View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Archivio istituzionale della ricerca - Università di Brescia

Estimation Procedures for Latent Variables Models with Psychological Traits

Anna Simonetto
Dipartimento di Metodi Quantitativi, Università degli Studi di Brescia
C.da Santa Chiara, 50 – 25122 Brescia – simonett@eco.unibs.it

Keywords: Measurement Error Model, Structural Equation Models, Item Response Theory, Latent Variable Model

1. Categorical Data Analysis and Measurement

The starting point for this thesis is a concrete problem: to measure, using statistical models, aspects of subjective perceptions and assessments, and to understand their dependencies. We can identify two main approaches to analyzing multivariate latent aspects taking into account the categorical nature of the observed variables (Cagnone et al., 2010): the Underlying Variable Approach (UVA) and the Item Response Theory (IRT). We study some parameter estimators of regression models with variables affected by measurement errors.

The UVA assumes that the observed categorical outcomes are incomplete observations of unobserved continuous variables: underlying each of the categorically observed variables Y_j' there is a continuous variable Y_j^* which is actually measuring the underlying latent factors Θ not directly observable. We assume the linear factor analysis model for the partially observed variables $Y_j^* = \Lambda\Theta + \epsilon$. One of the most used family of models that belongs to UVA framework is the Structural Equation Model (SEM).

The second approach is based on Item Response Model (IRM): we describe, through a nonlinear monotonic function, the association between a respondent's underlying latent trait level and the probability of a particular item response. In an IRM we find two kinds of parameters, one describes the qualities of the subject under investigation (ability), and the other relates to the characteristics of each item (difficulty). Within the framework of IRT, several models have been proposed to synthesize data obtained from a questionnaire producing an objective measure of the latent construct. The Rating Scale Model (RSM), used in this thesis, focuses on the use of Likert scales in psychometrics. The two main features of this model are that items have the same number of categories and the difference between any given threshold location and the mean of the threshold locations is equal or uniform across items.

2. The Estimation procedures

We focused on two aspects of latent variable models with psychological traits: to obtain "good" measures and to assess the dependence relationships between the constructs represented by these measures are. These two objectives may be combined into a single estimation procedure where all the model parameters are estimated simultaneously (One-step Procedure), or developed sequentially one at a time (Two-step Procedure).

2.1 The One-step Procedure (1SP)

The One-step procedure, combined with the UVA, involves the simultaneous estimation of all model parameters through the implementation of a SEM. We have two components: the Structural Model, showing potential causal dependencies between endogenous and exogenous variables, and the Measurement model, showing the relations between latent variables and their indicators. In the simple case of two latent factors with 2 categorical indicators, we have:

the Structural Model	$\theta_1 = \zeta_1$	$\theta_2 = \beta\theta_1 + \zeta_2$
the Measurement Model	$Y_1^* = \lambda_1\theta_1 + \epsilon_1$	$Y_3^* = \lambda_3\theta_2 + \epsilon_3$
	$Y_2^* = \lambda_2\theta_1 + \epsilon_2$	$Y_4^* = \lambda_4\theta_2 + \epsilon_4$

where $\epsilon_j \sim N(0,1)$. For each ordinal indicator Y_j we have

$$y_j = \begin{cases} c_j - 1 & if \quad \tau_{j,c_j-1} < y_j^* \\ c_j - 2 & if \quad \tau_{j,c_j-2} < y_j^* \leq \tau_{j,c_j-1} \\ \vdots & \\ 0 & if \quad y_j^* \leq \tau_{j,1} \end{cases}$$

It is important to underlie that all the model error terms, ζ_h and ϵ_j, are considered to be uncorrelated with each other and with other variables in the model. We are interested in the estimation of the standardized regression coefficient β and of the threshold parameters $\tau_{h,k}$. We implemented two different estimation methods: the SEM standard (SEMstd) and the SEM with IRT approach (SEMirt). SEMstd is the simple SEM just seen in previous equations. SEMirt is a version of previous the model, inspired by the work of Gibbons et al. (2007), that introduces the structure of IRM in SEM. With this model, we can estimate the standardized regression coefficient β, the threshold parameters $\tau_{h,k}$ and, in addition to the previous model, the item difficulty parameters.

2.2 The Two-step Procedure (2SP)

This procedure is based on the IRT approach, which focuses on observed variables. We define two different method for this procedure: the RSM Linear Regression Model (RSM-LRM) and the RSM-LRM with measurement error (RSM-LRMme).

The first step of both methods is the same: for each construct θ_j we apply a RSM to estimate the measure X_j with its reliability, using the standard errors of the person parameters. The second step, in which we estimate the dependence relationships between the two constructs, changes between the two methods: in RSM-LRM we assume the simple linear regression model without measurement errors $X_2 = \beta X_1 + \zeta_2$, while in the RSM-LRMme we assume a linear regression model with variables affected by measurement errors:

the Structural Model	$\theta_2 = \beta\theta_1 + \zeta_2$	
the Measurement Model	$X_1 = \gamma_1\theta_1 + \delta_1$	$X_2 = \gamma_2\theta_2 + \delta_2$

with the two measurement error variance estimates with the *Rasch Person Reliability Index* obtained in the I step together with the two measures.

