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MULTIDIMENSIONAL AND FUZZY MEASURES OF POVERTY AT REGIONAL LEVEL IN MOZAMBIQUE[1]

This study provides a step-by-step account of how fuzzy measures of non-monetary deprivation and also monetary poverty may be constructed at the regional level, based on the Mozambican Household Budget Survey 2008-09 (IOF08). To our knowledge, this is the first attempt to apply Fuzzy Set Theory to poverty measurement in Mozambique.

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The dataset we used is the most recent budget survey available for Mozambique and it is representative of the national, regional (North, Centre, South), provincial and urban/rural level. In order to construct a Fuzzy Set index of poverty, monetary as well as non-monetary indicators are considered, and two different measures of deprivation are subsequently constructed: the Fuzzy Monetary (FM) and Fuzzy Supplementary (FS).

Keywords: poverty, regional measures, fuzzy sets, Mozambique

1. Introduction

Mozambique is among the poorest countries in the world, with a per capita income level of approximately $428, ranking 197 out of 210 countries (World Bank, 2010). After the end of the civil war in 1992, Mozambique underwent a process of sustained growth and poverty reduction that led the country to be considered as a success story by the World Bank and international donors (World Bank, 2008).

Nevertheless, poverty levels remain very high and poor living conditions are widespread throughout the country. The process of poverty reduction has been deeply monitored and analysed by three official national assessments (MPF, 1998; MPF, 2004; MPD-DNEAP, 2010) and several other studies by both Mozambican and international analysts (Hanlon, 2007; Castel-Branco, 2010; Ossemane, 2010; Van den Boom, 2011).

What emerges from the three main household surveys conducted in the 1996-2008 period and from other field-specific surveys, is that Mozambican citizens substantially improved their situation with respect to some non-monetary dimensions: access to education and health services, household asset ownership, and quality of housing. On the other hand, monetary poverty remained fairly stable between 2002 and 2008: the Head Count Ratio slightly increased from a value of 54.1 % in 2002-03 to 54.7 % in 2008-09. However, it is important to note that this stabilisation followed a sharp fall from its previous (69 %) levels in 1996-97.

In the Third National Poverty Assessment (MPD-DNEAP, 2010) an analysis of both monetary and non-monetary poverty is outlined. The Mozambican Government and international donors invested considerably in reducing non-monetary poverty. In particular, education and health are considered key intervention areas, and progressively more people have been granted access to schools and health facilities in urban as well as rural areas (Chao and Kostermans, 2002; Government of Mozambique, 2005; Republic of Mozambique, 2006). Nonetheless, monetary poverty did not decrease between 2002 and 2008. At provincial level, the Southern provinces and some of the rural areas in the North experienced a sharp fall in their Head Count Ratio, while Central regions witnessed an increase. Nationwide, rural poverty increased from 55.3 % in 2002-03 to 56.9 % in 2008-09, whereas urban poverty decreased from 51.5 % to 49.6 % in the same period.

For non-monetary dimensions, each of the three dimensions considered (housing conditions, ownership of durable goods, and access to public goods and services) is separately compared in 2008-9 with the same dimension six years before but without computing a general composite welfare indicator. The results indicate that on average housing conditions improved between 2002 and 2008, though differences at sub-national level remain high nowadays. Ownership of durable goods also improved: the percentage of households owning a radio, a TV, a fridge, a mobile, a telephone, a car, and a bike or motorbike increased by 5.7 points. Turning to the access to public goods and services, it emerges that access to education peaked such that in 2008-09 more than 76 % of all children aged 6-13 were attending school which reflects a big jump if compared to a figure of 66.8 % in 2002-03. Moreover, geographic inequality in access to education decreased over time and access to health facilities improved. At the same time, other non-monetary dimensions of deprivation did not improve in a substantial way: access to safe water and chronic malnutrition, for example, remained more or less stable[1].

The present paper intends to give more information to policy makers about poverty situation in the country; this is done by estimating fuzzy multidimensional measures at both national and regional level. The work proceeds as follows: in Section 2 we illustrate the concept of multidimensional poverty, as well as the Fuzzy Set technique and its application to poverty estimation. In Section 3 we introduce the dataset that is used throughout the study, while in Section 4 we set out the empirical analysis and the resulting poverty estimates at the regional level. Section 5 concludes.

[1] Chronic malnutrition (stunting) is still suffered by 46.4 % of under-five children, which is among the highest percentages in the world (WHO, 2011).

2. Multidimensional poverty and fuzzy set theory

In order to understand poverty and social exclusion, it is necessary to consider deprivation simultaneously in different terms (i.e. as low income as well as different non-monetary aspects of deprivation). The need to adopt a multidimensional approach has been noted, among others, by Kolm (1977); Atkinson and Bourguignon (1982); Maasoumi (1986); Tsui (1995); Sen (1999). Moreover, the multidimensional nature of poverty is a widely recognised fact, not only by the international scientific community, but also by many official statistical agencies (e.g. Eurostat, Istat) as well as by international institutions (United Nations, World Bank).

In the present work we go beyond the conventional study of poverty based simply on the poor/non-poor dichotomy defined in relation to a chosen poverty line. Instead, poverty and multidimensional deprivation are treated as matters of degree based on the individual's position in the distribution of income and other aspects of living condition. State of deprivation is thus seen in the form of 'fuzzy sets' to which all members of the population belong yet to varying degrees. This fact brings with it more complete and realistic view of the phenomenon but also an increased complexity at both the conceptual and the analytical levels.

A number of authors have applied the concepts of fuzzy sets to the analysis of poverty and living conditions (Chiappero Martinetti, 1994; Vero and Werquin, 1997, inter alia). Our application is based on the specific methodology developed by Cerioli and Zani (1990), Cheli (1995), Cheli and Lemmi (1995), Cheli and Betti (1999), Betti et al. (2002), Betti et al. (2004), Betti et al. (2006), Betti and Verma (2008).

Under the so-called traditional approach, poverty is characterized by a simple dichotomization of the population into poor and non poor defined in relation to a chosen poverty line, z. This approach presents two main limitations: firstly, it is unidimensional (i.e. it refers to only one proxy of poverty, namely low income or consumption expenditure), and secondly it reduces the population to a simple dichotomy. However, poverty is a much more complex phenomenon that is not formed solely of its monetary dimension but must also take account non-monetary indicators of living condition. Moreover it is not an attribute that characterises an individual as being either present or absent, but is rather a difficult to define predicate that manifests itself in different shades and degrees.

The fuzzy approach considers poverty as a matter of degree rather than an attribute that is simply present or absent for individuals in the population. In this case, two additional aspects have to be introduced:

1. The choice of membership functions (m.f.), i.e. quantitative specifications of individuals' or households' degrees of poverty and deprivation;

2. The choice of rules for the manipulation of the resulting fuzzy sets.

The traditional approach can be seen as a special case of the fuzzy approach, where the membership function may be seen as $\mu_i^H = 1$ if $y_i < z$, $\mu_i^H = 0$ if $y_i \geq z$, where y_i is the income of individual i and z is the poverty line.

