For generating the full sky map, Eq. (11) has been evaluated without any other approximation and required more than 3000 Nambu-Goto string simulations to fill the whole comoving volume between the observer and the last scattering surface. Details on the numerics can be found in Ringeval & Bouchet (2012).

This method therefore includes all string effects from the last scattering surface till today, but does not include the Doppler contributions induced by the strings into the plasma prior to recombination. As a result, our full sky map represents the ISW contribution from strings, which is dominant at large and small scales but underestimates the signal on intermediate length scales, as can be directly checked on the power spectrum (see Fig. 5). We therefore expect the string searches based on this map to be less constraining than those using the power spectrum, but certainly more robust as any line-like gravitating object should generate such a signal.

Calibration and training for the non-Gaussian searches of Sect. 4 have required the generation of new full sky and statistically independent cosmic string maps. The numerical challenges underlying the  $N_{\text{side}} = 4096$  map (Ringeval & Bouchet 2012) are such that it was numerically too expensive to create another one of the same kind. At this resolution, the computa-



**Fig. 6.** All sky Mollweide projection of the simulated cosmic strings CMB sky after convolution by a Gaussian beam of  $5'$  resolution. The color scale indicates the range of  $\Delta T/T/(G\mu/c^2)$  fluctuations.



**Fig. 7.** A  $20^\circ$  gnomonic projection patch extracted from the full sky map and zooming into string induced temperature steps (see Fig. 6). Applying the spherical gradient magnitude operator enhances the temperature steps, and thus the string locations, even more (right).

tions typically require 800 000 cpu-hours, so we have chosen to generate three new maps at a lower resolution of  $1.7'$ , i.e.,  $N_{\text{side}} = 2048$ . Unfortunately, at this lower resolution, the simulated string maps, hereafter referred to as raw maps, exhibit a strong aliasing at small scales that could have induced spurious systematics even after convolution with the *Planck* beam.

This aliasing concerns pixel-sized structures and comes from the method used to numerically evaluate Eq. (11). In order to spare computing time, the signal associated with each pixel is only computed at the centroid direction  $\hat{n}$ . This has the effect of including some extra power associated with string small-scale structure that is below the pixel angular size, thereby aliasing



the map. In order to address this problem, we have used semi-analytical methods to design an optimal anti-aliasing filter, both in harmonic space and in real space. As discussed in Fraisse et al. (2008); Bevis et al. (2010), the small scale angular temperature power spectrum slowly decays as a power law  $\ell^{-p}$  such that any deviations from this behaviour can only come from the aliasing. For each  $N_{\text{side}} = 2048$  raw map, we have performed a multi-parameter fit of the power spectrum, and of the one-point distribution function, to extract, and then removes, its small scale aliasing contribution. In order to validate the procedure, we have checked that the power spectrum of each of the filtered maps matches the one associated of the raw  $N_{\text{side}} = 4096$  map, the latter being also being affected but at half the scale. In Fig. 5, we have plotted the power spectra of one of the  $N_{\text{side}} = 2048$  maps before and after convolution with our anti-aliasing filter. As expected, it matches with the one extracted from the  $N_{\text{side}} = 4096$  map (here truncated at  $\ell = 4096$ ). Finally, in order to include the effects associated with the HEALPix pixelization scheme, the anti-aliased maps have been convolved with the HEALPix pixel window function before being used for further processing.

In total, this method has provided four theoretical full sky string maps that have been used in the string searches we will discuss in Sect. 4. As an illustration, we have represented in Fig. 6, one of the filtered string map after convolution by a Gaussian beam of  $\text{FWHM} = 5'$ . The color scale traces the relative temperature anisotropies  $\Delta T/T$ , divided by the string tension  $G\mu/c^2$ . The anisotropy patterns may look Gaussian at first because most of the string signatures show up on the smallest length scales. In Fig. 7, we have plotted a gnomonic projection representing a field of view of  $20^\circ$ , in which the temperature steps are now clearly apparent. The right panel of Fig. 7 represents the magnitude of the spherical gradient, which enhances the steps.

Finally, in order to provide a much larger statistical sample beyond only four string realisations, we have also produced a collection of 1000 small angle patches ( $7.2'$ ) of the CMB sky derived in the flat sky approximation (Stebbins 1988; Hindmarsh 1994; Stebbins & Veeraraghavan 1995; Bouchet et al. 1988; Fraisse et al. 2008). Although the large-scale correlations are lost, these maps have been shown to accurately reproduce various analytically expected non-Gaussian string effects such as the one-point and higher  $n$ -points functions by Takahashi et al. (2009); Hindmarsh et al. (2009, 2010); Regan & Shellard (2010); Yamauchi et al. (2010b,a); Ringeval (2010).

### 3. Power spectrum constraints on cosmic strings and other topological defects

In order to compute constraints on cosmic string scenarios we just add the angular power spectrum to that for an simple adiabatic model—which assumes that they are uncorrelated— with the fraction of the spectrum contributed by cosmic strings being  $f_{10}$  at  $\ell = 10$ . This parameter is then added as an extra parameter to the standard six parameter fit using COSMOMC and the *Planck* likelihood described in Planck Collaboration XV (2013). We use a Flat  $\Lambda$ CDM cosmology defined through the physical densities of baryons,  $\Omega_b h^2$ , and cold dark matter,  $\Omega_c h^2$ , the acoustic scale,  $\theta_{MC}$ , the amplitude,  $A_s$  and spectral index,  $n_s$  of density fluctuations and the optical depth to reionization  $\tau$ . The Hubble constant is a derived parameter and is given by  $H_0 = 100 h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . We use the same priors on the cosmological and nuisance parameters as are used in Planck Collaboration XVI (2013) and use *WMAP* polarization data to help fix  $\tau$ . In addition to just using the *Planck* data, we have also

**Table 2.** 95% upper limits on the constrained parameter  $f_{10}$  and the derived parameter  $G\mu/c^2$  for the five defect models discussed in the text. We present limits using *Planck* and polarization information from *WMAP* (*Planck* + WP), and from also including high  $\ell$  CMB information from ACT and SPT (*Planck* + WP + highL).

Defect type	<i>Planck</i> +WP		<i>Planck</i> +WP+highL	
	$f_{10}$	$G\mu/c^2$	$f_{10}$	$G\mu/c^2$
NAMBU	0.015	$1.5 \times 10^{-7}$	0.010	$1.3 \times 10^{-7}$
AH-mimic	0.033	$3.6 \times 10^{-7}$	0.034	$3.7 \times 10^{-7}$
AH	0.028	$3.2 \times 10^{-7}$	0.024	$3.0 \times 10^{-7}$
SL	0.043	$11.0 \times 10^{-7}$	0.041	$10.7 \times 10^{-7}$
TX	0.055	$10.6 \times 10^{-7}$	0.054	$10.5 \times 10^{-7}$

added high- $\ell$  CMB data from SPT and ACT to obtain stronger constraint (Sievers et al. 2013; Hou et al. 2012).

