Embedded Morphological Dilation Coding for 2D and 3D images

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ABSTRACT

Current wavelet-based image coders obtain high performance thanks to the identification and the exploitation of the statistical properties of natural images in the transformed domain. Zerotree-based algorithms, as "Embedded Zerotree Wavelets" (EZW) and "Set Partitioning In Hierarchical Trees" (SPIHT), offer high Rate-Distortion (RD) coding performance and low computational complexity by exploiting statistical dependencies among insignificant coefficients on hierarchical subband structures. Another possible approach tries to predict the clusters of significant coefficients by means of some form of morphological dilation. An example of a morphology-based coder is the "Significance-Linked Connected Component Analysis" (SLCCA) that has shown performance which are comparable to the zerotree-based coders but is not embedded. A new embedded bit-plane coder is proposed here based on morphological dilation of significant coefficients and context based arithmetic coding. The algorithm is able to exploit both intra-band and inter-band statistical dependencies among wavelet significant coefficients. Moreover, the same approach is used both for two and three-dimensional wavelet-based image compression. Finally the algorithms are tested on some 2D images and on a medical volume, by comparing the RD results to those obtained with the state-of-the-art wavelet-based coders.

Keywords: 2D and 3D image coding, wavelet transform, embedding, arithmetic coding, morphology.

1. INTRODUCTION

In the last years the need to store and communicate large amounts of visual data has required the study of new compression techniques. Nowadays, there exists a rich literature and standardization activities regarding image coding. Among the best two-dimensional (2D) compression schemes, wavelet based zerotree coding offers high Rate-Distortion (RD) performance with low computational complexity. This system is composed of three stages: wavelet transform, zerotree-based quantization and entropy coding. The success of the zerotree-based approach is due to the statistical exploitation of the inter-subband dependency among insignificant wavelet coefficients, which is pointed out by the reorganization of coefficients in subband tree structures. Examples of coding techniques that use the zerotree idea are "Embedded Zerotree Wavelet" coding (EZW)[1] and "Set Partitioning In Hierarchical Trees" (SPIHT).[2] Moreover EZW and SPIHT coders have been extended to the third dimension[3–5] obtaining satisfactory results in terms of RD and complexity performance for volumetric data.

In addition to the zerotree idea other statistical dependencies in the wavelet domain can be highlighted. For example the recent "Embedded Block Coding with Optimized Truncation" (EBCOT)[6] algorithm, adopted in the JPEG2000 standard, combines RD optimization, block coding and context-based arithmetic coding in an original way, which has demonstrated a good correspondence to the statistical nature of wavelet data. Such statistically matched behavior is the basis which allows to reach a low-complexity and progressive high-performance coder. The zerotree idea can also be reformulated in a dual approach in order to define the possibility to find structures of significant coefficients in the wavelet domain. Thus, an alternative way, in order to explore the statistical dependencies among wavelet coefficients, can be the prediction of the significance and the coding of a significance-map by a morphological dilation approach. The basic reason that justify the use

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of a morphology-based approach is the hypothesis of the presence of energy clusters in wavelet subbands.[7] A recently proposed algorithm based on morphological representation of wavelet data, the "Significance-Linked Connected Component Analysis" (SLCCA),[8] has shown compression performance similar to the best zerotree-based algorithms. However SLCCA does not provide an embedded bit-stream.

We propose here a new embedded bit-plane coder based on morphological dilation of significant coefficients which is able to exploit both intra-band and inter-band statistical dependencies among wavelet significant coefficients. Moreover we extend our algorithm to three-dimensional image compression. The coding results of the Lena, Barbara and Goldhill images for 2D applications and of the test medical volume CT_SKULL[3–5] for 3D applications are often sensibly superior to the state-of-the-art compression performance.

The rest of the paper is organized as follow. In Section II the statistical properties of wavelet-transformed natural images and their exploitation in zerotree-based and morphological algorithms are first discussed. In Section III our Embedded Morphological Dilation Coder EMDC is presented in detail. In Section IV the entropy coder is described. In Section V an experimental performance evaluation respect to the state-of-the-art wavelet-based coders is presented. Finally, the conclusions of the paper are in Section VI.

2. ZEROTREE AND MORPHOLOGICAL CLUSTERING APPROACHES

2.1. Statistical properties of wavelet coefficients

Recognizing the statistical properties of wavelet-transformed natural images and building a statistical model of these data is a fundamental task in the design of a high-performance wavelet-based coder. We resume here some important features of wavelet-transformed coefficients of natural images:

1. spatial-frequency localization;
2. decay of magnitude of wavelet coefficients across subbands;
3. magnitude correlation of coefficients at same spatial location in consecutive decomposition levels[1];
4. magnitude correlation of neighboring coefficients within a subband[9];
5. clustering of significant coefficients (bigger than a certain threshold) within the subbands.[7,8]

The first property is a feature of the wavelet transform and means that each wavelet coefficient refers to a specific area of the original data (i.e. it is spatially localized) and represents information in a certain frequency range (i.e. it is frequency localized). The other properties are due to the high-order statistical characteristics of natural images. In fact, the non-stationary nature of an highly correlated image source (natural image) reflects on the non-stationary nature of the weakly correlated transformed coefficients. This situation has led to the significant - non significant dichotomy and to the exploitation of high-order statistical properties of the wavelet coefficients both in the quantization and entropy coding steps. The various wavelet-based coders try to exploit some of the mentioned statistical characteristics in order to obtain good RD performance with low computational complexity.