An early attempt to incorporate the concept of poverty as a matter of degree at methodological level was made by Cerioli and Zani (1990) who drew inspiration from the theory of Fuzzy Sets initiated by Zadeh (1965). Subsequently, Cheli and Lemmi (1995) proposed the so called Totally Fuzzy and Relative (*TFR*) approach in which the m.f. is defined as the distribution function $F(y_i)$ of income, normalised (linearly transformed) so as to equal 1 for the poorest and 0 for the richest person in the population.

2.1 Income poverty: the Fuzzy Monetary (FM) measure

In the present study we make use of a fuzzy monetary indicator as found in Betti et al. (2012). The proposed *FM* indicator is defined as a combination of the $(1 - F_{(M),i})$ indicator, namely the proportion of individuals richer than individual i (Cheli and Lemmi, 1995), and of the $(1 - L_{(M),i})$ indicator, namely the share of the total income received by all individuals richer than individual i (Betti and Verma, 2008). Formally:

$$\mu_i = FM_i = \left(1 - F_{(M),i}\right)^{\alpha-1}\left(1 - L_{(M),i}\right) = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma \mid y_\gamma > y_i}{\sum_{\gamma=2}^{n} w_\gamma \mid y_\gamma > y_1}\right)^{\alpha-1}\left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma y_\gamma \mid y_\gamma > y_i}{\sum_{\gamma=2}^{n} w_\gamma y_\gamma \mid y_\gamma > y_1}\right), \quad (1)$$

where y_γ is the income, $F_{(M),i}$ is the income distribution function, w_γ is the sample weight of individual of rank γ ($\gamma = 1,\ldots, n$) in the ascending income distribution, $L_{(M),i}$ represents the value of the Lorenz curve of income for individual i. The parameter α is estimated so that the overall *FM* indicator (which is calculated simply as the weighted mean of the individual FM_i), is equal to the Head Count Ratio computed for the official poverty line.

2.2 Non-monetary poverty: the Fuzzy Supplementary (FS) measure

In addition to the level of monetary income, the standard of living of households and individuals can be described by a host of indicators, such as housing conditions, possession of durable goods, health conditions, education, perception of hardship. To quantify and put together diverse indicators of deprivation several steps are necessary. In particular, decisions are required for assigning numerical values to the ordered categories, weighting the score to construct composite indicators, choosing their appropriate distributional form and scaling the resulting measures in a meaningful way.

First, from the large set which may be available, a selection has to be made of indicators which are substantively meaningful and worthwhile for the analysis of interest. Secondly, it is useful to group different indicators into statistical components (or dimensions) in order to reduce dimensionality. Whelan et al. (2001) suggest, as a first step in an analysis of life-style deprivation, to systematically examine the range of deprivation items to see whether the items cluster into distinct groups. Factor analysis can be used to identify such clusters of interrelated variables. To quantify and put together diverse indicators several steps are necessary.

1. Identification of items;
2. Transformation of the items into the [0, 1] interval;
3. Exploratory and confirmatory factor analysis;
4. Calculation of weights within each dimension (each group);
5. Calculation of scores for each dimension;
6. Calculation of an overall score and the parameter α;
7. Construction of the fuzzy deprivation measure in each dimension (and overall).

Aggregation over a group of items in a particular dimension h (h = 1, 2, ..., m) is given by a weighted mean taken over j items: $s_{hi} = \frac{\sum w_{hj} \cdot s_{hj,i}}{\sum w_{hj}}$ where w_{hj} is the weight of the j-th deprivation variable in the h-th dimension. An overall score for the i-th individual is calculated as the unweighed mean:

$$s_i = \frac{\sum_{h=1}^{m} s_{hi}}{m}. \qquad (2)$$

Then, we calculate the *FS* indicator for the i-th individual over all dimensions as:

$$FS_i = (1 - F_{(S),i})^{\alpha - 1} (1 - L_{(S),i}). \qquad (3)$$

As for the *FM* indicator, the estimates of α is determined so as to make the overall non-monetary deprivation rate (which is calculated simply as the weighted mean of the individual FS_i) numerically identical to the Head Count Ratio computed for the official poverty line. The parameter α estimated is then used to calculate the *FS* indicator for each dimension of deprivation separately. The *FS* indicator for the h-th deprivation dimension and for the i-th individual is defined as combination of the $(1 - F_{(S),hi})$ indicator and the $(1 - L_{(S),hi})$ indicator.

$$\mu_{hi} = FS_{hi} = \left(1 - F_{(S),hi}\right)^{\alpha-1} \left(1 - L_{(S),hi}\right) =$$

$$= \left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma} \mid s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^{n} w_{h\gamma} \mid s_{h\gamma} > s_{h1}} \right]^{\alpha-1} \left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma} s_{h\gamma} \mid s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^{n} w_{h\gamma} s_{h\gamma} \mid s_{h\gamma} > s_{h1}} \right],$$

$$h = 1, 2, ..., m; \quad i = 1, 2, ..., n; \quad \mu_{hn} = 0. \qquad (4)$$

The $(1 - F_{(S),hi})$ indicator for the i-th individual is the proportion of individuals who are less deprived, in the h-th dimension, than the individual concerned. $F_{(S),hi}$ is the value of the score distribution function evaluated for individual i in dimension h and $w_{h\gamma}$ is the sample weight of the i-th individual of rank γ in the ascending score distribution in the h-th dimension.

The $(1 - L_{(S),hi})$ indicator is the share of the total lack of deprivation score assigned to all individuals less deprived than the person concerned. $L_{(S),hi}$ is the value of the Lorenz curve of score in the h-th dimension for the i-th individual.

As for the Fuzzy Monetary and the Fuzzy Supplementary indicators, the overall index corresponding to each dimension FS_h is calculated simply as the weighted mean of the individual FS_{hi}. Here it is interesting to note that the overall ranking of the *FS* indicator cannot directly be obtained from the rankings in each dimension; however, the ranking obtained with FS_i is consistent with the ranking obtained from FS_{hi}[1].

3. Data

The dataset used in the study is the Mozambican Household Budget Survey 2008-09

[1] A possible alternative definition of the overall Fuzzy Supplementary indicator could be the simple average of the corresponding indicators. An advantage would be that the overall indicators would fulfill consistency properties with respect to decomposition (Chakravarty et al., 1998; among others). A drawback would be that the weighted mean of the individual would not be equal to the Fuzzy Monetary and the Head Count Ratio indicators.

(IOF08) (Inquerito aos Agregados Familiares sobre Orçamento Familiar 2008-09), a nationally representative household survey conducted by the National Institute of Statistics (INE). The IOF08 was conducted from August 2008 to September 2009. The survey has a stratified structure with three steps of selection: i) selection of the primary sampling units (PSUs), ii) selection of the enumeration areas[1] within the PSUs, and iii) selection of the households within the enumeration areas. Twenty-one strata were constructed, one for each urban/rural sample of the 11 provinces of Mozambique (the province of Maputo City does not have a rural area). The IOF08 has a sample size of 10,832 households and it is representative at the national, regional (North, Centre, South), provincial and urban/rural level. The survey includes information on general characteristics of the individuals and of the households, on daily, monthly and durable goods final consumption expenditures, own consumption, transfers and gifts. Supplementary information for the IOF08 can be found in (INE, 2010; MPD-DNEAP, 2010).