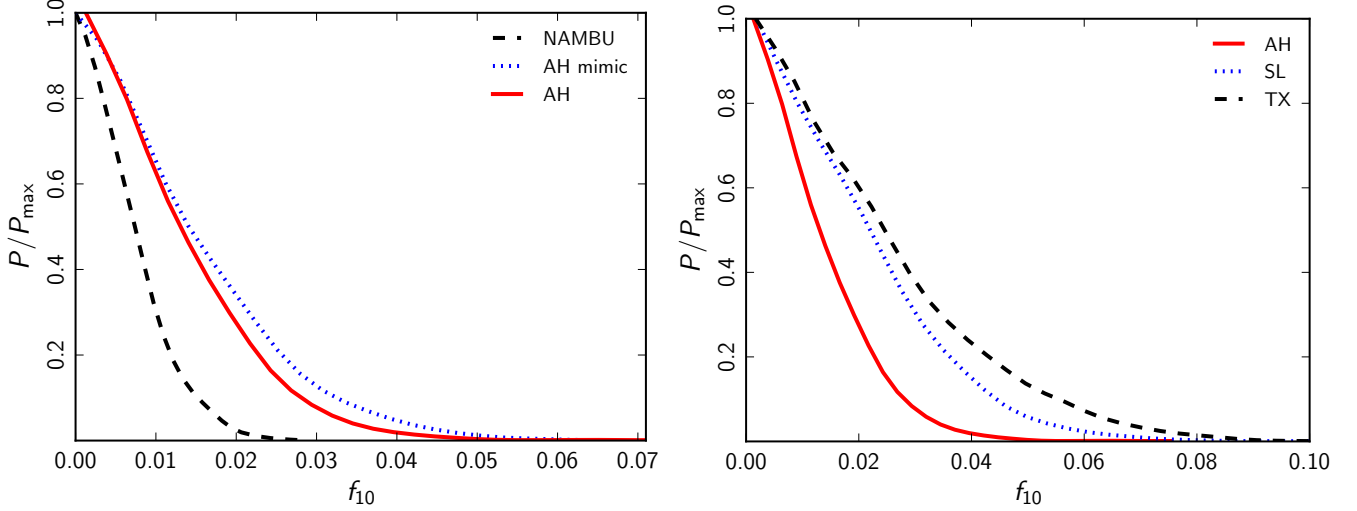
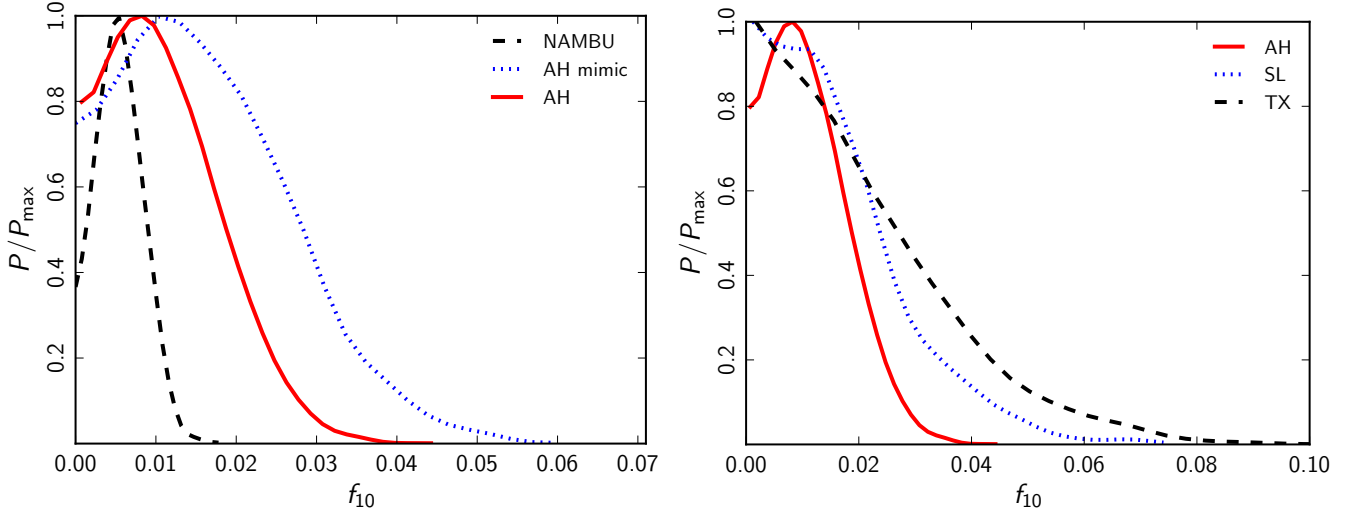
For the USM-based models we use the approach used in Battye et al. (2006) and Battye & Moss (2010). We find that the constraints on the standard six parameters are not significantly affected by the inclusion of the extra string parameter and that there are no significant correlations with other parameters (see Table 3). For the case of *Planck* data only and using the NAMBU model we find that  $G\mu/c^2 < 1.5 \times 10^{-7}$  and  $f_{10} < 0.015$ , whereas for the AH-mimic model we find that  $G\mu/c^2 < 3.6 \times 10^{-7}$  and  $f_{10} < 0.033$ , with all the upper limits being at 95% confidence level. The 1D marginalized likelihoods for  $f_{10}$  are presented in the upper panels of Fig. 8. The differences between the upper limits for the NAMBU and AH-mimic models is compatible with those seen previously using *WMAP* 7-year and SDSS data (Battye & Moss 2010). The upper limits from this version of the *Planck* likelihood are better than those computed from *WMAP*7+SPT (Dvorkin et al. 2011) and *WMAP*7+ACT (Dunkley et al. 2011) and are significantly better than those from *WMAP*7+SDSS (Battye & Moss 2010). Based on the *Planck* “Blue Book” values for noise levels we predicted (Battye et al. 2008) a limit of  $G\mu/c^2 < 6 \times 10^{-8}$ , while the present limit is about a factor of two worse than this. The main reason for this is that the projected limit ignored the need for nuisance parameters to model high  $\ell$  foregrounds and that not all the frequency channels have been used. The corresponding limits for the AH model are  $f_{10} < 0.028$  and  $G\mu/c^2 < 3.2 \times 10^{-7}$ .

There is now very little degeneracy between the  $f_{10}$  and  $n_s$  parameters, something that was not the case for *WMAP* alone (Battye et al. 2006; Bevis et al. 2008; Urrestilla et al. 2011). This has implication for supersymmetric hybrid inflation models as discussed in Battye et al. (2010) that typically require  $n_s > 0.98$ . The simplest versions of these models appear to be ruled out. The strongest correlation using the NAMBU and AH *mimic* models is between  $f_{10}$  and  $\Omega_b h^2$  as illustrated in Fig. 10. In addition, we find agreement with Lizarraga et al. (2012), that there are significant correlations between the amount of strings  $f_{10}$  in the AH model and the number of relativistic degrees of freedom  $N_{\text{eff}}$  as well as between  $f_{10}$  and the primordial helium abundance  $Y_{\text{He}}$ . We leave a detailed study of these correlations to later work.

In Fig. 8 we also present the 1D marginalized likelihoods for the texture and semilocal string models (compared to the AH field theory strings). The resulting constraints on the  $f_{10}$  parameter are given in Table 2 as well. For the conversion into constraints on  $G\mu/c^2$  we have that for semilocal strings

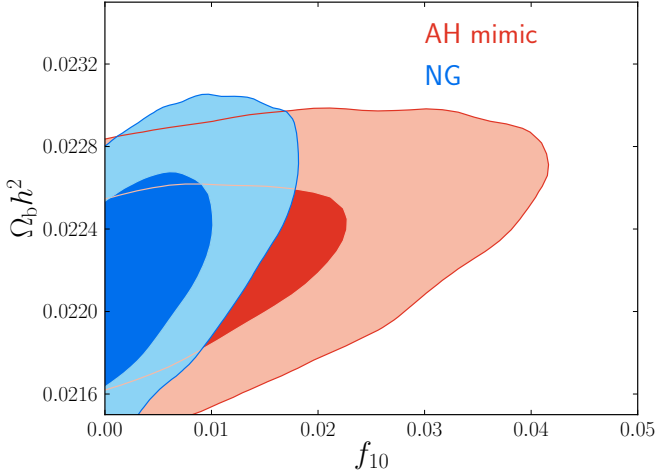
**Table 3.** Constraints on the fitted cosmological parameters in the case of *Planck* alone. It is clear from this that the fitted parameters are not significantly affected by the inclusion of defects.

Parameter	NAMBU	AH mimic	AH	SL	TX
$\Omega_b h^2$	$0.0223 \pm 0.0003$	$0.0223 \pm 0.0003$	$0.0223 \pm 0.0003$	$0.0223 \pm 0.0003$	$0.0223 \pm 0.0003$
$\Omega_c h^2$	$0.119 \pm 0.003$	$0.119 \pm 0.003$	$0.119 \pm 0.003$	$0.119 \pm 0.003$	$0.119 \pm 0.003$
$\theta_{MC}$	$1.0415 \pm 0.0006$	$1.0415 \pm 0.0006$	$1.0415 \pm 0.0006$	$1.0415 \pm 0.0006$	$1.0415 \pm 0.0006$
$\tau$	$0.089 \pm 0.013$	$0.090 \pm 0.013$	$0.090 \pm 0.013$	$0.090 \pm 0.013$	$0.088 \pm 0.014$
$\log(10^{10} A_s)$	$3.080 \pm 0.027$	$3.080 \pm 0.026$	$3.081 \pm 0.025$	$3.081 \pm 0.025$	$3.078 \pm 0.028$
$n_s$	$0.961 \pm 0.007$	$0.963 \pm 0.008$	$0.963 \pm 0.008$	$0.964 \pm 0.007$	$0.965 \pm 0.008$
$H_0$	$68.4 \pm 1.3$	$68.3 \pm 1.2$	$68.3 \pm 1.3$	$68.2 \pm 1.2$	$68.3 \pm 1.2$
$G\mu/c^2$	$< 1.5 \times 10^{-7}$	$< 3.6 \times 10^{-7}$	$< 3.2 \times 10^{-7}$	$< 1.10 \times 10^{-6}$	$< 1.06 \times 10^{-6}$
$f_{10}$	$< 0.015$	$< 0.033$	$< 0.028$	$< 0.043$	$< 0.055$


**Fig. 8.** Marginalized constraints on  $f_{10}$  for topological defects from *Planck* data plus polarization from *WMAP* (*Planck*+WP). The left panel show constraints on cosmic strings, with NAMBU in black dashed, AH-mimic in blue dotted and AH in red solid. The right panel show the constraints on SL (blue dotted) and TX (black dashed) compared to AH (again solid red).

