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AUTOMATIC PROCEDURES AS HELP FOR OPTIMAL CAM DESIGN

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ABSTRACT

In this work we suggest a synthesis of recent results obtained on the application of soft-computing techniques to solve typical automatic machines design problems. Particularly, here we show an optimization method based on the application of a specialized algorithms ruled by a generalized software procedures, which appears able to help the mechanical designer in the first part of the design process, when he has to choose among different wide classes of solutions. In this frame, among the different problems studied, we refer here about the choice of the best class of motion profiles, to be imposed to a cam follower, which must satisfy prefixed design specifications. A realistic behaviour of the system is considered and the parameter model identification is set up by a soft computing procedure. The design, based on theoretical knowledge, sometimes is not sufficient to fulfil desired dynamical performances, in this situation, a residual optimization is achieved with the help of another optimizing method.

The problem of a cam-follower design is presented. A class of motion profiles and the best theoretical motion profile is selected by an evolutionary algorithm. A realistic model is considered and its parameter identification is achieved by a genetic algorithm. The residual optimization is achieved by a servomotor optimized by another genetic algorithm. Evolutionary approach is used during all the design process and, as was shown, it allows really interesting performance in terms of simplicity of the design process and in terms of performance of the product.

INTRODUCTION

During his own activity the designer of automatic machines has to solve different problems: he has to design his machine according to

- specific limits (working time, stroke…)
- some parameter that can change in a smaller range
- some parameter that give the idea of well designed machine

From a mathematical view point he has to find the best solution under a series of constrain, more or less wide: he has to find the solution of a constrained maximization/minimization problem.

Sometimes in this operation (at least during the first phase of his work) he is aided by some table or graph that suggest him the best choice. As an example, consider Figure 1 in which the main characteristics of typical mechanical transmissions are given: note how picture specifies some boundaries, which circumscribe the utilization of the different devices. By such an aid, also a non-expert designer is guided through a good choice of the right transmission for the particular problem he is facing.

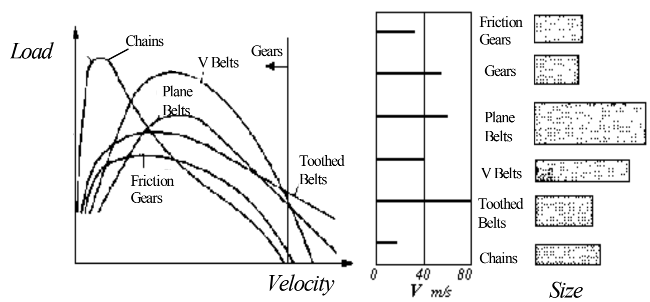

FIG. 1. Synthetic indications for the choice of a mechanical transmission [1].

In other cases the process is based on designer experience or reiterated analysis. In this second case the work is long and tedious.

In this framework we have tried to solve the designer constrain optimization problem by the use of a specific mathematical tool called genetic algorithm: among possible classes of solutions, the method is able to select the best ones according to pre-defined goals. Moreover, by changing the goals, it is possible to automatically create tables or graphs, just like the one above described, which define the limits of good utilization of particular solutions.

It was explained that we have to solve a constrain optimization problem. There are many mathematical tool to solve this specific problem, but we have decided to implement and use a genetic algorithm in particular for one reason. Genetic algorithm at the end of the optimization phase gives a family of solutions; the designer can select his solution inside this family considering also other parameters that can guides through the best machine. This scenario reflects the real situation: in a very wide range of real situation he can select from different solutions (Figure 1), all of them suitable from the specific problem.

In this paper, the procedure will be presented both from a general point of view and by the results obtained by its application on a practical example dealing with the motion planning of a cam-follower mechanism.

NOMENCLATURE

Genetic algorithm variables.

n: number of the elements of a vector. $x_1, x_2, ..., x_n$: elements of a vector ("genes")

$\mathbf{x}$: vector having $x_1, x_2, ..., x_n$ as elements.

$F = F(\mathbf{x}) = F(x_1, x_2, ..., x_n)$:generic functional depending on vector $\mathbf{x}$ (on variables $x_1, x_2, ..., x_n$).

F^* : value of the functional to be reached (goal).