2.2. The zerotree approach

Zerotree-based coders, as EZW[1] and SPIHT,[2] basically exploit the magnitude decay of the wavelet coefficients across the subbands by the use of subband trees collection structure, where the children of a node are the coefficients at the same spatial location in the finer scale of a subband decomposition. In fact, although in this kind of structure the linear correlation between a parent and its children is irrelevant, there is a relevant dependency among their magnitudes. Experiments on natural images showed a correlation between the square magnitude of a child and its parent between 0.2 and 0.6 with a strong concentration around 0.35.[1] Besides, the magnitude of wavelet coefficients typically decays along the parent-children hierarchy.[10] So large collection of coefficients, which can be organized in multiresolution trees, are composed of coefficients smaller than their progenitors. This behavior can be described by coding the sub-trees composed of all insignificant coefficients with a single symbol (zerotree) or by using similar set-partitioning techniques. Other theoretical justifications of the performance of zerotree-based coders can be found for example in.[11]

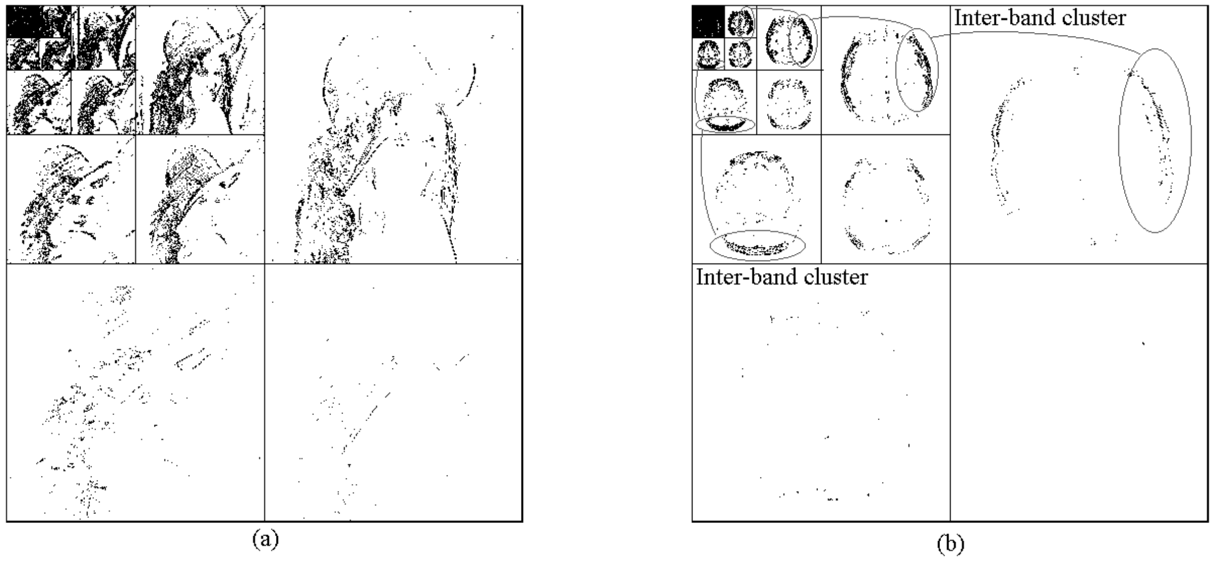

Figure 1. Significance-map for four-level decomposition. (a) Image Lena. (b) A medical image with indicated two inter-band clusters.

2.3. The clustering of significant coefficients

Even if the zerotree-based algorithms are very efficient, they don't exploit an important statistical property of wavelet-transformed natural images: the clustering of significant coefficients. This property can be very useful for compression purposes because it allows, given a small number of significant coefficients, to identify the areas where the greatest part of significant coefficients may be found. The clustering tendency of significant wavelet coefficients is due, as stated above, to the characteristics of natural images. In fact these kind of data are typically composed of large homogeneous or textured regions delimited by edge discontinuities. Homogeneous regions mostly consist of low frequency components and so the coefficients associated with these areas in the medium-high frequency subbands typically have low magnitude. Textured regions consist of a mixture of low and high frequency coefficients. Edges are mostly composed of high frequency components and so they generate a relatively small number of coefficients with high magnitude in the medium-high frequency subbands. These coefficients have a spatial distribution which depends on the local direction of the object boundaries in the original image with respect to the spatial orientation of the high and low-pass subband filters. The clustering of significant coefficients can be empirically observed in Fig.1–a where the clusters within a wavelet subband mainly correspond to the edges and secondly to the textured areas in the original image. We can also observe that these clusters tend to be localized at same spatial positions and filtering orientations at each decomposition level.[7] In fact, since the clusters are due to spatial edges and textured areas, we expect some energy concentration in these zones at all scales. For example in the presence of a vertical edge we expect to find clusters in all subbands with horizontal high-pass filtering and vertical low-pass filtering. We call inter-band clusters these sets of clusters which can be linked together through some form of hierarchical collection structures (see Fig.1–b).