**Fig. 9.** Marginalized constraints on  $f_{10}$  for topological defects with high- $\ell$  CMB data from SPT and ACT added to the *Planck* + WP constraints data (compare with constraints shown in Fig. 8). The left panel show constraints on cosmic strings, with NAMBU in black dashed, AH-mimic in blue dotted and AH in red solid. The right panel show the constraints on SL (blue dotted) and TX (black dashed) compared to AH (solid red).

$G\mu_{10}/c^2 = 5.3 \times 10^{-6}$  and for global texture  $G\mu_{10}/c^2 = 4.5 \times 10^{-6}$ , cf Urrestilla et al. (2008). We notice that, as expected for a fixed

$G\mu$ , semilocal strings lead to significantly less anisotropies than cosmic strings (a factor of about 8 in the  $C_\ell$ ), and texture are sim-



**Fig. 10.** Marginalized likelihoods for  $f_{10}$  and in the  $f_{10}$ - $\Omega_b h^2$  plane for the NAMBU model in blue and the AH *mimic* model in red using *Planck* + WP. This is the strongest correlation with any of the standard cosmological parameters

ilar to the semilocal strings. We thus expect significantly weaker constraints on  $G\mu$  for the SL and TX models, especially since in addition the constraints on  $f_{10}$  for these models are weaker. Indeed we find a 95% limit of  $G\mu/c^2 < 1.10 \times 10^{-6}$  for semilocal strings and  $G\mu/c^2 < 1.06 \times 10^{-6}$  for global textures.

#### 4. Non-Gaussian searches for cosmic strings

Cosmic strings and other topological defects generically create non-Gaussian signatures in the cosmic microwave sky, counterparts of their inevitable impact on the CMB power spectrum. This is a critical test of differentiating defects from simple inflation, while offering the prospect of direct detection. Searches for these non-Gaussian defect signatures are important for two key reasons: on the one hand, constraints from the CMB power spectrum can be susceptible to degeneracies with cosmological parameters in the standard concordance model; on the other hand, any apparent defect detection in the power spectrum should have a well-defined prediction in higher-order correlators or other non-Gaussian signals, and vice versa. Non-Gaussian tests can also be used to distinguish cosmic defects from residual foregrounds or systematic contributions. Below we will present results from NG tests that seek strings in multipole space (bispectrum) and in real space (Minkowski functionals), as well as hybrid methods (wavelets).

##### 4.1. Foregrounds, systematics and validation

It is well-known that the microwave sky contains not only the CMB signal but also emission from different astrophysical contaminants. In particular, point source emission is expected to be a special cause of confusion for cosmic defects, notably those with high resolution signatures, such as cosmic strings. In addition, systematic effects may also be present in the maps at a certain level. Therefore, before claiming a cosmological origin of a given detection, alternative extrinsic sources should be investigated and discarded. This can be done by performing a number of consistency checks in the data, most of which are common to the other non-Gaussianity papers, where they are discussed in greater detail. Here, we provide a brief summary of the main issues.

Foreground-cleaned CMB maps are provided using four different component separation techniques (for further details, see Planck Collaboration XII 2013): SMICA (semi-blind approach); NILC (internal linear combination in needlet space); SEVEM (internal template fitting); and Commander/Ruler (C-R, parametric method). To demonstrate the robustness of a particular result, it should be replicated with at least two different cleaned CMB maps. The adoption of different masks that exclude different regions of the sky (ranging from more aggressive to more conservative) has been used to test the stability of non-Gaussian estimates. Another interesting test is the use of cleaned maps at different frequencies (for instance, those provided by the SEVEM foreground separation technique). A given detection should be consistent at all frequencies, since the behaviour of contaminants and systematic effects will, in general, vary with frequency. A further test is the study of noise maps constructed from the difference between two *Planck* maps (either at the same or at different frequencies) smoothed to the same resolution. These maps will not contain the CMB signal and, therefore, any NG detection should vanish on them. The opposite would indicate that the claimed result is due to foreground residuals or to the presence of systematic effects.

In order to validate the various non-Gaussian methods that are described below, we instituted a series of *Planck String Challenges*. These were blind tests employing post-recombination string simulation maps with an unknown  $G\mu/c^2$ , co-added to a Gaussian CMB map, together with standard masks and increasingly more realistic noise models. For calibrating the non-Gaussian tests, several different string simulations were also provided. In addition, 1000  $\Lambda$ CDM Gaussian maps, with simulated noise and using the same mask, were provided for analysis purposes, notably for determining the variance of different techniques. The aim has been to determine the sensitivity of the proposed non-Gaussian tests and to see if the  $G\mu/c^2$  in the string challenge map could be measured accurately. The results from these challenges were an important part of the validation for each of the methods described below. *Planck* simulation pipelines for each of the component separation methods were also used to estimate realistic foreground residuals and were used to validate non-Gaussian pipelines and to remove string signal bias.

##### 4.2. Cosmic string bispectrum

###### 4.2.1. Modal bispectrum methods

The CMB bispectrum is the three point correlator of the  $a_{\ell m}$  coefficients,  $B_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} = a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$ . For the purposes of a search for cosmic strings we assume the network cumulatively creates a statistically isotropic signal, that is, we can employ the angle-averaged reduced bispectrum  $b_{\ell_1 \ell_2 \ell_3}$ , defined by

$$b_{\ell_1 \ell_2 \ell_3} = \sum_{m_i} h_{\ell_1 \ell_2 \ell_3}^{-2} \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} B_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3}, \quad (12)$$

where  $h_{\ell_1 \ell_2 \ell_3}$  is a weakly scale-dependent geometrical factor and  $\mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3}$  is the well-known Gaunt integral over three  $Y_{\ell m}$  that can be expressed in terms of Wigner-3j symbols. The CMB bispectrum  $b_{\ell_1 \ell_2 \ell_3}$  is defined on a tetrahedral domain of multipole triples  $\{\ell_1 \ell_2 \ell_3\}$  satisfying both a triangle condition and  $\ell < \ell_{\max}$  set by the experimental resolution. When seeking the string bispectrum  $b_{\ell_1 \ell_2 \ell_3}^{\text{string}}$  in the *Planck* data, we employ the following es-

timator to find or limit its amplitude:

$$\mathcal{E} = \frac{1}{\tilde{N}^2} \sum_{\ell_i m_i} \frac{\mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} b_{\ell_1 \ell_2 \ell_3}^{\text{string}}}{\tilde{C}_{\ell_1} \tilde{C}_{\ell_2} \tilde{C}_{\ell_3}} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}, \quad (13)$$

where we assume a nearly diagonal covariance matrix  $C_{\ell_1 m_1, \ell_2 m_2} \approx C_\ell \delta_{\ell_1 \ell_2} \delta_{m_1 - m_2}$  and we modify  $C_\ell$  and  $b_{\ell_1 \ell_2 \ell_3}$  appropriately to incorporate instrument beam and noise effects, as well as a cut-sky. To simplify Eq. (13), we have ignored the ‘‘linear term’’ (which is included in the analysis). A much more extensive introduction to bispectrum estimation can be found in Planck Collaboration XXIV (2013).

