V Workshop on Fiscal Federalism: The Political Economy of Decentralization

Barcelona, June 13-14, 2013



PROGRAM: June 13, 2013 – LOCATION: Faculty of Economics and Business of the University of Barcelona (Av. Diagonal, 696 - Barcelona) The event will take place at Avinguda Diagonal 696 BUILDING

08:45 - 09:15	Opening address: Albert Solé-Ollé (Universitat de Barcelona & IEB) (Room: Sala de Juntes)	
09:15 - 10:00	Plenary Session: "Political Reform in China: Elections, Public Goods and Income Distribution" - Gerard Padró-i-Miquel ((London School of Economics) (Room: Sala de Juntes)
10:00 - 10:30	Coffee break	
	Session 1a - ALLOCATION OF TRANSFERS (Room: Sala de Juntes)	Session 1b - GENDER (Room: E)
10:30 - 12:15		
	Chair: Pilar Sorribas-Navarro (Universitat de Barcelona & IEB)	Chair: Monica Martinez-Bravo (CEMFI)
	"Fiscal-Capacity Equalization-Grants with Taxpayers' Lobbying"	"What Happens When a Woman Wins an Election? Evidence from Close Races in Brazil"
	Alejandro Esteller-Moré (Universitat de Barcelona & IEB), Umberto Galmarini (Università dell'Insubria), Leonzio Rizzo	Fernanda Brollo (University of Alicante), Ugo Troiano (University of Alicante)
	(Università di Ferrara)	Discussant: Johanna Rickne
	Discussant: Michaela Redoano	
	"Condina the Darie Hamas District True Dise in Transform to Halian Musicina Microsoftian"	Gender Quotas and the Unsis of the Mediocre Man: I neory and E Vidence from Sweden
	Sensing the Pork Home: Birth Lown Blas in Transfers to Italian Municipalities	Imotry Besiev (London School Economics), Olle Folke (Columbia University, SIPA & Research Institute for Industrial
	Peripe Carozzi (CEIWIFI), Luca Repetito (CEIWIFI)	Economics, Totsien Person (TES, Stockholm Oniversity), Johanna Rickne (Research institute for industrial economics
	Discussant, Filar Sofilaashavano	a oppsate center to tado otado otados
	Session 2a - DECENTRALIZATION (Room: Sala de Juntes)	Session 2h - POLITICAL COMPETITION (Room: E)
12.15 - 14.00		
12.10 14.00	Chair: Federico Revelli (University of Torino)	Chair: Albert Solé-Ollé (Universitat de Barcelona & JEB)
	"Partial Fiscal Decentralization and Public-Sector Heterogeneity: Theory and Evidence from Norway"	"Divided We Reform? Evidence from US Welfare Policies"
	Lars-Erik Borge (Norwegian University of Science and Technology), Jan K. Brueckner (University of California, Irvine)	Andreas Bernecker (University of Mannheim)
	Jorn Rattsø (Norwegian University of Science and Technology)	Discussant: Albert Solé-Ollé
	Discussant: Federico Revelli	
		"Seat Competitiveness and Redistricting: Evidence from Voting on Municipal Mergers"
	"Political Economy of Centralized Redistribution and Local Government Fiscal Structure"	Ari Hyytinen (University of Jyväskylä & Yrjö Jahnsson Foundation), Tuukka Saarimaa (Government Institute for Economic
	Stephen Calabrese (Carnegie Mellon University)	Research VATT), Janne Tukiainen (Government Institute for Economic Research VATT)
	Discussant: Amedeo Piolatto	Discussant: Olle Folke
14:00 - 15:30	WORKING Lunch (Location: Hall Aula Magna)	
	Session 3a - EFFECTS OF TRANSFERS I (Room: Sala de Juntes)	Session 3b - MERGERS (Room: E)
15:30 - 17:15		
	Chair: Daniel Wilson (Fed San Francisco)	Chair: Daniel Montolio (Universitat de Barcelona & IEB)
	The Long-run and intergenerational Education impacts of intergovernmental transfers	Local Representation and Strategic Voting: Evidence from Electoral Boundary Reforms
	Inneu de Carvano Finito (international vionetary Fund & Georgetown Oniversity School of Foreign Service in Ostor), Stanhag Litaghia (Liniversitet Damagu, Esbarg and Bagaglago CSE).	Tourka Saarmaa (Government institute for Economic Research VATT), Janne Tukianen (Government institute for Economic Research VATT)
	Qatal), Stephan Lascing (Universitat Fompeu Fabra and Barcelonia GSE)	Europhic Research (a Porto
	Discussant. Damei Wilson	Discussant. Lucated un one
	"The Effects of Unemployment Benefits on Migration in Lagging Areas"	"Cooperation among Local Governments to Deliver Public Services: a "Structural" Bivariate Response Model"
	Jordi Jofre-Monsenv (IEB and University of Barcelona)	Edoardo di Porto (Universita di Roma). Vincent Merlin (CREM CNRS & University of Caen Basse Normandie). Sonia
	Discussant: Stephan Litschig	Paty (Université de Lyon)
		Discussant: Tuukka Saarimaa
17:15 – 18:00	Plenary Session: "Will the German Debt Brake Succeed? Survey Evidence from State Politicians" - Eckhard Janeba (Un	iversity of Mannheim) (Room: Sala de Juntes)
20:30	Dinner	

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PROGRAM: June 14, 2013 – LOCATION: Faculty of Economics and Business of the University of Barcelona (Av.Diagonal, 696 - Barcelona) The event will take place at Avinguda Diagonal 696 BUILDING

00.00 40.45	Session 4a - EFFECTS OF TRANSFERS II (Room: Sala de Juntes)	Session 4D - DECENTRALIZATION (ROOM: E)
09:00 - 10:45		
	Chair: Teresa Garcia-Mila (Universitat Pompeu Fabra)	Chair: Massimo Bordignon (Catholic University, Milan & CESito)
	"Are State Governments Roadblocks to Federal Stimulus Spending? Evidence From Highway Grants in the 2009	"The Political Economy of Pricing and Investment in Congestible Facilities in a Federal State"
	Recovery Act"	Bruno De Borger (University of Antwerp), Stef Proost (University of Antwerp)
	Sylvain Leduc (Fed San Francisco), Daniel Wilson (Fed San Francisco)	Discussant: Federico Boffa
	Discussant: Teresa García-Milà	
		"Centralization and Accountability: Theory and Evidence from the Clean Air Act"
	"Resource Windfalls and Public Goods: Evidence From a Policy Reform"	Federico Boffa (Università di Macerata & IEB). Amedeo Piolatto (Universitat de Barcelona & IEB)
	Michele Valsecchi (University of Gothenburg) Ola Olsson (University of Gothenburg)	Giacomo A M Ponzetto (CREL Universitat Pompeu Fabra & Barcelona GSE)
	Discussant: Faline Carrotal (CEME)	Discussant: Massimo Bordignon
10.45 11.15		Discussant. Massino Doragion
10.45 - 11.15		
44.45 42.45	Session 5a - FISCAL RULES (ROOM: Sala de Juntes)	Session 50 - ACCOUNTABILITY (Room: E)
11:15 - 13:45		
	Chair: Dirk Foremny (Universität de Barcelona & IEB)	Chair: Amedeo Piolatto (Universitat de Barcelona & IEB)
	"Spending within Limits: Evidence from Municipal Fiscal Restraints"	"Political Competition, Tax Salience and Accountability: Theory and Some Evidence from Italy"
	Leah Brooks (Division of Research and Statistics Federal Reserve Board of Governors), Yosh Halberstam (University of	Emanuele Bracco (University of Lancaster), Francesco Porcelli (University of Warwick), Michaela Redoano (University of
	Toronto), Justin Phillips (Columbia University)	Warwick)
	Discussant: Dirk Foremny	Discussant: Lars-Erik Borge
	"Tax Limits and Local Democracy"	"The Role of Local Officials in New Democracies: Evidence from Indonesia"
	Federico Revelli (University of Torino)	Mónica Martínez-Bravo (CEMFI)
	Discussant: Leah Brooks	Discussant: Michele Valsecchi
	"Sovereign Bond Market Reactions to Fiscal Rules and No-Bailout Clauses – The Swiss Experience"	"Dress-Up Contest: A Dark Side of Fiscal Decentralization"
	Lars P. Feld (Walter Eucken Institut & University of Freiburg). Alexander Kalb (BavernLB. Department of Risk & Sector	Ruixin Wang (Tilburg University). Wendun Wang (CentER, Department of Econometrics and Operations Research.
	Analysis) Marc-Daniel Moessinger (Center for European Economic Research ZEW) Steffen Osterloh (German Council	Tilburg University)
	of Economic Experts)	Discussant: Rafeel Parchet
	Discussant' Gilberto Turati	
13:45 - 15:00	WORKING Lunch (Location: Hall Aula Magna)	
10.40 10.00	Session 6a - POLITICAL SELECTION (Room: Sala de Juntes)	Session 6h - TAY COMPETITION (Room: E)
15.00 - 16.15		
13.00 - 10.43	Chair: Giacomo Ponzetto (CREI)	Chair: Alajandro Estallar-Morá (Universitat de Barcelona & IER)
	"Fiscal Enderslipm and Political Solution"	"Are Local Tax Dates Strategic Complements or Strategic Substitutes?"
	A social exerting and Following Selection	Are Local rack rates of a tegic complements of strategic sousands:
	Massimo Boroignon (Catholic University, Milan & CESito), Matteo Gamaleno (University of Warwick), Gilberto Turati	Raphael Parchet (University of Basel / University of Lausanne)
	(University of 1 urin & CIFREL, Catholic University, Milan)	Discussant: Jordi Joffe-Monseny
	Discussant: Glacomo Ponzetto	
		"City Competition for the Creative Class"
	"Preterential Voting and the Selection of Party Leaders: Evidence from Sweden"	I hiess Buettner (Friedrich-Alexander-University (FAU) Nuremberg), Eckhard Janeba (University of Mannheim)
	Olle Folke (Columbia University, SIPA & Research Institute for Industrial Economics), Torsten Persson (IIES, Stockholm	Discussant: Stephen Calabrese
	University), Johanna Rickne (Research Institute for Industrial Economics & Uppsala Center for Labor Studies)	
	Discussant: Monica Martinez-Bravo	
16:45 - 17:30	Plenary Session: "Local Governance and Economic Development: Strengthening State Capacity at the Local Level" - Fre	derico Finan (University of California, Berkeley) (Room: Sala de Juntes)
17:30	Farewell drink (Location: Hall Aula Magna)	

Fiscal-Capacity Equalization-Grants with Taxpayers' Lobbying

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Abstract

The economic analysis of tax-base equalization-grants from central to local governments suggests that the transfer mechanism distorts fiscal policies by providing incentives to local governments to set excessively high tax rates. In this paper, we extend the analysis by allowing taxpayers to lobby the policy makers for reductions of their own tax burdens. In principle, the distortions spurring from the lobbying activity should mitigate those caused by the equalization program. In contrast, we show that taxpayers' lobbying amplifies the distortions of the equalization mechanism. The degree of fiscal equalization can then be adjusted to alleviate the efficiency costs of lobbying.

Keywords: Tax-base equalization-grants, Local fiscal policy, Special interest groups.

JEL Codes: H77, D72, H21.

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1 Introduction

Transfer programs from central to sub-national governments, aimed at fiscal equalization, represent an important feature of public sector finance in many states with multiple levels of government. The rationale for fiscal equalization comes from the fact that local tax bases, as well as local expenditure needs, are not generally uniformly distributed in the various jurisdictions. As a result, large disparities in the burden of taxation, as well as large disparities in public expenditure benefits, for otherwise similar individuals living in different regions would emerge at the local level, unless some corrective policies are undertaken. The primary role of equalization grants is then that of reducing the differentials in fiscal capacities and expenditure needs of the various local jurisdictions, thus enhancing the working of the federation both on equity and efficiency grounds.¹

A particular, but relevant, type of fiscal equalization programs, namely those aimed at tax-base equalization, has received a great deal of attention both in the theoretical and in the empirical literature. The interest lies in the fact that programs of this type are run in some important federal countries, like Canada and Germany. The aim of tax-base equalization is that of reducing the disparities of regional per capita tax bases (fiscal capacities) on own-source tax revenues.² In general terms, each local government is entitled to a grant which depends on the difference between the average (of all local governments) standard tax revenue and its standard tax revenue. However, standard tax revenues depend on effective tax bases, which in turn depend on effective tax rates. Therefore, if local tax bases are elastic, the equalization program gives the local governments an incentive to set tax rates that are higher than the optimal ones. The argument, clearly exposed by Smart (1998), is simple. An increase in local tax rates, by reducing local tax bases, increases the gap between the average and the local government standard tax revenues, thereby augmenting the grant from the central government. For empirical analysis of this issue, see Esteller-Moré and Solé-Ollé (2002) and Smart (2007) for Canada, and Buettner (2006) for Germany.