3. The Simulation Design and Results

We created many different simulated datasets in order to evaluate the obtained estimates, knowing the real value of the parameter of interest β. We imagined two scenarios: the first with only two latent factors and a dependence relationship of the second versus the first; in the second one we have considered three latent factors, where one is dependent from the other two. We change the number of the indicators for the independent construct and the structural error variance of the dependent latent variable. So, we created 35 different parameter configurations and, for each configuration, we simulated 500 samples of 1,000 response patterns.
To compare the results obtained with the four estimation methods, we evaluate: the Relative Bias $RB(\hat{\beta}) = (\beta - \hat{\beta})/\beta$; the Relative Standard Error, $RSE(\hat{\beta}) = SE(\hat{\beta})/\beta$; the Relative Root Mean Square Error, $RMSE(\hat{\beta}) = \left[RB(\hat{\beta})^2 + RSE(\hat{\beta})^2\right]^{1/2}$. Because of we find that the results for the first and the second scenario are analogous, we report a summary of both. All reported results refer to the estimated regression coefficient β.

Table 1: *Percentage RMSE for the case with 3 latent factors*

		Structural error variance				
β	*Method*	**10%**	**30%**	**50%**	**70%**	**90%**
0.30	SEMstd	8.24	10.42	13.57	18.93	35.30
	SEMirt	9.32	10.40	13.56	18.88	35.26
	RSM-LRMme	9.75	11.96	15.43	21.76	39.03
	RSM-LRM	17.65	20.07	23.13	27.52	39.17
0.67	SEMstd	3.02	4.11	5.48	8.09	15.67
	SEMirt	3.02	4.12	5.48	8.07	15.56
	RSM-LRMme	3.39	4.57	6.49	9.75	17.97
	RSM-LRM	17.03	18.71	20.44	22.59	26.55
0.90	SEMstd	1.41	2.42	3.53	5.53	11.34
	SEMirt	1.39	2.43	3.54	5.53	11.32
	RSM-LRMme	1.69	3.10	4.93	7.64	13.78
	RSM-LRM	18.31	19.50	20.76	22.24	25.05

We observe that the results of RSM-LRM show a strong negative bias, consistent with the theory of measurement errors. It is interesting to note that all results of the two 1SPs are practically identical, though they represent, conceptually, two very different methods. They show a distortion of reduced entity (in absolute value less than 1%). It is also interesting to note that in the case with more indicators for the independent latent factor the distortion of the RSM-LRMme is less than the case with few indicators, consistent with IRT. The RSM-LRM shows $RSE(\hat{\beta})$ lower than other methods, while

RSM-LRMme is the less precise. For all the 4 methods, the standard errors increase as the variance of the structural error term of the dependent latent variable increase. Observing the relative $RMSE(\hat{\beta})$, we note the importance of considering, in the estimation procedure, the measurement errors that affect the variables. In fact, the RSM-LRM, presents a very high relative $RMSE(\hat{\beta})$ (up to 10 times the other method values). It is due to the strong bias, not sufficiently compensated by the good accuracy in estimating. The results of the two 1SPs have the lowest RMSE. The SEM-LRMme has a slightly higher $RMSE(\hat{\beta})$, although the discrepancy with the 1SP results is never more than 3 percentage points.
We also compared the values of the classical *Cronbach's alpha* and the *Rasch Person Reliability Index*, and we have seen that they are perfectly consistent.

4. Concluding remarks

A first consideration is about the RSM-LRM method: our simulation showed that the bias of estimator for the parameter of interest is very strong.
Remembering that one of our goals was to implement a two-step procedure efficient and precise, we focus the attention on the RSME index (that combines an assessment of bias and efficiency). The 2SP has a slight distortion and a loss of efficiency, but its estimates are coherent with that providing by the 1SP and often the difference with them is really very small (at maximum 4.3 percentage points). It is a very interesting result, that provides a useful tool for future analysis starting to real data.
We have repeatedly stressed the advantages of IRM (greater flexibility of analysis, reliability analysis, possibility of verification of hypothesized relations), but we did not know what was the price to pay in terms of loss of efficiency and distortion. Given this simulation data, we could say that, for the cases presented, the 2SP results sufficiently precise and unbiased. Obviously the choice of which procedure to implement is the prerogative of the researcher and it depends strongly of the purposes of its analysis, but for the cases described in the simulation study, both the two approaches could be used to obtain statistically significant results.

References

Cagnone, S., Mignani, S., and Moustaki, I. (2010) Latent variable models for ordinal data, in: *Statistical Methods for the Evaluation of Educational Services and Quality of Products, Contributions to Statistics*, Monari, P., Bini, M., Piccolo, D., and Salmaso, L., pp. 17-28. Physica-Verlag HD.

Carroll, R.J., Ruppert, D., Stefanski, L.A., Crainiceanu, C.M. (2006) *Measurement Error in Nonlinear Models: A Modern Perspective*, 2nd Ed., Chapman & Hall.

Gibbons R. D., Bock R. D., Hedeker D., Weiss D. J., Segawa E., Bhaumik D. K., Kupfer D. J., Frank E. , Grochocinski V. J., Stover A. (2007) Full-Information Item Bifactor Analysis of Graded Response Data, *Applied Psychological Measurement*, 31(1), pp. 4-19.

Linacre J.M. (1997) KR-20 / Cronbach Alpha or Rasch Reliability: Which Tells the "Truth"?, Rasch Measurement Transaction*s*, 11:3 p. 580-1, 1997.

Muthén, B. (1984) A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators, *Psychometrika*, 49, 115-132.