A key step in observational searches for non-separable bispectra, such as those induced by cosmic strings (denoted by  $b_{\ell_1 \ell_2 \ell_3}^{\text{string}}$ ), is to expand it into separable modes (Fergusson & Shellard 2009; Fergusson et al. 2010a) taking the signal-to-noise-weighted form:

$$\frac{b_{\ell_1 \ell_2 \ell_3}^{\text{string}}}{\sqrt{C_{\ell_1} C_{\ell_2} C_{\ell_3}}} = \sum_n \beta_n^Q Q_n(\ell_1, \ell_2, \ell_3), \quad (14)$$

where the modes  $Q_n(\ell_1, \ell_2, \ell_3) = \frac{1}{6}[\bar{q}_p(\ell_1) \bar{q}_r(\ell_2) \bar{q}_s(\ell_3) + \text{perms.}]$  are constructed from symmetrized products (the  $n$  label a distance-ordering for the triples  $\{prs\}$ ). The product basis functions  $Q_n(\ell_1, \ell_2, \ell_3)$  are not in general orthogonal, so it is very useful to construct a related set of orthonormal mode functions  $R_n(\ell_1, \ell_2, \ell_3)$  such that  $\langle R_n, R_p \rangle = \delta_{np}$ . Substituting the separable mode expansion (14) reduces the estimator (13) to the simple form

$$\mathcal{E} = \frac{1}{\tilde{N}^2} \sum_n \beta_n^Q \beta_n^Q, \quad (15)$$

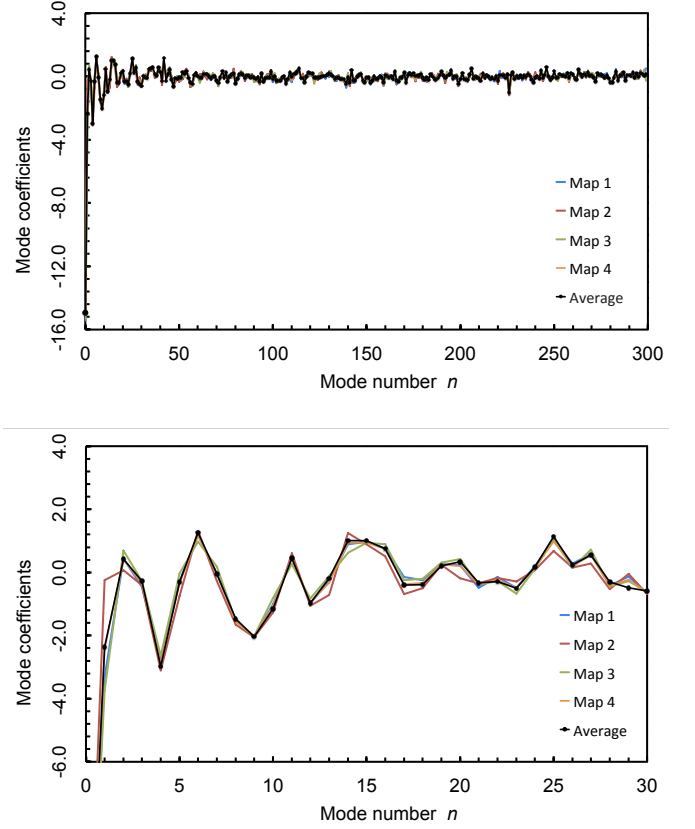
where the  $\beta_n^Q$  coefficients are found by integrating products of three *Planck* maps filtered using the basis function  $\alpha_n^Q$  (an efficient product with each map multiplied by the separable  $q_r(\ell)$ ). We can validate this estimator by using the modal methodology to create CMB map realisations for cosmic strings from the predicted  $\beta_n^Q$  with a given  $G\mu/c^2$  (see Fergusson et al. 2010a). It is easy to show that the expectation value for  $\beta_n^R$  for such an ensemble of maps in the orthogonal basis should be

$$\langle \beta_n^R \rangle = \alpha_n^R. \quad (16)$$

Alternatively, we can exploit this fact by reconstructing the  $\alpha_n^R$  from given CMB map realisations created directly from string simulations, an approach we will adopt here.

#### 4.2.2. Post-recombination string bispectrum

In order to estimate the string bispectrum at *Planck* resolution we employed the modal reconstruction method Eq. (16) on the post-recombination string challenge simulations described in Sect 2.5. These string maps include the accumulated line-like discontinuities induced by the string network on CMB photons propagating from the surface of last scattering to the present day. This work does not include recombination physics, that is, contributions from the surface of last scattering that will increase the string anisotropy signal substantially. As discussed previously, there are four full-sky maps of two different resolutions, which were provided for the purpose of calibrating *Planck* searches for cosmic strings (Ringeval & Bouchet 2012). For the modal analysis, we have adopted the hybrid polynomial basis augmented with a local shape mode (in total with  $n_{\text{max}} = 600$  modes), as



**Fig. 11.** Coefficients  $\alpha_n^R$  Eq. (14) for the hybrid Fourier mode expansion of the cosmic string bispectrum Eq. (14). The average value  $\langle \alpha_n^R \rangle$  (bold black line) is in remarkable agreement with all four string simulations as can be seen for  $n < 30$  (lower panel), with each exhibiting better than a 97% correlation overall. *Figure to be updated imminently.* ]

well as a hybrid Fourier basis ( $n_{\text{max}} = 300$ ), which are both described in Planck Collaboration XXIV (2013).

To extract the string bispectrum in a *Planck*-realistic context, we chose a fairly high non-Gaussian signal with  $G\mu/c^2 = 1 \times 10^{-6}$ . The normalized string maps were added to noise maps generated by the component separation pipelines of SMICA, NILC and SEVEM, creating twelve sets of 200 simulated string realisations. Each of these maps was then filtered using the modal estimator to find the  $\beta_n^R$  coefficients appropriate for each component-separation method. After averaging each set of modal coefficients  $\alpha_n^R = \langle \beta_n^R \rangle$  over the different (unmasked) noise realisations, we found remarkable consistency between the estimated  $\beta_n^R$  for the four string simulations as shown in Fig. 11 for the Fourier modes. Agreement was good across all the 300  $\alpha_n^R$  modes determined, as shown in detail for  $n=1-30$  (see the lower panel of Fig. 11).

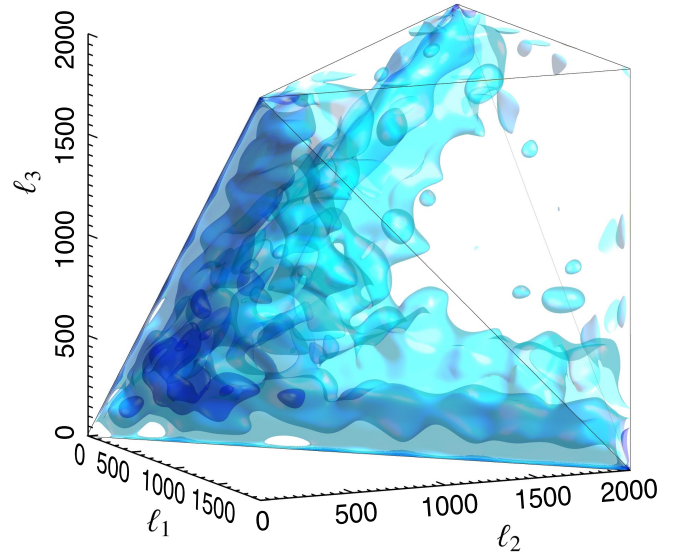
Quantitatively, the different string simulations produced bispectrum shapes that had above 97% correlations with each other (i.e.,  $\sum_n N^{-1} \alpha_n^{(1)} \alpha_n^{(2)} > 0.97$  for  $N^2 = \sum_n \alpha_n^{(1)2} \sum_n \alpha_n^{(2)2}$ ). The overall integrated bispectrum amplitudes was consistent to within 4%. Despite only four string map simulations, these are small errors relative to experimental and theoretical uncertainties. This robustness indicates that the overall string bispectrum signal at *Planck* resolution is a statistical summation of very many contributions from the millions of strings between the observer and the last scattering surface. To ensure the bispectrum

$C_l$  weighting was not significantly affected by the presence of the large string signal, we repeated the modal extraction procedure for  $G\mu/c^2 = 5 \times 10^{-7}$  (the string bispectrum amplitude reduced by a factor of 8). For the same string simulation, the shape correlations for different  $G\mu/c^2$  were 99.4% or above and the amplitude scaled as expected with  $(G\mu/c^2)^3$  to within 2%. The string bispectrum shown in Fig. 11 is well converged with random errors from the averaging procedure small relative to the actual signal. We conclude that, assuming the physics and numerical accuracy of the string simulations that are available, we have extracted a string bispectrum of sufficient accuracy for the present non-Gaussian analysis.

The overall three-dimensional reconstruction of the string bispectrum shape is shown in Fig. 12, normalized in the usual way to approximately illustrate the signal-to-noise expected (that is, removing an overall  $l^{-4}$  scaling, by dividing by the constant Sachs-Wolfe bispectrum shape). The first point to note is that the bispectrum is negative, reflecting the underlying string velocity correlations and curvature correlations that have created it; in the expanding universe, curved strings preferentially collapse, creating a negative temperature fluctuation towards the centre and a positive signal outside. In the overall spectrum, the  $n=0$  mode is dominant, but it is modulated by other modes providing further interesting structure that could be described as broad arms extending along each axis (see Fig. 11); although somewhat “squeezed”, the correlation with the local model is low. The string simulation power spectrum shown in Fig. 5 can be understood to be quantitatively modulating the string bispectrum away from the axes, with the signal slowly decaying beyond (say)  $l_1, l_2 > 500$  in the  $l_3$  direction.