Following this line of research, some authors (see, e.g., Koethenbuerger, 2002; Bucovetsky and Smart, 2006; Rizzo, 2008), have observed that the distorting incentives of fiscal equalization should be confronted with the incentives of opposite sign arising from horizontal tax competition among local governments. Clearly, if the two effects cancel out each other, the conditions for allocative efficiency might be restored. However,

¹See Dahlby and Wilson (1994) for the analysis of the optimal design of grant schemes based on the equalization of the social marginal cost of raising tax revenue across jurisdictions.

 $^{^{2}}$ Another type of fiscal equalization programs are those based on the equalization of tax revenues (e.g., the German interstate transfer system). In this paper, we focus only on tax-base equalization.

subcentral fiscal policies suffer also from sources of distortions which originate from the political process. Indeed, as forcefully pointed out by Boadway (2004, p. 244): "... the agenda involved in taking political considerations into account in the design of an equalization system is a daunting one, and has yet to be exploited." In this paper, we take on this research agenda, by focusing on the incentives offered to local policy makers by taxpayers' lobbying.

Our premise is that local tax policies, even in the absence of equalization, can be distorted by the activities of pressure groups that lobby the policy makers for lower tax rates. If tax-base equalization is added to the picture, incentives for higher taxes come into play. Since the two sources of distortion push tax policy towards opposed directions, one could be lead to expect that tax policy in the presence of both distortions turns out to be more efficient than in the presence of only one distortionary source. In contrast to this presumption, we show that, in general, lobbying by taxpayers exacerbates the distortions arising from tax-base equalization programs.

Our analysis is based on a simple public finance model, in which a set of identical local authorities belonging to a federation finance local public expenditure by taxing incomes accruing to two types of production factors, while the central authority runs a transfer program aimed at tax-base equalization. In order to focus on the interplay between the distorting incentives of lobbying and those of fiscal equalization on local fiscal policy, we abstract from horizontal tax competition issues, by assuming that the owners of the taxed production factors supply only in their region of residence, and that they are immobile across regions. Moreover, by focusing on the case of identical regions, our analysis addresses only the efficiency aspects of lobbying and fiscal equalization, abstracting from their equity implications.

We take as benchmark case the optimal policy that would be set by a benevolent local policy maker in the absence of lobbying and fiscal equalization. In this case, optimality is defined by a standard Ramsey rule, by which relative tax rates are a function of the relative supply elasticities of the taxed production factors, with tax rates inversely related to elasticities. We then derive 'modified' Ramsey rules under the incentives of lobbying and/or fiscal equalization, in order to asses the directions of the distortions. Our analysis shows that lobbying and fiscal equalization do not simply pull tax policy towards different directions. Rather, they impact on fiscal policy in ways which are fundamentally different. While fiscal equalization lowers the perceived *absolute levels* of the marginal cost of public funds, thereby providing incentives to raise taxes above their optimal levels, lobbying usually distorts the *relative* marginal cost of public funds, thereby giving incentives to set a distorted tax structure.

The different impact that lobbying and fiscal equalization have on the marginal

costs of public funds is the key factor in driving our results. Concerning the structure of taxation, while lobbying by two antagonist groups which are *equally powerful* has no impact on fiscal policy (since there is no impact on the relative marginal cost of public funds), tax-base equalization at *uniform* rates provides incentives to raise taxes above their optimal levels (since it lowers the absolute levels of the marginal costs of public funds). Lobbying by two groups which are *not equally powerful* leads the policy maker to reduce taxation on the more powerful group and to raise it on the less powerful group (since the perceived marginal cost of taxation is lower on the former group than on the latter). In contrast, the impact of *non-uniform* fiscal equalization (which, in addition to lowering the absolute levels of the marginal costs of public funds, it also changes their relative structure), can take a variety of forms, depending on the elasticity of substitution between the taxed production factors. For instance, fiscal equalization of only one tax base leads to tax increases on both tax bases in the case of a Cobb-Douglas production function; instead, with a linear or a Leontief technology, while the tax rate of the equalized tax base increases, that of the non-equalized tax base decreases.

Lobbying and fiscal equalization bear different implications also with respect to the level of local public good provision. By distorting the *relative* marginal costs of public funds, which are an increasing and convex function of tax rates, lobbying increases the *average* marginal cost of public funds, and therefore its final effect is to lower public good provision. On the contrary, fiscal equalization always lowers the *average* marginal cost of public funds, thereby giving incentives to raise taxation and public good production.

Although our analysis suggests that taxpayers' lobbying for lower taxation does not generally offset the pressures for higher taxes which are caused by fiscal equalization, it also shows that by properly differentiating the degrees of equalization on local tax bases the central government in charge of the equalization mechanism can improve the efficiency of local fiscal policies which are distorted by the pressure of lobbying groups. In particular, when taxation is distorted in favor of a particularly powerful lobby group, it is efficiency enhancing to set a higher degree of fiscal equalization on the tax base of the more powerful group than on that of the less powerful group, because the incentives provided by fiscal equalization offset those provided by lobbying.

The theoretical literature on fiscal equalization transfers can be traced back to the works of Buchanan and Goetz (1972), Flatters, Henderson and Mieszkowski (1974), Boadway and Flatters (1982). In these works, like in part of the subsequent literature, emphasis is given to inefficiencies in the allocation of the population as a consequence of local fiscal externalities in the presence of perfect mobility of the population. In this setting, the role of transfers is to internalize the fiscal externalities. More recent

contributions, e.g. Dahlby (1996), Sato (2000) and Albouy (2012), extend the previous models by assuming imperfect mobility of the population and distortionary taxation.

While the literature which is more closely related to the object of this paper, surveyed above, addresses the efficiency aspects of fiscal equalization, other strands of the literature focus on different aspects. Lockwood (1999) and Bordignon, Manasse and Tabellini (2001) focus on redistribution among regions in a setting of asymmetric information in which, because of moral hazard and adverse selection, the optimal equalization transfers are second best. Kotsogiannis and Schwager (2008) examine how fiscal capacity equalization impacts on the accountability of local politicians. Recent empirical research stresses the importance of political incentives on the allocation of grants: Dahlberg and Johansson (2002) focus on electoral incentives, while Levitt and Snyder (1995), Larcinese, Rizzo and Testa (2006), and Solé-Ollé and Sorribas-Navarro (2008), focus on the partisan alignment hypothesis.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 examines the incentive effects of fiscal equalization on local public policy in the absence of lobbying by taxpayers, which is introduced in the following Section 4. A numerical example, presented in Section 5, illustrates how tax-base equalization and taxpayers' lobbying impact on efficiency. Section 6 concludes.

2 The model

Our theoretical framework combines some features of the models developed by Persson and Tabellini (2000, chapter 12) for the analysis of the political-economic determinants of capital and labor incomes taxation, with some features of the models used by Smart (1998) to examine the distortions on local tax policy that may be caused by tax-revenue equalization mechanisms.

2.1 The economy

Consider a federation composed of a given number of identical local jurisdictions. In each region, two types of goods are produced: a private good, Y, and a local public good, G. Good Y is produced using two factors of production: labor, L, and capital, K. The production function, Y = F(K, L), is quasi-concave and linearly homogeneous (i.e., it exhibits constant returns to scale). As for the public good, it is produced using the private good as the only input, by means of a linear technology that transforms one unit of good Y into one unit of good G. The private good serves also the role of consumption good and that of *numeraire* good. All markets are perfectly competitive. The population resident in each region is composed of two types of agents: 'workers', that supply labor, and 'capitalists', that supply capital. Both groups are composed of identical agents. Let n^{w} , $0 < n^{w} < 1$, and $n^{c} = 1 - n^{w}$, be the mass of the group of workers and capitalists, respectively. Also, all agents are assumed to be immobile, which means that they supply their own production factor only in their region of residence.

The individuals belonging to the group of capitalists take their economic decisions over a two-period span. In the first period, they decide how to allocate an exogenous endowment of a given commodity between consumption, \tilde{x}^c , and capital supply (or savings), \tilde{k} .³ Normalizing the endowment to unity, the first period budget constraint is $\tilde{x}^c = 1 - \tilde{k}$. In the second period, capitalists consume the principal plus interest income, net of taxation. The second period budget constraint is $x^c = (1 + (1 - t)r)\tilde{k}$, where x^c is second period consumption, r is the interest rate (or the rental price of one unit of capital), and t is the ad valorem tax rate on interest income, $t \in [0, 1]$.

Preferences of capitalists are represented by the utility function:

$$u^{\mathsf{c}}(\tilde{x}^{\mathsf{c}}, x^{\mathsf{c}}, G) = \zeta(\tilde{x}^{\mathsf{c}}) + x^{\mathsf{c}} + b(G), \tag{1}$$

where the strictly concave function $\zeta(\tilde{x}^c)$ represents the utility of first period consumption, and the strictly concave function b(G) represents the utility of the public good, which is enjoyed in the second period. Quasi linearity of the utility function (1) implies that all income effects fall on the demand for second-period consumption.

Taking G, t and r as given, the representative capitalist solves:

$$\max_{\tilde{k}} \zeta(1-\tilde{k}) + (1+(1-t)r)\tilde{k}.$$
(2)

The capital supply function, $\tilde{k}^{s}(r,t)$, that solves problem (2) is defined by the first order condition:⁴

$$\zeta_x(1-k) = 1 + (1-t)r.$$
(3)

To ensure an interior solution, we assume that $\lim_{\tilde{k}\to 1} \zeta_x(1-\tilde{k}) = +\infty$, $\zeta_x(1) = 0$. It is immediate to see that capital supply is increasing in r, and decreasing in t, since $\tilde{k}_r^s = -(1-t)/\zeta_{xx} > 0$, $\tilde{k}_t^s = r/\zeta_{xx} < 0$.

Workers supply labor and consume only in the second period. Their preferences are represented by the utility function:

$$u^{w}(x^{w}, h, G) = x^{w} + \phi(h) + b(G),$$
(4)

³For capitalists, all first period variables are denoted by a 'tilde'.

⁴Throughout the paper, subscripts denote partial derivatives.

where x^{w} and h are consumption and leisure, respectively, and the strictly concave function $\phi(h)$ represents the utility of leisure time. Quasi linearity of the utility function (4) in x^{w} implies the absence of income effects both on labor supply⁵ and on the demand for the public good; all income effects fall on the demand for the consumption good.

The endowment of time is normalized to unity; hence the time constraint is h+l = 1, where l is labor supply. Let w denote the wage rate. Labor income, wl, is taxed at a proportional tax rate $\tau, \tau \in [0, 1]$. The individual budget constraint is: $x^{w} = (1-\tau)wl$. Therefore, taking G, τ and w as given, the representative worker solves:

$$\max_{l} \ (1-\tau)wl + \phi(1-l).$$
(5)

The labor supply function, $l^{s}(w, \tau)$, that solves problem (5) is defined by the first order condition:

$$(1-\tau)w = \phi_h(1-l).$$
 (6)

To ensure an interior solution, we assume that $\lim_{l\to 1} \phi_h(1-l) = +\infty$, $\phi_h(1) = 0$. It is immediate to see that labor supply is increasing in w, and decreasing in τ , since $l_w^s = -(1-\tau)/\phi_{hh} > 0$, $l_\tau^s = w/\phi_{hh} < 0$.

Next we turn to the production sector. A large number of profit-maximizing competitive firms, the mass of which is normalized to one, demand labor (in the second period) and capital (in the first period) to produce the private good (in the second period). There is no depreciation of capital input during the one-period production. Therefore, taking r and w as given, the representative firm maximizes its profits by solving the problem:

$$\max_{K,L} F(K,L) - wL - rK.$$
(7)

Given that the production function is assumed to be homogeneous of degree one, the first order conditions of problem (7):

$$F_K(K,L) = r, (8)$$

$$F_L(K,L) = w, (9)$$

define only the optimal capital-labor ratio as a function of production factors relative gross prices. The supply of output is perfectly elastic, with zero profits in equilibrium.

 $^{{}^{5}}$ The quasi-linearity assumption is made for analytical convenience and can be relaxed. Note also that empirical estimates of the impact of income taxes on labor supply find weak income effects (see, e.g., Gruber and Saez, 2002).

Remark 1 The focus on workers' and capitalists' incomes taxation, while not void of some practical interest, is primarily adopted as a useful illustrative device. In fact, problems (2) and (5) defining, respectively, the supply of capital and labor inputs are formally equivalent. This means that the model can accommodate a wide variety of circumstances, each one fitting a specific context of interest. For instance, the model can be framed in terms of two types of labor inputs (e.g., skilled and unskilled), or different types of capital inputs (like land, or natural resources, in which case one should accommodate the model by setting inelastic supplies of the production factor). Therefore, the key feature of our theoretical framework does not lie in the specific example we consider. Rather, it lies in the fact of allowing for two distinct local tax bases on factor incomes, which are linked by means of a production function.