$\mathbf{x}^* = (x_1^*, x_2^*, ..., x_n^*)$: solution vector that satisfies $F(\mathbf{x}^*) = F^*$.

$\mathbf{x}_j^i$: j-th vector of the i-th generation.

m: number of vectors in a generation.

$\mathbf{X}^i = (\mathbf{x}_1^i, \mathbf{x}_2^i, ..., \mathbf{x}_m^i)$: set of vectors; i-th generation.

$\varepsilon_j^i = \left|F^* - F(\mathbf{x}_j^i)\right|$: error with regard to the goal when using vector j-th of generation i-th.

$A_j^i = \frac{b_j^i}{b^i}$ where $b_j^i = \frac{1}{\varepsilon_j^i}$ and $b^i = \sum_{j=1}^{m} b_j^i$: fitness index of vector $\mathbf{x}_j^i$.

Variables for motion definition

t: time variable.

s: generic displacement in time t.

S: total displacement.

T: total time to perform displacement S.

T*, S*: precision point co-ordinates.

1 THE GENETIC ALGORITHM AND THE EVOLUTION STRATEGY USED

The method discussed enters the category of the ”genetic algorithms”, so called because they follow some procedures which appear in Darwin’s doctrine of evolution, basically explaining that every creature or individual must adapt to the environment, being otherwise its progeny doomed to extinction. It possible to say that the genetic process optimizes the species because only the best elements of each generation can survive.

In this context, we propose here, by using a so called "evolution strategy" (see [4-5] for a deeper knowledge of the subject), an extension of the classic genetic algorithm, devising the parallel evolution of more than one species which periodically are compared each other as regard to a pre-established goal. The relative comparison among the elements of different species is a sort of "fight" inside the same environment: each species tries to survive in the environment and, at the same time, pushes the others to extinction. In this way, at the end of the process, or we will have only one species surviving, or the contemporary presence of more than one species living in equilibrium in the same environment.

In the case of a numerical application, it is possible to build the following scheme: the "environment" is a functional where the "creature" is the functional variable (in general a vector), and the goal to be reached "to survive" is a pre fixed value of the functional itself.

Mathematically, consider functional

$$F = F(\mathbf{x}) = F(x_1, x_2, ..., x_n)$$

The elements of vector x are named "genes".

Let's consider a pre-fixed value F* of the functional; the aim of the typical genetic algorithm is to find vector $x^* = (x^*_1, x^*_2, \dots, x^*_n)$, so that $F(x^*) = F^*$.

To reach the goal, let's define a set of vectors $X^1 = (\mathbf{x}_1^1, \mathbf{x}_2^1, ..., \mathbf{x}_m^1)$. X^1 is the first family (generation) of vectors by means of which we try to find the solution.

For every vector forming the first generation, it is now possible to define an index by means of which we evaluate the closeness of the vector itself to the solution.

A possible index is the inverse of the error, so defined:

$$b_1^1 = \frac{1}{\varepsilon_1^1} \quad ; \quad b_2^1 = \frac{1}{\varepsilon_2^1} \quad ; \quad ... \quad ; \quad b_m^1 = \frac{1}{\varepsilon_m^1} \tag{1}$$

Where

$$\varepsilon_1^1 = \left|F^* - F(\mathbf{x}_1^1)\right|; \varepsilon_2^1 = \left|F^* - F(\mathbf{x}_2^1)\right|; ...; \varepsilon_m^1 = \left|F^* - F(\mathbf{x}_m^1)\right| \tag{2}$$

are the errors evaluated in correspondence to all the vectors of the first generation, as regard to the pre-fixed functional value F*.

It is useful to normalize the index above defined as follows:

$$A_j^1 = \frac{b_j^1}{b^1}, \text{ where } b^1 = \sum_{j=1}^{m} b_j^1 \tag{3}$$

So, at every vector $\mathbf{x}_j^1$ of the first generation we have linked a value A_j^1 named "fitness index": a vector with a high fitness index will have good possibilities of surviving in the next generation, while a vector with a low fitness index will have poor possibilities of surviving in the next generation.

From the definition, it is easy to verify that

$$\sum_{1}^{m} A_j^1 = 1 \tag{4}$$

It is easy to understand the parents selection process by looking at the next figure (Fig. 2).