2.4. The morphological approach

A method for exploiting the clustering of significant coefficients is the morphological dilation.[7,8] Before describing this technique we give a brief review of some aspects of mathematical morphology. A survey of the application of morphology to multi-dimensional signal processing may be found in.[12] We consider a set $\mathcal{S}$ of points in a N-dimensional structure and a structuring element $\mathcal{B}$ that defines a morphology-based distance. The morphological dilation of set $\mathcal{S}$ by $\mathcal{B}$ is the union of all points falling under the structuring element $\mathcal{B}$ when it is centered at each point of $\mathcal{S}$. In our context the set $\mathcal{B}$ consists of the already identified significant coefficients and through the morphological dilation we want to find new significant coefficients. With iterative

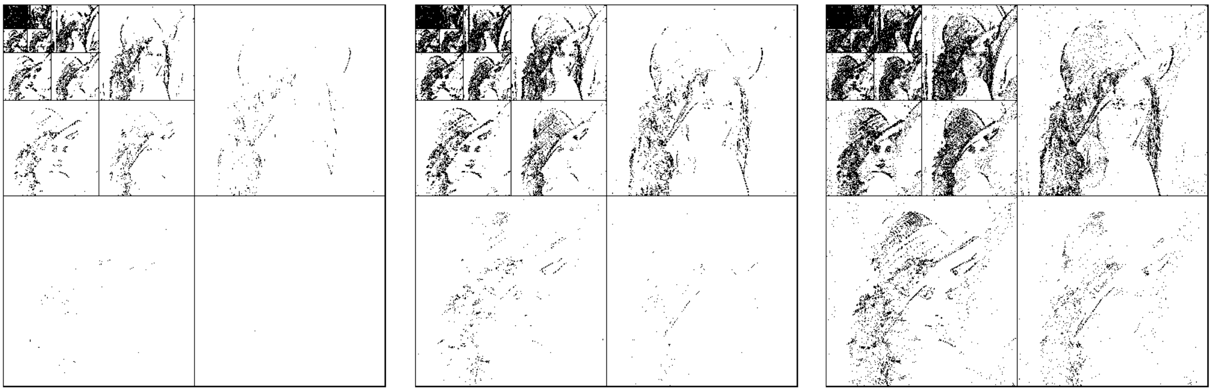

Figure 2. Significance-maps of a four-level decomposition on Lena obtained with three successive thresholds: 32, 16 and 8.

morphological dilation we refer to a sequence of morphological dilations, applied to new significant coefficients. The iteration finishes when a dilation does not find new significant coefficients. Moreover, we use the term inter-band expansion to indicate a sort of morphological dilation performed from parents towards their children in the tree collection structures. Now we can see how this concepts can be used in significance-map coding.

We consider the wavelet significant coefficients with respect to a certain threshold. As seen these coefficients tend to form clusters within the various subbands, linked in inter-band clusters. If we know the position of a very small number of significant coefficients, one for each inter-band cluster, we can identify many other significant coefficients by applying an iterative morphological dilation and an inter-band expansion. This way, we can map a great number of significant coefficients which result to be collected in inter-band clusters. To do this we have to analyze a relative small number of coefficients and explicitly code a single coefficient position for each inter-band cluster.

2.5. Our embedded morphological approach

Our primary objective is to introduce a morphological approach within an embedded framework. Considering a set of decreasing thresholds (typically the decreasing powers of two), the growing of the clusters of significant coefficients can be observed at the same time (Fig.2). If at each step (i.e. for each threshold) a morphological dilation and an inter-band expansion is applied to the already significant-marked coefficients, the large part of significant coefficients can be predicted, with a high probability, by analyzing a relative little number of the total of the wavelet coefficients. In fact, at each threshold level, it is reasonable to look for new significant coefficients inside the already detected clusters. When these clusters have been processed the attention can be moved in other regions in order to find new clusters or to identify isolated significant coefficients. Using this approach two important effects can be obtained:

1. The regions with a higher expected percentage of significant coefficient are analyzed first. This means that the significance-map coding bit-stream is statistically ordered by distortion reduction. A sort of implicit rate-distortion optimization is obtained, without any computational complexity increment or the need to transmit auxiliary information.