Concerning socioeconomic status, we use data on (real) per capita daily consumption, available from the IOF08. Such variable is used by the Government for official analyses of poverty which makes our results immediately comparable to existing ones. This measure of income also considers the inflation that occurred during the implementation of the surveys, the different values of the Metical — the Mozambican currency — in different periods of the year, and spatial differences in price levels among different provinces and rural/urban areas.[2]

In order to compute a measure for non-monetary poverty we use information on ownership of durable goods, housing quality, health status and education level.

3.1 Problems with the data

The IOF08 is a very rich and detailed dataset. It has been carefully designed and implemented, to the effect of providing reliable information and statistical results. Nonetheless, a few problems with the data were encountered while conducting analysis on multidimensional poverty. In particular, it was found that sampling weights were not calibrated at the household level following a non-response or other problems occurred in the surveying process. Moreover, such weights ranged from 54.6 to 93,452.2, and as a result of this, a few households with very high weights significantly influenced statistical results.

In terms of household real consumption — the variable used to assess socioeconomic status — we conformed to official analyses that divide it by the number of household members, and on the basis of such a variable we estimated poverty rates. However, this overlooks issues of intra-household allocation of resources and economies of scale, which might considerably matter when dealing with poverty estimates in a country whose average household size is approximately made up of six members. Indeed, the Head Count Ratio computed dividing household consumption per adult equivalent produces very different poverty estimates where the percentage of poor was 36.8 % vs. 54.69 %.

4. Empirical analysis and results

In this section we describe the steps involved in the measurement of multidimensional poverty in Mozambique at national, provincial and urban/rural level, as outlined in previous sections. This is followed by an analysis of the results for Mozambique's monetary poverty and the different dimensions of non-monetary deprivation.

As introduced in Section 3, the Fuzzy Monetary measures, *FM*, are based on a household's real consumption divided by its size. Real consumption is obtained by taking into account regional differences in price levels, inflation and seasonal fluctuations. In order to obtain *FM*, we need to take into account both the proportion of households richer than each particular household and the cumulative share of consumption such richer households receive. Finally, the resulting distribution is transformed such that its mean is equal to the Head Count Ratio: this ensures comparability between the two measures and the two approaches, namely the traditional and the multidimensional one.

As for the Fuzzy Supplementary measures, we use information about thirty-two basic items, as described in Section 3. The deprivation dimensions are initially determined using an exploratory factor analysis: this procedure permits to describe the variability among observed variables — our basic items — in terms of a lower number of unobserved, uncorrelated variables, which are called factors. In the exploratory factor analysis the observed variables are expressed as a linear combination of the underlying factors, without any a priori assumption about the factor structure.

The results of the exploratory factor analysis are then calibrated according to the literature and

[1] An enumeration area (EA) represents the area assigned to each enumerator for distributing questionnaires to households and it is the smallest building block of the geographical frame for the Mozambican Household Budget Surveys.

[2] This is the same methodology used in official analyses of poverty.

to the experience acquired during the fieldwork in Mozambique.

Finally, a confirmatory factor analysis was performed by imposing a priori assumptions on the underlying factor structure. This allowed us to test whether the proposed calibration of initial items into a lower number of dimensions made statistical sense.

After these preliminary steps, thirty-two basic indicators were grouped into six dimensions, roughly corresponding to: i) housing conditions; ii) more widespread and affordable durable goods; iii) less common, more expensive durable goods; iv) housing quality; v) income-related deprivation; vi) health and education.

For what concerns the aggregation of different indicators in each single dimension, a weighting procedure was carried out, as described in Section 2. Depending on the distribution of each indicator in the population and its correlation with other indicators in the same dimension, we constructed item-specific composite weights with equal value for all households in the population. The item-specific weights, W_j, are composed of two parts: Wa_j, which is an inverse function of the percentage of people deprived in item j, and Wb_j, an inverse function of the correlation between item j and all the other items in the same dimension. For each dimension we have that $W_j = Wa_j \times Wb_j$.

Intuitively, the first component of the weights, Wa_j, takes into account that if a high percentage of people possess j, then the few who do not possess j are very deprived; the second component, Wb_j, tries to achieve parsimony assigning a lower weight to items that are highly correlated in the same dimension (e.g. high-quality walls and high-quality roof in the 'housing conditions' dimension).[1]

The result is the identification of six different fuzzy supplementary measures, one for each dimension: *FS*1, *FS*2, *FS*3, *FS*4, *FS*5, *FS*6. Subsequently, we aggregate the different non-monetary dimensions into a single composite Fuzzy Supplementary poverty indicator, *FS*. This is done by assigning equal weights to each supplementary dimension, based on the assumption that all dimensions are equally important in determining supplementary deprivation. The resulting *FS* distribution is also scaled so that its mean is equal to the Head Count Ratio, as we did for the monetary poverty indicator, *FM*. The rescaling ensures that the traditional and the fuzzy indicators are comparable.

4.1 Poverty estimates at national level

As previously outlined, the overall Fuzzy Monetary (*FM*) and Fuzzy Supplementary (*FS*) dimensions are constructed such that their mean is equal to the official Head Count Ratio so they do not convey additional information to our analysis at national level (*Head Count Ratio* = *FM* = *FS* = 54.69 %). Hence, in this subsection we only focus on the values of the supplementary dimensions *FS*1–*FS*6. From Figure 1, it can be seen that the factor with highest level of deprivation is *FS*3 which corresponds to less common, expensive durable goods. Most Mozambicans do not possess any of the items included in this dimension and a level of deprivation of about 0.75 is thus reasonable. Conversely, the deprivation value for less expensive durable goods (*FS*2) is lower, showing that some durable goods — especially mobile phones and bikes — are becoming more common in the country.

The level of deprivation for housing conditions (*FS*1) is also very high (0.53), and reflects the fact that many households lack basic facilities in their dwellings. Even so, the proportion of households lacking decent household quality (*FS*4) is significantly lower (0.31). Income-related deprivation (*FS*5) appears to be relatively low: this result is probably influenced by the inclusion of a dummy for whether someone in the household had a job (formal or informal) or not. Since most of the households interviewed (about 98 %) had a member with a formal or informal job, the entire dimension was pushed towards low levels of deprivation (0.12). When this variable is eliminated from the *FS*5 dimension, the average deprivation raises significantly (0.64). This was then taken into account in the following analyses.

Finally, the result for health and education (*FS*6) shows that education and health conditions in Mozambique are improving. However, one needs to be warned that the relatively low average value of deprivation for this dimension (0.32) is likely to be affected by the low level of deprivation characteristic of chronic illnesses and ability to read and write. Indeed, the level of child malnutrition in Mozambique is still among the highest in the world (WHO, 2011).

4.2 Poverty estimates at provincial level

When fuzzy set poverty analysis is carried out at sub-national level then it becomes evident how the inclusion of multiple dimensions substantially increases the amount of available information.

[1] This weighting system has been officially adopted by Eurostat (2002) in the 2nd Social Report on Income, Poverty and Social Exclusion for comparative analysis. It has been widely adopted also in other fields; for instance Aassve et al. (2007) have studied the effects of marital disruption on economic well-being, while Betti et al. (2011) have constructed a fuzzy indicator of educational mismatch for university graduates.