Remark 2 The existence of two distinct groups, each one composed of identical agents, implies that it is not efficient to employ distortionary income taxes to finance the public good. A uniform lump sum tax (if between-groups redistribution is not an issue), or a pair of group-specific lump sum taxes (if also some between-groups redistribution is to be achieved), would be first best efficient. However, following part of the literature (e.g., Smart, 1998), we take the identical agents setup as a convenient shortcut for representing a more realistic, but also more complex world, of heterogenous individuals, in which distortionary income taxes are to be employed because of the lack of information on individual skills and endowments that would allow for the use of personalized first best lump sum taxes.

2.2 Market equilibrium

Using the market clearing condition $n^{c}\tilde{k}^{s} = K$ to substitute for $\tilde{k}^{s} = K/n^{c}$ into Eq. (3), and using the market clearing condition $n^{w}l^{s} = L$ to substitute for $l^{s} = L/n^{w}$ into Eq. (6), we obtain the following equations:

$$\zeta_x (1 - K/n^c) = 1 + (1 - t)r, \tag{10}$$

$$\phi_h (1 - L/n^{\mathsf{w}}) = (1 - \tau)w. \tag{11}$$

The equilibrium of labor and capital markets is then defined by solving the equation system (8), (9), (10) and (11) in the unknown r, w, K and L. Notice that, by Walras law, if the labor and capital markets clear, also the market for the private good clears.⁶

⁶By adding the second period aggregate budget constraints of capitalists and workers we get $n^{c}x^{c} + n^{w}x^{w} = (1+r)n^{c}\tilde{k} + wn^{w}l - trn^{c}\tilde{k} - \tau wn^{w}l$. By adding the local government budget constraint, $G = trn^{c}\tilde{k} + \tau wn^{w}l$, we get $n^{c}x^{c} + n^{w}x^{w} + G = (1+r)n^{c}\tilde{k} + wn^{w}l$. Using the fact that, when the

Denote the equilibrium prices and quantities as $w^*(\tau, t)$, $r^*(\tau, t)$, $L^*(\tau, t)$, $K^*(\tau, t)$. Let:

$$\Delta = (1 - \tau) n^{w} \zeta_{xx} F_{LL} + (1 - t) n^{c} \phi_{hh} F_{KK} + \phi_{hh} \zeta_{xx} > 0.$$
(12)

In Appendix A.1, it is shown that the labor income tax τ impacts on market equilibrium as follows:

$$w_{\tau}^{*} = \Delta^{-1} n^{w} w^{*} \zeta_{xx} F_{LL} > 0, \qquad K^{*} r_{\tau}^{*} = -L^{*} w_{\tau}^{*} < 0, \tag{13}$$

$$K_{\tau}^* = \Delta^{-1} n^{\mathsf{w}} n^{\mathsf{c}} w^* (1-t) F_{LL}(L^*/K^*) < 0, \tag{14}$$

$$L_{\tau}^* = w_{\tau}^* / F_{LL} + K_{\tau}^* (L^* / K^*) < 0.$$
⁽¹⁵⁾

The taxation of labor income bears a negative impact on both labor and capital utilization, with a negative impact on production. Note that while an increase in τ reduces the equilibrium gross interest rate r^* , it increases the equilibrium gross wage w^* , meaning that the tax on labor income is partially shifted to producers. The first negative term in Eq. (15) accounts for the substitution of labor for capital into production as a consequence of labor taxation; this term vanishes if the production function is of Leontief type.

The tax on interest income t impacts on market equilibrium in a specular, way (see Appendix A.1):

$$r_t^* = \Delta^{-1} n^c r^* \phi_{hh} F_{KK} > 0, \qquad L^* w_t^* = -r_t^* K^* < 0, \tag{16}$$

$$L_t^* = \Delta^{-1} n^{\mathsf{w}} n^{\mathsf{c}} r^* (1 - \tau) F_{KK}(K^*/L^*) < 0, \tag{17}$$

$$K_t^* = r_t^* / F_{KK} + L_t^* (K^* / L^*) < 0.$$
(18)

2.3 Income and welfare levels

Denote as

$$R^{*}(\tau,t) = r^{*}(\tau,t)K^{*}(\tau,t), \qquad W^{*}(\tau,t) = w^{*}(\tau,t)L^{*}(\tau,t),$$
(19)

the per capita gross income of capitalists and workers, respectively. Using the results derived above, it is immediate to see that

$$R_{\tau}^{*} = r_{\tau}^{*}K^{*} + r^{*}K_{\tau}^{*} < 0, \qquad W_{t}^{*} = w_{t}^{*}L^{*} + w^{*}L_{t}^{*} < 0, \tag{20}$$

capital and the labor markets clear, $n^{c}\tilde{k} = K$ and $n^{w}l = L$, and the fact that Y = rK + wL, we finally obtain that $n^{c}x^{c} + n^{w}x^{w} + G = K + Y$, i.e., market clearing in the market for the private good.

i.e., the labor (resp. interest) income tax has a negative impact on capitalists (resp. workers) income. On the contrary, the sign of the direct impacts is in general ambiguous, since:

$$R_t^* = r_t^* K^* + r^* K_t^* = \Delta^{-1} n^c r^* \phi_{hh} (F_{KK} K^* + r^*) + r^* L_t^* (K^*/L^*),$$
(21)

$$W_{\tau}^{*} = w_{\tau}^{*}L^{*} + w^{*}L_{\tau}^{*} = \Delta^{-1}n^{w}w^{*}\zeta_{xx}(F_{LL}L^{*} + w^{*}) + w^{*}K_{\tau}^{*}(L^{*}/K^{*}).$$
(22)

Using the first order conditions (8) and (9), it is immediate to see that sufficient conditions for $R_t^* < 0$ and $W_{\tau}^* < 0$ are $F_{KK}K + F_K > 0$ and $F_{LL}L + F_L > 0$, respectively (these conditions hold, for instance, when the production function is of Cobb-Douglas type). The case $R_t^* > 0$ and $W_{\tau}^* > 0$ may occur when a large degree of complementarity between labor and capital into production (as in the limit case of Leontief technology) is coupled with sufficiently inelastic supplies of the two factors. Note, however, that aggregate income, $R^* + W^*$, is always decreasing in both tax rates. This means that when the direct effects R_t^* and W_{τ}^* are positive, then the indirect ones, R_{τ}^* and W_t^* , which are always negative, outweighs the former.

Next we turn to welfare levels. By inserting the equilibrium quantities into the respective utility functions (1) and (4), the per capita welfare of capitalists and workers can be written as:

$$v^{c}(\tau, t, G) = \zeta (1 - K^{*}/n^{c}) + (1 + (1 - t)r^{*})(K^{*}/n^{c}) + b(G),$$
(23)

$$v^{\mathsf{w}}(\tau, t, G) = \phi(1 - L^*/n^{\mathsf{w}}) + (1 - \tau)w^*(L^*/n^{\mathsf{w}}) + b(G).$$
(24)

By differentiating Eqs. (23) and (24) with respect to the tax rates, and by applying the envelope theorem, we see that both the labor and the interest income tax bear a negative impact both on workers and on capitalists:⁷

$$n^{c}v_{\tau}^{c} = (1-t)K^{*}r_{\tau}^{*} < 0, \tag{25}$$

$$n^{c}v_{t}^{c} = ((1-t)r_{t}^{*} - r^{*})K^{*} < 0,$$
(26)

$$n^{\mathsf{w}}v_{\tau}^{\mathsf{w}} = ((1-\tau)w_{\tau}^{*} - w^{*})L^{*} < 0, \tag{27}$$

$$n^{\mathsf{w}}v_t^{\mathsf{w}} = (1-\tau)L^*w_t^* < 0.$$
⁽²⁸⁾

3 Optimal local government's fiscal policy

In this section, we set the benchmark by considering the choices taken by benevolent local policy makers. The distortionary incentives that may arise from taxpayers' lobbying and/or from tax-revenue equalization grants are examined in Section 4.

⁷To see that Eq. (26) is negative, use Eqs. (12) and (16) to show that $(1-t)r_t^* - r^* = -\Delta^{-1}(\phi_{hh} + n^w(1-\tau)F_{LL})\zeta_{xx}r^* < 0$. To see that Eq. (27) is negative, use Eqs. (12) and (13) to show that $(1-\tau)w_{\tau}^* - w^* = -\Delta^{-1}(\phi_{hh} + n^c(1-t)F_{KK})\zeta_{xx}w^* < 0$.

We assume that each local government takes a Utilitarian objective function, by which workers and producers welfare levels are linearly added. Social welfare is then defined as:

$$\tilde{V}(\tau, t, G) = n^{c} v^{c}(\tau, t, G) + n^{w} v^{w}(\tau, t, G).$$
⁽²⁹⁾

Letting public good expenditure to be defined from the local government budget constraint,

$$\tilde{G}(\tau, t) = \tau W^*(\tau, t) + t R^*(\tau, t),$$
(30)

the problem solved by the benevolent local policy maker is that of maximizing Eq. (29) subject to Eq. (30).

Using Eqs. (25)–(28), and differentiating Eq. (30), the first order conditions with respect to the tax rates are:

$$b_G(\tilde{G}) = -\frac{\tilde{V}_{\tau}}{\tilde{G}_{\tau}} \quad \Rightarrow \quad b_G(\tilde{G}) = -\frac{(1-t)K^*r_{\tau}^* + ((1-\tau)w_{\tau}^* - w^*)L^*}{W^* + \tau W_{\tau}^* + tR_{\tau}^*}, \tag{31}$$

$$b_G(\tilde{G}) = -\frac{\tilde{V}_t}{\tilde{G}_t} \quad \Rightarrow \quad b_G(\tilde{G}) = -\frac{(1-\tau)L^* w_t^* + ((1-t)r_t^* - r^*)K^*}{R^* + \tau W_t^* + tR_t^*}.$$
 (32)

By using Eqs. (13)–(15), (16)–(18), (20)–(22), these first order conditions can be written as:

$$b_G = \frac{(\tau - t)L^* w_\tau^* + W^*}{W^* + (\tau - t)w_\tau^* L^* + \tau w^* w_\tau^* / F_{LL} + (\tau w^* L^* / K^* + tr^*) K_\tau^*},$$
(33)

$$b_G = \frac{(t-\tau)K^*r_t^* + R^*}{R^* + (t-\tau)r_t^*K^* + tr^*r_t^*/F_{KK} + (tr^*K^*/L^* + \tau w^*)L_t^*}.$$
(34)

By equating the right hand sides of Eqs. (33) and (34), we get the equation that defines the optimal tax structure, for given level of public good provision. Using Eqs. (13)-(15) and (16)-(18), after some manipulations, we get:

$$\frac{t}{\tau} = \frac{n^{\mathsf{w}}w^*K^*\zeta_{xx}}{n^{\mathsf{c}}r^*L^*\phi_{hh}}.$$
(35)

By using the fact that:

$$\zeta_{xx} = -\frac{(1-t)n^{c}r^{*}}{K^{*}\varepsilon^{k}}, \qquad \phi_{hh} = -\frac{(1-\tau)n^{w}w^{*}}{L^{*}\varepsilon^{l}}, \tag{36}$$

where $\varepsilon^k \ge 0$ and $\varepsilon^l \ge 0$ are the elasticities of capital and labor supplies, respectively, we finally obtain the following relation between the two tax rates:

$$\frac{t}{1-t}\varepsilon^k = \frac{\tau}{1-\tau}\varepsilon^l.$$
(37)

This is a Ramsey type formula for efficient taxation, which shows $\varepsilon^l \gtrless \varepsilon^k$ implies $t \gtrless \tau$. That is, the less elastic tax base should bear a lower tax rate than the more elastic tax base, since in this way the excess burden for financing the public good is minimized.

Remark 3 The assumptions of (i) a Utilitarian social welfare function and (ii) a constant marginal utility of income for the agents belonging to the groups of workers and capitalists, imply that the optimal tax policy derived above serves only efficiency considerations. Distributional considerations between the two groups of taxpayers play no role.

4 Taxpayers' lobbying and fiscal equalization

In this Section we examine how fiscal policy may be influenced by taxpayers' lobbying and by the incentives offered by a mechanism of equalization transfers.

4.1 The lobbying game

We assume that the lobbying activity takes a 'legal' and 'public' form, in which each group of taxpayers (by means of an association representing their interests) makes monetary offers to the policy maker (in the form of campaign contributions, for instance) conditional on changes in fiscal policy. This 'buying influence' approach for modelling lobbying behavior has been popularized in the context of 'common agency' (many principals, one agent) games by Dixit et al. (1997) and Grossman and Helpman (1994, 2001), building on previous work by Bernheim and Whinston (1986a, 1986b).