2. The wavelet coefficients can be partitioned in two subsets corresponding to regions where there are clusters of significant coefficients and regions where we expect to find only a few isolated significant coefficients. As shown in two experiments exposed in[7] the entropy of the composite model obtained through this partition is widely inferior to the global entropy of the wavelet coefficients. This fact can be exploited during the entropy coding step.

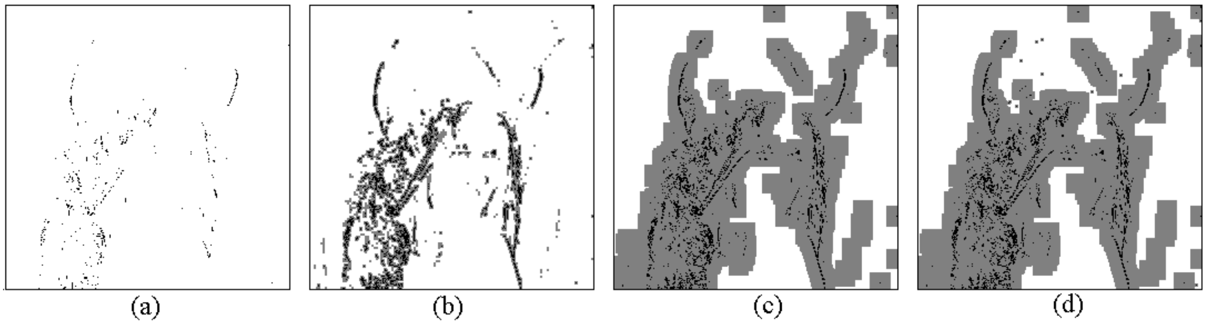

Figure 3. Different phases in significance-map coding of a Lena's subband. In black significantly-marked coefficients, in gray insignificant ones within the area considered for inspection. (a) Initial significance-map for a threshold equal to 16. (b) Significance-map after intra-band morphological dilation and inter-band expansion. (c) Significance-map after additional-morphological dilation. (d) Final significance-map after explicit position coding.

3. EMDC: EMBEDDED MORPHOLOGICAL DILATION CODING

Like in EZW and SPIHT, the proposed progressive algorithm EMDC is a bit-plane coder composed of two iterative steps: a sorting pass (or significance-map coding) that identifies significant coefficients (i.e. larger than a given threshold) and codes their position, and a refinement pass that adds to the precision of the significantly-marked coefficients. Initially the threshold is set to the largest power of two which is smaller than the largest magnitude of the wavelet coefficients. Once a scanning strategy is defined, the positions of significant coefficients are coded in the sorting pass, then a refinement pass follows to complete the bit-plane coding. At this point the threshold is halved and the scanning restarts. This way an embedded bit-stream is obtained. The coding of the significance map is a crucial task in this kind of scheme. The main idea we propose for obtaining an efficient coding of the significance-map is to look for new significant coefficients near the already significantly-marked ones within the same subband (intra-band morphological dilation), and among their sons (inter-band expansion) in the corresponding tree structure. More precisely, the significance-map coding of each subband is subdivided in four phases: intra-band morphological dilation, inter-band expansion, additional morphological dilation and explicit position coding.

1. **Intra-band morphological dilation**: the coefficients near the significantly-marked ones are analyzed through an iterative morphological dilation; this operation is repeated each time a new significant coefficient is found also during the other three phases. We tested different structuring elements for this morphological dilation and we obtained best results using a 3x3 square for images and a 3x3x3 cube for volumes.
2. **Inter-band expansion**: the sons of the significantly-marked coefficients are analyzed.
3. **Additional morphological dilation**: a morphological dilation is further performed around coefficients analyzed in previous phases and resulted insignificant; this operation is repeated until two successive scans do not lead to the detection of any new significant coefficient. Even in this case the structuring element used is a 3x3 square in 2D and a 3x3x3 cube in 3D.
4. **Explicit position coding**: the position of remaining significant coefficients is explicitly sent.

Intra-band morphological dilation and inter-band expansion allow to explore the already-identified clusters, the additional morphological dilatation analyze the coefficients on the border of clusters and finally the explicit position coding allow to find new clusters (especially at the start of the coding) and code the position of isolate significant coefficients. The four steps are ordered in term of their efficiency considering a statistical measure of distortion reduction per coding bit. For this reason the significance-map is not encoded subband by subband

but step by step: first the intra-band morphological dilation is performed on each subband, then the inter-band expansion is performed and so on. With this method the compression ratio at the end of a sorting pass is substantially unaltered but the performance at intermediate points is superior. *
Fig.3 shows, with an example, how the EMDC algorithm works during the various phases within a subband.

3.1. EMDC data structures

For implementation purposes the significance/insignificance information of each wavelet subband (l, i) (where l is the decomposition level and i the subband number) are stored into two sets of lists:

- the lists of significant coefficients ($\mathrm{LSC}_{l,i}$), containing all significantly-marked coefficients;
- the lists of insignificant coefficients ($\mathrm{LIC}_{l,i}$), containing the coefficients analyzed in the present sorting pass which resulted insignificant. These lists are used for the additional morphological dilation.