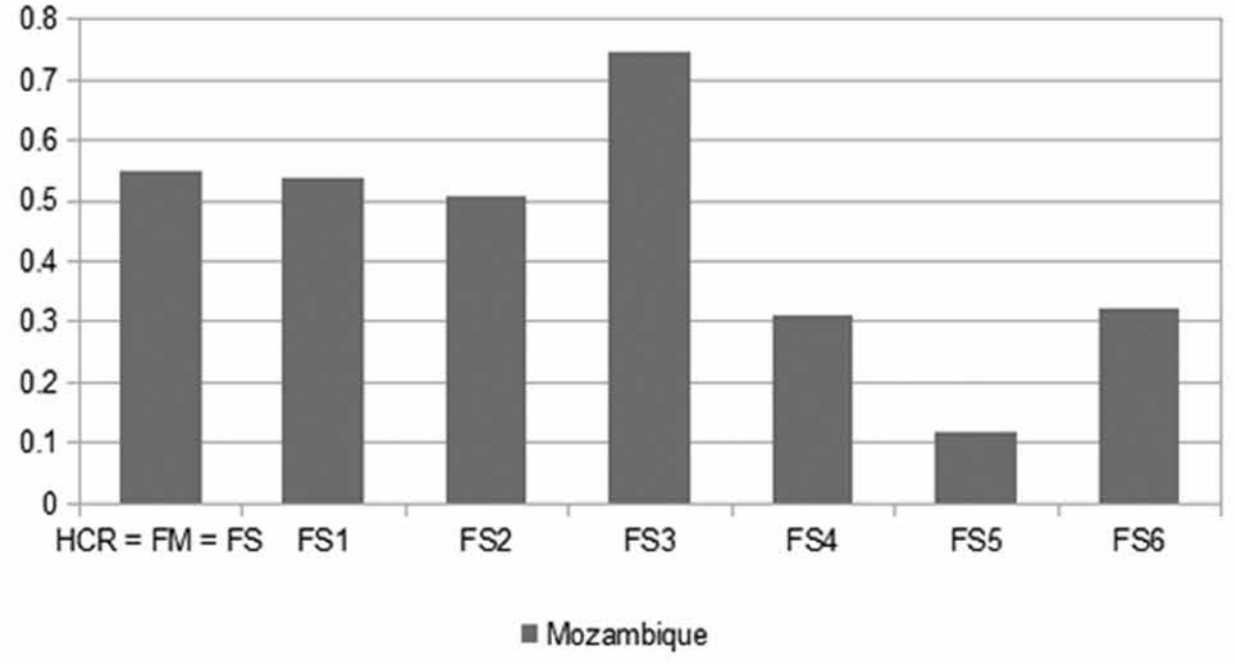


Fig. 1. *Deprivation by dimension, national level*

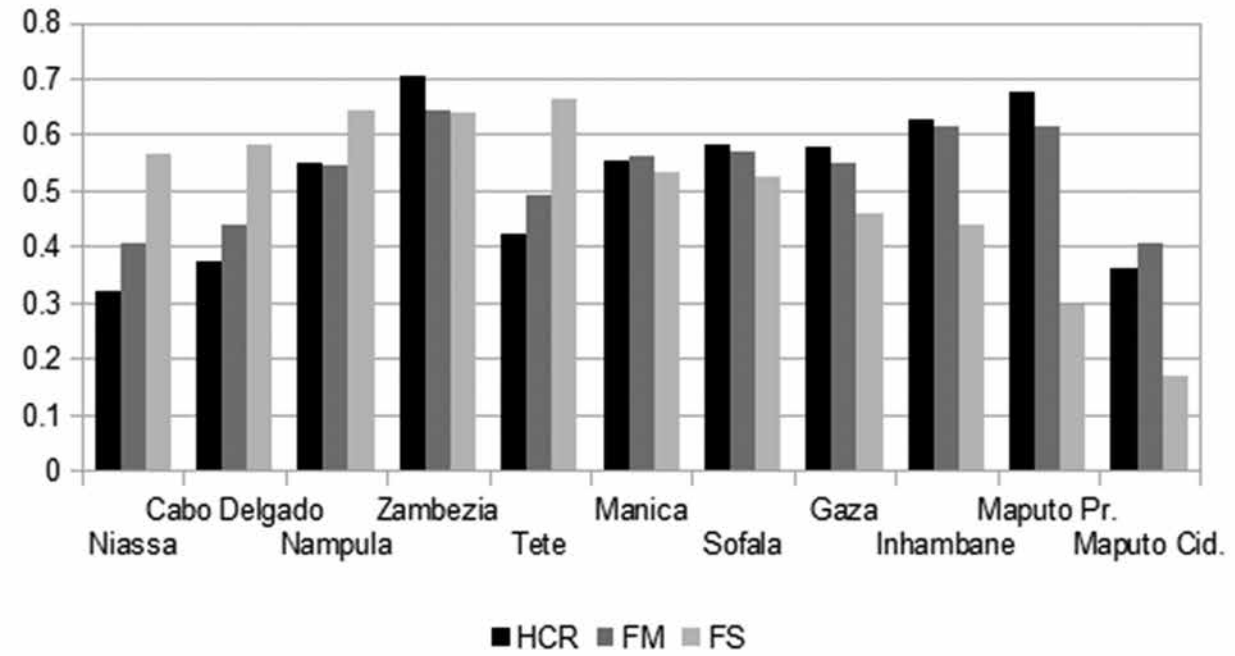


Fig. 2. *Head Count Ratio (HCR), Fuzzy Monetary (FM) and Fuzzy Supplementary (FS), by province*

Mozambique is divided into eleven provinces[1]. These territories are quite heterogeneous with regard to economic development, culture, ethnic and linguistic composition. Consequently, huge differences in poverty rates exist among different zones and provinces in Mozambique. Even though some insights emerged from the official Head Count reports, the multidimensional analysis of poverty we undertook using Fuzzy Set Theory allows us to highlight important characteristics that would otherwise go unnoticed in a traditional poverty assessment.

In particular, by looking at the Fuzzy Monetary (*FM*) and Fuzzy Supplementary (FS) statistics presented in Figure 2 it appears striking that some of the provinces with low rates of monetary poverty are also much more deprived in other dimensions, and the converse is also true. The Northern

[1] The eleven provinces of Mozambique are grouped into three bigger zones: the North, which includes the provinces of Niassa, Cabo Delgado and Nampula; the Centre, with the provinces of Zambezia, Tete, Manica and Sofala; the South, containing the provinces of Gaza, Inhambane, Maputo Province and Maputo City.

provinces (Niassa, Cabo Delgado, Nampula) and the Central province of Tete, all have much higher Fuzzy Supplementary (*FS*) averages with respect to their Fuzzy Monetary (*FM*) ones. The other Central provinces (Zambezia, Manica and Sofala) have similar statistics in both the *FM* and *FS* dimension, while the Southern provinces show *FS* averages that are lower than their respective *FM* averages.