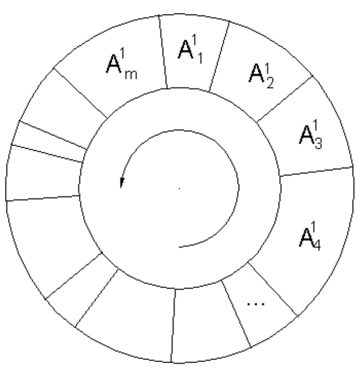

FIG. 2. Random determination of the "parents" from a generation.

We rotate the wheel two times, the first one for the mother generation and the second for the father generation. It is obvious that the probability to select the bigger square is higher then the probability to select a smaller square; in other words it is easier to select vectors having a higher value of fitness.

These two selected vectors are named "parents" and form a couple of vectors from which a new vector originates, the "son". The son is generated from the parents by means of three basic mechanisms (Fig. 3) acting on the elements (genes) of the two vectors: the inheritance, the cross-over, and the mutation.

The inherited gene directly passes from the parent to the son, without changing its value or its position in the vector. The cross-over mechanism determines the passage of a group of genes from one parent to the son, but in different positions. The mutation causes the presence of a random gene in the son. The three mechanisms are active under a random flag, so that the chances of the son turning out a perfect copy of one of the parents is as likely as it being completely different from them both.

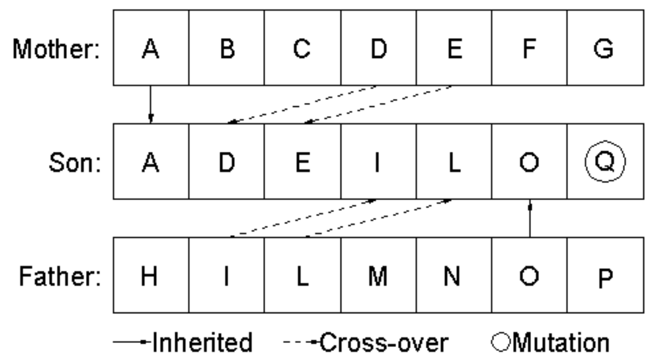

FIG. 3. The mechanisms for the generation of a son vector

The son's generation is iterated repeatedly in order to define a new generation X^2. Starting from this new family, the phases described above for the first generation are repeated. The procedure stops when a vector with fitness near 1 (inside a tolerance pre-fixed by the user) is obtained.

Now let's imagine that the evolution towards the goal contemporaneously happens on more than one species. Periodically, after a pre-established number of iterations (or generations) a pre-fixed number of vectors are drawn from each population and "fight" each other: that is, a classification is made on all the selected individuals according to their own fitness index, and only the first ones (according to a pre-fixed number) come back to the original populations; the other ones are discarded from the evolution. It comes that the population which remains with a low number of individuals will have difficulties in its evolution and it is doomed to extinction.

At the end of the process we can have two cases:

1) only one of the populations has defeated all the others and, continuing with the evolution, its creatures go towards the goal;

2) two or more families establish an equilibrium (no one could defeat all the others) and the evolution continues in a parallel way towards the goal (Fig. 4).

It is clear that this kind of procedure gives two results: it is able to identify the best (or more robust) population(s), and, inside each population it will be possible to recognize "the champion", that is the element which is nearest to the pre-established goal.

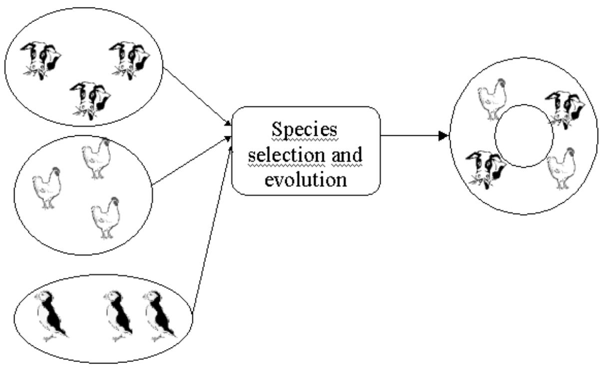

FIG. 4. The equilibrium of more than one species.