Besides, the same information is also stored in a wavelet coefficients matrix (WCM) in order to allow an immediate knowledge of the status of the coefficients: significantly-marked ("S"), insignificantly-marked ("I") or not yet analyzed ("NA") in the current sorting pass. Finally we define a data structure to store each subband status $\mathrm{SS}(l, i)$, i.e. the next step that has to be executed in the significance-map coding. Possible values in this data structure are: morphological dilation("MD"), inter-band expansion ("IE"), additional morphological dilation ("AMD"), explicit position coding ("EPC"), sorting pass terminated ("SPT") and all coefficients are significant ("ACS").

3.2. EMDC algorithm

The coding algorithm is virtually the same for 2D and 3D image coding. To know if in a subband there are new (not already marked) significant coefficients (with respect to the current 2^n threshold) we use the function $\mathcal{S}_n(l, i)$. This function is equal to 0 if all significant coefficients in the (l, i) subband are already been found, while it is equal to 1 if there are significant coefficients to find. The scanning of the wavelet coefficients, if not re-routed by the morphological operations, is performed by row and the subband scanning is from low to high resolution.

1. Initialization: MAX = maximum magnitude of wavelet coefficients; output $n = \lfloor \log_2(MAX) \rfloor$, set the threshold to 2^n, set all the $\mathrm{LSC}_{l,i}$ and $\mathrm{LIC}_{l,i}$ to empty lists, set all elements of WCM to "NA" and all subband status to "MD" *.
2. Sorting Pass:
   - for each subband (l, i) with status not equal to "ACS" do:
     - if $\mathcal{S}_n(l, i) = 0$ output 0 and set $\mathrm{SS}(l, i)$ to "SPT";
     - else output 1 and if $\mathrm{LSC}_{l,i}$ in not empty set $\mathrm{SS}(l, i)$ to "MD", otherwise if $l \geq 1$ and $\mathrm{LSC}_{l-1,i}$ is not empty set $\mathrm{SS}(l, i)$ to "IE" and otherwise set $\mathrm{SS}(l, i)$ to "EPC".

   (a) **Intra-band morphological dilation**: for each subband (l, i) with $\mathrm{SS}(l, i)$ equal to "MD":
   - for each entry in the $\mathrm{LSC}_{l,i}$ analyze the adjacent coefficients in the same subband: for each coefficient of coordinates $\overline{x}$ do:
     - if $\mathrm{WCM}(\overline{x})$ is not equal to "NA" pass to the next coefficient;
     - else if the coefficient is significant (bigger than 2^n): output 1, output its sign, add it to the end of $\mathrm{LSC}_{l,i}$ and set $\mathrm{WCM}(\overline{x})$ to "S";
     - if it is not significant: output 0, add it to the end of $\mathrm{LIC}_{l,i}$ and set $\mathrm{WCM}(\overline{x})$ to "I";
   - if $\mathcal{S}_n(l, i) = 0$ then set $\mathrm{SS}(l, i)$ to "SPT" and output 0;

*Actually is sufficient that the subband status are different from "ACS".

- else output 1 and if $l \geq 1$ and $\text{LSC}_{l-1,i}$ in not empty set $\text{SS}(l,i)$ to "IE", otherwise set $\text{SS}(l,i)$ to "AMD".

(b) **Inter-band expansion**:for each subband (l,i) with $\text{SS}(l,i)$ equal to "IE":

- For each entry in the father-subband $\text{LSC}_{l-1,i}$ the four (eight in 3D) sons are analyzed: for each coefficient of coordinates $\overline{x}$ do:
  - if $\text{WCM}(\overline{x})$ is not equal to "NA" pass to the next coefficient;
  - else if the coefficient is significant: output 1, output its sign, add it to the end of $\text{LSC}_{l,i}$, set $\text{WCM}(\overline{x})$ to "S" and immediately perform an intra-band dilation (a) on the new coefficient added to LSC;
  - if it is not significant: output 0, add it to the end of $\text{LIC}_{l,i}$ and set $\text{WCM}(\overline{x})$ to "I";
- if $\mathcal{S}_n(l,i) = 0$ then set $\text{SS}(l,i)$ to "SPT" and output 0;
- else output 1 set $\text{SS}(l,i)$ to "AMD".

(c) **Additional morphological dilation**:for each subband (l,i) with $\text{SS}(l,i)$ equal to "AMD":

- For each entry in $\text{LIC}_{l,i}$ of the subband except those included in this additional morphological dilation analyze the adjacent coefficients in the same subband: for each coefficient of coordinates $\overline{x}$ do:
  - if $\text{WCM}(\overline{x})$ is not equal to "NA" pass to the next coefficient;
  - if the coefficient is significant: output 1, output its sign, add it to the end of $\text{LSC}_{l,i}$, set $\text{WCM}(\overline{x})$ to "S" and immediately perform an intra-band dilation (a) on the new significant coefficient;
  - if it is not significant: output 0, add it to the end of $\text{LIC}_{l,i}$ and set $\text{WCM}(\overline{x})$ to "S";
- Repeat the additional morphological dilation on the new entry of $\text{LIC}_{l,i}$ until two successive scans do not lead to new significant coefficients;
- if $\mathcal{S}_n(l,i) = 0$ then set $\text{SS}(l,i)$ to "SPT" and output 0;
- else output 1 and set $\text{SS}(l,i)$ to "EPC".