The analysis of Fuzzy Supplementary dimensions indicates that the South is generally more developed than the Centre and the North, with Maputo City being much less deprived than all other provinces. These characteristics remained hidden using the standard poverty Head Count analysis. This is probably due to various causes: first, consumption is highly dependent on temporary and/or seasonal fluctuations — e.g. a bad harvest in 2008, — while other dimensions as those included in the computation of the Fuzzy Supplementary statistics are more robust to such changes. Indeed, buying an asset, a durable good or investing in education requires an evaluation of a household's economic status that is only partially related to the level of income/consumption in a given year. Moreover, a large part of the Mozambican population has consumption levels that are close to the poverty line, hence even small fluctuations can alter the poverty Head Count statistics in a substantial way. This is one of the main drawbacks of using a dichotomous index like the Head Count Ratio for the analysis of a complex phenomenon such as poverty. In fact, poverty Head Count analyses based on Mozambican Budget Surveys generally yielded strange or non-robust results, with strong fluctuations in the Head Count Ratio and re-ranking of poor and rich provinces (Van den Boom, 2011; pp. 7-8).

A deeper investigation into supplementary factors yields additional results. As for housing conditions (*FS*1), we can identify three distinct groups of provinces on the basis of their *FS*1 averages: the Central provinces (Zambezia, Tete, Manica and Sofala) and the province of Nampula are the most deprived in this dimension with an average of about 0.60 for Nampula, Manica and Sofala but roughly 0.70 for Zambezia and Tete. In the second group, with an average deprivation of about 0.40, we find two Northern provinces (Niassa and Cabo Delgado) and two Southern provinces (Gaza and Inhambane). Finally, the least deprived provinces are again Maputo Province and Maputo City, the latter with an average level of deprivation of 0.03.

In the *FS*2 dimension we put together some durable goods that are more widespread than others, like mobile phones, bikes and motorbikes, radios, watches and TVs. Indeed, most provinces show similar average levels of deprivation in this dimension, ranging within the 0.44–0.55 interval where Nampula is the most and Niassa and Manica the least deprived with 0.60, 0.36, and 0.33 scores respectively.

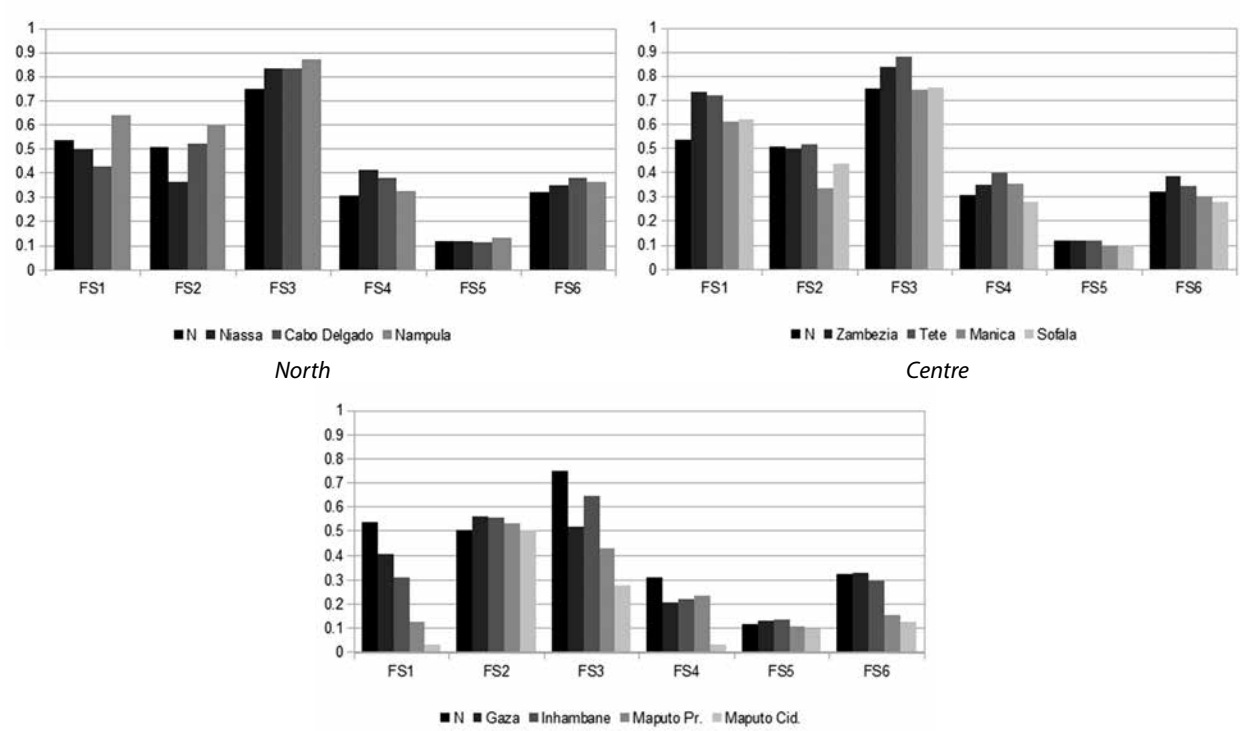
The *FS*3 dimension, instead, consists of those durable goods that are less affordable and thus less common among Mozambicans such as cars, fridges or freezers, irons, computers, printers, other tools, and sewing machines. As evidenced in the analysis at national level, this is the factor for which average levels of deprivation are highest. This is particularly true in the North and in the Centre, where five provinces (Niassa, Cabo Delgado, Nampula, Zambezia and Tete) have average values that exceed 0.80, in contrast to other Central provinces (Manica and Sofala) with values around 0.75 which perform a little better. Once again, the Southern provinces of Gaza, Maputo Province and Maputo City have much lower deprivation levels with scores 0.52, 0.43, and 0.28 respectively, confirming the finding that Southern provinces are less deprived than Northern and Central provinces in various dimensions.

As for access to safe water, energy sources for cooking, in-house lighting and the likes — included in the *FS*4 dimension, — we find that the average level of deprivation is relatively low. For Northern and Central provinces it ranges between 0.28 for Sofala and 0.41 for Niassa, while all Southern provinces perform comparatively better.

As presented in Subsection 4.1, the *FS*5 dimension (income-related deprivation) is the one for which average levels of deprivation are lowest. In this case, there are no noticeable differences between provinces. However, when the variable "formal or informal job" is taken out, then it emerges that there is a group of provinces including Manica, Sofala, Maputo, and Maputo City, with average deprivation values between 0.40 and 0.50 whilst all other provinces perform comparatively worse with values of around 0.65-0.75.

Finally, the last supplementary dimension (*FS*6) takes into account education measured as education level and ability to read and write and health measured as child malnutrition and chronic illnesses. In this case, Maputo Province and Maputo City record an average level of 0.12-0.15, while the estimated values for other provinces range between 0.28 for Sofala and 0.38 for Zambezia, which amounts to more than twice the level of deprivation of the two most Southern provinces.