2 THE CASE STUDIED

In the definition of the motion profile of a cam-follower system, we generally must obtain a pre-fixed displacement S in a pre-fixed time T, with null velocity at the beginning and at the end of the movement. Often, it is also necessary to respect other constraints as the passing of the follower through a precision point P defined by a specified displacement S* in time T*.

In Figure 5 this situation is depicted. We will consider the follower passing through the precision point when the error e = |s(T*) - S*| will result less than a predetermined value.

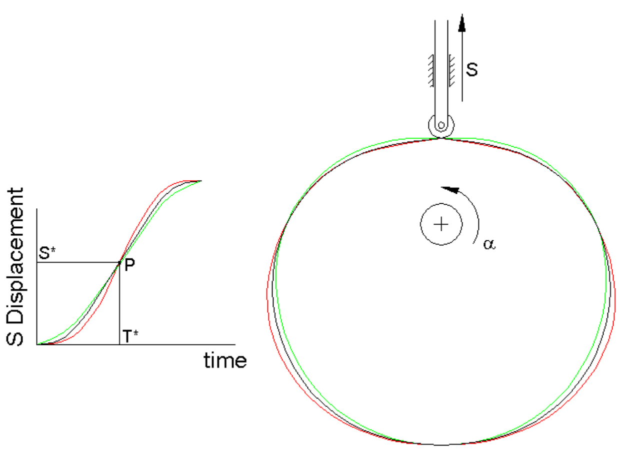

FIG. 5. The displacement profile through a precision point: different motion laws are possible

It is clear that, among different types of parametric function s(t), commonly used in these kinds of mechanical design problems, it will be possible to find the class which better satisfies the above given constraints.

The analyzed motion functions, which are also the most commonly used in cams design, are:

1) The "**modified trapezoidal motion**" (also known as "seven segments motions"). They are largely described in specialistic literature [1-3] and greatly used in technical field, mainly because it is possible to obtain very smooth profiles on the velocity diagrams.
They allow to define the movement on a total displacement S, in total time T, by means of seven coefficients which represent the time percentages as regard the total time of the acceleration diagram partitions, as depicted in Figure 6. The velocity and displacement trends are obtained by integration of the acceleration diagram on which it is possible to impose some constraints (the continuity and null values at the motion ends, for instance).

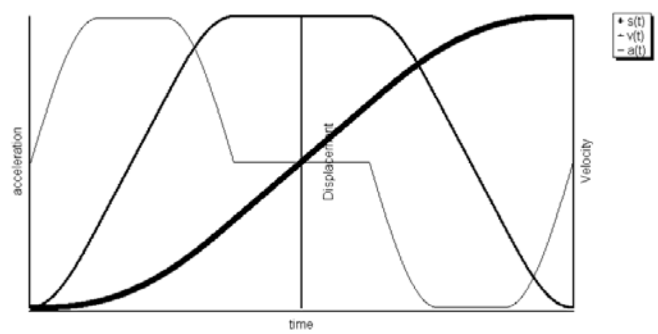

FIG. 6. Acceleration, velocity and displacement of modified trapezoidal motion

The mathematical relationships to define this category of motion planning can be easily found in literature: we will only say here that this kind of acceleration profile is made up by an alternation of harmonic and constant segments.

2) The **trigonometric profiles** (Figure 7) are defined as a combination of a pre-fixed number k of harmonic functions having periods multiple of T (total motion time) which satisfy the typical boundary conditions at the ends of the movement (displacement S in time and null velocity at time T and null velocity at time 0). The main advantage of this class of motion is the possibility, during the motion definition, to shut out a-priori the harmonic functions having their own frequency close to the natural frequency of the system, avoiding so the resonance condition.

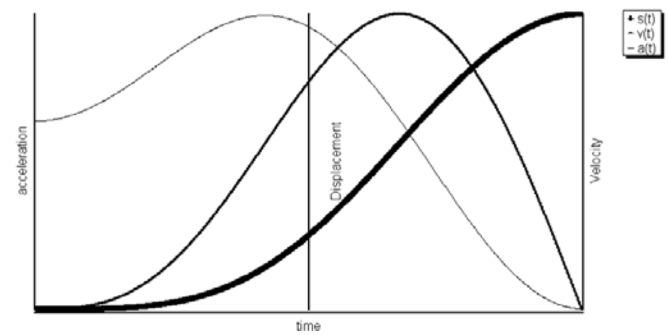

FIG. 7. Acceleration, velocity and displacement of trigonometric motion

3) The **polynomial motions** (Figure 8) are formed by a polynomial function, whose coefficients can be determined imposing the boundary conditions on displacement, velocity and acceleration. The main characteristic of these profiles is the possibility to increase the number of boundary conditions simply by increasing the polynomial degree.