The lobbying game evolves along two stages. In the first, the taxpayers' associations (the principals of the game) present to the policy maker a menu of offers, that is a series of contributions, each one associated to a given fiscal policy, from which the policy maker, upon acceptance of the offer in the second stage of the game, can make the preferred choice. The game is solved backward. Formally, we assume that each lobby group presents to the policy maker *truthful*, or *compensating*, contribution functions, in the way defined by Dixit et al. (1997). A truthful contribution function is shaped along an indifference curve of the lobby group, so that any given change in the policy instrument brings about a corresponding change in the contribution that is equal to the change in the payoff of the lobby group. Truthful contribution functions (not necessarily truthful) of the other principals always contains a truthful contribution schedule (Dixit et al., 1997, proposition 2).

The truthful contributions offered by workers and capitalists are:

$$C(\tau, t, G; \lambda^{\mathsf{w}}, \bar{\pi}^{\mathsf{w}}) = \lambda^{\mathsf{w}} \max\left\{0, n^{\mathsf{w}} v^{\mathsf{w}}(\tau, t, G) - \bar{\pi}^{\mathsf{w}}\right\},\tag{38}$$

$$C(\tau, t, G; \lambda^{\mathbf{c}}, \bar{\pi}^{\mathbf{c}}) = \lambda^{\mathbf{c}} \max\left\{0, n^{\mathbf{c}} v^{\mathbf{c}}(\tau, t, G) - \bar{\pi}^{\mathbf{c}}\right\},\tag{39}$$

respectively, where $\bar{\pi}^{w} \geq 0$ and $\bar{\pi}^{c} \geq 0$ are scalars representing the net payoff of the lobby group.⁸ The parameters $\lambda^{w} \in [0, 1]$ and $\lambda^{c} \in [0, 1]$ capture, in a reduced form, how effective is each group in its lobbying activity. At one end, the closer to zero the parameter λ is, the less effective is the group, say because many of its members behave as free riders. At the other end, when the group is very strong because most of its members actively participate in the lobbying process, then λ is close to one. That is, the parameter λ represents the degree of influence that the lobby group is able to exert on the policy maker. Note finally that each contribution schedule is conditional on all fiscal policy instruments, and not only of the tax directly affecting the group.

Under the influence of lobbying, the objective function of the policy maker is then equal to:

$$\tilde{V}(\tau, t, G) + C(\tau, t, G; \lambda^{\mathsf{w}}, \bar{\pi}^{\mathsf{w}}) + C(\tau, t, G; \lambda^{\mathsf{c}}, \bar{\pi}^{\mathsf{c}}),$$
(40)

where $\tilde{V}(.)$ is the social welfare function defined in Eq. (29).

4.2 Tax-base equalization-grants

Public good expenditure is now defined by the following local government budget constraint:

$$G(\tau, t; \bar{e}, \alpha^l, \alpha^k) = \tau W^* + tR^* + e(\tau, t; \bar{e}, \alpha^l, \alpha^k),$$
(41)

where $e(\tau; \bar{e}, \alpha^l, \alpha^k)$ is an unconditional grant from the central government.

As for the tax-base equalization mechanism, it takes the following form:

$$e(\tau; \bar{e}, \alpha^l, \alpha^k) = \bar{e} + \alpha^l \bar{\tau} \left(\bar{W}^* - W^* \right) + \alpha^k \bar{t} \left(\bar{R}^* - R^* \right), \tag{42}$$

where $\bar{e} \geq 0$, $\alpha^l \in [0, 1]$, $\alpha^k \in [0, 1]$. The term \bar{e} represents the lump sum component of the grant.⁹ The other terms represent the equalization of fiscal capacities, with α^l and α^k the parameters expressing the degree of equalization of local tax bases, respectively labor and interest income. In the latter case, given the average tax base of all regions, \bar{R}^* , and the average tax rate, \bar{t} , the equalization grant covers the share α^k of the gap between the standardized average tax revenue, $\bar{t}\bar{R}^*$, and the standardized tax revenue, $\bar{t}R^*$, of the local government. The same mechanism applies to labor taxation.

⁸We solve only for the second stage of the lobby game, in which fiscal policy is set under the incentives of the contribution function. We do not solve for the first stage of the game, in which the scalars $\bar{\pi}^w$ and $\bar{\pi}^c$ are determined. The reason is that we are only interested in the efficiency consequencies of lobbying, and not on its distributional impact.

⁹The lump sum component \bar{e} of the transfer is a measure of the vertical fiscal gap.

Notice that an increase in one of the tax rates, by reducing both tax bases W^* and R^* , determines, *coeteris paribus*, an increase of the grant. That is, the equalization of each one of the tax bases provides an increase to over-tax both tax bases.

Remark 4 In our framework with identical regions, the equilibrium is always symmetric, with $e(.) = \bar{e}$ for all regions, since $\bar{\tau} = \tau$, $\bar{W}^* = W^*$, $\bar{t} = t$, $\bar{R}^* = R^*$, for all regions. That is, in equilibrium there is no redistribution of resources among identical regions. Hence, in principle fiscal equalization should not be used, since it is costly in terms of efficiency. However, the focus of the paper is precisely the evaluation of the interplay between the distortions arising from fiscal equalization with those arising from taxpayers' lobbying, leaving aside distributional considerations.

4.3 Local government's fiscal policy

Ignoring the non-negativity constraint on the contribution function, in stage two of the lobby game the local policy maker chooses the tax rates (τ, t) and public good supply G to maximize the objective function:

$$V(\tau, t, G) = (1 + \lambda^{c})n^{c}v^{c}(\tau, t, G) + (1 + \lambda^{w})n^{w}v^{w}(\tau, t, G),$$

$$(43)$$

subject to the budget constraint (41) and to the equalization transfer mechanism (42).

Note that the optimal fiscal policy derived in Section 3 is encompassed as a special case by setting $\lambda^{c} = \lambda^{w} = 0$ and $\alpha^{l} = \alpha^{k} = 0$. If $\lambda^{c} \neq \lambda^{w}$, $\lambda^{c} \ge 0$, $\lambda^{w} \ge 0$, then the policy maker maximizes a distorted social welfare function.

Using Eqs. (25)–(28), and Eqs. (13) and (16), the impact of a change in the tax rates on social welfare can be expressed as:

$$V_{\tau} = \left[(1 - \tau)(1 + \lambda^{w}) - (1 - t)(1 + \lambda^{c}) \right] L^{*} w_{\tau}^{*} - (1 + \lambda^{w}) W^{*}, \tag{44}$$

$$V_t = -\left[(1-\tau)(1+\lambda^{\rm w}) - (1-t)(1+\lambda^{\rm c})\right] K^* r_t^* - (1+\lambda^{\rm c}) R^*.$$
(45)

By differentiating Eq. (41) subject to Eq. (42) we get:

$$G_{\tau} = W^* + (\tau - \alpha^l \bar{\tau}) W_{\tau}^* + (t - \alpha^k \bar{t}) R_{\tau}^*, \tag{46}$$

$$G_t = R^* + (\tau - \alpha^l \bar{\tau}) W_t^* + (t - \alpha^k \bar{t}) R_t^*.$$
(47)

In computing these expressions, we made the 'small jurisdictions assumption', which implies that the local government takes the average tax rates $\bar{\tau}$ and \bar{t} as given when setting its own tax rates.

In a symmetric equilibrium (recall that in our model with identical regions there are only symmetric equilibria), $\tau = \bar{\tau}$ and $t = \bar{t}$. Hence, we can write Eqs. (46) and

(47) as:

$$G_{\tau} = W^* + \tau \delta^l W^*_{\tau} + t \delta^k R^*_{\tau}, \tag{48}$$

$$G_t = R^* + \tau \delta^l W_t^* + t \delta^k R_t^*, \tag{49}$$

where we define

$$\delta^l = 1 - \alpha^l, \qquad \delta^k = 1 - \alpha^k,$$

to shorten notation. Also, let:

$$\bar{\lambda} = n^{\mathbf{c}}\lambda^{\mathbf{c}} + n^{\mathbf{w}}\lambda^{\mathbf{w}}.$$

By substituting the expression for G from Eq. (41) into Eq. (43) for social welfare, and then differentiating with respect to the tax rates, we obtain the following pair of first order conditions, which define the equilibrium fiscal policy:

$$(1+\bar{\lambda})b_G = -\frac{V_\tau}{G_\tau},\tag{50}$$

$$(1+\bar{\lambda})b_G = -\frac{V_t}{G_t}.$$
(51)

As we show in Appendix A.2, these first order conditions can be manipulated to obtain the following equations:

$$(1+\bar{\lambda})b_{G} = \frac{-\left[(1-\tau)(1+\lambda^{w})-(1-t)(1+\lambda^{c})\right]L^{*}w_{\tau}^{*}+(1+\lambda^{w})W^{*}}{W^{*}+(\tau\delta^{l}-t\delta^{k})w_{\tau}^{*}L^{*}+\tau\delta^{l}w^{*}w_{\tau}^{*}/F_{LL}+(\tau\delta^{l}w^{*}L^{*}/K^{*}+t\delta^{k}r^{*})K_{\tau}^{*}},$$

$$(52)$$

$$(1+\bar{\lambda})b_{G} = \frac{\left[(1-\tau)(1+\lambda^{w})-(1-t)(1+\lambda^{c})\right]K^{*}r_{t}^{*}+(1+\lambda^{c})R^{*}}{R^{*}+(t\delta^{k}-\tau\delta^{l})r_{t}^{*}K^{*}+t\delta^{k}r^{*}r_{t}^{*}/F_{KK}+(t\delta^{k}r^{*}K^{*}/L^{*}+\tau\delta^{l}w^{*})L_{t}^{*}}.$$

$$(53)$$

By equating the right hand sides of Eqs. (52) and (53), we get an equation that allows for the determination of the relative tax rates, i.e., one tax rate as a function of the other one. By comparing the resulting expression with that given in Eq. (37), one should be able to assess how lobbying and fiscal equalization distort the tax rates. By considering also the local government budget constraint (41), one should then be able to solve also for the absolute tax rates and the level of public good supply, and therefore to assess whether lobbying and fiscal equalization determine under- or over-provision of the public good. Unfortunately, a full characterization of the answers to the above questions is not possible at this level of generality. Hence, in what follows we look at some specific but important cases, and then return to the general formulation by looking at some numerical simulations. But before doing this, we state a useful result that can be obtained from Eqs. (52) and (53). **Proposition 1** If workers and capitalists are equally effective in lobbying, i.e., $\lambda^{w} = \lambda^{c}$, then lobbying has no impact on the fiscal policy of the local government.

The proof follows immediately by setting $\lambda^{w} = \lambda^{c} = \lambda = \overline{\lambda}$ into Eqs. (52) and (53); then the factor $(1 + \lambda)$ appears on both sides of the equations and can be simplified.

Lobbying is effective in distorting fiscal policy only when the two groups of taxpayers are not equally strong in making pressure on the policy maker.

Lobbying without fiscal equalization

Consider first the special case in which there is no fiscal equalization (i.e, $\alpha^l = \alpha^k = 0$, which implies $\delta^l = \delta^k = 1$), but there is lobbying. Then the formula (37) linking tax rates at the equilibrium fiscal policy is amended as follows:

$$\frac{t}{1-t}(1+\lambda^{\mathbf{w}})\varepsilon^{k} + \lambda^{\mathbf{c}} = \frac{\tau}{1-\tau}(1+\lambda^{\mathbf{c}})\varepsilon^{l} + \lambda^{\mathbf{w}}.$$
(54)

As expected, this formula shows that taxation tilts in favor of the more powerful group. In formal terms, let (τ^0, t^0) be the set of equilibrium tax rates that can occur in the absence of lobbying and of fiscal equalization, i.e., all pairs of tax rates satisfying Eq. (37). Let $(\tau^{\rm L}, t^{\rm L})$ be the set of equilibrium tax rates that can occur in the presence of lobbying but in the absence of fiscal equalization, i.e., all pairs of tax rates satisfying Eq. (54). It is then immediate to see that $t^{\rm L} \leq t^0$ for $\tau^{\rm L} = \tau^0$ whenever $\lambda^{\rm c} \geq \lambda^{\rm w}$.¹⁰ The more powerful lobby group succeeds in lowering its own tax rate at the expense of the other group. This distortion in the tax mix, by determining an increase in the cost of public funds, results in a lower level of tax revenue and public good provision with respect to the optimal level.