What this subsection makes clear is that the analysis of dimensions other than consumption

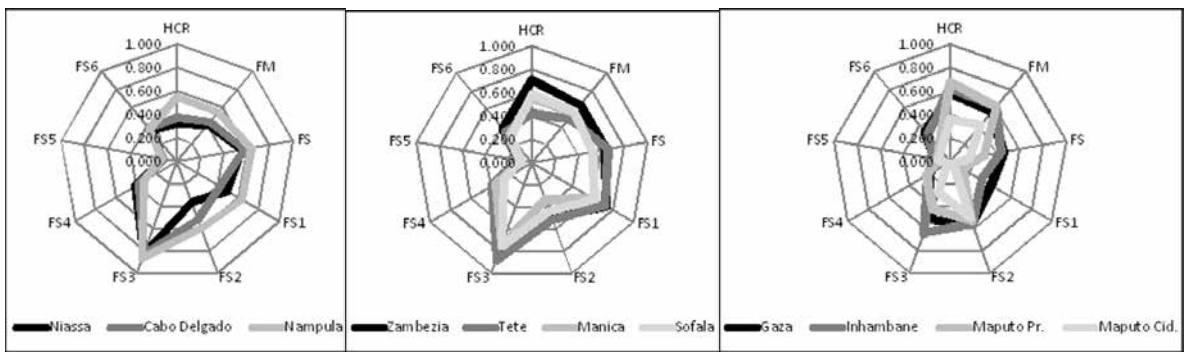


North *Centre*

South

Note: FS1 = housing conditions; FS2 = more widespread and affordable durable goods; FS3 = less common, more expensive durable goods; FS4 = housing quality; FS5 = income-related deprivation; FS6 = health and education.

Fig. 3. *Fuzzy Supplementary dimensions (FS1-FS6), by region and province*



North *Centre* *South*

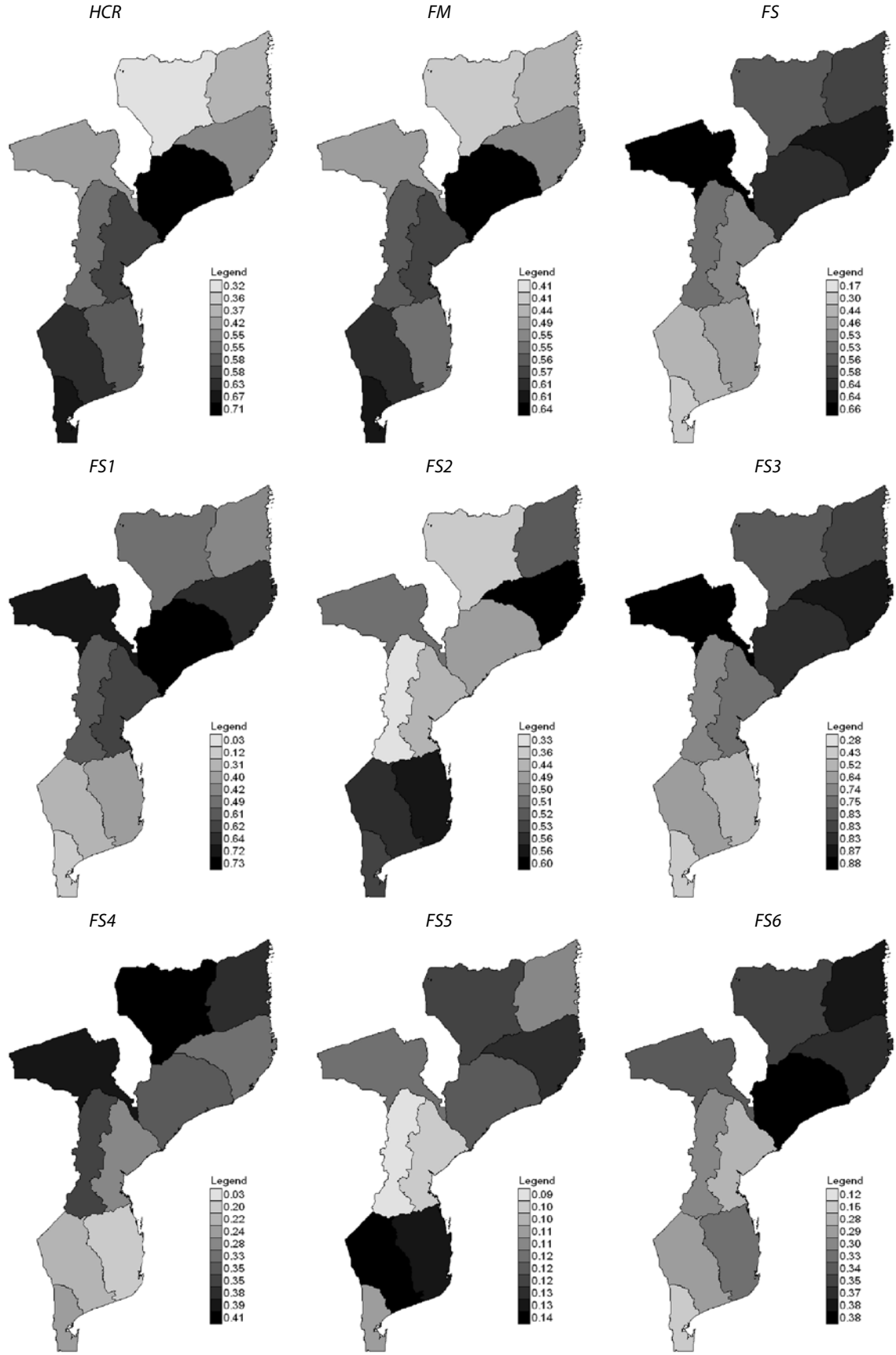
Note: HCR = Head Count Ratio; FM = Fuzzy Monetary; FS = Fuzzy Supplementary; FS1 = housing conditions; FS2 = more widespread and affordable durable goods; FS3 = less common, more expensive durable goods; FS4 = housing quality; FS5 = income-related deprivation; FS6 = health and education.

Fig. 4. *Deprivation by dimension and province*

substantially improves the mapping of provincial differences regarding poverty. In particular, the higher level of development of the Southern provinces distinctly surfaced in more than one dimension (*FS*, and particularly *FS*1, *FS*3, *FS*4, *FS*6). At the same time, understanding which factors are most influential upon deprivation, yields a deeper insight about which characteristics are more unequally distributed throughout the country.

While Figure 2 presents the Head Count Ratio, Fuzzy Monetary and Fuzzy Supplementary averages for all provinces, which highlights the differences that exist between monetary and overall non-monetary deprivation, in Figure 3, instead, all the different supplementary dimensions are shown, divided by region and province. In Figure 4 both the monetary and individual non-monetary dimensions are shown for each region and province on a net graph. This kind of graph provides additional information about the overall condition of each province compared to other provinces in the same region.

Finally, Figure 5, which reports maps of deprivation by dimension, is particularly informative since it allows a comparison of all provinces in all dimensions, and reveals the gap between the



Note: HCR = Head Count Ratio; FM = Fuzzy Monetary; FS = Fuzzy Supplementary; FS1 = housing conditions; FS2 = more widespread and affordable durable goods; FS3 = less common, more expensive durable goods; FS4 = housing quality; FS5 = income-related deprivation; FS6 = health and education.