(d) **Explicit position coding**:for each subband (l,i) with $\text{SS}(l,i)$ equal to "EPC":

- The position of remaining significant coefficients is explicitly sent as a binary number using a sufficient number of digits considering the number of not analyzed coefficients in the subband; also in this case the significant coefficients are added to $\text{LSC}_{l,i}$ and an intra-band dilation(a) is performed after each position coding;
- A special symbol (for example the position -1) is sent when all significant coefficients in the subband (l,i) have been identified.

3. Refinement Pass: for each entry in all LSC's, except those included in the last sorting pass, output the n-th most significant bit;

4. Set the data structures for next sorting pass:
   - Empty all $\text{LIC}_{l,i}$;
   - all elements in WCM equal to "I" are set to "NA";
   - if a subband (l,i) has all coefficients significantly-marked then $\text{SS}(l,i)$ is set to "ACS".

5. Quantization-Step Update: decrement n by 1 (the threshold is halved) and repeat from step 2.

Note that the first significant coefficient is always coded by an explicit position coding. The algorithm can be stopped in any moments, at the desired rate or distortion.

4. ENTROPY CODING IN EMDC

In order to increase the coding efficiency, the bit-stream produced by the progressive quantization is entropy coded using a context-based adaptive arithmetic coder, based on.[13] For this purpose we consider the EMDC algorithm as a binary non-stationary symbol source with memory. First we identify seven main contexts in the structure of EMDC (*structural contexts*): sign bits, refinement bits, four contexts for the four phases of significance-map coding and a context for the output of the $\mathcal{S}_n(l,i)$ function. Moreover, inside some structural-context we locate some sub-contexts (*proper contexts*) in order to better exploit the statistical relationships between the wavelet coefficients. We use the same set of contexts for 2D and 3D applications.

4.1. Significance-map coding contexts

As seen in Section II there is a correlation between magnitude of the wavelet coefficients at same spatial and frequency location in consecutive resolution levels and between neighboring coefficients within a same subband. This means that there is also a correlation between the significance value of these coefficients. This correlation can be exploited by the arithmetic coding through the identification of contexts to be associated to the significance-map structure. In order to consider and exploit the causal dependencies between two consecutive bit-plane related significance-maps, we propose (see also[5]) to make use of three significance values, distinguishing not only between significant and insignificant coefficients, but also between previously-significant (marked as significant in a previous sorting pass) and newly-significant ones (marked significant in the current sorting pass).

For the significance-map coding we identify 12 different contexts:

- 7 contexts for intra-band morphological dilation:
  - 1 context for the first level (lower resolution) subband;
  - 6 (3x2) contexts for the other subbands depending on the significance value of the father (three possible values: previously-significant, newly-significant and insignificant) and the presence of at least one significantly-marked coefficient among the adjacent ones towards the low-pass filtering direction (two possible values, except for the HH subband's coefficients which have a unique value);
- 2 contexts for inter-band expansion, depending on the significance value of the father (previously-significant or newly-significant);
- 1 context for additional morphological dilation;
- 1 context for explicit position coding and 1 context for the output of the $\mathcal{S}_n(l,i)$ function: these bits are not compressed, so these contexts are not adaptive and are set as binary and equiprobable.

4.2. Refinement bits compression

Refinement bits may be compressed because the typical subband coefficient histogram of each bit-plane interval is unbalanced towards lower values and so it is more probable that a coefficient lies in the inferior half-interval rather than in the superior one.[5] This unbalance is more pronounced for the first refinement bits and decreases as the refinement proceeds. For this reason we create two sub-contexts for refinement bits compression: a context for the first refinement bit of each coefficients and another one for the other refinement bits. In this way the refinement bits are entropy compressed by about 8%, in a bit-rate range from 0.25 to 1.0 bpp.

4.3. Sign bits compression

As stated in[6] sign bits may be compressed as well, because adjacent wavelet coefficients in mixed lowpass/highpass subbands (e.g. LH and HL in 2D) exhibit substantial statistical dependencies. In fact, along the low-pass filtering direction, neighboring coefficients exhibit, with a high probability, the same sign; while along the high-pass filtering direction they often have opposite signs. For exploiting this statistical evidence we make use of the two functions $l(\overline{x})$ and $h(\overline{x})$, that summarize the information about the significantly-marked adjacent coefficients of the coefficient $\overline{x}$, respectively towards the high-pass and the low-pass directions. These functions assume value 1 if the majority of adjacent coefficients is positive, -1 if it is negative and 0 if a sign predominance is not present. 9 (3x3) contexts are thus obtained, depending on the value of the functions $l(\overline{x})$ and $h(\overline{x})$. We note that only the sign of coefficients in mixed lowpass/highpass subbands are entropy coded. For the other sign bits we use an additional non-adaptive and equiprobable context. We experimented that sign bits may be compressed of about 10% for 2D images while in the 3D case the coding gain may even reach 40%.