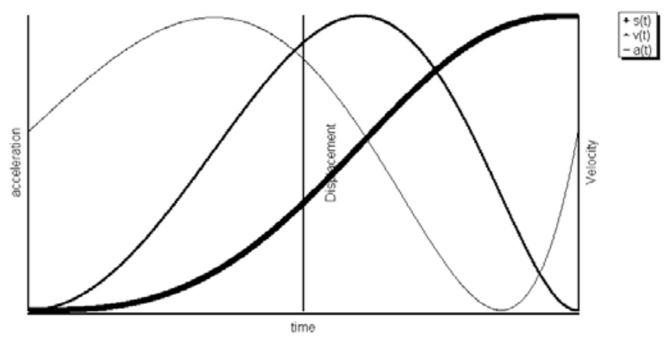

FIG. 8. Acceleration, velocity and displacement of polynomial motion

To operate a selection on the three s(t) motion functions in order to determine what is the best to guarantee the passage with the best precision (that is minimising error e = |s(T*) - S*|) through point P = (T*, S*), contemporaneously fulfilling other boundary conditions, we have used a strategy based on the above described genetic procedure.

3 RESULTS OF THE SELECTIVE ALGORITHM

The results of the use of the genetic algorithm procedure on the problem above stated are summed up in Fig. 9. (Total time T and total displacement S have been normalized to 1).
The graph has to be read as follows: if precision point P=(T*; S*) falls inside the white zone, it is advisable to use the "seven segment" motion profile to gain it (that is, it will be easier to find inside the "seven segment" motion class a motion profile which will get the solution with high precision and contemporaneously fulfills the above stated boundary conditions). If precision point co-ordinates fall in dark grey zone it will be advisable to use the "polynomial profile"; if point P falls inside the dotted area, the best choice to obtain the highest precision in passing through the point is the "trigonometric profile"; finally, in the dark grey zone we have an equilibrium, so that no one of the profiles is the best (inside the genetic procedure, in this case we have the cohabitation of all the three populations which cannot overcome each other).

The black zones cannot be reached by any of the profile; the genetic algorithm does not converge towards a solution for anyone of the profiles.

In conclusion, we can say that we have tried to use a genetic algorithm ruled by an evolution strategy to select possible classes of solutions inside a problem having several parameters and boundary conditions to be fulfilled.

The shown application, even if inside a very narrow and specific problem (however very felt by mechanical designers), has given satisfactory results, not only from the procedure point of view (which has resulted very easy to be implemented), but also from the particular obtained result; in fact, the graph above shown can be a substantial and actual help to the designer in the choice of the motion class.

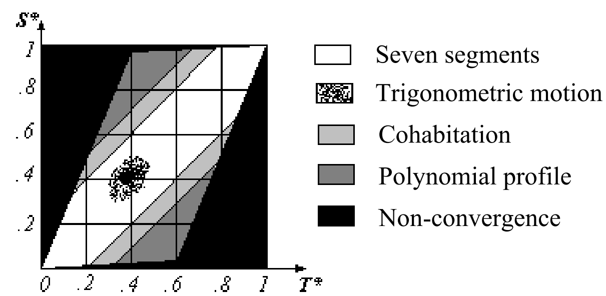

FIG. 9. Motion profiles choice for the passage through a precision point

4 REALISTIC BEHAVIOUR OF THE SYSTEM

Some experimental tests, executed by means of an accelerometer fixed on the rotating table, have suggested that the peak value of the table acceleration is remarkably higher than the theoretical one. This kind of behaviour becomes more and more evident when the inertial load is high.

Two phenomena are particularly evident: the maximum negative amplitude of the table acceleration is about twice than the theoretical one; there are important residual vibrations at the end of the rising phase.