The CMB bispectrum and trispectrum induced by the post-recombination gravitational effects of cosmic strings have been estimated analytically (Hindmarsh et al. 2009, 2010; Regan & Shellard 2010). With simplifying assumptions, these predicted that the constant mode would be dominant with a broad central “equilateral” peak, but not the substructure observed in Fig. 12. In terms of missing physics in this post-recombination string bispectrum, we expect the recombination signal to lie in the range  $\ell = 200 - 1000$  (shown in the full NAMBU power spectrum in Fig. 3) and to significantly enhance the the overall amplitude of the bispectrum (see also Landriau & Shellard 2011, where recombination physics is included).

The correlation of the post-recombination string bispectrum with standard primordial shapes is small, because it does not contain an oscillatory component from the transfer functions. The local, equilateral and orthogonal bispectrum models correlate with strings at about 6%, 11% and 12%, respectively. The late-time CMB ISW-lensing bispectrum  $b_{\ell_1 \ell_2 \ell_3}^{\text{ISW}}$  is also mainly negative, but it is much more squeezed/flattened and correlates at only about 11% with the present string bispectrum, so the predicted ISW signal should provide a small positive bias for string detection of about  $0.44 \sigma$  (see below). Diffuse point-source contamination is of greater concern because at  $\ell_{\text{max}} = 2000$  this has a 40% anti-correlation with strings (for the simple Poisson-distributed point source template with  $b_{\ell_1 \ell_2 \ell_3}^{\text{PS}} = \text{constant}$ ). This close relationship with point sources requires a joint analysis. Other foreground contamination must also be considered, as we shall discuss, and for this we rely on realistic simulated foreground residuals provided by the *Planck* component separation pipelines (see Planck Collaboration XII 2013).



**Fig. 12.** Modal reconstruction of the post-recombination string bispectrum Eq. (14) extracted from *Planck* resolution map simulations. This is a 3D view of the allowed tetrahedral set of multipoles  $(\ell_1, \ell_2, \ell_3)$  showing isosurfaces of the bispectrum density with darker blue for more negative values (it is normalized relative to the constant SW bispectrum).

#### 4.2.3. *Planck* string bispectrum results

Using the modal bispectrum estimator, we have searched for the string bispectrum in the *Planck* CMB maps obtained using the foreground-separation techniques SMICA, NILC and SEVEM. We note that the modal estimator has passed through the full validation suite of NG tests described in the Planck Collaboration XXIV (2013), where further details about the analysis can be found. In summary, we have used the standard U73 mask, which includes a Galactic cut and a conservative point source mask, together with “inpainting” as in the  $f_{\text{NL}}$  analysis (essentially apodizing the mask). Together with the foreground separated maps, realistic noise simulations were used to determine the estimator’s linear correction term and to determine the bispectrum variance, which was very nearly optimal. For calibration purposes, we always compare to the string model with tension  $G\mu/c^2 = 1 \times 10^{-6}$ , for which can choose to define and normalise a string bispectrum parameter  $f_{\text{NL}} = 1$  at  $\ell_{\text{max}} = 2000$ . The standard deviation  $\Delta f_{\text{NL}} = 0.2$  obtained from this *Planck* analysis would imply  $G\mu/c^2 = 1 \times 10^{-6}$  string detection at  $5\sigma$ . Instead, defining an  $F_{\text{NL}}$  normalized relative to the local model, we could expect to measure  $F_{\text{NL}} = 31.6 \pm 6.3$ . The strong scaling of the bispectrum amplitude on the string tension  $\propto (G\mu/c^2)^3$ , implies a given measurement yields  $G\mu/c^2 = (f_{\text{NL}})^{1/3} \times 10^{-6}$ .

The results of the string bispectrum estimation for each of the SMICA, NILC and SEVEM maps are shown in Table 4. Given that the *Planck* data exhibit significant detections of both ISW lensing and residual point source signals, we also quote their measurements. In the first instance, we have undertaken an independent analysis of each map, which showed no evidence for a cosmic string signal (all estimates were within  $1\sigma$ ). However, given the significant measurement of diffuse point sources and their strong anti-correlation with the string bispectrum, we have also undertaken a joint analysis of the *Planck* data (which in this case is the same as marginalising over the point source sig-

**Table 4.** Modal bispectrum analysis of foreground-separated SMICA, NILC and SEVEM maps showing  $f_{\text{NL}}$  from strings, ISW-lensing and diffuse point sources. Three values for  $f_{\text{NL}}$  are given from independent analysis, joint point source/string analysis after ISW-lensing subtraction, and joint analysis after both ISW-lensing and foreground residual subtraction. Resulting 95% confidence limits for  $G\mu/c^2$  are also given.

Method	Bispectrum signal type	Independent analysis $f_{\text{NL}}$	ISW subtract Joint $f_{\text{NL}}$	ISW/FG res. Joint $f_{\text{NL}}$
SMICA	Lensing ISW	$0.75 \pm 0.37$	–	–
	Diff. PS $\times 10^{28}$	$1.05 \pm 0.32$	$1.35 \pm 0.34$	$1.40 \pm 0.34$
	Cosmic strings	$0.19 \pm 0.20$	$0.50 \pm 0.21$	$0.37 \pm 0.21$
	$G\mu/c^2$ (95%)	$8.4 \times 10^{-7}$	$9.7 \times 10^{-7}$	$9.3 \times 10^{-7}$
NILC	Lensing ISW	$0.91 \pm 0.36$	–	–
	Diff. PS $\times 10^{28}$	$1.16 \pm 0.32$	$1.44 \pm 0.34$	$1.44 \pm 0.34$
	Cosmic strings	$0.13 \pm 0.20$	$0.46 \pm 0.21$	$0.23 \pm 0.21$
	$G\mu/c^2$ (95%)	$8.1 \times 10^{-7}$	$9.6 \times 10^{-7}$	$8.7 \times 10^{-7}$
SEVEM	Lensing ISW	$0.6 \pm 0.36$	–	–
	Diff. PS $\times 10^{28}$	$1.07 \pm 0.35$	$1.33 \pm 0.38$	–
	Cosmic strings	$0.10 \pm 0.20$	$0.38 \pm 0.21$	–
	$G\mu/c^2$ (95%)	$7.9 \times 10^{-7}$	$9.3 \times 10^{-7}$	–

nal). Before doing so, we subtract the expected ISW lensing signal that provides a  $f_{\text{NL}} = 0.09$  string bias. The third column in Table 4 shows that marginalising point sources enhances the string signal up to the  $2\sigma$  level for all component separation methods; essentially the constant mode becomes more strongly negative once the measured point sources are removed.

Other foregrounds, such as dust emission, could potentially produce spurious string signals if not subtracted properly. Foreground contamination has been extensively simulated within the *Planck* pipeline and foreground residual maps are provided by each component separation team. We have analysed the residual maps provided by both SMICA and NILC to seek evidence of correlations with the string bispectrum. The SMICA combined-residual map, without point sources and analysed with realistic noise, produces a string bias of  $f_{\text{NL}} = 0.23$ , which after ISW subtraction becomes  $f_{\text{NL}} = 0.14$  (relative to a variance  $\Delta f_{\text{NL}} = 0.20$ ). After both ISW and foreground residual subtraction, a joint analysis with point sources yields a SMICA string signal  $f_{\text{NL}} = 0.37 \pm 0.21$ . The apparent string bias in NILC from residual foregrounds was even higher  $f_{\text{NL}} = 0.22$  (after ISW subtraction), meaning a joint analysis obtained  $f_{\text{NL}} = 0.23 \pm 0.21$  (see Table 4).

We conclude, given our present understanding of point sources and foregrounds, that there does not appear to be significant evidence for a string bispectrum signal in the *Planck* nominal mission maps, so we infer the following post-recombination bispectrum constraint on strings (from  $f_{\text{NL}} = 0.30 \pm 0.21$ ):

$$G\mu/c^2 < 8.8 \times 10^{-7} \quad (95\% \text{ confidence}). \quad (17)$$

The susceptibility of the string bispectrum to point source and other foreground contamination deserves further investigation and will require improved characterisation of the diffuse point source bispectrum (beyond the simple Poisson model), as well as identification of other foreground residuals generating a small string bias.