4.4. Adaptivity of the model

Because of the non-stationarity of the EMDC source we use an adaptive model to obtain better compression performance. By reducing the number of bits used to store symbol frequencies (that we call frequency-bits), we can emphasize the adaptivity of the model making the arithmetic coding more reactive with respect to fluctuations in the wavelet coefficients statistics. However, we must use a sufficient number of frequency-bits in order to obtain an enough accurate approximation of the actual frequencies of the symbols.[13] This is especially true for contexts where the probabilities of the two symbols are very different (in our case the morphological dilation context presents this feature). So we had to slightly modify the arithmetic coder[13] in order to make possible the use of a different number of frequency-bits for different contexts. To do this, it is sufficient to insert a frequency-bits field in the context record, and then use, in each context, the proper frequency-bits number. We tested various combination of frequency-bits obtaining best results using 10 bits in 2D and 15 bits in 3D for the additional morphological dilation context, and 6 bits for all the other contexts both in 2D and in 3D.

5. EXPERIMENTAL RESULTS

5.1. 2D Performance

In the case of 2D coding we tested the EMDC algorithm on the Lena, Barbara and Goldhill images. We used a five-level wavelet decomposition with 9/7-tap filters.[14] We compared our coder with respect to the SPIHT[2] and EBCOT[6] embedded coders[†] and the morphological coder SLCAA.[8] As shown in Tab.1 the EMDC algorithm obtains the best performance in almost all cases. In Fig.4 the RD performance of our algorithm compared to SPIHT and EBCOT on the Goldhill image is shown. In Fig.5 some pictorial coding results are shown for the Lena image at various bit-rates.

5.2. 3D Performance

For 3D applications we tested the EMDC algorithm on the 256x256x128 medical test volume CT_SKULL and compared the coding results with respect to others state-of-art coders.[3–5] We used a 3D separable four-level decomposition wavelet transform with different combination of biorthogonal filters which have been selected in each spatial direction between the 9/7-tap and the 10/18-tap[15] ones. Our results are sensibly superior to those shown by other coders both in terms of mean PSNR on the whole volume, worst slice PSNR and in terms of mean slice based PSNR fluctuations[16] among near slices. The results are summarized in Tab.2. With mean PSNR fluctuation we refer to the mean value of the maximum slice based PSNR difference calculated for a sliding window of 10 slices. In Fig.6 the PSNR calculated on each slice along the z direction is shown for two different target rates: 0.1 and 0.5 bpp. In Fig.7 some pictorial coding results are shown for the worst slice (nr.72) of the CT_SKULL volumetric data-set, at various bit-rates.

6. CONCLUSIONS

In this paper a new embedded bit-plane coder, called Embedded Morphological Dilation Coder (EMDC), has been proposed. This algorithm is based on a morphological dilation of significant coefficients which exploits inter-band and intra-band statistical dependencies among significant coefficients and their tendency to form clusters. Some features of the proposed morphological approach, which allow to improve the coding performance, can be highlighted: the sorting pass bit-stream of each bit-plane has been structured in order to obtain a good rate-distortion characteristic and an embedded bit-stream; moreover, the wavelet coefficients have been subdivided in two regions with very different significant coefficients frequencies, in order to obtain a highly efficient context-based arithmetic coding. The EMDC algorithm has been tested both for 2D and 3D applications showing performance that are often sensibly superior with respect to the state-of-the-art embedded coders.

[†]We consider here the almost-embedded version called "Generic" in the original paper.

Table 1: PSNR results on 2D images, measured in dB.

Image	Rate(b/p)	SPIHT[2]	SLCCA[8]	EBCOT[6]	2D-EMDC
Lena	0.25	34.11	34.28	34.16	34.50
	0.50	37.21	37.35	37.29	37.57
	1.00	40.41	40.47	40.48	40.50
Barbara	0.25	27.58	28.18	28.40	28.42
	0.50	31.39	31.89	32.29	32.16
	1.00	36.41	36.69	37.11	37.35
Goldhill	0.25	30.56	30.60	30.59	30.75
	0.50	33.13	33.26	33.25	33.45
	1.00	36.55	36.66	36.59	36.97

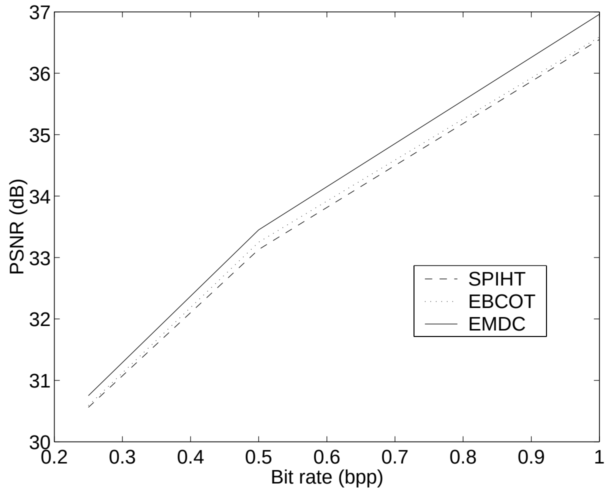

Figure 4: 2D coding results on the Goldhill image.