Fig. 5. *Maps of deprivation, by dimension*

Table

Deprivation by dimension, provincial and urban/rural level

	Mozambique		Niassa		Cabo Delgado		Nampula		Zambezia		Tete	
	urban	rural	urban	rural	urban	rural	urban	rural	urban	rural	urban	rural
Head Count Ratio	0.4962	0.5691	0.4224	0.2892	0.4429	0.3552	0.499	0.5667	0.6362	0.718	0.5302	0.4015
SE	0.0164	0.0170	0.0512	0.0388	0.0866	0.0382	0.0548	0.0343	0.0680	0.0381	0.0798	0.0502
mean FM	0.5042	0.5656	0.4664	0.3893	0.4922	0.4239	0.5105	0.5607	0.5985	0.6503	0.5462	0.4808
SE	0.0114	0.0108	0.0221	0.0300	0.0588	0.0226	0.0400	0.0204	0.0485	0.0243	0.0478	0.0309
mean FS	0.339	0.6378	0.3887	0.6165	0.4358	0.623	0.4901	0.7072	0.4557	0.6743	0.479	0.6926
SE	0.0103	0.0067	0.0313	0.0301	0.0387	0.0191	0.0312	0.0155	0.0457	0.0119	0.0709	0.0190
mean FS1	0.2596	0.6545	0.3521	0.5357	0.3099	0.4538	0.4646	0.7068	0.4923	0.7756	0.4927	0.7588
SE	0.0113	0.0094	0.0269	0.0359	0.0232	0.0233	0.0325	0.0160	0.0543	0.0147	0.0958	0.0274
mean FS2	0.487	0.5119	0.3531	0.3639	0.5145	0.5183	0.5847	0.602	0.4285	0.5068	0.5599	0.5053
SE	0.0117	0.0087	0.0232	0.0267	0.0527	0.0227	0.0437	0.0159	0.0278	0.0159	0.0497	0.0323
mean FS3	0.5248	0.8421	0.6586	0.8822	0.6328	0.8862	0.7224	0.9333	0.7072	0.8581	0.6481	0.9191
SE	0.0133	0.0095	0.0334	0.0240	0.0675	0.0192	0.0385	0.0113	0.0484	0.0319	0.0779	0.0161
mean FS4	0.1317	0.3848	0.1711	0.4779	0.1951	0.4258	0.1637	0.3953	0.184	0.3768	0.1743	0.432
SE	0.0119	0.0136	0.0475	0.0493	0.0434	0.0418	0.0454	0.0357	0.0401	0.0280	0.0500	0.0410
mean FS5	0.1075	0.1195	0.1069	0.1221	0.1251	0.1099	0.1205	0.133	0.1064	0.1191	0.0866	0.1199
SE	0.0033	0.0024	0.0103	0.0079	0.0127	0.0064	0.0092	0.0046	0.0127	0.0072	0.0136	0.0075
mean FS6	0.2135	0.3699	0.2355	0.3821	0.2972	0.3969	0.2951	0.3945	0.2769	0.4039	0.2978	0.3525
SE	0.2135	0.3699	0.0221	0.0178	0.0232	0.0204	0.0161	0.0164	0.0349	0.0262	0.0371	0.0160
n	5222	5609	384	430	240	540	570	1005	336	1187	192	576

	Manica		Sofala		Gaza		Inhambane		Maputo Pr.		Maputo Cid.
	urban	rural	urban	rural	urban	rural	urban	rural	urban	rural	urban
Head Count Ratio	0.4537	0.5828	0.5005	0.6293	0.4049	0.6267	0.4836	0.6784	0.6366	0.7633	0.3615
SE	0.0505	0.0543	0.0721	0.0643	0.0429	0.0592	0.0612	0.0376	0.0276	0.0502	0.0239
mean FM	0.4818	0.5864	0.4865	0.6197	0.4459	0.5786	0.5059	0.6533	0.5914	0.6649	0.4054
SE	0.0322	0.0314	0.0474	0.0535	0.0268	0.0412	0.0427	0.0277	0.0195	0.0308	0.0182
mean FS	0.2888	0.6111	0.2849	0.6751	0.2969	0.5045	0.3566	0.469	0.2401	0.43	0.1672
SE	0.0255	0.0264	0.0318	0.0362	0.0269	0.0276	0.0545	0.0275	0.0134	0.0427	0.0066
mean FS1	0.2387	0.7322	0.2507	0.8436	0.185	0.4609	0.1875	0.3525	0.0584	0.2748	0.0323
SE	0.0314	0.0363	0.0404	0.0279	0.0276	0.0361	0.0534	0.0357	0.0058	0.0514	0.0029
mean FS2	0.3591	0.3263	0.3906	0.4639	0.4695	0.5817	0.5061	0.5738	0.5049	0.5902	0.5031
SE	0.0320	0.0230	0.0334	0.0626	0.0325	0.0287	0.0271	0.0288	0.0244	0.0520	0.0137
mean FS3	0.4919	0.823	0.5364	0.8831	0.4025	0.5494	0.56	0.6752	0.3523	0.5958	0.2768
SE	0.0350	0.0305	0.0488	0.0235	0.0363	0.0357	0.0466	0.0355	0.0219	0.0616	0.0159
mean FS4	0.0615	0.4509	0.0414	0.4196	0.1017	0.2327	0.1958	0.2272	0.1987	0.3213	0.0327
SE	0.0234	0.0519	0.0109	0.0588	0.0220	0.0348	0.0908	0.0496	0.0268	0.1058	0.0042
mean FS5	0.0775	0.0955	0.0989	0.0937	0.1164	0.1357	0.1285	0.1381	0.1096	0.0976	0.0979
SE	0.0069	0.0072	0.0147	0.0080	0.0076	0.0050	0.0089	0.0050	0.0082	0.0151	0.0069
mean FS6	0.2123	0.3311	0.1703	0.3435	0.2185	0.3567	0.2499	0.3099	0.1248	0.2145	0.1234
SE	0.0228	0.0195	0.0216	0.0367	0.0263	0.0382	0.0260	0.0200	0.0151	0.0380	0.0087
n	336	468	527	324	336	467	382	432	720	180	1199

Note: standard errors (SE) are computed using Jackknife Repeated Replication (JRR) in the form proposed by Verma and Betti (2011).

Centre-North and the South for supplementary dimensions of deprivation as clearly evident.

4.3 Multidimensional poverty estimates by province and area of residence (urban/rural)

In what follows, we present multidimensional deprivation as estimated by province and by area of residence (urban/rural). The huge differences in poverty estimates that exist between urban and rural areas at both national and sub-national level in Mozambique were already introduced in Section 1. Nonetheless, unexpected results emerge from the analysis of supplementary dimensions of deprivation (Table 1): when these are introduced, the urban/rural deprivation gap widens substantially, contrasting with the official analyses based on consumption that estimate a differential of about seven percentage points. Indeed, at national level the Head Count Ratio of rural and urban areas for 2008-09 is about 56.9 % and 49.6 %, respectively (MPD-DNEAP, 2010).

However, when supplementary dimensions of deprivation are considered, a different picture also emerges for Mozambique. The aggregated Fuzzy Supplementary (*FS*) deprivation level for urban areas is 0.34, whereas the one for rural areas exceeds 0.64. Such difference is due to the urban/rural gap found in the underlying supplementary dimensions. In particular, housing conditions (*FS*1), possession of less common, more expensive durable goods (*FS*3), housing quality (*FS*4) and — to a lesser extent — health and education (*FS*6) all show very different deprivation levels for urban and rural areas.