Fiscal equalization without lobbying

Next we examine the special case in which there is no lobbying (i.e., $\lambda^{c} = \lambda^{w} = 0$; however, note that Proposition 1 implies that the same results hold in the case of equally effective groups, i.e., $\lambda^{c} = \lambda^{w} > 0$), but there is fiscal equalization (i.e., $\alpha^{l} > 0$, $\alpha^{k} > 0$, which implies $\delta^{l} < 1$, $\delta^{k} < 1$). In this case, the formula linking in equilibrium the two tax rates turns out to be as follows:

$$\frac{t}{\tau} = \frac{(n^{\mathsf{w}}F_{LL}\zeta_{xx} + n^{\mathsf{c}}F_{KK}\phi_{hh})L^{*}\alpha^{l} - n^{\mathsf{w}}w^{*}\zeta_{xx}(1-\alpha^{l})}{(n^{\mathsf{w}}F_{LL}\zeta_{xx} + n^{\mathsf{c}}F_{KK}\phi_{hh})K^{*}\alpha^{k} - n^{\mathsf{c}}r^{*}\phi_{hh}(1-\alpha^{k})} \left(\frac{K^{*}}{L^{*}}\right).$$
(55)

¹⁰More precisely, $t^{\rm L} \leq t^0$ for $\tau^{\rm L} = \tau^0$ iff $\lambda^{\rm c} \geq \lambda^{\rm w}$ for tax rates such that tax revenue is on the increasing side of the Laffer curve. At the bliss point of the Laffer curve, it is $t^{\rm L} = t^0$, $\tau^{\rm L} = \tau^0$, for all $\lambda^{\rm c} > 0$, $\lambda^{\rm w} > 0$.

This expression obviously reduces to Eq. (35) for $\alpha^l = \alpha^k = 0$ (no fiscal equalization). It then shows that the equilibrium tax structure is always $t = \tau$ under full tax-base equalization (i.e., $\alpha^l = \alpha^k = 1$). That is, under full equalization the policy maker has no incentives to differentiate the tax rates according to the elasticities of the factor supplies, and therefore she sets a uniform tax rate on the two tax bases. The result is due to the fact that the higher the degree of equalization the lower is the elasticity of tax bases perceived by the policy maker; with full equalization, tax bases are perceived as inelastic, and therefore their elasticities play no role in determining the chosen tax structure.

Concerning the level of taxation and public good supply, fiscal equalization determines the well-known incentive to overtaxation, since it reduces the perceived cost of public funds by the policy maker.

Additional interesting aspects on the relation between fiscal equalization and the structure of the equilibrium tax policy are illustrated in the numerical simulation in Section 5 below, where we consider what happens when the two tax bases are equalized at different rates.

Linear production function

Thanks to its analytical simplicity, the case of a linear production technology allows for the joint analysis of the impact on fiscal policy of both lobbying and fiscal equalization.

With perfect substitutability between capital and labor, the production function is linear in input quantities, with constant marginal products, $F_L = \bar{F}_L > 0$, $F_K = \bar{F}_K > 0$; hence, $F_{LL} = F_{KK} = 0$. The equilibrium gross factor prices, $w^* = \bar{F}_L$, $r^* = \bar{F}_K$, are independent of tax rates, and each tax rate bears a negative impact only on the equilibrium quantity of the taxed input. That is, $w^*_{\tau} = r^*_{\tau} = K^*_{\tau} = 0$, $L^*_{\tau} = n^{w}w^*/\phi_{hh} < 0$, and $w^*_t = r^*_t = L^*_t = 0$, $K^*_t = n^c r^*/\zeta_{xx} < 0$.

Taxation impacts on the objective function of the policy maker and on public good provision as follows:

$$V_{\tau} = -(1 + \lambda^{w})W^{*}, \quad G_{\tau} = W^{*} + \tau \delta^{l}W_{\tau}^{*}, \quad W_{\tau}^{*} = -\frac{W^{*}\varepsilon^{l}}{1 - \tau},$$
$$V_{t} = -(1 + \lambda^{c})R^{*}, \quad G_{t} = R^{*} + t\delta^{k}R_{t}^{*}, \quad R_{t}^{*} = -\frac{R^{*}\varepsilon^{k}}{1 - t}.$$

Provided that $W^* > 0$, $R^* > 0$, the first order with respect to the tax rates can then

be written as:

$$b_G(G) = \frac{1+\lambda^{\mathrm{w}}}{1+\bar{\lambda}} \frac{1}{1-\frac{\tau}{1-\tau}(1-\alpha^l)\varepsilon^l},\tag{56}$$

$$b_G(G) = \frac{1+\lambda^c}{1+\bar{\lambda}} \frac{1}{1-\frac{t}{1-t}(1-\alpha^k)\varepsilon^k},\tag{57}$$

where public good provision is determined by the budget constraint:

$$G = \tau \bar{F}_L L^*(\tau) + t \bar{F}_K K^*(t) + \bar{e}.$$
(58)

By equating the right hand sides of Eqs. (56)–(57), we get the following equation, which defines the structure of the equilibrium tax rates:

$$\frac{t}{1-t}(1+\lambda^{\mathsf{w}})(1-\alpha^k)\varepsilon^k + \lambda^{\mathsf{c}} = \frac{\tau}{1-\tau}(1+\lambda^{\mathsf{c}})(1-\alpha^l)\varepsilon^l + \lambda^{\mathsf{w}}.$$
(59)

The structure and the interpretation of Eq. (59) are similar to that of Eq. (54), with the added feature of fiscal equalization. While the latter equation contains the effective factor-supplies elasticities, the former contains those *perceived* by the policy maker as a result of the fiscal equalization mechanism.

Concerning the equilibrium tax structure, the 'modified' Ramsey formula (59) then shows that, *coeteris paribus*, taxation is lower for the production factor (i) which is more elastic in supply, (ii) which is supplied by the more powerful lobby group, and (iii) which tax base is subjected to the lower degree of equalization.

Eqs. (56)–(57) also allow for a clear evaluation of how lobbying and fiscal equalization impact on total tax revenue and therefore on public good supply. In fact, for each tax rate the equation equates the marginal benefits of public goods (on the left hand side) to the perceived marginal cost of public funds (on the right hand side). The latter depends on two factors: the relative intensity of lobbying (the first factor on the right hand side of the equation) and the perceived elasticity of the tax base (the second factor on the right hand side of the equation). Eqs. (56)–(57) clearly show that lobbying impacts on the *relative* weights of the marginal costs of public funds (provided that $\lambda^{w} \neq \lambda^{c}$), since by definition it is:

$$n^{\mathbf{w}}\frac{1+\lambda^{\mathbf{w}}}{1+\bar{\lambda}} + n^{\mathbf{c}}\frac{1+\lambda^{\mathbf{c}}}{1+\bar{\lambda}} = 1,$$

for all $(\lambda^{w}, \lambda^{c})$. Lobbying then determines an increase in the *average* marginal cost of public funds, because marginal costs are an increasing and convex function of tax rates and the equilibrium tax mix is distorted away from its optimal level.

On the contrary, fiscal equalization, by lowering the perceived elasticity of tax bases, always reduces both the individual and the average marginal costs of public funds, therefore giving incentives to the policy maker to raise taxation and public good production.

Overall, our analysis then shows that taxpayers' lobbying for lower taxation does not counteract the distortions for higher taxation caused by fiscal equalization, because the former tends to distort the tax mix whereas the latter tends to cause over-taxation. However, as we show by means of a numerical simulation in Section 5 below, the degrees of fiscal equalization of the two tax bases can be used to reduce the distortions caused by lobbying.

Inelastic supply of a production factor

Consider now the case in which one of the production factors is fixed in supply. Land as capital input, or some types of natural resources, could fit into such a special case. In the context of our model, to fix ideas suppose that the supply of capital is inelastic, while that of labor continues to have a positive elasticity.

In the absence of both lobbying and fiscal equalization, it is then optimal, in general, to tax the rent accruing to the fixed factor at the maximum feasible level (100%, or a lower level below it, if there is a political or legal constraint imposing an upper ceiling on the tax rate), and then to use residually the tax on the income of the elastic factor to equate the marginal benefits of public goods with the marginal cost of public funds. The explanation of this result is that while the capital tax is an efficient lump sum tax, that on labor is distortionary.

The fact that the tax rate on capital income is already at its maximum level drives all the results when lobbying and/or fiscal equalization are added to the picture. In the absence of fiscal equalization, only capitalists have a direct incentive to lobby, with the intent of inducing the policy maker to reduce the tax on capital and to increase that on labor. Workers have no incentive to lobby, but only if capitalists do not lobby, since the tax on capital cannot be further increased. However, if capitalists do lobby, then also workers are forced to lobby, in order to limit the influence of the antagonist group.

In the absence of lobbying, tax-base equalization of the tax on the fixed factor (capital) gives an incentive to local policy makers to over-tax the elastic factor (labor), since interest income is decreasing in the level of labor income taxation. This is the type of result that emerges in the model set up by Smart (1998, Section 4.2) to analyze the distortions arising from the equalization of resource taxes, an issue which is highly debated in the context of the mechanism of fiscal equalization among Canadian Provinces.

These results show that the direction of the distortion in tax policy is always the same for lobbying and tax-base equalization. If the suppliers of the inelastic production factor (capital, in our example) are more powerful lobbyists than the suppliers of the elastic factor (labor, in our example), then the local policy maker is induced to lower the efficient tax on the inelastic factor and to raise the distortionary tax on the elastic factor. This distortion is reinforced by the incentives offered by fiscal equalization. If, on the contrary, workers are the more powerful lobbyists, then lobbying has no impact on fiscal policy (since the tax on capital is already at its maximum level), while fiscal equalization tends to distort the labor tax upward. It is also possible to show that, when capitalists are more powerful lobbyists than workers, an increase in the degree of equalization of the tax on capital income is bad for efficiency,¹¹ a result that bears some interest for the ongoing debate about the equalization of resource taxes in Canada (Smart, 2005, 2006).

5 A numerical example

The purpose of the numerical example is that of shedding light on some issues that could not be addressed in general terms in the previous Section. In particular, the interplay between lobbying and fiscal equalization in distorting local fiscal policy, and the impact of these distortions on public good supply.

For capitalists, the utility of first period consumption \tilde{x}^{c} is specified as:

$$\zeta(\tilde{x}^{c}) = \tilde{x}^{c} - A^{c} \frac{\bar{\varepsilon}^{k}}{1 + \bar{\varepsilon}^{k}} (1 - \tilde{x}^{c})^{(1 + \bar{\varepsilon}^{k})/\bar{\varepsilon}^{k}}, \qquad \bar{\varepsilon}^{k} > 0, \ A^{c} > 0.$$

Therefore, in terms of capital supply \tilde{k} we have:

$$\begin{split} \zeta(1-\tilde{k}) &= 1-\tilde{k} - A^{c} \frac{\bar{\varepsilon}^{k}}{1+\bar{\varepsilon}^{k}} \tilde{k}^{(1+\bar{\varepsilon}^{k})/\bar{\varepsilon}^{k}}, \\ \zeta_{x}(\tilde{k}) &= 1 + A^{c} \tilde{k}^{1/\bar{\varepsilon}^{k}}, \qquad \zeta_{xx}(\tilde{k}) = -\frac{A^{c}}{\bar{\varepsilon}^{k}} \tilde{k}^{(1-\bar{\varepsilon}^{k})/\bar{\varepsilon}^{k}} \end{split}$$

Capital supply is of constant elasticity, $\varepsilon^k = \overline{\varepsilon}^k$. In fact, from the first order condition

$$1 + (1-t)r = 1 + A^{\mathbf{c}}\tilde{k}^{1/\bar{\varepsilon}^k},$$

we obtain capital supply and its elasticity with respect to the interest rate r as:

$$\tilde{k}^s(r,t) = ((1-t)r/A^c)^{\bar{\varepsilon}^k}, \qquad \varepsilon^k = \frac{\tilde{k}_r^s r}{\tilde{k}^s} = \bar{\varepsilon}^k.$$

¹¹The details of this result, which is obtained by means of numerical simulations, are available from the authors upon request.

For workers, the utility of leisure h is specified as:

$$\phi(h) = 1 - A^{\mathsf{w}} \frac{\overline{\varepsilon}^l}{1 + \overline{\varepsilon}^l} (1 - h)^{(1 + \overline{\varepsilon}^l)/\overline{\varepsilon}^l}, \qquad \overline{\varepsilon}^l > 0, \ A^{\mathsf{w}} > 0.$$

Therefore, in terms of labor supply l we have:

$$\begin{split} \phi(1-l) &= 1 - A^{\mathsf{w}} \frac{\bar{\varepsilon}^l}{1+\bar{\varepsilon}^l} l^{(1+\bar{\varepsilon}^l)/\bar{\varepsilon}^l},\\ \phi_h(l) &= A^{\mathsf{w}} l^{1/\bar{\varepsilon}^l}, \qquad \phi_{hh}(l) = -\frac{A^{\mathsf{w}}}{\bar{\varepsilon}^l} l^{(1-\bar{\varepsilon}^l)/\bar{\varepsilon}^l}. \end{split}$$

Labor supply is of constant elasticity, $\varepsilon^l = \overline{\varepsilon}^l$. In fact, from the first order condition

$$(1-\tau)w = A^{\mathsf{w}}l^{1/\bar{\varepsilon}^l},$$

we obtain labor supply and its elasticity with respect to the wage rate w as:

$$l^{s}(w,\tau) = ((1-\tau)w/A^{w})^{\overline{\varepsilon}^{l}}, \qquad \varepsilon^{l} = \frac{l_{w}^{s}w}{l^{s}} = \overline{\varepsilon}^{l}.$$

Benefits of public good provision are specified as:

$$b(G) = B \frac{\eta}{\eta - 1} G^{(\eta - 1)/\eta}, \qquad \eta > 0, \ \eta \neq 1, \ B > 0,$$

$$b_G(G) = B G^{-1/\eta}, \qquad b_{GG} = -\frac{B}{\eta} G^{-(1+\eta)/\eta},$$

where η is a measure of the the 'elasticity' of the demand for public goods.