Figure 5. Coding results for Lena. (a) Original image. (b) Compressed image at 0.5 bpp. (c) Compressed image at 0.25 bpp.

Table 2: Mean PSNR, worst slice PSNR and mean PSNR fluctuation on the 128 slices volume CT_SKULL.

Rate (b/p)	EZW[3]	3DSPIHT[4]	I3DSPIHT[5]	3D-EMDC (9/7+10/18)	3D-EMDC (10/18 taps)
0.1	$\simeq$ 32.5	33.99	34.07	35.38	35.38
	29.8	29.2	30.98	32.31	32.62
	$\simeq$ 2.5	$\simeq$ 6.0	2.28	2.25	2.04
0.5	$\simeq$ 42	42.89	43.67	44.77	44.56
	39.5	37.5	41.64	42.88	42.87
	$\simeq$ 2.5	$\simeq$ 4.0	2.39	2.09	1.99

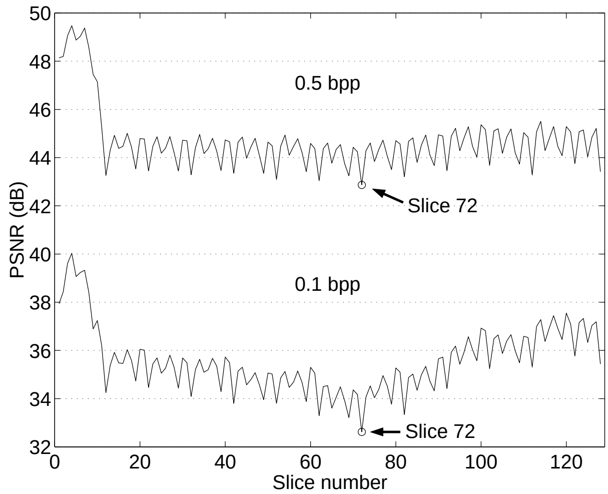

Figure 6. Coding results for CT_SKULL with 10/18 filters: slice based PSNR for 0.5 and 0.1 bpp along the z direction.

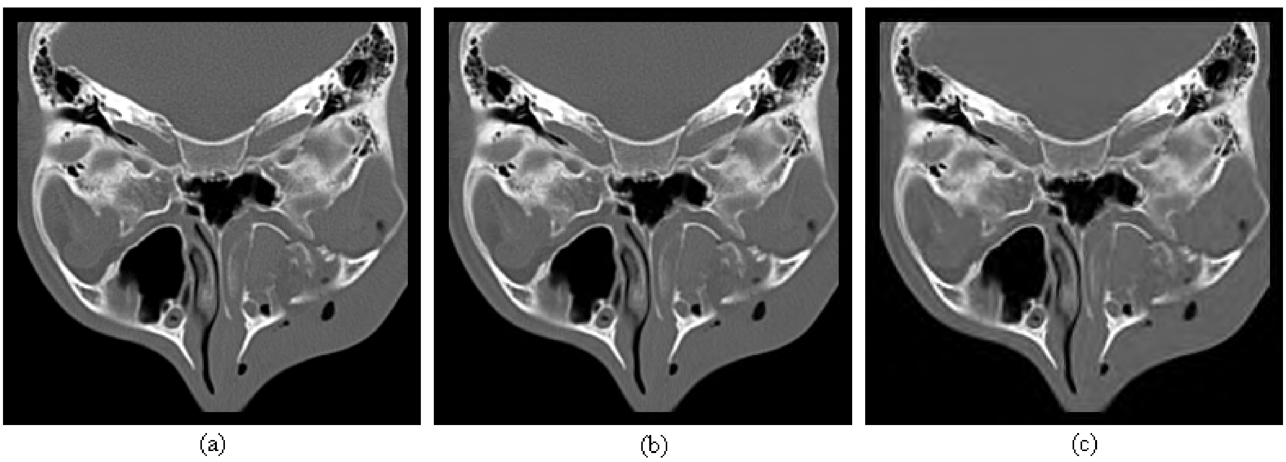

Figure 7. Coding results for the CT_SKULL volume. (a) Original slice n.72. (b) Slice n.72 of the compressed volume at 0.5 bpp. (c) Slice n.72 of the compressed volume at 0.1 bpp.

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