The string bispectrum constraint Eq. (17) is a conservative upper limit on the string tension  $G\mu/c^2$  because we have not included recombination contributions. Although this constraint is

weaker than that from the power spectrum, it is an independent test for strings and the first quantitative string bispectrum limit to date. This should be considerably improved in future by inclusion of recombination physics and more precise foreground analysis. A comparison with the power spectrum amplitude indicates the string bispectrum should rise by  $(2)^{3/2}$ , which, together with the full mission data, would see the sensitivity improve by a factor of two (allowing constraints around  $G\mu/c^2 < 4 \times 10^{-7}$ ). We note that the bispectrum is not the optimal non-Gaussian test for strings, because the string signal is somewhat suppressed by symmetry (the bispectrum cancels for straight strings). This fact motivates further study of the trispectrum, for which the *Planck* sensitivity is forecast to be  $\Delta G\mu/c^2 \approx 1 \times 10^{-7}$  (Fergusson et al. 2010b), as well as joint analysis of polyspectra.

#### 4.3. Steerable wavelet searches for cosmic strings

Wavelets offer a powerful signal analysis tool due to their ability to localise signal content in scale (cf. frequency) and position simultaneously. Consequently, wavelets are well-suited for detecting potential CMB temperature contributions due to cosmic strings, which exhibit spatially localised signatures with distinct frequency content. Wavelets defined on the sphere are required to analyse full-sky *Planck* observations (see, for example, Freeden & Windheuser 1997; Wiaux et al. 2005; Sanz et al. 2006; McEwen et al. 2006; Starck et al. 2006; Marinucci et al. 2008; Wiaux et al. 2008).

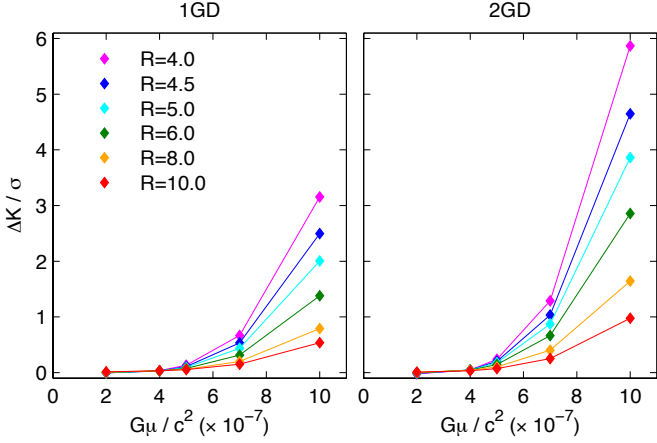
We perform an analysis using the steerable wavelets on the sphere constructed by Wiaux et al. (2005). Here we exploit steerability to dynamically adapt the orientations analysed to the underlying data, performing frequentist hypothesis testing. We apply the first (1GD) and second (2GD) Gaussian derivative steerable wavelets, defined on the sphere through a stereographic projection, in order to search for cosmic strings in the *Planck* data. A steerable wavelet is a directional filter whose rotation by  $\chi \in [0, 2\pi)$  about itself can be expressed in terms of a finite linear combination of non-rotated basis filters. Thus, the analysis of a signal with a given steerable wavelet  $\Psi$  naturally identifies a set of wavelet coefficients,  $W_{\Psi}(\omega_0, \chi, R)$ , which describe the local features of the signal at each position  $\omega_0$  on the sphere, for each orientation  $\chi$  and for each physical scale  $R$ . Several local morphological properties can be defined in terms of the wavelet coefficients (Wiaux et al. 2008), including the signed-intensity,

$$I(\omega_0, R) \equiv W_{\Psi}(\omega_0, \chi_0, R). \quad (18)$$

This quantity represents the value of the wavelet coefficient at the local orientation  $\chi_0(\omega_0, R)$  that maximizes the absolute value of the wavelet coefficient itself.

The presence of a cosmic string signal in the CMB is expected to leave a non-Gaussian signature that induces a modification in the distribution of  $I(\omega_0, R)$  with respect to the lensed Gaussian case. We calibrated the dependence of these signatures on the string tension using four simulations of the cosmic string contribution (Ringeval & Bouchet 2012) combined with a large set of lensed Gaussian CMB realizations, along with a realistic description of the *Planck* instrumental properties (refer to Planck Collaboration XII (2013)).

A wide range of string tension values were explored,  $G\mu/c^2 \in [2.0 \times 10^{-7}, 1.0 \times 10^{-6}]$ , considering several wavelet scales,  $R = [4.0, 4.5, 5.0, 6.0, 8.0, 10.0]$  arcmin. We choose the wavelet scale range as a trade off between the signal-to-noise ratio of the string contribution and the small scale foreground contamination. In fact, the wavelet for the smallest scale considered in this analysis peaks at  $\ell = 1300$ , while extending at



**Fig. 13.** Deviation of the kurtosis of the signed-intensity as a function of  $G\mu/c^2$ , normalized to the standard deviation of CMB and noise simulations:  $\Delta K/\sigma = (K(R, G\mu/c^2) - K(R, G\mu/c^2 = 0))/\sigma$ . The left panel shows results for the 1GD wavelet and the right shows the 2GD wavelet. Each curve corresponds to a wavelet scale,  $R$  (arcmin), included in the analysis. The final sensitivity of the method is determined by combining the two wavelets and all the scales.

higher multipoles with a broad distribution. We use maps at an HEALPix resolution of  $N_{\text{side}} = 2048$ , including multipoles till  $\ell_{\text{max}} = 2500$ . We analyse the simulations with the same U73 mask on the *Planck* CMB map (refer to Planck Collaboration XII 2013), which masks both diffuse and compact foregrounds, leaving 73% of the sky remaining for further analysis (refer to discussion in Sect. 4.2.3).

The string non-Gaussian signatures are characterized in terms of the kurtosis of the signed-intensity  $I(\omega_0, R)$  in Eq. (18) at the different scales  $R$  and for both the 1GD and 2GD wavelets. The averaged results from the non-Gaussian simulations were used to model the distribution of the kurtosis as functions of  $G\mu/c^2$ , i.e.,  $K(R, G\mu/c^2)$ . Other statistics, such as the skewness and the Higher-Criticism, have also been explored. We found that the kurtosis sensitivity to the string tension is higher than the alternative measures. In Fig. 13, we show the difference between the average kurtosis at several  $G\mu/c^2$  values and the average kurtosis for  $G\mu/c^2 = 0$ , normalized to the standard deviation of the simulations. On the given range of scales, the 2GD wavelet appears to be more sensitive to the string signal. The final sensitivity of the method in recovering the string tension from simulated data is established through a goodness-of-fit test, performed jointly on the two wavelets for all the scales, and taking into account the correlations by means of a covariance matrix estimated from CMB and noise simulations. From the distribution of the  $G\mu/c^2$  values recovered from simulations the null hypothesis can be rejected at 95% CL for  $G\mu/c^2 > 7 \times 10^{-7}$ .

The procedure described above was applied to the SEVEM CMB cleaned map, observing no evidence of a string signal at any of the scales studied. This would imply a wavelet constraint on the string tension at 95% CL,  $G\mu/c^2 < 7 \times 10^{-7}$ , however, we note that the signed intensity kurtosis of the data is smaller than that of the model even for small values of  $G\mu/c^2$ , and it is not fully compatible with it. A similar behavior is observed for the skewness of  $I(\omega_0, R)$ . We evaluated the impact on the estimator of unresolved point sources, both from radio galaxies and sub-millimetre star-forming galaxies, using simulations of these astrophysical components processed through the compo-

nent separation pipelines (Planck Collaboration XII 2013). We found that at the smallest wavelet scale considered in this analysis,  $R = 4$  arcmin, these residual foregrounds induce a shift in the kurtosis,  $\Delta K/\sigma = 0.03$ , i.e., a bias that is negligible for the present analysis. This shift increases to more than  $\Delta K/\sigma = 0.3$  when extending the analysis to wavelet scales as small as  $R = 2.5$  arcmin. However, the shift induced by unresolved point sources increases the kurtosis of the signed intensity, so it could not reconcile the tension between the model and the data pointed out above. Validation tests performed on the String Challenge data set demonstrated that the sensitivity of the estimator can improve to  $\Delta G\mu/c^2 \approx 4 \times 10^{-7}$  for small values of  $G\mu/c^2$  when wavelet small scales are included. For this reason, we believe it could be a powerful tool in constraining strings, even though further investigation is needed to reliably modeling the string signal, the foreground contribution and the noise properties, in order to fully exploit the sensitivity of the *Planck* data.