Given these specifications, there is a perfect symmetry between the individual welfare levels of workers and capitalists, but for the elasticity of supply, that can differ between the two groups. In fact:

$$\begin{aligned} v^{\mathbf{w}}(\tau,t,G) &= (1-\tau)wl^* + \phi(1-l^*) + b(G) = \\ &= (1-\tau)wl^* + 1 - A^{\mathbf{w}} \frac{\bar{\varepsilon}^l}{1+\bar{\varepsilon}^l} (l^*)^{(1+\bar{\varepsilon}^l)/\bar{\varepsilon}^l} + b(G), \\ v^{\mathbf{c}}(\tau,t,G) &= (1+(1-t)r)\tilde{k}^* + \zeta(1-\tilde{k}^*) + b(G) = \\ &= (1-t)r\tilde{k}^* + 1 - A^{\mathbf{c}} \frac{\bar{\varepsilon}^k}{1+\bar{\varepsilon}^k} (\tilde{k}^*)^{(1+\bar{\varepsilon}^k)/\bar{\varepsilon}^k} + b(G). \end{aligned}$$

The production function takes the Constant Elasticity of Substitution (CES) form:

$$F(K,L) = A \left[\xi L^{-\rho} + (1-\xi)K^{-\rho} \right]^{-\frac{1}{\rho}}, \qquad A > 0, \ 0 < \xi < 1, \ -1 < \rho.$$

The parameter ρ is related to the elasticity of substitution between labor and capital as $\sigma = 1/(1 + \rho)$. Three standard specifications belonging to this class of production functions which are considered in the numerical simulations are the following:

$$\rho \to 0, \quad \sigma \to 1, \quad F = A L^{\xi} K^{1-\xi}, \quad \text{Cobb-Douglas},$$

 $\rho \to +\infty, \quad \sigma \to 0, \quad F = A \min \{\xi L, (1-\xi)K\}, \quad \text{Leontief},$
 $\rho \to -1, \quad \sigma \to +\infty, \quad F = A [\xi L + (1-\xi)K], \quad \text{Linear.}$

At one end, with Leontief technology there is perfect complementarity between capital and labor. At the other end, with Linear technology the two inputs are perfect substitutes into production.

In the numerical simulation, for given model's parameters, the solution of a sixequation system — Eqs. (10) and (11) defining the optimal supplies of labor and capital, Eqs. (8) and (9) defining the optimal capital/labor ratio, Eqs. (52) and (53) defining the equilibrium fiscal policy — is computed in the unknowns L, K, w, r, τ and t. Given the equilibrium tax rates, public good supply is then computed from the budget constraint (41).¹² In all the numerical simulations, we set $A^{w} = A^{w} = A = 5$, $\xi = B = n^{w} = .5$, $\eta = .8$.

The first set of simulations is shown in Table 1. In columns I-XII, the elasticity of supply is the same, $\bar{\varepsilon}^k = \bar{\varepsilon}^w = .5$, for capital and labor, whereas in columns XIII-XXIV we consider a higher elasticity for capital, $\bar{\varepsilon}^k = .6 > .4 = \bar{\varepsilon}^w$. In columns I-IV and XIII-XVI, the production function is of Cobb-Douglas type ($\sigma = .9999$), in columns V-VIII and XVII-XX it is linear ($\sigma = 100$), and finally in columns IX-XII and XXI-XXIV it is of Leontief type ($\sigma = .0099$). For each specification of the supplies elasticities and of the elasticity of factors substitution, four types of equilibrium fiscal policies are computed: no lobbying and no fiscal equalization ($\lambda^w = \lambda^c = \alpha^l = \alpha^k = 0$), lobbying by capitalists only in the absence of fiscal equalization ($\lambda^w = 0, \lambda^c = .5, \alpha^l = \alpha^k = 0$), uniform fiscal equalization of 75% of tax bases in the absence of lobbying ($\lambda^w = \lambda^c = 0, \alpha^l = \alpha^k = .75$), and finally equalization of capital tax base only in the absence of lobbying ($\lambda^w = \lambda^c = 0, \alpha^l = 0, \alpha^l = 0, \alpha^k = .75$). The results shown in Table 1 can be summarized as follows:

- The lobbying group is more effective in obtaining a reduction of its own tax rate when the production function is Leontief (see columns X and XXII), whereas it causes a larger increase in the tax levied on the non-lobbying group when the production function is linear (see columns VI and XVIII).
- In the absence of lobbying, a uniform level of fiscal equalization distorts tax rates upwards, it preserves the ranking of tax rates if the elasticities of supplies are

¹²The Excel spreadsheet used for the numerical simulation is available from the authors upon request.

different, but the gap between the tax rates is larger the larger is the elasticity of substitution between the production factors.

• In the absence of lobbying, if only one tax base is equalized (however, the result holds in general for different degrees of equalization), then (a) both tax rates are distorted upwards under Cobb-Douglas technology (see columns IV and XVI), (b) the tax rate of the equalized tax base is distorted upward whereas that of the non-equalized tax base is distorted downward under linear technology (see columns VIII and XXX), (c) the tax rate of the equalized tax base is distorted upward under lownward under Leontief technology (see columns XII and XXIV). The apparently odd result (c) is due to the fact that with a Leontief technology tax bases are increasing in their own tax rates whereas are decreasing in the tax rate of the other tax base.

The second set of simulations, which are shown in Table 2, examines the interplay between lobbying and fiscal equalization. Columns I-X refer to linear technology, while columns XI-XX to Cobb-Douglas one. In columns I-V and XI-XV the elasticities of factor supplies are identical ($\bar{\varepsilon}^k = \bar{\varepsilon}^w = .5$) whereas in columns VI-X and XVI-XX they are different ($\bar{\varepsilon}^k = .6, \bar{\varepsilon}^w = .4$). In the case of linear technology, the simulations show that by properly setting the degrees of fiscal equalization it is possible to dampen the distortions caused by lobbying. In fact, column II shows that lobbying by capitalists distorts tax rates, by lowering t from 31.8 to 24.3%, and by increasing τ from 31.8 to 39.6%. By equalizing 50% of the capital tax base, and by not equalizing the labor income tax base, column III then shows that the tax rate t raises to 30.4% while the tax rate τ lowers to 36.5%, with a welfare gain with respect to the previous situation. Column IV then shows that it does not pay to increase the equalization rate to 70%in order to realign tax rates at the same level, as implied by the Ramsey rule in the absence of distortions. More importantly, column V shows that fiscal equalization at the uniform level of 25% underperforms, in terms of social welfare, than the nonuniform rates of 50% for capital and zero for labor. Columns VI-X of Table 2 show that in the case of linear technology the same results hold also when the elasticities of supplies are not uniform. The remaining part of the Table, columns XI-XX, show instead that the above result vanishes in the presence of a Cobb-Douglas technology, which shows a much lower degree of input substitutability than the linear one. The simulations show that the equalization of the tax base belonging to the lobby group has the effect of raising *both* tax rates, which precludes the possibility of correcting the distortions caused by lobbying by means of differentiated fiscal equalization.

6 Concluding remarks

In this paper, we analyze how the incentives arising from tax-base equalization-grants interact with the incentives arising from taxpayers' lobbying in distorting the fiscal policy of the recipient local governments. Although lobbying and fiscal equalization provide incentives that push in opposed directions — the former for lower, and the latter for higher, taxation — our analysis shows that in general the two sources of distortion reinforce each other in terms of efficiency losses. At the most, the central authority can differentiate the degrees of equalization of the local tax bases to partially compensate for heterogeneous abilities of the lobby groups in distorting local fiscal policies.

Our analysis is cast in a framework of local tax bases which are immobile across the local jurisdictions. In future research, it could be interesting to examine how the results would be affected by allowing factors mobility, which would introduce horizontal tax competition among local governments. Another interesting extension of the analysis would be to characterize the optimal degree of fiscal equalization of the two tax bases, for a given average degree of fiscal equalization; the question then would be the characterization of the optimal combination of fiscal equalization that minimizes the efficiency loss of the transfer program, for a given level of average equalization. Finally, another interesting topic would be to address the trade-off between efficiency and equity in fiscal equalization, by introducing into the analysis heterogeneous local jurisdictions.

Mathematical appendix

A.1 The impact of tax rates on market equilibrium

By substituting the equilibrium prices and quantities, $w^*(\tau, t)$, $r^*(\tau, t)$, $L^*(\tau, t)$ and $K^*(\tau, t)$, into the equation system (8), (9), (10) and (11), we get the identities:

$$\zeta_x (1 - K^*/n^c) \equiv 1 + (1 - t)r^*, \tag{A.1}$$

$$\phi_h(1 - L^*/n^{\mathbf{w}}) \equiv (1 - \tau)w^*,$$
(A.2)

$$F_L(K^*, L^*) \equiv w^*, \tag{A.3}$$

$$F_K(K^*, L^*) \equiv 1 + r^*.$$
 (A.4)

Applying the implicit function theorem, by totally differentiating Eqs. (A.1)-(A.4) with

respect to τ , we get:

$$-\zeta_{xx}K_{\tau}^{*}/n^{c} = (1-t)r_{\tau}^{*}, \tag{A.5}$$

$$-\phi_{hh}L_{\tau}^{*}/n^{w} = (1-\tau)w_{\tau}^{*} - w^{*}, \qquad (A.6)$$

$$F_{LK}K_{\tau}^{*} + F_{LL}L_{\tau}^{*} = w_{\tau}^{*}, \tag{A.7}$$

$$F_{KK}K_{\tau}^{*} + F_{KL}L_{\tau}^{*} = r_{\tau}^{*}.$$
(A.8)

Using the following properties of the production function, which is assumed to be linearly homogeneous,

$$F_{LK} = -\frac{K}{L}F_{KK}, \qquad F_{KL} = -\frac{L}{K}F_{LL}, \qquad \frac{F_{KK}}{F_{LL}} = \left(\frac{L}{K}\right)^2, \tag{A.9}$$

by solving the equation system (A.5)–(A.8) for the unknown w_{τ}^* , r_{τ}^* , L_{τ}^* and K_{τ}^* , we obtain the partial derivatives (13)–(15) shown in the text.

Applying the implicit function theorem, by totally differentiating Eqs. (A.1)–(A.4) with respect to t, we get:

$$-\zeta_{xx}K_t^*/n^{\rm c} = (1-t)r_t^* - r^*, \tag{A.10}$$

$$-\phi_{hh}L_t^*/n^{w} = (1-\tau)w_t^*, \tag{A.11}$$

$$F_{LK}K_t^* + F_{LL}L_t^* = w_t^*, (A.12)$$

$$F_{KK}K_t^* + F_{KL}L_t^* = r_t^*. (A.13)$$

Using properties (A.9) of the production function, by solving the equation system (A.10)–(A.13) for the unknown w_t^* , r_t^* , L_t^* and K_t^* , we obtain the partial derivatives (16)–(18) shown in the text.

A.2 The first order conditions for optimal fiscal policy

The elements of Eqs. (50) and (51) can be manipulated as follows. Partial derivatives:

$$G_{\tau} = W^* + \tau \delta^l (w_{\tau}^* L^* + w^* L_{\tau}^*) + t \delta^k (r_{\tau}^* K^* + r^* K_{\tau}^*),$$

$$G_t = R^* + \tau \delta^l (w_t^* L^* + w^* L_t^*) + t \delta^k (r_t^* K^* + r^* K_t^*),$$

can be written as:

$$G_{\tau} = W^* + (\tau \delta^l - t \delta^k) w_{\tau}^* L^* + \tau \delta^l w^* L_{\tau}^* + t \delta^k r^* K_{\tau}^*,$$

$$G_t = R^* + (t \delta^k - \tau \delta^l) r_t^* K^* + \tau \delta^l w^* L_t^* + t \delta^k r^* K_t^*.$$

These in turn can be written as:

$$G_{\tau} = W^* + (\tau \delta^l - t \delta^k) w_{\tau}^* L^* + \tau \delta^l w^* w_{\tau}^* / F_{LL} + (\tau \delta^l w^* L^* / K^* + t \delta^k r^*) K_{\tau}^*,$$

$$G_t = R^* + (t \delta^k - \tau \delta^l) r_t^* K^* + t \delta^k r^* r_t^* / F_{KK} + (t \delta^k r^* K^* / L^* + \tau \delta^l w^*) L_t^*.$$

Finally, using the latter expressions we get Eqs. (52) and (53).