In order to explain the actual behaviour of the mechanism, a proper mathematical model has been studied but its parameter could be correctly and easily evaluated only when this system is physically realized. It particularly considers:

* the characteristic of the motor (which can explain the speed fluctuations due to the load changes);
* the presence of clearance in the gear speed reducer (particularly high in applications with many serial reduction stages);
* the elasticity and structural damping of the mechanical components.

The motor is described by its characteristic function $T_m = T_m(\dot{\gamma})$.

The clearances of the gear speed reducer and the elasticity of the mechanical members are simulated by means of elastic elements, as depicted in Fig. 10. Backlash g_1 is due to the tolerances between the teeth of the speed reducer; its value is particularly important when the transmission ratio is high. The coupling between the table and the indexing mechanism output shaft mainly causes clearance g_2, which is generally very close to zero. Finally, the damping variables have been considered as scalar constant values, as well as all the moments of inertia.

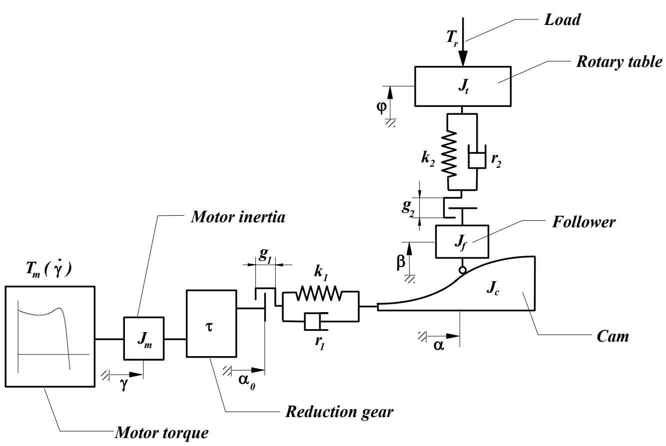

FIG. 10. Equivalent scheme of the indexing mechanism

According to the definitions given above, referring to the symbols of Fig. 10, the set of differential equations which describes the dynamical behaviour of the mechanism appears as follows:

$$\begin{cases} \ddot{\alpha} = \dfrac{M_1 + \beta' M_2 - J_f \beta' \beta'' \dot{\alpha}^2}{J_c + J_f \beta'^2} \\ \ddot{\varphi} = -\dfrac{T_r + M_2}{J_t} \\ \ddot{\gamma} = \dfrac{T_m - \tau M_1}{J_m} \end{cases} \tag{5}$$

where the dot symbol is used for time derivatives while the prime notation is utilized for derivatives calculated with respect to the cam rotation angle α Clearly, the relationship between angles β and α is linked to the cam profile inside the indexing mechanism, in accordance with the well known formulas [1-3]:

$$\beta = \beta(\alpha) \quad \dot{\beta} = \beta'(\alpha)\dot{\alpha} \quad \ddot{\beta} = \beta''(\alpha)\dot{\alpha}^2 + \beta'(\alpha)\ddot{\alpha} \tag{6}$$

Functions M_1 and M_2 describe the sum of the effects of a compliance component (elasticity and backlash) with a damping element. Their analytical expressions are:

$$M_1 = \begin{cases} 0 & |\alpha_0 - \alpha| \le g_1/2 \\ k_1(\alpha_0 - \alpha - g_1/2) + r_1(\dot{\alpha}_0 - \dot{\alpha}) & \alpha_0 - \alpha > g_1/2 \\ k_1(\alpha_0 - \alpha + g_1/2) + r_1(\dot{\alpha}_0 - \dot{\alpha}) & \alpha_0 - \alpha < g_1/2 \end{cases} \tag{9.1}$$

$$M_2 = \begin{cases} 0 & |\varphi - \beta| \le g_2/2 \\ k_2(\varphi - \beta - g_2/2) + r_2(\dot{\varphi} - \dot{\beta}) & \varphi - \beta > g_2/2 \\ k_2(\varphi - \beta + g_2/2) + r_2(\dot{\varphi} - \dot{\beta}) & \varphi - \beta < g_2/2 \end{cases} \tag{9.2}$$

5 PARAMETER IDENTIFICATION

It is possible to observe that the proposed mathematical model utilizes some parameters which are generally known a-priori, directly from the manufacturers (the cam profile of the indexing mechanism, all the moments of inertia of the various components, the backlash inside the speed reducer, the motor characteristic), and other parameters which are typically unknown, such as the compliance characteristics of the two joints (k_1 , r_1 , k_2 , r_2).