Finally, we note that we have also endeavoured to study the simulated string maps using spherical wavelets, making an extension of previous work (Wiaux et al. 2008; Hammond et al. 2009) to compute the Bayesian posterior distribution of the string tension. Both the spherical and steerable wavelet methods offer good prospects for improved non-Gaussian string constraints from the *Planck* full mission data.

#### 4.4. Real space tests for cosmic strings

##### 4.4.1. Minkowski functionals method

Minkowski Functionals (MFs) describe morphological properties of the CMB field, and can be used as generic estimators of non-Gaussianities. They have long been considered to constrain cosmic string physics, for example on gradient temperature maps (see e.g., Gott et al. 1990). Indeed they have sensitivity to non-Gaussianity sourced by strings at all orders (i.e., including the kurtosis or trispectrum) and they could prove to be a powerful tool to constrain topological defects in general. For the sake of brevity and conciseness, precise definitions of MFs and analytic formulations are presented in Planck Collaboration XXIII (2013) and, here, we only review how MFs can be used to constrain the string energy density  $G\mu/c^2$ , following the method discussed in Ducout et al. (2012), for the local model  $f_{\text{NL}}^{\text{local}}$ .

We measure the four normalized<sup>2</sup> functionals  $v_k$  ( $k = 0, 3$ ) (respectively Area, Perimeter, Genus and  $N_{\text{cluster}}$ ), computed on  $n_{\text{th}} = 26$  thresholds  $v$ , between  $v_{\text{min}} = -3.5$  and  $v_{\text{max}} = +3.5$  in units of the standard deviation of the map. The curves are combined into one vector  $y$  (of size  $n = 104$ ).

The principle is then to compare measurement of MFs on *Planck* data  $\hat{y}$  to measurements on NG simulations including a string tension level  $G\mu/c^2$ , that is, we determine  $\hat{y}(G\mu/c^2)$  numerically. These simulations reproduce the same systematics (noise, beam) and the same processing (filter, masks) as the data. Assuming that MFs are multi-variate Gaussians, we obtain a posterior distribution for  $G\mu/c^2$  through a  $\chi^2$  test :

$$P(G\mu/c^2 | \hat{y}) \propto \exp \left[ -\frac{\chi^2(\hat{y}, G\mu/c^2)}{2} \right], \quad (19)$$

<sup>2</sup> Raw Minkowski functionals  $V_k$  depend on the Gaussian part of fields through a normalization factor  $A_k$ , that is a function only of the power spectrum shape. We therefore normalize functionals  $v_k = V_k/A_k$  to focus on non-Gaussianity, see Planck Collaboration XXIII (2013) and references therein.

with

$$\chi^2(\hat{y}, G\mu/c^2) \equiv [\hat{y} - \bar{y}(G\mu/c^2)]^T C^{-1} [\hat{y} - \bar{y}(G\mu/c^2)]. \quad (20)$$

The covariance matrix  $C$  is computed from  $10^4$  Gaussian simulations<sup>3</sup> because, given the existing stringent constraints on cosmic strings, this should be accurate without biasing results. The cosmic string MF curve  $\bar{y}(G\mu/c^2)$  is calibrated on  $10^3$  realistic lensed *Planck* simulations, to which we have added a string component at a specified level. These simulations take into account the asymmetry of beams and the component separation process (FFP6 simulations, see Planck Collaboration ES (2013) for a detailed description). For the string component, we had at our disposal only two high resolution string simulations (Ringeval & Bouchet 2012), so our model is the averaged curve obtained from this combination of *Planck* and string simulations.

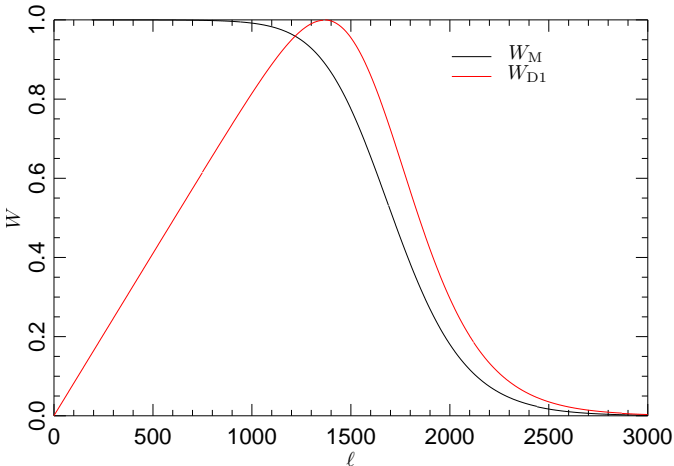
Due to the nonlinear dependence of MFs on  $G\mu/c^2$  and the small number of string simulations, the posterior distribution is quite complex and noisy. For this reason, we evaluated the posterior at  $n_{\text{NL}} = 51$  values of  $G\mu/c^2$ , between 0 and  $10 \times 10^{-7}$ , to obtain our *Planck* estimate for  $G\mu/c^2$ . This estimate is stable and has been validated in realistic conditions with the *Planck* String Challenges described above, and for which we found consistent results with the underlying (unknown)  $G\mu/c^2$ .

#### 4.4.2. Minkowski functionals results

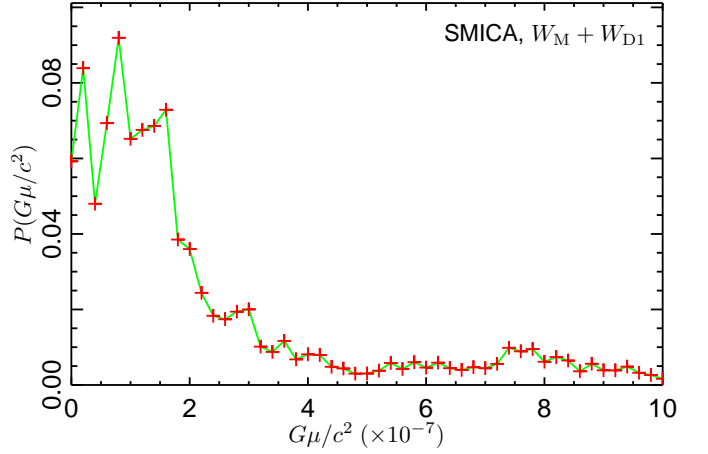
For the constraint on  $G\mu/c^2$ , we analysed the foregrounds separated SMICA map at  $N_{\text{side}} = 2048$  and  $\ell_{\text{max}} = 2000$ , using the U73 mask ( $f_{\text{sky}} = 73\%$  of the sky is unmasked). The smallest point sources holes were inpainted. We applied two specific Wiener filters to the map, designed to enhance the information from the map itself ( $W_M$ ) and from the gradients of the map ( $W_{D1} = \sqrt{\ell(\ell+1)}W_M$ ). The filters are shown in Fig. 14.

Additionally, we estimated the average impact of some residual foregrounds and secondaries (FG) on  $G\mu/c^2$ , using the linear properties of MFs and foregrounds models processed through the *Planck* simulation pipeline (FFP6 simulations, see Planck Collaboration ES (2013)). Uncorrelated (Poissonian) unresolved point sources (PS), Cosmic Infrared Background (CIB) and

<sup>3</sup> The Gaussian simulations endeavour to incorporate realistic noise from the *Planck* data, but only the effective isotropic beam of the component separation method.



**Fig. 14.** The two Wiener filters,  $W_M$  and  $W_{D1}$  used to constrain  $G\mu/c^2$  with Minkowski Functionals.



**Fig. 15.** Posterior distribution of the parameter  $G\mu/c^2$  obtained with Minkowski functionals. This estimate takes into account the lensing of the data, but not the effects of foreground residuals.

Sunyaev-Zeldovich cluster<sup>4</sup> (SZ) signals can be introduced as a simple additive bias  $\Delta\bar{y}^{\text{PS},\dots}$  on MF curves following:

$$\hat{y} = \hat{y}^{\text{FGsubtracted}} + \Delta\bar{y}^{\text{PS}} + \Delta\bar{y}^{\text{CIB}} + \Delta\bar{y}^{\text{SZ}}. \quad (21)$$

These biases are obtained as an average from 100 simulations, however, these do not comprehensively cover all the different component contributions in the actual *Planck* data.