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	I	II	III	IV	V	VI	VII	VIII	IX	Х	XI	XII
I^3	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
ε^k	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
λ^c	0,00	0,50	0,00	0,00	0,00	0,50	0,00	0,00	0,00	0,50	0,00	0,00
λ^w	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
α^k	0,00	0,00	0,75	0,75	0,00	0,00	0,75	0,75	0,00	0,00	0,75	0,75
α^I	0,00	0,00	0,75	0,00	0,00	0,00	0,75	0,00	0,00	0,00	0,75	0,00
t	0,3181	0,1769	0,3890	0,3477	0,3181	0,1923	0,3890	0,5458	0,3181	0,1392	0,3890	0,0093
τ	0,3181	0,4475	0,3890	0,3477	0,3181	0,4522	0,3890	0,2311	0,3181	0,4364	0,3890	0,4834
G	0,4644	0,4532	0,5375	0,4964	0,4644	0,4485	0,5375	0,5042	0,4644	0,4640	0,5375	0,4649
r	2,5000	2,3393	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	1,9885	2,5000	1,7287
w	2,5000	2,6717	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	3,0126	2,5000	3,2737
K	0,2919	0,3103	0,2764	0,2855	0,2919	0,3178	0,2764	0,2383	0,2919	0,2926	0,2764	0,2926
L	0,2919	0,2717	0,2764	0,2855	0,2919	0,2617	0,2764	0,3100	0,2919	0,2914	0,2764	0,2908
V tilde	-0,7592	-0,7719	-0,7729	-0,7618	-0,7592	-0,7772	-0,7729	-0,7957	-0,7592	-0,7596	-0,7729	-0,7602
σ	0,9999	0,9999	0,9999	0,9999	100	100	100	100	0,0099	0,0099	0,0099	0,0099
	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXII	XXIII	XXIV
۲^۱	XIII 0,40	XIV 0,40	XV 0,40	XVI 0,40	XVII 0,40	XVIII 0,40	XIX 0,40	XX 0,40	XXI 0,40	XXII 0,40	XXIII 0,40	XXIV 0,40
٤^۱ ٤^k	XIII 0,40 0,60	XIV 0,40 0,60	XV 0,40 0,60	XVI 0,40 0,60	XVII 0,40 0,60	XVIII 0,40 0,60	XIX 0,40 0,60	XX 0,40 0,60	XXI 0,40 0,60	XXII 0,40 0,60	XXIII 0,40 0,60	XXIV 0,40 0,60
ε^Ι ε^κ λ^c	XIII 0,40 0,60 0,00	XIV 0,40 0,60 0,50	XV 0,40 0,60 0,00	XVI 0,40 0,60 0,00	XVII 0,40 0,60 0,00	XVIII 0,40 0,60 0,50	XIX 0,40 0,60 0,00	XX 0,40 0,60 0,00	XXI 0,40 0,60 0,00	XXII 0,40 0,60 0,50	XXIII 0,40 0,60 0,00	XXIV 0,40 0,60 0,00
ε^Ι ε^k λ^c λ^w	XIII 0,40 0,60 0,00 0,00	XIV 0,40 0,60 0,50 0,00	XV 0,40 0,60 0,00 0,00	XVI 0,40 0,60 0,00 0,00	XVII 0,40 0,60 0,00 0,00	XVIII 0,40 0,60 0,50 0,00	XIX 0,40 0,60 0,00 0,00	XX 0,40 0,60 0,00 0,00	XXI 0,40 0,60 0,00 0,00	XXII 0,40 0,60 0,50 0,00	XXIII 0,40 0,60 0,00 0,00	XXIV 0,40 0,60 0,00 0,00
ε^Ι ε^k λ^c λ^w α^k	XIII 0,40 0,60 0,00 0,00 0,00	XIV 0,40 0,60 0,50 0,00 0,00	XV 0,40 0,60 0,00 0,00 0,75	XVI 0,40 0,60 0,00 0,00 0,75	XVII 0,40 0,60 0,00 0,00 0,00	XVIII 0,40 0,60 0,50 0,00 0,00	XIX 0,40 0,60 0,00 0,00 0,75	XX 0,40 0,60 0,00 0,00 0,75	XXI 0,40 0,60 0,00 0,00 0,00	XXII 0,40 0,60 0,50 0,00 0,00	XXIII 0,40 0,60 0,00 0,00 0,75	XXIV 0,40 0,60 0,00 0,00 0,75
ε^l ε^k λ^c λ^w α^k α^l	XIII 0,40 0,60 0,00 0,00 0,00 0,00	XIV 0,40 0,60 0,50 0,00 0,00 0,00	XV 0,40 0,60 0,00 0,00 0,75 0,75	XVI 0,40 0,60 0,00 0,00 0,75 0,00	XVII 0,40 0,60 0,00 0,00 0,00 0,00	XVIII 0,40 0,60 0,50 0,00 0,00 0,00	XIX 0,40 0,60 0,00 0,00 0,75 0,75	XX 0,40 0,60 0,00 0,00 0,75 0,00	XXI 0,40 0,60 0,00 0,00 0,00 0,00	XXII 0,40 0,60 0,50 0,00 0,00 0,00	XXIII 0,40 0,60 0,00 0,00 0,75 0,75	XXIV 0,40 0,60 0,00 0,00 0,75 0,00
ε^l ε^k λ^c λ^w α^k α^l	XIII 0,40 0,60 0,00 0,00 0,00 0,00 0,2734	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,75	XVI 0,40 0,60 0,00 0,00 0,75 0,00 0,3086	XVII 0,40 0,60 0,00 0,00 0,00 0,00 0,2712	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341	XIX 0,40 0,60 0,00 0,00 0,75 0,75 0,75	XX 0,40 0,60 0,00 0,00 0,75 0,00 0,5045	XXI 0,40 0,60 0,00 0,00 0,00 0,00 0,2777	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167	XXIII 0,40 0,60 0,00 0,00 0,75 0,75 0,75	XXIV 0,40 0,60 0,00 0,00 0,75 0,00 0,0086
ε^I ε^k λ^c λ^w α^k α^l τ	XIII 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022	XVI 0,40 0,60 0,00 0,00 0,75 0,00 0,3086 0,3766	XVII 0,40 0,60 0,00 0,00 0,00 0,2712 0,3583	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970	XIX 0,40 0,60 0,00 0,00 0,75 0,75 0,3329 0,4281	XX 0,40 0,60 0,00 0,00 0,75 0,00 0,5045 0,2763	XXI 0,40 0,60 0,00 0,00 0,00 0,00 0,2777 0,3657	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912	XXIII 0,40 0,60 0,00 0,00 0,75 0,75 0,3839 0,3844	XXIV 0,40 0,60 0,00 0,00 0,75 0,00 0,0086 0,5392
ε^l ε^k λ^c λ^w α^k α^l t τ	XIII 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608 0,4680	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953 0,4608	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022 0,5393	XVI 0,40 0,60 0,00 0,00 0,75 0,00 0,3086 0,3766 0,4961	XVII 0,40 0,60 0,00 0,00 0,00 0,00 0,2712 0,3583 0,4692	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970 0,4591	XIX 0,40 0,60 0,00 0,00 0,75 0,75 0,3329 0,4281 0,5397	XX 0,40 0,60 0,00 0,00 0,75 0,00 0,5045 0,2763 0,5030	XXI 0,40 0,60 0,00 0,00 0,00 0,00 0,2777 0,3657 0,4657	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912 0,4647	XXIII 0,40 0,60 0,00 0,00 0,75 0,75 0,75 0,3839 0,3844 0,5380	XXIV 0,40 0,60 0,00 0,00 0,75 0,00 0,0086 0,5392 0,4661
ε^I ε^k λ^c λ^w α^k α^l t τ G	XIII 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953 0,4608 2,4570	XV 0,40 0,60 0,00 0,75 0,75 0,75 0,3688 0,4022 0,5393 2,6802	XVI 0,40 0,60 0,00 0,00 0,75 0,00 0,3086 0,3766 0,4961 2,6467	XVII 0,40 0,60 0,00 0,00 0,00 0,00 0,2712 0,3583 0,4692 2,5000	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970 0,4591 2,5000	XIX 0,40 0,60 0,00 0,75 0,75 0,75 0,3329 0,4281 0,5397 2,5000	XX 0,40 0,60 0,00 0,75 0,00 0,5045 0,2763 0,5030 2,5001	XXI 0,40 0,60 0,00 0,00 0,00 0,00 0,2777 0,3657 0,4657 2,8748	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912 0,4647 2,3607	XXIII 0,40 0,60 0,00 0,75 0,75 0,75 0,3839 0,3844 0,5380 3,0800	XXIV 0,40 0,60 0,00 0,00 0,75 0,00 0,0086 0,5392 0,4661 2,1026
ε^I ε^k λ^c λ^w α^k α^l τ G r	XIII 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953 0,4608 2,4570 2,5438	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022 0,5393 2,6802 2,3319	XVI 0,40 0,60 0,00 0,00 0,75 0,00 0,3086 0,3766 0,4961 2,6467 2,3615	XVII 0,40 0,60 0,00 0,00 0,00 0,2712 0,3583 0,4692 2,5000 2,5000	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970 0,4591 2,5000 2,5000	XIX 0,40 0,60 0,00 0,75 0,75 0,75 0,3329 0,4281 0,5397 2,5000 2,5000	XX 0,40 0,60 0,00 0,75 0,00 0,5045 0,2763 0,5030 2,5001 2,5001	XXI 0,40 0,60 0,00 0,00 0,00 0,2777 0,3657 0,4657 2,8748 2,1257	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912 0,4647 2,3607 2,6394	XXIII 0,40 0,60 0,00 0,75 0,75 0,3839 0,3844 0,5380 3,0800 1,9214	XXIV 0,40 0,60 0,00 0,75 0,00 0,0086 0,5392 0,4661 2,1026 2,8980
ε^I ε^k λ^c λ^w α^k α^k α^l t τ G r w	XIII 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953 0,4608 2,4570 2,5438 0,3005	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022 0,5393 2,6802 2,3319 0,2610	XVI 0,40 0,60 0,00 0,75 0,00 0,3086 0,3766 0,4961 2,6467 2,3615 0,2736	XVII 0,40 0,60 0,00 0,00 0,00 0,00 0,2712 0,3583 0,4692 2,5000 2,5000 0,2728	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970 0,4591 2,5000 2,5000 0,3026	XIX 0,40 0,60 0,00 0,75 0,75 0,3329 0,4281 0,5397 2,5000 2,5000 0,2587	XX 0,40 0,60 0,00 0,75 0,00 0,5045 0,2763 0,5030 2,5001 2,5001 2,5000 0,2165	XXI 0,40 0,60 0,00 0,00 0,00 0,2777 0,3657 0,4657 2,8748 2,1257 0,2951	XXII 0,40 0,60 0,50 0,00 0,00 0,1167 0,4912 0,4647 2,3607 2,6394 0,2959	XXIII 0,40 0,60 0,00 0,75 0,75 0,3839 0,3844 0,5380 3,0800 1,9214 0,2796	XXIV 0,40 0,60 0,00 0,75 0,00 0,0086 0,5392 0,4661 2,1026 2,8980 0,2958
ε^Ι ε^k λ^c λ^w α^k α^Ι τ τ G r w K L	XIII 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807 0,3105	XIV 0,40 0,60 0,50 0,00 0,00 0,00 0,1288 0,4953 0,4608 2,4570 2,5438 0,3005 0,2903	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022 0,5393 2,6802 2,3319 0,2610 0,3000	XVI 0,40 0,60 0,00 0,75 0,00 0,3086 0,3766 0,3766 0,4961 2,6467 2,3615 0,2736 0,3066	XVII 0,40 0,60 0,00 0,00 0,00 0,00 0,2712 0,3583 0,4692 2,5000 2,5000 0,2728 0,3173	XVIII 0,40 0,60 0,50 0,00 0,00 0,00 0,1341 0,4970 0,4591 2,5000 2,5000 0,3026 0,2879	XIX 0,40 0,60 0,00 0,75 0,75 0,75 0,3329 0,4281 0,5397 2,5000 2,5000 0,2587 0,3030	XX 0,40 0,60 0,00 0,75 0,00 0,5045 0,2763 0,5030 2,5001 2,5000 0,2165 0,3329	XXI 0,40 0,60 0,00 0,00 0,00 0,2777 0,3657 0,4657 2,8748 2,1257 0,2951 0,2960	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912 0,4647 2,3607 2,6394 0,2959 0,2955	XXIII 0,40 0,60 0,00 0,75 0,75 0,75 0,3839 0,3844 0,5380 3,0800 1,9214 0,2796 0,2809	XXIV 0,40 0,60 0,00 0,00 0,008 0,5392 0,4661 2,1026 2,8980 0,2958 0,2949
ε^Ι ε^k λ^c λ^w α^k α^l t τ G r w K L V tilde	XIII 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807 0,3105 -0,7458	XIV 0,40 0,60 0,50 0,00 0,00 0,1288 0,4953 0,4608 2,4570 2,5438 0,3005 0,2903 -0,7592	XV 0,40 0,60 0,00 0,00 0,75 0,75 0,3688 0,4022 0,5393 2,6802 2,3319 0,2610 0,3000 -0,7592	XVI 0,40 0,60 0,00 0,75 0,00 0,3086 0,3766 0,4961 2,6467 2,3615 0,2736 0,2736 0,3066 -0,7478	XVII 0,40 0,60 0,00 0,00 0,00 0,2712 0,3583 0,4692 2,5000 2,5000 0,2728 0,3173 -0,7422	XVIII 0,40 0,60 0,50 0,00 0,00 0,1341 0,4970 0,4591 2,5000 2,5000 0,3026 0,2879 -0,7618	XIX 0,40 0,60 0,00 0,75 0,75 0,75 0,3329 0,4281 0,5397 2,5000 2,5000 0,2587 0,3030 -0,7543	XX 0,40 0,60 0,00 0,75 0,00 0,5045 0,2763 0,5030 2,5001 2,5000 0,2165 0,3329 -0,7770	XXI 0,40 0,60 0,00 0,00 0,00 0,2777 0,3657 0,4657 2,8748 2,1257 0,2951 0,2951 0,2960 -0,7530	XXII 0,40 0,60 0,50 0,00 0,00 0,00 0,1167 0,4912 0,4647 2,3607 2,6394 0,2959 0,2955 -0,7534	XXIII 0,40 0,60 0,00 0,75 0,75 0,75 0,3839 0,3844 0,5380 3,0800 1,9214 0,2796 0,2809 -0,7663	XXIV 0,40 0,60 0,00 0,00 0,75 0,00 0,0086 0,5392 0,4661 2,1026 2,8980 0,2958 0,2958 0,2949 -0,7539