So, one of the main difficulties to use the model is to correctly esteem the value of these parameters. To solve this problem we have tried to use an identification technique based on a genetic algorithm which finds out the optimum set of parameters as regard a particular index which can measure the "distance" between the actual behaviour of the device and the simulated one (Figure 11).

Let's consider the actual behaviour of an indexing mechanism (for example, by the acceleration measured on the table) and the simulated result obtained by the mathematical model with the use of a particular set of parameters. After a sampling of the two graphs, it is possible to define, as goodness index, a value related with the number of points of the simulated plot which fall inside a pre-fixed area of tolerance drawn on the graph describing the actual behaviour of the device (Fig. 12). The percentage of the points falling inside the area upon the total number of examined points, determines the goodness index value. At the extremes, if all the considered points will fall inside the pre-defined area, we will say that we have a very good coincidence between the simulated and actual results (goodness index value = 100%); on the contrary, if we will obtain all the points outside the tolerance, than the index value will be zero. A big difficulty in this way of proceeding lies in the fact that it is possible to obtain very close goodness index values with different combinations of the parameters (non-coincident solutions).

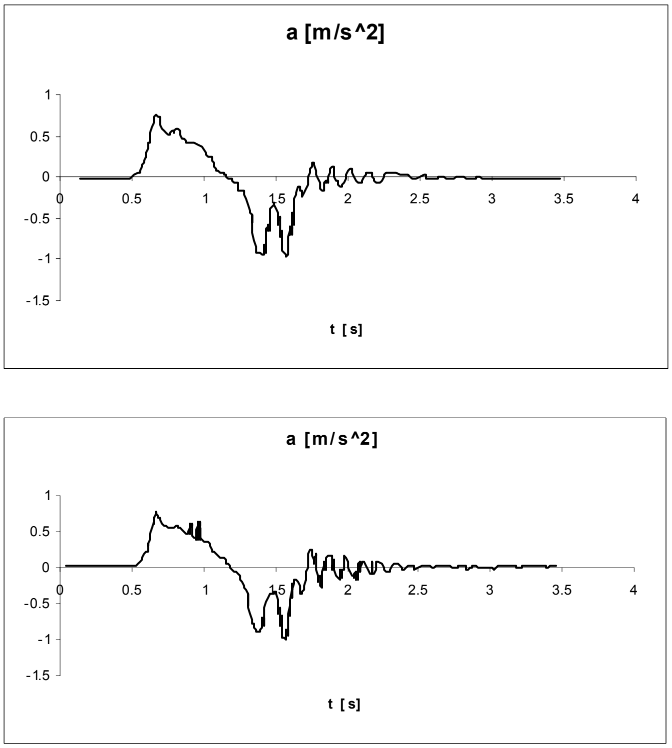

FIG. 11. Example of simulated acceleration (first graph) and measured acceleration (second graph)

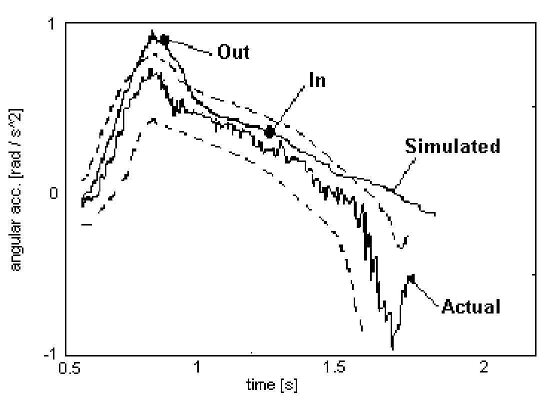

FIG. 12. Definition of goodness index

The genetic algorithm does not get a single solution, but acts in terms of families of solutions. In this way, if there are several sets of combinations of parameters, which offer very similar values of the goodness index, the genetic algorithm finds all of them. Among the different solutions, it will be the user who, by the direct comparison of the simulated and the actual graph, will select the one that appears the best for his own purpose.