We eventually obtain the posterior distribution of  $G\mu/c^2$ , and we integrate it to report confidence intervals. Results are summarized in Table 5, for raw data (lensing subtracted) and foreground subtracted data (PS, CIB and SZ subtracted). The discrepancy between the two filters can be explained because the derivative filter  $W_{D1}$  scans smaller scales than  $W_M$  so it is more easily biased by foreground residuals. Given the remaining foreground uncertainties, we take the most conservative MF constraint for the cosmic string contribution to the *Planck* data to be

$$G\mu/c^2 < 7.8 \times 10^{-7} \quad \text{at } 95\% \text{ C.L.}$$

The corresponding posterior is presented in Fig. 15.

Some caveats need to be mentioned that may influence these results. First, for the MF method itself, an important limitation is the small number of string simulations used to calibrate the estimator. The estimator appears to be mostly sensitive to low-redshift strings (infinite strings, with redshifts between 0 and 30), and this is affected by cosmic variance. As low-redshift string simulations are much faster to produce than complete simulations back to recombination, it should be possible to improve the robustness of the constraint using these relatively soon. Secondly, the impacts of the different point-source foreground components (here, PS, CIB and SZ) have been evaluated by averaging over 100 *Planck* simulation maps for which the modelling is only partial. The precise contributions of these different components needs to be investigated in more detail for the *Planck* data. Fortunately, using the linearity of MFs for these contributions it will be possible to jointly estimate these as their characterisation improves in future studies. Finally, the impact of Galactic residuals should also be assessed in further detail, especially for the filter  $W_{D1}$  that we have observed to be less robust against residuals than the  $W_M$  filter.

With advances in studying these experimental effects there are good prospects for the full mission data, the sensitivity

<sup>4</sup> The SZ signal does not include the SZ-lensing NG contribution.



**Table 5.** MFs constraints obtained on  $G\mu/c^2$ , at the 95% C.L. These results are obtained on the SMICA map with the U73 mask ( $f_{\text{sky}} = 0.73$ ). The “Raw map” result includes only the lensing contribution to the data, while the “Foreground subtracted map” includes the lensing, Poissonian point sources, CIB and SZ clusters contributions.

$G\mu/c^2$	$W_M$	$W_M + W_{\text{DI}}$
Raw map	$< 6.8 \times 10^{-7}$	$< 7.8 \times 10^{-7}$
FG subtracted map	$< 6.0 \times 10^{-7}$	$< 3.6 \times 10^{-7}$

of the MFs estimator should improve substantially, with simulations forecasting possible MF cosmic string constraints of  $G\mu/c^2 < 3 \times 10^{-7}$  at the 95% C.L. We note that further real space analysis of string map simulations has been undertaken with scaling indices of the pixel temperature distribution (see, e.g., Räth et al. 2011). Extensions calculating a set of anisotropic scaling indices along predefined directions appear to offer good prospects for string detection.

## 5. Conclusions

We have reviewed the signatures induced by cosmic strings in the CMB and searched for these in the *Planck* data, resulting in new more stringent constraints on the dimensionless string tension parameter  $G\mu/c^2$ . A pre-requisite for accurate constraints on cosmic strings is a quantitative understanding of both cosmological string network evolution and the effects they induce in the CMB. These are computationally demanding problems but progress has been made recently on several fronts: First, high resolution simulations of Nambu-Goto strings have yielded robust results for the scale-invariant properties of string networks on large scales, while there has been increasing convergence about small-scale structure and loops (for which the CMB predictions are less sensitive). Secondly, post-recombination gravitational effects of strings have been incorporated into full-sky *Planck* resolution CMB temperature maps that are important for validating non-Gaussian search methods. Finally, fast Boltzmann pipelines to calculate CMB power spectra induced by causal sources have been developed and tested at high resolution, whether for field theory simulations of strings or textures or for models of Nambu-Goto strings. Three-dimensional field theory simulations of vortex-strings at sufficient resolution should, in principle, converge towards the one-dimensional Nambu-Goto string simulations, but testing this is not numerically feasible at present. For this reason, we believe it is prudent to also include constraints on field theory strings (labelled  $G\mu_{\text{AH}}$ ), thus encompassing cosmic string models for which radiative effects are important at late times (such as global strings). We believe this brackets the important theoretical uncertainties that remain, that is, we have used the best available information to constrain both Nambu-Goto strings (NAMBU) and field theory strings (AH). This work has also obtained more stringent constraints on semilocal strings and global textures.

### 5.1. Cosmic string constraints and the CMB power spectrum

Accurate forecasts for the CMB power spectrum induced by cosmic strings are more difficult to compute than their equivalent

for simple adiabatic inflationary scenarios. It requires knowledge of the source, quantified by the unequal-time correlator (UETC) of the defect stress-energy tensor, from well before recombination to the present day, which is not computationally feasible. Fortunately, we can exploit scale-invariant string evolution to extrapolate the results of simulations with substantially smaller dynamic range. We use two methods to obtain predictions for the UETCs. An unconnected segment model (USM) is used to model the properties of an evolving string network, determining its density from an analytic one-scale evolution model, and the sources are coupled to the line-of-sight Boltzmann solver CMBACT. A second independent pipeline measures the UETCs directly from string simulations in the abelian-Higgs field theory, passing these to a modified form of the CMBEASY Boltzmann code. The resulting Nambu-Goto and abelian-Higgs string CMB power spectra are illustrated in Fig. 3. Free parameters in the USM model can be chosen to phenomenologically match the field theory UETCs (denoted the AH-mimic model) and the comparison is also shown in Fig. 3, validating the two independent pipelines.

To compute constraints on cosmic string scenarios we have added the angular power spectrum to that for a simple adiabatic model, assuming that they are uncorrelated, with the fraction of the spectrum contributed by cosmic strings being  $f_{10}$  at  $\ell = 10$ . This has been added to the standard 6 parameter fit using COSMOMC with flat priors. For the USM models we have obtained the constraint for the Nambu-Goto string model

$$G\mu/c^2 < 1.5 \times 10^{-7}, \quad f_{10} < 0.015, \quad (22)$$

while for the abelian-Higgs field theory model we find,

$$G\mu_{\text{AH}}/c^2 < 3.2 \times 10^{-7}, \quad f_{10} < 0.028. \quad (23)$$

The marginalized likelihoods for  $f_{10}$  and in the  $f_{10}$ - $\Omega_b h^2$  plane were presented in Fig. 10. With *Planck* nominal mission data these limits are already about a factor of two more stringent than the comparable *WMAP* 7-year string constraints and these *Planck* limits improve further with the inclusion of high- $\ell$  data.

### 5.2. Non-Gaussian searches for cosmic strings

Complementary searches for non-Gaussian signatures from cosmic strings were performed and we have reported constraints from the string bispectrum, steerable wavelets and Minkowski functionals. These methods participated in the *Planck* String Challenges and have undergone non-Gaussian validation tests.

The post-recombination string bispectrum has been reconstructed and calibrated from string-induced CMB maps using a modal estimator. String Challenge analysis with *Planck*-realistic noise simulations and mask indicated a nominal mission sensitivity of  $\Delta G\mu/c^2 \approx 5.8 \times 10^{-7}$ . Analysis of SMICA, NILC and SEVEM foreground-separated maps has yielded  $f_{\text{NL}} = 0.37 \pm 0.21$  for the string bispectrum shape, which translates into a bispectrum constraint on the string tension,  $G\mu/c^2 < 8.8 \times 10^{-7}$  (95% CL). Steerable wavelet methods have been calibrated on string simulation maps added to Gaussian CMB maps with realistic noise and masking, showing a sensitivity of up to  $\Delta G\mu/c^2 \approx 4 \times 10^{-7}$ . The string signal was shown to have greater impact on the kurtosis of the signed-intensity than on its skewness, and no evidence of a string signal was found in the *Planck* data. Minkowski functionals have been applied to string simulation maps in a *Planck*-realistic context, computing the four functionals—area, perimeter, genus and  $N_{\text{cluster}}$ —after application of Wiener filters. Using these distributions, a Bayesian estimator has been constructed to constrain the string tension.

Analysis of the SMICA foreground-cleaned maps yielded a MF constraint of  $G\mu/c^2 < 7.8 \times 10^{-7}$  (95 % CL).

Non-Gaussian searches for strings are complementary to the power spectrum analysis and yield constraints as low as  $G\mu/c^2 < 7.8 \times 10^{-7}$ , though we note the potential impact of foreground residuals in limiting current precision. These are conservative upper bounds because they only include post-recombination string contributions, unlike the string power spectrum analysis. Having such a broad suite of tools, ranging from multipole space, through wavelets, to real space detection methods, allows cross-validation and reinforces the conclusion that there is at present no evidence for cosmic strings in the *Planck* nominal mission data.

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