Table 1: Numerical simulations

	I	II	III	IV	V	VI	VII	VIII	IX	X
۱^3	0,50	0,50	0,50	0,50	0,50	0,40	0,40	0,40	0,40	0,40
٤^k	0,50	0,50	0,50	0,50	0,50	0,60	0,60	0,60	0,60	0,60
λ^c	0,00	0,25	0,25	0,25	0,25	0,00	0,25	0,25	0,25	0,25
λ^w	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
α^k	0,00	0,00	0,50	0,70	0,25	0,00	0,00	0,50	0,90	0,25
α^I	0,00	0,00	0,00	0,00	0,25	0,00	0,00	0,00	0,00	0,25
t	0,3181	0,2429	0,3040	0,3459	0,2412	0,2712	0,1897	0,2462	0,3599	0,1827
τ	0,3181	0,3964	0,3650	0,3451	0,4406	0,3583	0,4385	0,4104	0,3620	0,4832
G	0,4644	0,4590	0,4813	0,4941	0,4770	0,4692	0,4677	0,4861	0,5136	0,4850
r	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000
w	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000	2,5000
К	0,2919	0,3076	0,2949	0,2859	0,3080	0,2728	0,2908	0,2784	0,2524	0,2923
L	0,2919	0,2747	0,2817	0,2861	0,2644	0,3173	0,3008	0,3068	0,3166	0,2910
V tilde	-0,7592	-0,7653	-0,7609	-0,7614	-0,7706	-0,7422	-0,7488	-0,7444	-0,7494	-0,7548
σ	100	100	100	100	100	100	100	100	100	100
	XI	XII	XIII	XIV	xv	XVI	XVII	XVIII	XIX	XX
اد ۱^ع	XI 0,50	XII 0,50	XIII 0,50	XIV 0,50	XV 0,50	XVI 0,40	XVII 0,40	XVIII 0,40	XIX 0,40	XX 0,40
<u>د</u> ٤^۱ ٤^k	XI 0,50 0,50	XII 0,50 0,50	XIII 0,50 0,50	XIV 0,50 0,50	XV 0,50 0,50	XVI 0,40 0,60	XVII 0,40 0,60	XVIII 0,40 0,60	XIX 0,40 0,60	XX 0,40 0,60
ε^I ε^I ε^k λ^c	XI 0,50 0,50 0,00	XII 0,50 0,50 0,25	XIII 0,50 0,50 0,25	XIV 0,50 0,50 0,25	XV 0,50 0,50 0,25	XVI 0,40 0,60 0,00	XVII 0,40 0,60 0,25	XVIII 0,40 0,60 0,25	XIX 0,40 0,60 0,25	XX 0,40 0,60 0,25
ε^I ε^I ε^k λ^c λ^w	XI 0,50 0,50 0,00 0,00	XII 0,50 0,50 0,25 0,00	XIII 0,50 0,50 0,25 0,00	XIV 0,50 0,25 0,00	XV 0,50 0,50 0,25 0,00	XVI 0,40 0,60 0,00 0,00	XVII 0,40 0,60 0,25 0,00	XVIII 0,40 0,60 0,25 0,00	XIX 0,40 0,60 0,25 0,00	XX 0,40 0,60 0,25 0,00
ε^I ε^k λ^c λ^w α^k	XI 0,50 0,50 0,00 0,00 0,00	XII 0,50 0,50 0,25 0,00 0,00	XIII 0,50 0,50 0,25 0,00 0,50	XIV 0,50 0,50 0,25 0,00 0,10	XV 0,50 0,50 0,25 0,00 0,10	XVI 0,40 0,60 0,00 0,00 0,00	XVII 0,40 0,60 0,25 0,00 0,00	XVIII 0,40 0,60 0,25 0,00 0,50	XIX 0,40 0,60 0,25 0,00 0,10	XX 0,40 0,60 0,25 0,00 0,10
ε^I ε^k λ^c λ^w α^k α^l	XI 0,50 0,50 0,00 0,00 0,00 0,00	XII 0,50 0,50 0,25 0,00 0,00 0,00	XIII 0,50 0,25 0,00 0,50 0,00	XIV 0,50 0,25 0,00 0,10 0,00	XV 0,50 0,25 0,00 0,10 0,10	XVI 0,40 0,60 0,00 0,00 0,00 0,00	XVII 0,40 0,60 0,25 0,00 0,00 0,00	XVIII 0,40 0,60 0,25 0,00 0,50 0,00	XIX 0,40 0,60 0,25 0,00 0,10 0,00	XX 0,40 0,60 0,25 0,00 0,10 0,10
ε^I ε^k λ^c λ^w α^k α^I	XI 0,50 0,50 0,00 0,00 0,00 0,00 0,3181	XII 0,50 0,50 0,25 0,00 0,00 0,00 0,2384	XIII 0,50 0,50 0,25 0,00 0,50 0,00 0,2469	XIV 0,50 0,25 0,00 0,10 0,00 0,2400	XV 0,50 0,25 0,00 0,10 0,10 0,2428	XVI 0,40 0,60 0,00 0,00 0,00 0,00 0,2734	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938	XX 0,40 0,60 0,25 0,00 0,10 0,10 0,1986
ε^I ε^k λ^c λ^w α^k α^I t	XI 0,50 0,50 0,00 0,00 0,00 0,00 0,3181 0,3181	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976	XV 0,50 0,25 0,00 0,10 0,10 0,2428 0,4036	XVI 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418	XX 0,40 0,60 0,25 0,00 0,10 0,10 0,1986 0,4467
ε^I ε^K λ^c λ^w α^k α^I t τ	XI 0,50 0,00 0,00 0,00 0,00 0,3181 0,3181 0,4644	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976 0,4636	XV 0,50 0,25 0,00 0,10 0,10 0,2428 0,4036 0,4684	XVI 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397 0,4669	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692	XX 0,40 0,60 0,25 0,00 0,10 0,10 0,1986 0,4467 0,4747
ε^I ε^I ε^k λ^c λ^w α^k α^I t τ G	XI 0,50 0,50 0,00 0,00 0,00 0,00 0,3181 0,3181 0,4644 2,5000	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607 2,4065	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753 2,3987	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976 0,4636 2,4050	XV 0,50 0,25 0,00 0,10 0,10 0,2428 0,4036 0,4684 2,4025	XVI 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397 0,4669 2,5291	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791 2,5291	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692 2,5291	XX 0,40 0,60 0,25 0,00 0,10 0,10 0,1986 0,4467 0,4747 2,5291
ε^I ε^K λ^c λ^w α^k α^I t τ G	XI 0,50 0,00 0,00 0,00 0,00 0,00 0,3181 0,3181 0,4644 2,5000 2,5000	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607 2,4065 2,5971	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753 2,3987 2,6055	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976 0,4636 2,4050 2,5987	XV 0,50 0,25 0,00 0,10 0,2428 0,4036 0,4684 2,4025 2,6014	XVI 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397 0,4669 2,5291 2,4713	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791 2,5291 2,4713	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692 2,5291 2,4713	XX 0,40 0,60 0,25 0,00 0,10 0,10 0,1986 0,4467 0,4747 2,5291 2,4712
ε^I ε^k λ^c λ^w α^k α^l t G r w K	XI 0,50 0,50 0,00 0,00 0,00 0,3181 0,3181 0,4644 2,5000 2,5000 0,2919	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607 2,4065 2,5971 0,3027	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753 2,3987 2,6055 0,3005	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976 0,4636 2,4050 2,5987 0,3023	XV 0,50 0,25 0,00 0,10 0,2428 0,4036 0,4684 2,4025 2,6014 0,3016	XVI 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397 0,4669 2,5291 2,4713 0,2923	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791 2,5291 2,5291 2,4713 0,2900	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692 2,5291 2,4713 0,2919	XX 0,40 0,60 0,25 0,00 0,10 0,1986 0,4467 0,4747 2,5291 2,4712 0,2909
ε^Ι ε^κ λ^c λ^w α^k α^Ι τ G r W K L	XI 0,50 0,50 0,00 0,00 0,00 0,00 0,3181 0,3181 0,3181 0,4644 2,5000 2,5000 0,2919 0,2919	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607 2,4065 2,5971 0,3027 0,2805	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753 2,3987 2,6055 0,3005 0,2767	XIV 0,50 0,25 0,00 0,10 0,00 0,2400 0,3976 0,4636 2,4050 2,5987 0,3023 0,2798	XV 0,50 0,25 0,00 0,10 0,2428 0,4036 0,4684 2,4025 2,6014 0,3016 0,2785	XVI 0,40 0,60 0,00 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807 0,3105	XVII 0,40 0,60 0,25 0,00 0,00 0,00 0,1918 0,4397 0,4669 2,5291 2,4713 0,2923 0,2992	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791 2,5291 2,4713 0,2900 0,2968	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692 2,5291 2,4713 0,2919 0,2987	XX 0,40 0,60 0,25 0,00 0,10 0,1986 0,4467 0,4747 2,5291 2,4712 0,2909 0,2977
ε^I ε^k λ^c λ^w α^k α^I t τ G r W K L V tilde	XI 0,50 0,50 0,00 0,00 0,00 0,00 0,3181 0,3181 0,4644 2,5000 2,5000 0,2919 0,2919 -0,7592	XII 0,50 0,25 0,00 0,00 0,00 0,2384 0,3941 0,4607 2,4065 2,5971 0,3027 0,2805 -0,7634	XIII 0,50 0,25 0,00 0,50 0,00 0,2469 0,4124 0,4753 2,3987 2,6055 0,3005 0,2767 -0,7644	XIV 0,50 0,25 0,00 0,10 0,2400 0,3976 0,4636 2,4050 2,5987 0,3023 0,2798 -0,7635	XV 0,50 0,25 0,00 0,10 0,2428 0,4036 0,4684 2,4025 2,6014 0,3016 0,2785 -0,7637	XVI 0,40 0,60 0,00 0,00 0,00 0,2734 0,3608 0,4680 2,6293 2,3771 0,2807 0,3105 -0,7458	XVII 0,40 0,60 0,25 0,00 0,00 0,1918 0,4397 0,4669 2,5291 2,4713 0,2923 0,2992 -0,7502	XVIII 0,40 0,60 0,25 0,00 0,50 0,00 0,2025 0,4507 0,4791 2,5291 2,4713 0,2900 0,2968 -0,7506	XIX 0,40 0,60 0,25 0,00 0,10 0,00 0,1938 0,4418 0,4692 2,5291 2,4713 0,2919 0,2987 -0,7502	XX 0,40 0,60 0,25 0,00 0,10 0,1986 0,4467 0,4747 2,5291 2,4712 0,2909 0,2977 -0,7503

Table 2: Numerical simulations