6 RESIDUAL OPTIMIZATION OF THE SYSTEM

As it has been shown, the real behaviour can be dominated by effects which are impossible to consider during the design stage. In this situation, after the physical realization of the cam-follower system, it is possible to achieve a residual optimization. Usually cam-follower systems are moved by a constant speed motor; instead we will use a variable-speed servomotor and we will optimize the speed motion profile of the motor to achieve the desired dynamical behaviour of all the system (Fig. 13).

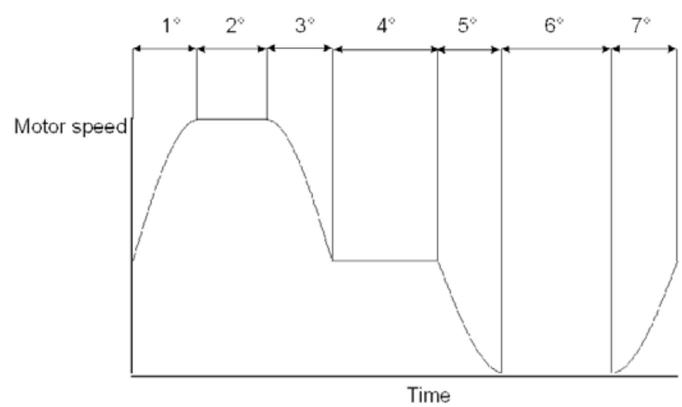

FIG. 13. Variable Motor Speed during the time cycle

So now we know the parameters of our mathematical model that allows to reproduce in an optimal way real behaviours of the system. It is easy to modify the seven segment motion lows coefficients of the variable speed servomotor and observe their effect on the motion of the follower. By the use of a genetic algorithm we change these coefficients to optimize the motion of the follower in such a way to reproduce the desired motion low of the follower

The optimization of the motor motion profile is achieved by a genetic algorithm and in Fig. 14 numerical results are showed.

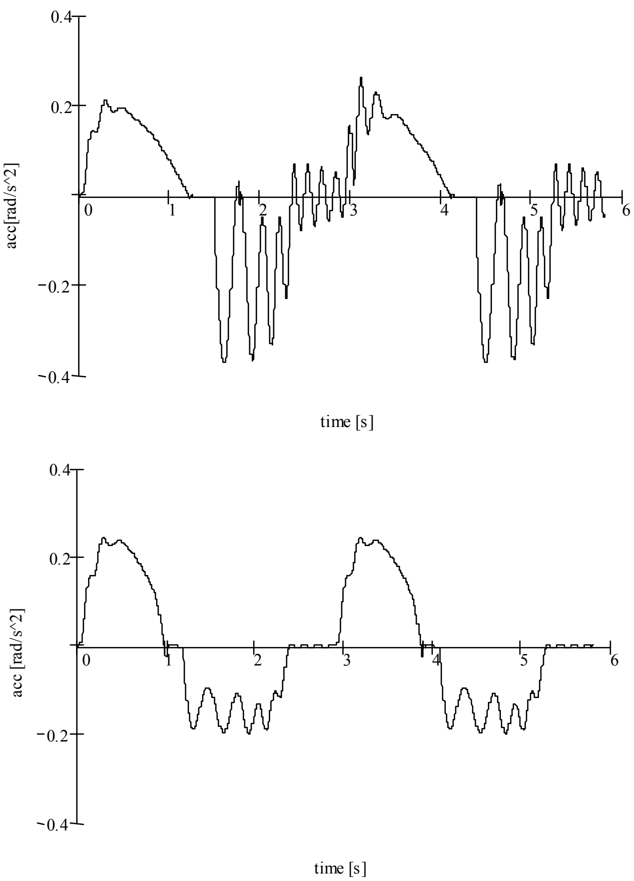

FIG. 14. Follower acceleration before and after an optimization

7 CONCLUSIONS

The problem of a cam-follower design is presented. A class of motion profiles and the best theoretical motion profile is selected by an evolutionary algorithm. A realistic model is considered and its parameter identification is achieved by a genetic algorithm. The residual optimization is achieved by a servomotor optimized by another genetic algorithm.

Evolutionary approach is used during all the design process and, as was shown, it allows really interesting performance in terms of simplicity of the design process and in terms of performance of the product.

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