For housing conditions (*FS*1) the urban deprivation level is 0.26, the rural one as high as 0.65. For more expensive durable goods (*FS*3) they are equal to 0.52 and 0.84 respectively. The values for the housing quality dimension (*FS*4) are 0.13 for urban areas and 0.38 for rural areas while those for the health and education dimension (*FS*6) are 0.21 for urban and 0.37 for rural. Much smaller differences exist in the more widespread durable goods (*FS*2) and income-related (*FS*5) deprivation dimensions.[1] The wide deprivation gap between urban and rural areas typical of most supplementary dimensions at national level is also reflected at the provincial one.

The central regions of Manica and Sofala exhibit the greatest difference between supplementary deprivation values in urban and rural areas. In the supplementary dimensions *FS*1 (housing conditions), *FS*3 (more expensive, less affordable durable goods) and *FS*4 (housing quality) such difference is conspicuous, ranging 30 to 60 percentage points. While the urban areas of these two provinces are among the less deprived areas of Mozambique in all dimensions, the opposite is true for their rural counterparts. The urban/rural deprivation gap for the dimensions *FS*1, *FS*3 and *FS*4 is substantial also for other provinces such as Niassa, Cabo Delgado, and especially Nampula, Zambezia, and Tete. Moreover, the Southern provinces of Gaza and Maputo Province also show significant differences between rural and urban areas.

Urban and rural deprivation levels are instead comparable for more widespread durable goods (*FS*2) and income-related (*FS*5) supplementary dimensions. Some of the rural areas score even better than their relative urban areas in *FS*2 (Tete, Manica). As pointed out in previous paragraphs, excluding the variable "formal or informal job" from the *FS*5 dimension changes the results for this dimension substantially. When this variable is excluded the difference between urban and rural areas increases largely for Niassa, Nampula, Tete, Sofala, Gaza and Inhambane.

For what concerns *FS*6 (health- and education-related indicators), rural areas are systematically more deprived than urban areas. This is plausible, as healthcare facilities and schools are more widespread in urban areas. The average gap between areas of residence amounts to more than ten percentage points, notwithstanding the commitment of the Mozambican government to increase the availability of health and education facilities in rural areas (Chao and Kostermans, 2002; Government of Mozambique, 2005; Republic of Mozambique, 2006).

As previously shown, both monetary deprivation dimensions — Head Count and Fuzzy Monetary — analysed at national level determine radically different results to non-monetary dimensions. This holds true also for the analysis at provincial and urban/rural level. From Head Count Ratio and Fuzzy Monetary estimates it turns out that the poorest region in Mozambique is the rural area of Maputo Province, while the rural areas of Niassa, Cabo Delgado and Tete are richer than their urban counterparts and present the same low deprivation levels of Maputo City, the capital. In these monetary dimensions the urban/rural deprivation gap of Manica and Sofala is not as wide as found in the supplementary dimensions, while the urban/rural Head Count Ratio gap of Gaza and Inhambane is found significantly wider.

Introducing supplementary dimensions to the analysis of poverty in Mozambique substantially increases the amount and quality of available in-

[1] Again, the urban/rural difference increases for dimension *FS*5 when the variable "formal or informal job" is not considered: in this case the average urban deprivation becomes 0.51, while the rural becomes one 0.70.

formation, providing figures that often contrast with the ones derived solely from monetary poverty estimates.

5. Conclusions

In this study we have shown how it is possible to construct poverty measures relative to monetary and non-monetary dimensions using Fuzzy Set Theory. We applied this technique to the Mozambican Household Budget Survey 2008-09 (IOF08) dataset, the most recent budget survey available for Mozambique.

Our main contribution to the analysis and measurement of poverty in Mozambique is twofold. On the one hand, we estimate a concept of poverty wider than monetary poverty, therefore involving supplementary dimensions. At the same time, we have obtained reliable estimates of poverty rates at sub-national and urban/rural level, by using the Jackknife Repeated Replications method to compute standard errors.

To our knowledge, this is the first study that applies Fuzzy Set Theory to the measurement of poverty in Mozambique. As a result, the figures provided in the study substantially increase the amount and quality of available information about Mozambican households' deprivation. Our estimates — especially those obtained for non-monetary dimensions — complement the ones derived solely from the Head Count Ratio. They also provide new evidence with respect to provincial and urban/rural deprivation levels.

With regards to monetary poverty, the Fuzzy Monetary estimates essentially confirm the official results obtained using the Head Count Ratio. In particular, the ranking of poorer and richer provinces remains unchanged, also when the analysis is carried out at the urban/rural level. This is due to both measures, the Head Count and Fuzzy Monetary, being based on consumption data.

Instead, innovative results come from the inclusion of six supplementary dimensions of deprivation in the analysis of poverty: housing conditions; more widespread and affordable durable goods; less common, more expensive durable goods; housing quality; income-related deprivation; health and education. When these dimensions are considered, some of the provinces showing relatively low Head Count Ratios are found to be among the most deprived with respect to supplementary dimensions of deprivation, and conversely. In particular, the Northern provinces and the Central province of Tete, all show much higher Fuzzy Supplementary (*FS*) averages with respect to their Fuzzy Monetary (*FM*) averages. The remaining Central provinces have similar statistics in both the *FM* and *FS* dimension, while the Southern provinces show lower *FS* averages than their respective *FM* averages.

The higher level of development of the Southern provinces distinctly becomes relevant to more than one supplementary dimension: housing conditions (*FS*1); less common, more expensive durable goods (*FS*3); housing quality (*FS*4); and, to a lesser extent, health and education (*FS*6).

Furthermore, in our analysis we point out that deprivation values found in urban and rural areas are very different. When we consider non-monetary dimensions of deprivation it emerges that the urban/rural gap is much wider than it appears from Head Count Ratio or Fuzzy Monetary statistics. The aggregated Fuzzy Supplementary deprivation level for urban areas is estimated to be 0.34, whereas the one for rural areas exceeds 0.64. Moreover, while the ranking of some rural area such as Cabo Delgado, Niassa, and Tete sensibly worsens, it improves for the provinces of Zambezia and Maputo Province).

One partial explanation of this large difference between the results for monetary and non-monetary poverty is that some of the items included in the supplementary analysis are non-essential items, like fridge, car or PC. For example, the highest average level of deprivation is found for *FS*3 (less common, expensive durable goods), as most Mozambicans do not possess any of the items included in this dimension, especially in the North and in the Centre. In fact it might be objected that the inclusion of these items in the analysis of poverty is not entirely justified. However, the said difference between monetary and non-monetary poverty is large also for those supplementary dimensions like housing conditions, housing quality, or health and education, which certainly denote a situation of deprivation.

Our results are particularly relevant since Mozambique is among the poorest countries in the world, given its per capita income level of approximately $428 (World Bank, 2010), and several donor countries and international agencies involved in poverty reduction plans. Accurate information and measurement of poverty at the local level is thus required and may be used to redirect funds.

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