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Equilibrium a la Cournot and Supply Function Equilibrium in Electricity Markets under Linear Price/Demand and Quadratic Costs

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Abstract

We present a comparison between the Cournot and the Supply Function Equilibrium models under linear price/demand, linear supply functions and quadratic costs.

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1 Introduction

The *Supply Function Equilibrium* (SFE) approach in an electric market takes place whenever firms compete in offering a schedule of quantities and prices. On the other hand, in the *Cournot equilibrium model* each firm chooses the optimal output by maximizing its own profit. It is well known that Cournot equilibrium is easier to compute than SFE because the mathematical structure of Cournot models turns out to be a set of algebraic equations, while the mathematical structure of SFE models turns out to be a set of differential equations (see e.g. Ventosa et al. [7]).

Bonenti and Zuanon [2] considered a simplified version of the Cournot equilibrium model under linear price/demand and quadratic costs presented by Ruiz et al. [5]. In particular, they presented the explicit expressions characterizing analytically the Cournot equilibrium values for the produced quantities, the market-clearing price and the profits whenever each generating company is characterized by a quadratic cost function and the price/demand function is linear.

In this paper, we consider the Supply Function Equilibrium in case that the *supply functions* $s_i(p) = \eta_i p$ are also linear and there are no other constraints. Starting from the linear demand function $D(p) = \frac{p-\gamma}{\beta}$, we use the classical first-order (unconstrained) optimality conditions (see e.g. Klemperer and Meyer [4] and Anderson and Hu [1]) at the market clearing price in order to determine the explicit expressions for the produced quantities, the optimal price and the profits. Finally, we make a comparison between the two models in the spirit of the considerations presented by Willems and Rumiantseva [8].

2 Comparison between Equilibrium à la Cournot and Supply function equilibrium

In the Cournot approach, n *generating companies* are considered, each of them producing a quantity of energy q_i. While each generating company affects the *market-clearing price*, it is assumed that the production level of any generator does not depend on the production levels of the remaining companies.

The generic i-th generating company is characterized by a quadratic cost function $C_i(q_i)$ defined as follows:

$$(1) \quad C_i(q_i) = a_i + \frac{1}{2} b_i q_i^2,$$

where the real parameters a_i and b_i are positive.

It is further assumed that the *price/demand function* (i.e., the *inverse demand function*) $p(q_1 + ... + q_n)$ is linear, i.e.

$$(2) \quad p(q_1 + ... + q_n) = \gamma + \beta(q_1 + ... + q_n)$$

with $\gamma > 0$ and $\beta < 0$, the following "simultaneous" maximum profit problem comes into consideration:

$$(3) \quad \begin{array}{ll} \max_{p,\, q_i} & \pi_i = p(q_1 + ... + q_n)q_i - C_i(q_i) = \\ \max_{p,\, q_i} & \left[(\gamma + \beta(q_1 + ... + q_n))q_i - a_i - \frac{1}{2} b_i q_i^2 \right] \qquad i = 1, ..., n, \end{array}$$

where π_i is the profit that is obtained by the i-th company.

Explicit solutions of problem (3) are obtained by considering the system of the partial derivatives of the profits equal to zero:

$$\begin{aligned}\frac{\partial \pi_i}{\partial q_i} &= \gamma + \beta \sum_{j \neq i} q_j + q_i(2\beta - b_i) = \gamma + \beta \sum_{j=1}^{n} q_j + q_i(\beta - b_i) = \\ &= p(\sum_{j=1}^{n} q_j) + q_i(\beta - b_i) = 0 \qquad i = 1, ..., n. \end{aligned} \tag{4}$$

Ruiz, Conejo, García-Bertrand [*Electric Power System Research* **78** (2008)] provide a nice expression of the solutions of system (4) (see also Bonenti and Zuanon [2]).

Indeed, the optimal quantities q_i^C to be produced, the market-clearing price p^C and the profits π_i^C are determined as follows:

$$q_i^C = \frac{\frac{\gamma}{\beta}}{\left(\frac{1}{\beta} - \sum_{j=1}^{n} \frac{1}{b_j - \beta}\right)(b_i - \beta)} \qquad i = 1, ..., n, \tag{5}$$

$$p^C = \frac{\frac{\gamma}{\beta}}{\frac{1}{\beta} - \sum_{j=1}^{n} \frac{1}{b_j - \beta}}, \tag{6}$$

$$\pi_i^C = \left(\frac{\frac{\gamma}{\beta}}{\left(\frac{1}{\beta} - \sum_{j=1}^{n} \frac{1}{b_j - \beta}\right)(b_i - \beta)}\right)^2 \left(\frac{b_i}{2} - \beta\right) - a_i \qquad i = 1, ..., n. \tag{7}$$

Let us now turn to the Supply Function Equilibrium in case that the *supply functions*

$$s_i(p) = \eta_i p \quad (i = 1, ..., n) \tag{8}$$

are linear and there are no other constraints (see e.g. Gao and Sheble [3]).

We consider the linear demand function (i.e,, precisely the inverse of (2))

$$D(p) = \frac{p - \gamma}{\beta} \tag{9}$$

and we assume that the cost functions are quadratic and expressed precisely by conditions (1).

We recall that first-order (unconstrained) optimality conditions are the following (see Klemperer and Meyer [4] and Anderson and Hu [1]):

$$(10)\quad s_i(p) = [p - C_i'(s_i(p))]\left[\sum_{j\neq i} s_j'(p) - D'(p)\right] \quad i = 1, ..., n,$$

at the *market clearing price* p, i.e., whenever

$$(11)\quad s_i(p) = D(p) - \sum_{j\neq i} s_j(p) \quad i = 1, 2, ..., n.$$

It is immediate to check that the system of equations (10) is equivalent to the following one:

$$(12)\quad \eta_i = [1 - b_i\eta_i]\left[\sum_{j\neq i} \eta_j - \frac{1}{\beta}\right] \quad i = 1, ..., n.$$

On the other hand, from the expressions (8) and (9) of the supply functions and respectively the demand function and the market clearing conditions (11), we immediately recover the following expression for the price:

$$(13)\quad p^{SFE} = \frac{\frac{\gamma}{\beta}}{\frac{1}{\beta} - \sum_{j=1}^{n} \eta_j}$$

The following proposition is an immediate consequence of a comparison between (6) and (13).

Proposition 1 *The optimal prices p^C and p^{SFE} are equal if and only if the following condition holds:*

$$(14)\quad \sum_{j=1}^{n} \frac{1}{b_j - \beta} = \sum_{j=1}^{n} \eta_j.$$

We recall that a case of particular relevance occurs whenever all but one the supply functions are zero. This means that there exists a generating company i such that $\eta_i \neq 0$ and $\sum_{j\neq i} \eta_j = 0$. In this case company i is acting as a *monopoly* and the system (12) reduces to the following equation:

$$(15)\quad \eta_i = \frac{1}{b_i - \beta}.$$

The following proposition summarizes the monopolistic situation in the supply function equilibrium approach.

Proposition 2 *If company i is acting as a monopoly, then*

$$(16) \quad p^{SFE} = \frac{\frac{\gamma}{\beta}}{\frac{1}{\beta} - \frac{1}{b_i - \beta}},$$

$$(17) \quad s_i(p^{SFE}) = \frac{\frac{\gamma}{\beta}}{(\frac{1}{\beta} - \frac{1}{b_i - \beta})(b_i - \beta)} = q_i^C.$$

On the other hand, easy computations preclude the choices $\eta_i = \frac{1}{b_i - \beta}$ for $i = 1, ..., n$, which seem to be naturally suggested by the consideration of the condition (14). Indeed, such a substitution would imply a monopoly with $\sum_{j \neq i} \frac{1}{b_j - \beta} = 0$ but this condition is impossible. Therefore, the following proposition holds.

Proposition 3 *It is never the case that*

$$\eta_i = \frac{1}{b_i - \beta} \qquad i = 1, ..., n$$

is a solution of the system (12).

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